

Dynamic Networks

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1. 09.05 Intro - “la raison d'être”, theory & challenges (Ismael)
2. 09.30 DN in navigation (Davide)
3. 09.45 Example 1-Nav (Jan)
4. 10.30 !!! COFFEE BREAK !!!
5. 11.00 DN with optical constrains (Davide)
6. 11.20 Example 2-Nav/lidar (Jan)
7. 11.35 DN in photogrammetry (Davide)
8. 11.50 Example 3-Nav/photo
9. 12.15 Example 4-UAV (Jan)

An extension to **Geodetic Networks**

Differences:

1. One body which *moves*.
2. Points are the position of that body as a function of *time*.
3. Sensor measurements have to do with the *dynamic* properties of the body (e.g., acceleration, velocity, ...).



Figure: The Swiss gravity network.

The literature is huge, I give the *oldest* papers that I know of.

1. Time dependent networks

Colomina, I., and M. Blázquez. "A unified approach to static and dynamic modelling in photogrammetry and remote sensing." *Proceedings of the XXth ISPRS Congress, Istanbul, Turkey*. 2004.

2. In terrestrial mobile mapping

Rouzaud, D., and Skaloud, J. "Rigorous integration of inertial navigation with optical sensors by dynamic networks." *Navigation* 58.2 (2011): 141-152.

The same concept is referred as *factor-graphs*, nowadays standard in Simultaneous Localization and Mapping (SLAM).

N.B. A DN is a factor-graph, a factor-graph is *sometimes* a DN

1. Least-squares network optimization

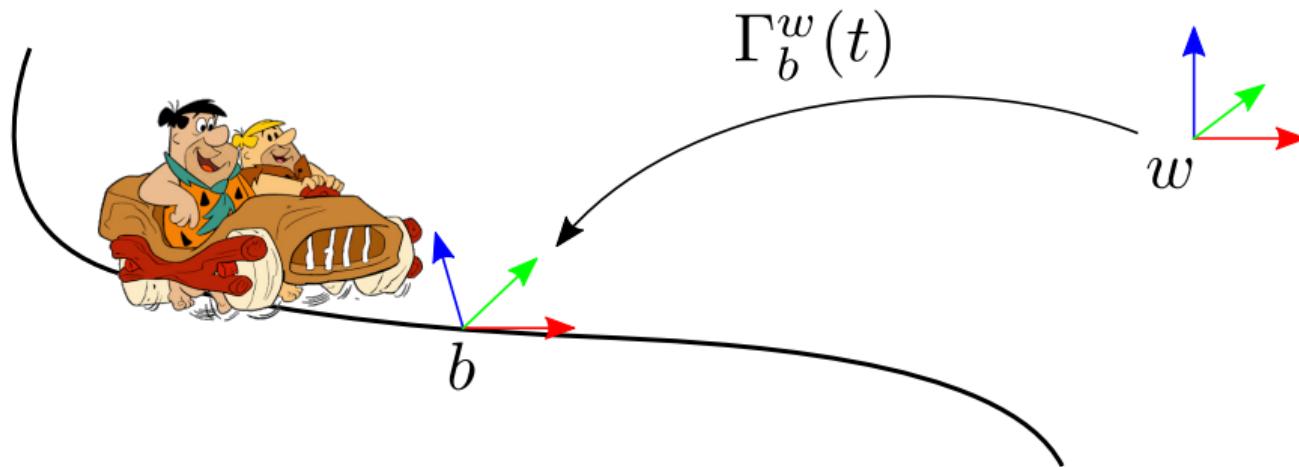
Kümmerle, R., et al. “g²o: A general framework for graph optimization.” *2011 IEEE International Conference on Robotics and Automation*. IEEE, 2011.

2. Georgia Tech Smoothing and Mapping (GTSAM)

Kaess, M., et al. “iSAM2: Incremental smoothing and mapping using the Bayes tree.” *The International Journal of Robotics Research* 31.2 (2012): 216-235.

3. Ceres Solver (Google)

To understand DNs we need to start from **navigation**.



1. Determine Fred's and Barney's **pose** $\Gamma_b^w(t) = [r^w(t), R_b^w(t)]$ from sensor measurements
2. (side-effect) determine other **parameters**, e.g., inertial sensor biases $bf(t)$ and $b\omega(t)$

1. We mount a set of sensors on b : $\{s_1, s_2, \dots, s_n\}$,
2. These sensors provide measurements, e.g., $z_{s_1,t}$,
3. These measurements are **noisy**,
4. We come up with a deterministic and stochastic model for z in the form

$$z_{s_j,t} = f_j \left(r^w(t), R_b^w(t), \underbrace{\dot{r}^w(t), \dot{R}_b^w(t)}_{\text{derivatives!}}, \dots, \underbrace{\xi_j(t)}_{\text{noise}} \right),$$

5. This gives $P(z_{s_j,t} | r^w(t), R_b^w(t), \dots)$,
6. We use this to determine $r^w(t), R_b^w(t)$, see later on.

The “state” (... steals money from people with taxes)

The **state**, \mathbb{X} , is the set of quantities that we need to *predict* the available sensor measurements (**Complete state assumption**, or **Markov assumption**).

What does it mean to “*predict*”?

$$\hat{z}_{s_j,t} = \mathbb{E}[z_{s_j,t} | \mathbb{X}]$$

N.B. If $\xi_j(t) \sim \mathcal{N}(0, \Sigma_j)$ and additive, i.e., $f_j(\mathbb{X}, \xi_j(t)) = f_j(\mathbb{X}) + \xi_j$

$$\hat{z}_{s_j,t} = f_j(\mathbb{X})$$

1. $\hat{\mathbb{X}}$ is what we can *hope* to estimate from the measurements that we have.
2. How can we formulate an estimator for $\hat{\mathbb{X}}$?

Unordered list of problems that you encounter in navigation:

1. defining f_j is already tricky in some cases,
2. f_j is typically non-linear in \mathbb{X} ,
3. ξ_j is not always Gaussian
4. \mathbb{X} is a non-Euclidean manifold, typically includes elements of SO(3) or SE(3)

