Project 7 – Vector Calculus

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Math 3030 –

Math Models for Computer Science

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Math-3030 – Math Models for CS

Project 7 – Vector Calculus

The problem:

Five points A, B, C, D1 and D2 in R3 are given.

1. Determine whether three points A, B, C are collinear (i.e. lie on the same line)
2. If A, B, C are not collinear, find the plane that contains these three points
3. Determine whether D1 and D2 are at the same side from the A, B, C-plane

Inputs:

1. Coordinates of point A, (xA, yA, zA)
2. Coordinates of point B, (xB, yB, zB)
3. Coordinates of point C, (xC, yC, zC)
4. Coordinates of point D1, (xD1, yD1, zD1)
5. Coordinates of point D2, (xD2, yD2, zD2)

Outputs:

1. Coordinates of points A, B, C
2. Determination of whether points A, B, C are collinear
3. Equation of the plane if not collinear
4. Determination of whether points D1, D2 are on the same side of A, B, C-plane

Solution:

1. First, determine if points A, B, and C are collinear.
   1. Find vectors AB, AC, and BC via dot product.
      1. AB = (B1-A1, B2-A2, B3-A3)
      2. AC = (C1-A1, C2-A2, C3-A3)
      3. CB = (B1-C1, B2-C2, B3-C3)
   2. Determine collinearity
      1. AB1 = AB2  = AB3

AC1 AC2 AC3

1. Second, find the plane that contains the points A, B, C.
   1. Take cross product of ab x bc.
      1. n: ( | ab2 ab3 | | ab1 ab3 | | ab1 ab2 |

| bc3 bc2 | - | bc3 bc1 | + | bc2 bc1 | )

* + - 1. n: ((ab2\*bc3 – ab3\*bc2), (ab1\*bc3 – ab3\*bc1), (ab1\*bc2 – ab2\*bc1))

1. Third, find if points D1, D2 are on the same side of the A, B, C-plane.
   1. n(x-x0) - n(y-y0) +n (z-z0)
      1. Find point p1
         1. p1 = n(d11-a1) - n(d12-a2) + n(d13-a3)
      2. Find point p2
         1. p2 = n(d21-a1) - n(d22-a2) + n(d23-a3)
      3. If p1 and p2 > 0, or p1 and p2 < 0, then p1 and p2 are on the same side of the plane

Trial Run 1: Collinear

Please enter the coordinates of point A:

Point A1: 1

Point A2: 2

Point A3: 3

Please enter the coordinates of point B:

Point B1: 4

Point B2: 5

Point B3: 6

Please enter the coordinates of point C:

Point C1: 7

Point C2: 8

Point C3: 9

Point A [1, 2, 3], Point B [4, 5, 6], and Point C [7, 8, 9], are Collinear.

Trial Run 2: Not Collinear, On same side of Plane

Please enter the coordinates of point A:

Point A1: 7

Point A2: 4

Point A3: 8

Please enter the coordinates of point B:

Point B1: 1

Point B2: 5

Point B3: 9

Please enter the coordinates of point C:

Point C1: 2

Point C2: 6

Point C3: 3

Please enter the coordinates of point D1:

Point D1.1:

1

Point D1.2:

2

Point D1.3:

3

Please enter the coordinates of point D2:

Point D2.1:

4

Point D2.2:

5

Point D2.3:

6

Point A [7, 4, 8], Point B [1, 5, 9], and Point C [2, 6, 3], are not Collinear, lie on the plane -4(x - 7) - 55(y - 4) + 31(x + 8), and are on the same side of the ABC-plane.

Trial Run 3: Not Collinear, Not on same side of Plane

Please enter the coordinates of point A:

Point A1: 7

Point A2: 48

Point A3: 1

Please enter the coordinates of point B:

Point B1: 5

Point B2: 9

Point B3: 2

Please enter the coordinates of point C:

Point C1: 6

Point C2: 3

Point C3: 3

Please enter the coordinates of point D1:

Point D1.1:

9

Point D1.2:

8

Point D1.3:

5

Please enter the coordinates of point D2:

Point D2.1:

6

Point D2.2:

4

Point D2.3:

2

Point A [7, 48, 1], Point B [5, 9, 2], and Point C [6, 3, 3], are not Collinear, lie on the plane 87(x - 7) - 9(y - 48) + -177(x + 1), and are not on the same side of the ABC-plane.