Algebra

Definition of Algebra

🔹 **Algebra** is a branch of mathematics that uses numbers, variables, and mathematical symbols to represent and analyze mathematical relationships and structures.

🔹 It allows us to **form and solve equations using symbols and unknown values**, which is not always possible with basic arithmetic (addition, subtraction, multiplication, division).

🔹 Using algebra, we can create **general formulas or equations** to solve complex problems efficiently.

Example:
If a number xx is multiplied by 5 and then 3 is added to get 23, we can represent it as: $5x + 3 = 23$
This is an **algebraic equation**, and algebra helps us solve for xx.

✅ **Important Branches of Algebra:**

1. **Elementary Algebra:** Deals with basic algebraic rules and equations.
2. **Abstract Algebra:** Focuses on mathematical structures like groups and rings.
3. **Linear Algebra:** Involves matrices, vector spaces, and transformations.

Algebra is widely used in real life, including **engineering, computer science, economics, physics, and machine learning**. 😊

Branches of Algebra with Examples

Algebra has several branches, each focusing on different mathematical concepts and structures. Here are the main branches with examples:

1️⃣Elementary Algebra

👉 **Deals with basic algebraic expressions, equations, and operations.**

✅ **Example:** Solve for xx in the equation: $5x + 3 = 18$

Solution: $5x = 18 - 3 \Rightarrow x = \frac{15}{5} = 3$

📌 Used in: Everyday calculations, simple problem-solving, and finance.

2️⃣Abstract Algebra

👉 **Studies advanced mathematical structures like groups, rings, and fields.**

✅ **Example:**

In **group theory**, the set of integers Z with addition forms a group because it satisfies closure, associativity, identity, and inverses.

📌 Used in: Cryptography, coding theory, and advanced mathematics.

3️⃣Linear Algebra

👉 **Focuses on vector spaces, matrices, and linear transformations.**

✅ **Example:** Solve the system of linear equations using matrices:

⟦{cases} $2x + 3y = 8$ \\ $x - y = 2$ ⟧{cases}

Using matrix representation:
$\llbracket\{bmatrix\}\ 2\ \&\ 3 \backslash\backslash\ 1\ \&\ -1\ \rrbracket\{bmatrix\}\ \llbracket\{bmatrix\}\ x \backslash\backslash\ y\ \rrbracket\{bmatrix\} = \llbracket\{bmatrix\}\ 8 \backslash\backslash\ 2\ \rrbracket\{bmatrix\}$

📌 Used in: Computer graphics, machine learning, and physics.

4️⃣ Boolean Algebra

👉 **Deals with binary variables (0 and 1) and logical operations.**
✅ **Example:**
If $A = 1, B = 0$, find $A \wedge B$ (AND operation). $1 \wedge 0 = 0$
📌 Used in: Digital circuits, computer logic, and programming.

5️⃣ Universal Algebra

👉 **Studies algebraic structures in a generalized way, beyond specific examples like groups or rings.**
✅ **Example:**
A **monoid** is an algebraic structure with a single associative binary operation and an identity element, such as natural numbers under addition.
📌 Used in: Theoretical computer science and advanced mathematics.

6️⃣ Commutative Algebra

👉 **Studies commutative rings, polynomials, and ideals.**
✅ **Example:**
The set of polynomials $Z[x]$ with integer coefficients forms a commutative ring since multiplication is commutative:
$a \cdot b = b \cdot a$

📌 Used in: Algebraic geometry and number theory.

7️⃣ Algebraic Geometry

👉 **Studies geometric properties using algebraic equations.**
✅ **Example:**
The equation of a circle:
$x^2 + y^2 = r^2$

📌 Used in: Robotics, physics, and cryptography.

8️⃣ Computational Algebra

👉 **Uses algorithms to solve algebraic problems efficiently.**

✅ **Example:**
Finding the greatest common divisor (GCD) of two numbers using **Euclidean Algorithm**.
For $\mathbf{GCD(48, 18)}$:
48 \div 18 = 2 \text{ remainder } 12 18÷12=1 remainder 618 \div 12 = 1 \text{ remainder } 6
12÷6=2 remainder 012 \div 6 = 2 \text{ remainder } 0

So, **GCD = 6**.
📌 Used in: Computer science and cryptography.

Conclusion
Each branch of algebra plays a crucial role in different fields, from basic problem-solving to advanced scientific research! 😊

Categorized into Different Types

Algebra is a branch of mathematics that deals with symbols, variables, and equations to represent relationships and solve problems. Basic algebra can be categorized into different types based on the nature of expressions, operations, and equations.

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1. Pre-Algebra

This is the foundational level of algebra that prepares for more advanced concepts. It includes:

- Basic arithmetic operations (+, -, ×, ÷)
- Factors and multiples
- Prime numbers
- Fractions, decimals, and percentages
- Basic properties of numbers (commutative, associative, distributive laws)

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2. Elementary Algebra

This introduces variables and basic algebraic operations. It includes:

- Solving simple equations (e.g., $x + 3 = 7$)
- Working with polynomials and algebraic expressions
- Laws of exponents
- Factoring expressions (e.g., $x^2 - 9 = (x - 3)(x + 3)$)
- Linear equations and inequalities

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3. Abstract Algebra (Modern Algebra)

This is an advanced branch that focuses on algebraic structures like groups, rings, and fields. Topics include:

- Groups and subgroups
- Rings and fields
- Vector spaces and modules
- Matrices and determinants

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4. Linear Algebra

This deals with vector spaces and linear equations. It includes:

- Systems of linear equations
- Matrices and matrix operations
- Determinants
- Eigenvalues and eigenvectors

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5. Boolean Algebra

A type of algebra used in logic and computer science, involving only two values: TRUE (1) and FALSE (0). It includes:

- Logical operators (AND, OR, NOT)
- Boolean functions and truth tables
- Simplifying logic expressions

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6. Commutative Algebra

This focuses on the study of commutative rings and their properties. It is useful in algebraic geometry and number theory. Topics include:

- Ideals and quotient rings
- Prime and maximal ideals
- Polynomial rings

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7. Abstract Linear Algebra

An advanced study of linear algebra concepts in a more theoretical way. Topics include:

- Inner product spaces
- Bilinear and quadratic forms
- Spectral theorem

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8. Universal Algebra

This studies algebraic structures in a broad, generalized way, including groups, rings, and fields.

Branches of Algebra:

Algebra can be categorized into several branches that deal with different aspects of algebraic theory and its applications.

1. **Elementary Algebra**:
   - This is the most basic form of algebra, focused on operations with numbers, variables, expressions, and simple equations.
   - **Topics**: Solving linear equations, inequalities, exponents, polynomials, and factoring.
2. **Abstract Algebra (Modern Algebra)**:
   - Focuses on algebraic structures such as **groups**, **rings**, **fields**, and **vectors**.
   - Deals with more abstract concepts that do not necessarily involve numbers directly but instead focus on the properties and operations of algebraic systems.

   Key topics in abstract algebra include:
   - **Group Theory**: The study of groups, which are sets with a single operation (e.g., integers under addition).
   - **Ring Theory**: Focuses on rings, which are sets equipped with two operations (addition and multiplication).
   - **Field Theory**: Concerns fields, which are rings with additional properties, like division.
   - **Linear Algebra**: The study of vector spaces and linear transformations.
3. **Linear Algebra**:
   - Studies vectors, vector spaces, and linear equations.
   - Focuses on solving systems of linear equations and working with matrices and determinants.
4. **Commutative Algebra**:
   - A subfield of algebra that studies commutative rings, which are rings in which multiplication is commutative (e.g., integers).
5. **Homological Algebra**:
   - Deals with algebraic structures in the context of categories and their relationships (especially used in higher mathematics).
6. **Universal Algebra**:
   - Studies algebraic structures and their properties in a more abstract way, dealing with operations on sets and systems that are governed by certain rules.
7. **Boolean Algebra**:
   - Focuses on the algebra of logical operations, such as AND, OR, and NOT.
   - Used in computer science, digital logic, and circuit design.
8. **Computational Algebra**:
   - Concerned with algorithms for solving algebraic problems, including polynomial factorization and solving systems of equations.
   - **Computer Algebra Systems (CAS)** are tools that automate symbolic mathematical computations.

Classification of Algebra:

Algebra can also be classified based on the complexity of the problems being solved and the types of numbers involved.

1. **Elementary Algebra**: Deals with simple equations and expressions involving real numbers.
2. **Advanced Algebra**: Involves more complex algebraic structures like matrices, complex numbers, and abstract structures such as groups, rings, and fields.
3. **Linear Algebra**: Specifically focuses on linear equations, vector spaces, matrices, and their applications.
4. **Abstract Algebra**: Focuses on the study of algebraic systems like groups, rings, and fields that do not depend on particular numbers but on the structure of the system itself.

Conclusion:

- **Algebra** starts with basic **expressions**, **equations**, and **polynomials** and moves into more advanced areas like **abstract algebra** and **linear algebra**.
- It has applications in many branches of mathematics and is foundational for fields like **geometry**, **calculus**, and **number theory**.
- The study of algebra can take different paths, whether through elementary principles, abstract structures, or specific applications in areas like computer science or engineering.

Prerequisites:

To study A-Level Mathematics effectively, you should have a strong foundation in the following topics, typically covered in **O-Level**, **IGCSE**, or equivalent high school curricula:

1. Algebra

- Basic algebraic expressions and identities
- Solving linear and quadratic equations
- Simultaneous equations (both linear and quadratic)
- Factorization, expansion, and simplification
- Inequalities and their graphical representation

2. Geometry and Trigonometry

- Basic properties of triangles, circles, and polygons
- Pythagoras' theorem
- Trigonometric ratios (sine, cosine, tangent)
- Basic trigonometric identities and solving simple equations
- Coordinate geometry of lines

3. Numbers and Arithmetic

- Operations with fractions, decimals, and percentages
- Ratio and proportion
- Surds and indices (exponents)
- Logarithms (basic properties)

4. Functions and Graphs

- Understanding the concept of a function
- Plotting and interpreting linear, quadratic, and cubic graphs
- Transformations of graphs (shifts, reflections, stretches)

5. Basic Calculus (Introductory Level)

- Understanding the concept of differentiation as the rate of change
- Introduction to integration as the reverse of differentiation

6. Probability and Statistics

- Basic probability rules
- Descriptive statistics (mean, median, mode, range)
- Simple data interpretation

Level of Algebra

📘 Step 1: Basic Algebra (Foundation)

এই ধাপটি অ্যালজেব্রার মৌলিক ধারণাগুলো বোঝার জন্য:

- **Numbers and Operations:** Whole numbers, fractions, decimals, negative numbers
- **Factors and Multiples:** HCF, LCM, prime factorization
- **Ratios and Proportions:** Simplifying ratios, solving proportion problems
- **Basic Algebraic Expressions:** Variables, constants, terms, coefficients
- **Simple Equations:** Linear equations with one variable
- **Exponents and Powers:** Laws of exponents

☞ **Reference:** তোমার A-Level Pure Mathematics 1 বই এবং A-Level Mathematics বইতে বেসিক টপিকস সুন্দরভাবে আছে।

🟢 Step 2: Intermediate Algebra (Pre-A Level)

এখন একটু কঠিন সমস্যার দিকে যাওয়া:

- **Linear Equations:** Two-variable systems of equations
- **Quadratic Equations:** Factoring, completing the square, quadratic formula
- **Inequalities:** Linear and quadratic inequalities
- **Polynomials:** Operations, factoring, division
- **Functions:** Linear, quadratic, and exponential functions
- **Coordinate Geometry:** Straight line, slope, midpoint, distance

☞ **Reference:** A-Level Pure Mathematics 1 বই এই পর্যায়ে খুব উপকারী।

⚫ Step 3: Advanced Algebra (A-Level Focus)

এখন A-Level কোর্সের মূল অ্যালজেব্রা টপিকগুলো:

- **Complex Numbers:** Basic operations, polar form
- **Sequences and Series:** Arithmetic, geometric progressions
- **Binomial Theorem:** Expansion of expressions
- **Matrices:** Operations, determinants, inverses
- **Vectors:** Basics and applications

☞ **Reference:** A-Level Pure Mathematics 2 & 3 এবং তোমার *Advanced Engineering Mathematics* বইটি।

📓 Step 4: Problem Solving and Past Papers

এই ধাপে বেশি করে প্র্যাকটিস:

- A-Level Past Papers সমাধান করো
- *2000 Solved Problems in Discrete Mathematics* বই থেকে অ্যালজেব্রার প্র্যাকটিস করো
- **Online Resources:** Examsolutions.net, Khan Academy

Definitions and Examples

Let's break down each step of algebra with clear definitions, characteristics, examples,

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📃 Step 1: Basic Algebra (Foundation)

This step builds your understanding of fundamental algebra concepts.

1. Numbers and Operations

- **Definition:** Numbers are mathematical objects used for counting, measuring, and labeling.
- **Types:**
  - **Whole Numbers:** $0,1,2,3,\ldots$
  - **Negative Numbers:** Numbers less than zero, e.g., -5
  - **Decimals:** Numbers with a decimal point, e.g., 3.75
  - **Fractions:** Parts of a whole, e.g., $\frac{3}{4}$

✅ **Example:**

$5+(-3)=2,\ \ 0.5\times\frac{2}{3}=\frac{1}{3}$

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2. Factors and Multiples

- **Definition:** Factors divide a number exactly, while multiples are products of a number.
- **HCF:** The highest common factor of two numbers.
- **LCM:** The lowest common multiple of two numbers.
- **Prime Factorization:** Expressing a number as a product of primes.

✅ **Example:**

For 12 and 18:

- Factors of $12: 1, 2, 3, 4, 6, 12$
- Factors of $18: 1, 2, 3, 6, 9, 18$
- **HCF:** 6, **LCM:** 36

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3. Ratios and Proportions

- **Ratio:** Comparison of two quantities, e.g., $3:4$.
- **Proportion:** Equality of two ratios, e.g., $\frac{3}{4}=\frac{6}{8}$.

✅ **Example:**

If the ratio of boys to girls is 3:5 and there are 24 girls, find the number of boys.

Solution: $\frac{3}{5}=\frac{x}{24}\Rightarrow x=\frac{3}{5}\times 24\Rightarrow 14.4\text{ (Rounded)}$

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4. Basic Algebraic Expressions

- **Definition:** An algebraic expression is a combination of variables, constants, and operators.
- **Components:**
  - **Variable:** A symbol representing an unknown value, like x.
  - **Constant:** A fixed value, like 5.
  - **Coefficient:** A number multiplying a variable, like 3 in 3x.
  - **Term:** A single variable or constant, like 4y.

✅ **Example:**

Expression: $3x + 5y - 7$

Here, 3 and 5 are coefficients, x and y are variables, and −7 is a constant.

5. Simple Equations

- **Definition:** An equation states that two expressions are equal.
- **Characteristics:** It usually contains an equals sign == and one or more variables.

✅ **Example:**

Solve $2x + 3 = 7$

Solution: $2x = 7 - 3 = 4$

$\Rightarrow x = \frac{4}{2} = 2$

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5. Exponents and Powers

- **Definition:** Exponents show how many times a base is multiplied by itself.
- **Laws of Exponents:**

1. $am \times an = am + n$
2. $\frac{am}{an} = am - n$
3. $(am)^n = amn$
4. $a^0 = 1$

✅ **Example:**

Step 2: Intermediate Algebra (Pre-A Level)

This stage involves more complex algebraic problems.

1. Linear Equations (Two Variables)

- **Definition:** Equations with two variables x and y, forming a straight line when graphed.
- **Form:** $ax + by = c$

✅ **Example:**

Solve the system, $2x + y = 10$: $x - y = 2$

Solution:

From $x - y = 2 \Rightarrow y = x - 2$

Substitute in $2x + y = 10$

$2x + (x - 2) = 10 \Rightarrow 3x - 2 = 10 \Rightarrow x = 4,\ y = 2$

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2. Quadratic Equations

- **Definition:** Equations of the form $ax^2 + bx + c = 0$.
- **Methods of solving:**
  - **Factoring:** $x^2 - 5x + 6 = (x - 2)(x - 3) = 0$
  - **Completing the Square:** $x^2 + 6x + 5 = (x + 3)^2 - 4$
  - **Quadratic Formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

✅ **Example:**

Solve, $x^2 - 4x - 5 = 0$.

Solution: $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)} = \frac{4 \pm \sqrt{36}}{2}$

Thus, $x = 5\ or\ x = -1$.

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3. Inequalities

- **Definition:** Expressions comparing values using $<, >, \leq, \geq <, >, \backslash leq, \backslash geq$.
- **Characteristics:** Flip the sign when multiplying or dividing by a negative number.

✅ **Example:**

Solve, $2x - 3 > 5$.

Solution: $2x > 8 \Rightarrow x > 4$

4. Polynomials

- **Definition:** Expressions with multiple terms, such as $3x^3 + 2x^2 - 5x + 7$.
- **Operations:** Addition, subtraction, multiplication, and division.

✅ **Example:**
$(x + 2)(x - 3) = x^2 - x - 6$

5. Functions

- **Definition:** A relation where each input has exactly one output.
- **Types:** Linear, quadratic, exponential.

✅ **Example:**
For $f(x) = 2x + 3,\ \text{Find } f(4).$
$f(4)\ =\ 2(4)\ +\ 3\ =\ 11$

6. Coordinate Geometry

- **Definition:** Study of points, lines, and shapes on a graph.
- **Key formulas:**
  - Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
  - Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
  - Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

✅ **Example:**
Find the distance between (1,2)$(1, 2)$ and (4,6)$(4, 6)$.
Solution: $d = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

🔵 Step 3: Advanced Algebra (A-Level Focus)

This step involves higher-level concepts for A-level exams.

1. Complex Numbers

- **Definition:** Numbers of the form $a + bia\ +\ bi$, where $i = \sqrt{-1}$.

✅ **Example:**
$(2 + 3i) + (1 - 4i) = 3 - i$

2. Sequences and Series

- **Definition:** Ordered lists of numbers with patterns.
- **Arithmetic Sequence:** $a,\ a + d,\ a + 2d,$ ...
- **Geometric Sequence:** $a,\ ar,\ ar^2,$ ...

✅ **Example:**
Find the 5th term of $3,\ 7,\ 11,$ ....
Solution: $a\ =\ 3,\ d\ =\ 4,\ a_5 = 3 + 4(4) = 19$

3. Binomial Theorem

- **Definition:** Expands expressions like $(a + b)^n$.

✅ **Example:**
Expand $(x + 1)^3$.
Solution: $(x + 1)^3 = x^3 + 3x^2 + 3x + 1$

4. Matrices

- **Definition:** Rectangular arrays of numbers.
- **Operations:** Addition, multiplication, determinant, inverse.

✓ **Example:**

$\llbracket\{bmatrix\}\ 1\ \&\ 2\ \backslash\backslash\ 3\ \&\ 4\ \rrbracket\ \{bmatrix\} + \llbracket\{bmatrix\}\ 5\ \&\ 6\ \backslash\backslash\ 7\ \&\ 8\ \rrbracket\ \{bmatrix\} = \llbracket\{bmatrix\}\ 6\ \&\ 8\ \backslash\backslash\ 10\ \&\ 12\ \rrbracket\ \{bmatrix\}$

5. Vectors

- **Definition:** Quantities with magnitude and direction.

✓ **Example:**

Find the magnitude of $v = 3\hat{\imath} + 4\hat{\jmath}$.

Solution: $\mathrm{db}\{v\}| = \sqrt{}\{3^2 + 4^2\} = \sqrt{}\{9 + 16\} = 5$

Elements of Algebra

Algebra is a broad field of mathematics that deals with **symbols**, **variables**, and the rules for manipulating these symbols to solve equations and understand relationships between quantities. Algebra is foundational to many areas of mathematics, and it can be broken down into several key elements, topics, and branches.

Elements of Algebra:

These are the building blocks of algebra that you'll typically learn in a progressive order:

1. **Basic Arithmetic Operations**:
   - Addition, subtraction, multiplication, and division.
   - Working with **numbers** and **fractions**.
   - Understanding **negative numbers** and **absolute value**.
2. **Variables and Expressions**:
   - **Variables** (like xx, yy, zz) represent unknown or changing quantities.
   - **Algebraic expressions**: Combinations of variables, constants, and operators (e.g., $3x + 5$).
3. **Equations**:
   - **Linear equations**: Equations involving variables raised to the first power (e.g., $2x + 3 = 7$).
   - **Solving equations**: Finding values of variables that make the equation true.
4. **Inequalities**:
   - Expressing relationships that are not necessarily equal, but greater than or less than (e.g., $x > 3$).
   - Solving **inequalities** and graphing the solutions on a number line.
5. **Polynomials**:
   - Expressions involving sums or differences of powers of variables (e.g., $x^2 + 2x + 1$).
   - Operations on polynomials: addition, subtraction, multiplication, division, and factoring.
6. **Factoring**:
   - Finding factors of algebraic expressions (e.g., factoring $x^2 + 5x + 6$ into $(x + 2)(x + 3)$).
   - Techniques include **factorization by grouping**, **difference of squares**, and **quadratic formula**.
7. **Rational Expressions**:
   - Expressions involving fractions with polynomials in the numerator and denominator.
   - Simplifying, adding, subtracting, multiplying, and dividing rational expressions.
8. **Exponents and Powers**:
   - Understanding powers (e.g., x^2) and the **laws of exponents**.
   - Working with scientific notation.
9. **Roots and Radicals**:
   - Working with square roots, cube roots, and higher-order roots.
   - Simplifying radical expressions.
10. **Functions**:
    - A function is a relation that assigns exactly one output to each input.
    - Common types of functions include **linear functions**, **quadratic functions**, and **polynomial functions**.

Types of Terms

In a mathematical expression, a **term** is a single mathematical entity that can be a constant, a variable, or a combination of both with mathematical operations. Here are the different types of terms found in mathematical expressions:

1. Constant Term

- A term that has only a number and no variable.
- Example: $5,\ -3, \frac{12}{4}$

2. Variable Term`

- A term that contains at least one variable.
- Example: $x,\ y,\ a^2,\ 3x,\ -4y^3$

3. Like Terms

- Terms that have the same variable raised to the same power.
- Example: $3x^2$ and $-5x^2$ are like terms because they both have x^2.

4. Unlike Terms

- Terms that have different variables or exponents.
- Example: $2x^2$ and $3x^3$ are unlike terms because their exponents differ.

5. Algebraic Term

- A term that consists of variables, constants, and coefficients.
- Example: $7x^3, -2y^5$

6. Numerical Term

- A term that consists only of numbers (constants).
- Example: $12,\ -7$

7. Polynomial Term

- A term in a polynomial expression, which includes coefficients and variables raised to non-negative integer exponents.
- Example: $4x^5,\ -3x^2,\ 7x$

8. Monomial Term

- A single term that is a product of a constant and variables with non-negative integer exponents.
- Example: $5x^3,\ -2y,\ 7z^2$

9. Binomial Term

- An expression with exactly two terms.
- Example: $3x\ +\ 5,\ x^2 - 4y$

10. Trinomial Term

- An expression with exactly three terms.
- Example: $x^2 + 3x + 2,\ a^3 - 4a + 7$

11. Rational Term

- A term that involves fractions or division with variables in the denominator.
- Example: $\frac{3}{x},\ \frac{5x}{y^2}$

12. Irrational Term

- A term that contains an irrational number like π\pi or 2\sqrt{2}.
- Example: $2\sqrt{3}x,\ \pi r^2$

Types of Expression

In mathematics, an **expression** is a combination of numbers, variables, and operators (such as +, -, ×, ÷, etc.). Expressions do **not** include an equality sign (=), unlike equations. Here are the different types of mathematical expressions:

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1. Algebraic Expressions

An expression that consists of variables, constants, and algebraic operations (+, -, ×, ÷, etc.).

a) Monomial Expression

- Contains only one term.
- Example: $3x,\ -5y^2,\ 7xyz$

b) Binomial Expression

- Contains exactly two terms.
- Example: $\mathrm{x} + 2\mathrm{y},\ 5\mathrm{a}^2 - 3\mathrm{b}$

c) Trinomial Expression

- Contains exactly three terms.
- Example: $\mathrm{x}^2 + 2\mathrm{x} + 3,\ \ \mathrm{a}^3 - 4\mathrm{a} + 7$

d) Polynomial Expression

- Contains one or more terms with variables raised to whole-number exponents.
- Example: $\mathrm{x}^4 + 3\mathrm{x}^2 - 2\mathrm{x} + 7$
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2. Arithmetic Expressions

- An expression that contains only numbers and arithmetic operations (no variables).
- Example: $5 + 3 \times 2,\ \frac{12}{4} - 3$

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3. Rational Expressions

- An expression that represents the ratio (fraction) of two polynomials.
- Example: $\frac{x+2}{x-1},\ \frac{3x}{y+5}$

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4. Irrational Expressions

- An expression that involves irrational numbers like $\sqrt{2}$ or π.
- Example: $\sqrt{x} + 3,\ \pi r^2$

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5. Exponential Expressions

- An expression where variables appear as exponents (powers).
- Example: $2^x,\ 5^{n+1},\ x^3$

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6. Logarithmic Expressions

- An expression that contains logarithms.
- Example: $\log_2 x\ ,\ \ln(y+1)$

7. Trigonometric Expressions

- An expression that involves trigonometric functions like sine, cosine, and tangent.
- Example: $\sin x + \cos x$, $\tan^2 \theta - 1$

8. Absolute Value Expressions

- An expression that contains the absolute value function |x||x|.
- Example: $|x-3|+2$, $|a+b|-5$

9. Determinant and Matrix Expressions

- Expressions that involve matrices or determinants.
- Example:

Types of Equation

In mathematics, an **equation** is a statement that shows the equality between two expressions using the equal sign (=). Equations are classified based on their form, the number of variables, and the types of operations involved.

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1. Based on the Number of Variables

a) Univariate Equation (One Variable)

- An equation that contains only one variable.
- Example: $x + 5 = 10,\ 3x^2 - 2x + 1 = 0$

b) Bivariate Equation (Two Variables)

- An equation that contains two variables.
- Example: $x + y = 7,\ 2x - 3y = 4$

c) Multivariable Equation (More than Two Variables)

- An equation that contains three or more variables.
- Example: $x + 2y + 3z = 10,\ a^2 + b^2 + c^2 = 1$

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2. Based on the Degree of the Equation

a) Linear Equation (Degree = 1)

- An equation where the highest power of the variable is 1.
- Example: $2x + 3 = 7,\ 3x - 4y = 5$

b) Quadratic Equation (Degree = 2)

- An equation where the highest power of the variable is 2.
- Standard form: $ax^2 + bx + c = 0$
- Example: $x^2 - 5x + 6 = 0$

c) Cubic Equation (Degree = 3)

- An equation where the highest power of the variable is 3.
- Standard form: $ax^3 + bx^2 + cx + d = 0$
- Example: $x^3 - 4x + 1 = 0$

d) Polynomial Equation (Degree ≥ 2)

- An equation where the highest power of the variable is greater than or equal to 2.
- Example: $x^4 - 3x^3 + 2x^2 - x + 5 = 0$

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3. Based on the Nature of the Equation

a) Identity Equation

- An equation that is **true for all values** of the variable.
- Example: $(a + b)^2 = a^2 + 2ab + b^2$

b) Conditional Equation

- An equation that is **true for specific values** of the variable.
- Example: $3x - 5 = 10$ (True only when x = 5)

c) Inconsistent Equation

- An equation that has **no solution**.
- Example: $x + 2 = x + 5$ (This is a contradiction because no real number satisfies it.)

4. Based on the Type of Operations

a) Algebraic Equation

- An equation that contains **only algebraic operations** (addition, subtraction, multiplication, division, exponents with whole numbers).
- Example: $x^2 - 4x + 3 = 0$

b) Transcendental Equation

- An equation that contains **non-algebraic operations** like logarithms, trigonometric functions, or exponentials.
- Example:
  - Logarithmic Equation: $\log x = 2$
  - Exponential Equation: $2^x = 8$
  - Trigonometric Equation: $\sin x = 0.5$

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5. Special Types of Equations

a) Diophantine Equation

- An equation where the solutions must be **integers**.
- Example: $x^2 + y^2 = z^2$ (Pythagorean Theorem)

b) Difference Equation

- An equation that expresses the relationship between **successive terms** in a sequence.
- Example: $a_n = 2a_{n-1} + 3$

c) Functional Equation

- An equation where the unknown is a **function**, not just a number.
- Example: $f(x + y) = f(x) + f(y)$

d) Differential Equation

- An equation that contains derivatives.
- Example: $\frac{dy}{dx} + 3y = e^x$

Types of Equations

সমীকরণকে ভেরিয়েবল, ঘাত, এবং রূপের ভিত্তিতে বিভিন্ন ভাগে ভাগ করা হয়।

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1. Based on Degree (ঘাতের ভিত্তিতে):

Type	Standard Form (প্রাথমিক রূপ)	Example (উদাহরণ)	Graph Shape (গ্রাফ আকার)
Linear Equation	$ax + b = 0$	$2x + 3 = 0$	সোজা রেখা (Straight Line)
Quadratic Equation	$ax^2 + bx + c = 0$	$x^2 - 4x + 4 = 0$	প্যারাবোলা (Parabola)
Cubic Equation	$ax^3 + bx^2 + cx + d = 0$	$x^3 - x = 0$	S-আকৃতি (S-Shaped Curve)
Quartic Equation	$ax^4 + bx^3 + cx^2 + dx + e = 0$	$x^4 - x^2 = 0$	W-আকৃতি (W-Shaped Curve)

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2. Based on Variables (ভেরিয়েবলের ভিত্তিতে):

Type	Example (উদাহরণ)	Description (বর্ণনা)
Univariate Equation	$x + 2 = 0$	একটি ভেরিয়েবল থাকে।
Bivariate Equation	$x + y = 5$	দুইটি ভেরিয়েবল থাকে।
Multivariate Equation	$x + y + z = 10$	তিন বা ততোধিক ভেরিয়েবল থাকে।

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3. Based on Function (ফাংশনের ভিত্তিতে):

Type	Example (উদাহরণ)	Usage (ব্যবহার)
Polynomial Equation	$x^3 - 4x + 1 = 0$	বীজগণিত সম্পর্ক বিশ্লেষণে।
Exponential Equation	$2^x = 16$	বৃদ্ধির সমস্যা সমাধানে।
Logarithmic Equation	$\log(x) = 2$	লগারিদমিক বিশ্লেষণে।
Trigonometric Equation	$\sin(x) = 0.5$	ত্রিকোণমিতির সমস্যায়।

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4. Based on Equality (সমানতার ভিত্তিতে):

Type	Example (উদাহরণ)	Description (বর্ণনা)
Conditional Equation	$x + 3 = 5$	নির্দিষ্ট শর্তে সত্য হয়।
Identity Equation	$(x + 1)^2 = x^2 + 2x + 1$	সব মানের জন্য সত্য হয়।
Inconsistent Equation	$x + 1 = x$	কোনো সমাধান থাকে না।

5. Based on System (সিস্টেমের ভিত্তিতে):

Type	Example (উদাহরণ)	Description (বর্ণনা)
Simultaneous Equation	$x + y = 5,\ x - y = 1$	একাধিক সমীকরণ একসাথে সমাধান করা হয়।
Differential Equation	$\frac{dy}{dx} + y = 0$	ফাংশনের পরিবর্তন বিশ্লেষণে ব্যবহৃত হয়।

Types of Function

In mathematics, a **function** is a relation between a set of inputs (domain) and a set of possible outputs (range), where each input has exactly one output. Functions can be classified based on their properties, behavior, and types of relationships between variables.

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1. Based on Mapping of Inputs to Outputs

a) One-to-One Function (Injective Function)

- Each input has a unique output, and no two inputs have the same output.
- Example: $f(x) = 2x + 3$

b) Many-to-One Function

- Multiple inputs can have the same output.
- Example: $f(x) = x^2$ (because $f(2) = 4$ and $f(-2) = 4$)

c) Onto Function (Surjective Function)

- Every element in the range is mapped by some element in the domain.
- Example: $f(x) = x^3$ maps all real numbers to real numbers.

d) Bijective Function (One-to-One and Onto Function)

- A function that is both injective and surjective.
- Example: $f(x) = x + 1$, where every input has a unique output, and all real numbers are covered.

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2. Based on Algebraic Operations

a) Polynomial Function

- A function that involves polynomials of varying degrees.
- Example: $f(x) = x^2 + 3x + 5$

b) Linear Function

- A function of the form $f(x) = ax + b$, where aa and bb are constants.
- Example: $f(x) = 2x + 3$

c) Quadratic Function

- A function of the form $f(x) = ax^2 + bx + c$.
- Example: $f(x) = x^2 - 4x + 7$

d) Cubic Function

- A function of the form $f(x) = ax^3 + bx^2 + cx + d$.
- Example: $f(x) = x^3 + 2x^2 - x + 1$

e) Rational Function

- A function that is a ratio of two polynomials.
- Example: $f(x) = \frac{x+1}{x-1}$

f) Radical Function

- A function that involves roots (square root, cube root, etc.).
- Example: $f(x) = \sqrt{x + 2}$

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3. Based on Nature of Exponents

a) Exponential Function

- A function where the variable is in the exponent.
- Example: $f(x) = 2^x$

b) Logarithmic Function

- A function that is the inverse of an exponential function.
- Example: $f(x) = \log_2(x)$

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4. Based on Trigonometry

a) Trigonometric Function

- Functions involving sine, cosine, tangent, etc.
- Example: $f(x) = \sin x,\ \cos x,\ \tan x$

b) Inverse Trigonometric Function

- The inverse of trigonometric functions.
- Example: $f(x) = \sin^{-1} x$

---

5. Based on Special Properties

a) Even Function

- Satisfies $f(x) = f(-x)$, symmetric about the **y-axis**.
- Example: $f(x) = x^2$

b) Odd Function

- Satisfies $f(-x) = -f(x)$, symmetric about the **origin**.
- Example: $f(x) = x^3$

c) Periodic Function

- A function that repeats its values over regular intervals.
- Example: $f(x) = \sin x,\ where\ f(x + 2\pi) = f(x)$

---

6. Based on Number of Variables

a) Single-Variable Function

- A function with only one input variable.
- Example: $f(x) = x^2 + 3x$

b) Multi-Variable Function

- A function with two or more input variables.
- Example: $f(x, y) = x^2 + y^2$

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7. Based on Special Cases

a) Constant Function

- A function that always returns the same output, regardless of the input.
- Example: $f(x) = 5$

b) Identity Function

- A function where the output is the same as the input.
- Example: $f(x) = x$

c) Piecewise Function

- A function defined by different expressions in different intervals.
- Example: $f(x)\ =\ \llbracket\{cases\}\, x\text{^}2, \&\, x\ <\ 0 \backslash\backslash\, 2x\ +\ 1, \&\, x\ \geq 0\ \rrbracket\ \{cases\}$

d) Step Function

- A function that jumps from one value to another.
- Example: The greatest integer function $f(x) = \lfloor x \rfloor$

Pure Mathematics (A-Level)

G1: Algebra

Algebra – P1

1. **Background Algebra**
    - Manipulating Algebraic Expression
      - You will often wish to tidy up an expression, or to rearrange it so that it is easier to read its meaning

    - Collecting Terms
      - Very often you just need to collect like terms together
      - Unlike terms cannot be added or subtracted, like ($4x + 4y$) remain as it is
      - Like terms can be added or subtracted, like ($4x + 4x = 8x$)
      - Like and unlike terms: $x,\ y,\ z,\ x^2,\ xy,\ yz,\ x^2y,\ 2x,\ 3y,\ 4xy,\ 12x^2$
      - Here like terms are – (x) And (2x) or (y) And (3y) or (xy) And (4xy) or (x2) And (12x2)
      - Here unlike ters are – (x) And (y), (x) And (z), (y) And (z), (x) And (x2), (z) And (xy), (xy) And (yz) etc

    - Removing Brackets
      - Sometimes you need to remove brackets before collecting like terms together
      - Terms outside the bracket called factor, when remove bracket then multiplies each term of inside the bracket with the factor

    - Factorization
      - Rewrite an expression as the **product** of two or more numbers or expressions, and is called **factor**.
      - This usually involves using **brackets** and is called **factorization**
      - Factorization may make an expression easier to use and help to interpret its meaning

    - Multiplications
      - Multiplication like ($axa = a^2$) or ($a\ x\ b\ =\ ab$)

    - Fractions
      - The rules for working fractions in algebra are exactly the same as those used in arithmetic
      - As in arithmetic, start by finding the common denominator
      - Then write each part as the equivalent fraction with that common denominator
      - If parts are related by multiplication of the expression, then term may be cancelled top and bottom as in arithmetic
      - When the numerator (top) and/or the denominator (bottom) are not factorized, first factorize them as much as possible to get common factors which can be cancelled

2. Linear Equations

- Finding the number of sides in a polygon with an angle sum of (1) 180 degree and (2)1080 degree
- Example 1.13:
  i. Equation: $S = 180(n - 1)$
  ii. Identity: $5(x - 1) = 5x - 5$
  To distinguish an identity from an equation, the symbol (≡) is sometimes used
  iii. Steps to solve an equations
    - The equation is $S = 180(n - 2)$
    - Submitting the value (180) for (S) gives $180 = 180(n - 2)$
    - Divided both sides by (180) $1 = 1(n - 2)$
    - Adding (2) to both sides $1 + 2 = (n - 2) + 2$
    - Simplify $3 = n - 2 + 2$
    - Tidy Up $3 = n$
- Example 1.14: $5(x - 3) = 2(x + 6)$
    - Open the brackets $5x - 15 = 2x + 12$
    - Subtract (2x) from both side $5x - 2x - 15 = 2x - 2x + 12$
    - Tidy Up $3x - 15 = 12$
    - Add (15) to both side $3x - 15 + 15 = 12 + 15$
    - Tidy Up $3x = 27$
    - Divided both side by (3) $\frac{3x}{3} = \frac{27}{3}$
    - Tidy Up $x = 9$
- Example 1.15: $\frac{1}{2}(x + 6) = x + \frac{1}{3}(2x - 5)$
    - Start by clearing the fraction.
    - Multiply both sides by 6 # $6x\frac{1}{2}(x + 6) = 6x + 6x\frac{1}{3}(2x - 5)$
    - Tidy up $3(x + 6) = 6x + 2(2x - 5)$
    - Open the brackets $3x + 18 = 6x + 4x - 10$
    - Subtract both side by (6x), (4x) and 18
      $3x - 6x - 4x + 18 - 18 = 6x + 4x - 10 - 6x - 4x - 18$
    - Tidy up $-7x = -28$
    - Divided both side by (-7) # $\frac{-7x}{-7} = \frac{-28}{-7}$
    - Tidy up $x = 4$
- **Exercises**

3. Changing the Subject of a Formula

- The area of a trapezium is given by $A = \frac{1}{2}(a+b)h$
- Here (a) and (b) the length of the parallel sides and (h) is the distance between them
  An **equation** like this is often called a **formula**
  The variable (A) is called the subject of this formula because it only appears once on its own on the left-hand side
- Often need to make one of the other variables the subject of a formula.
  In this case, the steps involved are just the same as those in solving an equation
- Example 1.16: Make (a) the subject in $A = \frac{1}{2}(a+b)h$
  - It is usually easiest if you start by arranging the equation so that the variable you want to be its subject is on the left-hand side
  - Start by clearing the fraction.
  - Multiply both sides by (2) # $2A = 2\,x\,\frac{1}{2}(a+b)h$
  - Tidy up $2A = (a+b)h$
  - Divided both side by (h) $\frac{2A}{h} = \frac{(a+b)h}{h}$
  - Tidy up $\frac{2A}{h} = a + b$
  - Subtract both side by (b) $\frac{2A}{h} - b = a + b - b$
  - Tidy up $\frac{2A}{h} - b = a$
  - Rearrange $a = \frac{2A}{h} - b$
- Example 1.17: Make (T) the subject in the simple interest formula $I = \frac{PRT}{100}$
  - Multiply both side by (100) $I100 = \frac{PRT}{100}x100$
  - Tidy up $I100 = PTR$
  - Divided both side by (P) and (R) $I\frac{100}{PR} = \frac{PRT}{PR}$
  - Tidy up $I\frac{100}{PR} = T$
  - Rearrange $T = I\frac{100}{PR}$
- Example 1.18: Make (x) the subject in the formula $v = w\sqrt{a^2 - x^2}$. Speed of an Oscillating point
  - Square both side $v^2 = w^2(a^2 - x^2)$
  - Divided both side by (w2) $\frac{v^2}{w^2} = \frac{w^2(a^2 - x^2)}{w^2}$
  - Tidy up $\frac{v^2}{w^2} = a^2 - x^2$
  - Add both side by (x2) $\frac{v^2}{w^2} + x^2 = a^2 - x^2 + x^2$
  - Tidy up $\frac{v^2}{w^2} + x^2 = a^2$
  - Subtract both side by (v2/w2) $\frac{v^2}{w^2} - \frac{v^2}{w^2} + x^2 = a^2 - \frac{v^2}{w^2}$
  - Tidy up $x^2 = a^2 - \frac{v^2}{w^2}$
  - Take the root of both side $x = \sqrt{a^2 - \frac{v^2}{w^2}}$
- Example 1.19: Make (m) the subject of the formula $mv = I + mu$. Momentum after an Impulse
  - Collecting terms in (m) on left-hand side $mv - mu = I$
  - Factorise the left-hand side $m(v - u) = I$
  - Divided both side by (v – u) $\frac{m(v-u)}{(v-u)} = \frac{I}{(v-u)}$
  - Tidy up $m = \frac{I}{(v-u)}$

4. Quadratic Equations

- Example (quadratic equation)
    - The length of a rectangular field is $40m$ greater than its width
    - And its area is $6000m^2$
    - Form an equation involving the length, (x)m, of the field
    - Solution –
    - Length of the area is $(x)m$
    - Width of the area is $(x - 40)m$
    - The area is $m^2 = x(x - 40)$
    - So, $6000 = x(x - 40)$
    - So, $6000 = x^2 - 40x$
    - So, $6000 - 6000 = x^2 - 40x - 6000$
    - So, $0 = x^2 - 40x - 6000$
    - Equation: $x^2 - 40x - 6000 = 0$

    - This equation, involving terms in (x2) and (x) as well as a (constant) term, is an example of **Quadratic Equation**
    - A **Linear Equation** in the variable (x) involves only terms (x) and (constant) terms
    - It is usual to write a quadratic equation with the right-hand side equal to (zero)
    - To solve, first factorize the left-hand side if possible

    - Solution $x^2 - 40x - 6000 = 0$
    - $x^2 + 60x - 100x - 6000 = 0$
    - $x(x + 60) - 100(x + 60) = 0$
    - $(x + 60)(x - 100) = 0$
    - So, $x + 60 = 0$
    - Or, $x - 100 = 0$
    - So, $x = -60$ (length cannot be negative)
    - Or, $x = 100$
    - So, length $(x) = 100m$
    - And width $(x - 40) = (100 - 40)m = 60m$
- Example 1.20: $xa + xb + ya + yb$ (a linear expression and factorization)
    - $x(a + b) + y(a + b)$
    - $(a + b)(x + y)$

    - The expression is now in the form of two factors, (x + y) and (a + b), this is answer
    - The same pattern is used for Quadratic Factorization.
    - But first you need to split the middle term into two parts and gives you four terms
- Example 1.21: $x^2 + 7x + 12$ (quadratic expression with all positive terms and factorization)
    - $x^2 + 4x + 3x + 12$
    - $x(x + 4) + 3(x + 4)$
    - $(x + 4)(x + 3)$

    - How to split the middle term, 7x, into 4x + 3x, rather than say 5x + 2x or 9x – 2x
    - Here, Middle term coefficient (7) and the last term constant (12), so should split the middle terms such the way, that produce the same value of middle coefficient by adding them and same as the constant by multiplication

- Example 1.22: $x^2 - 2x - 24$ (quadratic expression with all negative terms and factorization)
    - $x^2 - 6x + 4x - 24$
    - $x(x - 6) + 4(x - 6)$
    - $(x - 6)(x + 4)$
    - It makes no difference in case of factorization of $(+4x - 6x)$ or $(-6x + 4x)$
    - $x^2 - 2x - 24$
    - $x^2 + 4x - 6x - 24$
    - $x(x + 4) - 6(x + 4)$
    - $(x + 4)(x - 6)$
    - Above both are clearly same

    - To get back the to the original expression for checking, multiply the final answer
    - $(x + 4)(x - 6)$
    - $x^2 - 6x + 4x - 24$
    - $x^2 - 2x - 24$
- Example 1.23: $x^2 - 20x + 100$
    - $x^2 - 10x - 10x + 100$
    - $x(x - 10) - 10(x - 10)$
    - $(x - 10)(x - 10)$
    - $(x - 10)2$

    - The above expression was a perfect square and help to recognize the form of such expression $(x - y)^2$, here, $y = 10$
    - Equation: $(x - y)^2 = x^2 - 2xy + y2$
    - $x^2 - 2(x)(10) + 10(2)$
    - So $x^2 - 20x + 100$
- Example 1.25: $x^2 - 49$
    - $x^2 - 49$ can be written as $x^2 + 0x - 49$
    - $x^2 + 7x - 7x - 49$
    - $x(x + 7) - 7(x + 7)$
    - $(x + 7)(x - 7)$

    - The above expression is an example of the difference of two squares which may be written in more general form a $(x2 - 7(2))$. If here $y = 7$, then the form will be $(x2 - y2)$
    - Equation: $x2 - y2 = (x + y)(x - y)$
    - $x2 - y2$
    - $x2 - 7(2)$
    - $x2 - 49$
- Example 1.26: $6x2 + x - 12$
    - $6x2 - x - 12$ (first multiply the first coefficient and the constant, $12 \times 6 = 72$)
    - $6x2 - 9x + 8x - 12$ (now factorize)
    - $3x(2x - 3) + 4(2x - 3)$
    - $(2x - 3)(3x + 4)$
    - All the above examples are factorize using this method. There coefficient of the variable (x2) was (1) so multiplication with (1) and (constant) has not effect

- Example :
  - Before starting the procedure for factorizing a quadratic, you should always check that the terms do not have a common factor as for example in 2x2 – 8x + 6
  - This can be written as 2(x2 – 4x + 3)
  - Factorize 2(x2 – x – 3x + 3)
  - 2{ x(x – 1) – 3(x – 1) }
  - 2{ (x – 1)(x – 3) }
  - 2(x – 1)(x – 3)

  - Or 2x2 – 8x + 6 (check)
  - 2x2 – 6x – 2x + 6
  - 2x(x – 3) – x(x – 3)
  - (x – 3)(2x – x)

5. **Solving Quadratic Equations**
    -
    - Example 1.27 – 1.29
6. **Equations that Cannot be Factorized**
    - Introduction
    - Graphical Solution
    - Completing the Square
    - Example 1.30
7. **The Graphs of Quadratic Function**
    - 7i9/qaxc v/
    - Introduction and Explanation
    - Example 1.31
8. **The Quadratic Formula**
    - Introduction and Explanation
    - Example 1.32 – 1.33
9. **Simultaneous Equations**
    - Introduction
    - Example 1.34 – 1.36
10. **Inequalities**
    - Linear Inequalities
    - Example 1.37 – 1.40

Algebra – P2

1. Operations with Polynomials
2. Solutions of Polynomial Equations
3. The Modulus Function

Further Algebra – P3

4. The General Binomial Expansion
5. Review of Algebraic Fractions
6. Partial Fractions
7. Using Partial Fractions with the Binomial Expansion

G2: Series and Functions

Sequences and Series

1. Definitions and Notation
2. Arithmetic Progressions
3. Geometric Progressions
4. Binomial Expression

Functions

1. The Language of Functions
2. Composite Functions
3. Inverse Functions

G3: Co-Ordinate Geometry

Co-ordinate Geometry

1. Co-ordinates
2. Plotting, Sketching and Drawing
3. The Gradient of a Line
4. The Distance Between Two Points
5. The Mid-point of a Line Joining Two Points
6. The Equation of a Straight Line
7. The Intersection of Two Lines
8. Drawing Curves
9. The Intersection of a Line and a Curve

G4: Trigonometry

Trigonometry – P1

1. Trigonometry Background
2. Trigonometrical Functions
3. Trigonometrical Functions for Angles of Any Size
4. The Sine and Cosine Graphs
5. The Tangent Graph
6. Solving Equations Using Graphs of Trigonometrical Functions
7. Circular Measure
8. The Length of an Arc of a Circle
9. The Area of a Sector of a Circle
10. Other Trigonometrical Functions

Trigonometry – P2

11. Reciprocal Trigonometrical Functions
12. Compound-angle Formulae
13. Double-Angle Formulae
14. The Forms $r \cos(\theta \pm \alpha)$, $r \sin(\theta \pm \alpha)$
15. The General Solutions of Trigonometrical Equations

G5: Calculus

Differentiation – P1

1. The Gradient of a Curve
2. Finding the Gradient of a Curve
3. Finding the Gradient from first Principles
4. Differentiating by Using Standard Results
5. Using Differentiation
6. Tangents and Normals
7. Maximum and Minimum Points
8. Increasing and Decreasing Functions
9. Points of Inflection
10. The Second Derivative
11. Applications

12. The Chain Rule

Differentiation – P2

13. The Product Rule
14. The Quotient Rule
15. Differentiating Natural Logarithms and Exponentials
16. Differentiating Trigonometrical Functions
17. Differentiating Functions Defined Implicitly
18. Parametric Equations
19. Parametric Differentiation

Integration – P1

1. Reversing Differentiation
2. Finding the Area Under a Curve
3. Area as the Limit of a Sum
4. Aras Below the X Axis
5. The Area Between Two Curves
6. The Area Between a Curve and the Axis
7. The Reverse Chain Rule
8. Improper Integrals
9. Finding Volumes by Integration

Integration – P2

10. Integrals Involving the Exponential Function
11. Integrals Involving the Natural Logarithm Function
12. Integrals Involving Trigonometrical Functions
13. Numerical Integration

Further Integration – P3

14. Integration by Substitution
15. Integrals Involving Exponentials and Natural Logarithms
16. Integrals Involving Trigonometrical Functions
17. The Use of Partial Fractions in Integration
18. Integration by Parts
19. General Integration

G6: Others

Logarithms and Exponentials – P2

1. Logarithms
2. Exponential Functions
3. Modelling Curves
4. The Natural Logarithm Function
5. The Exponential Function

Numerical Solution of Equations – P2

1. Interval Estimation – Change-of-Sign Methods
2. Fixed-Point Iteration

Complex Numbers – P3

1. The Growth of the Number System
2. Working with Complex Numbers
3. Representing Complex Numbers Geometrically
4. Sets of Points in an Argand Diagram
5. The Modulus-Argument from of Complex Numbers
6. Sets of Points Using the Polar Form
7. Working with Complex Numbers in Polar Form
8. Complex Exponents
9. Complex Numbers and Equations

Differential Equations – P3

1. Forming Differential Equations from Rates of Change
2. Solving Differential Equations

G7: Vectors

Vectors – P1

1. Vectors in Two Dimensions
2. Vectors in Three Dimensions
3. Vector Calculations
4. The Angle Between Two Vectors

Vectors – P2

5. The Vector Equation of a Line
6. The Vector Equation of Two Lines
7. The Angle Between Two Lines
8. The Perpendicular Distance from a Point to a Line
9. The Vector Equation of a Plane

10. The Intersection of a Line and a Plane
11. The Distance of a Point from a Plane
12. The Angle Between a Line and a Plane
13. The Intersection of Two Planes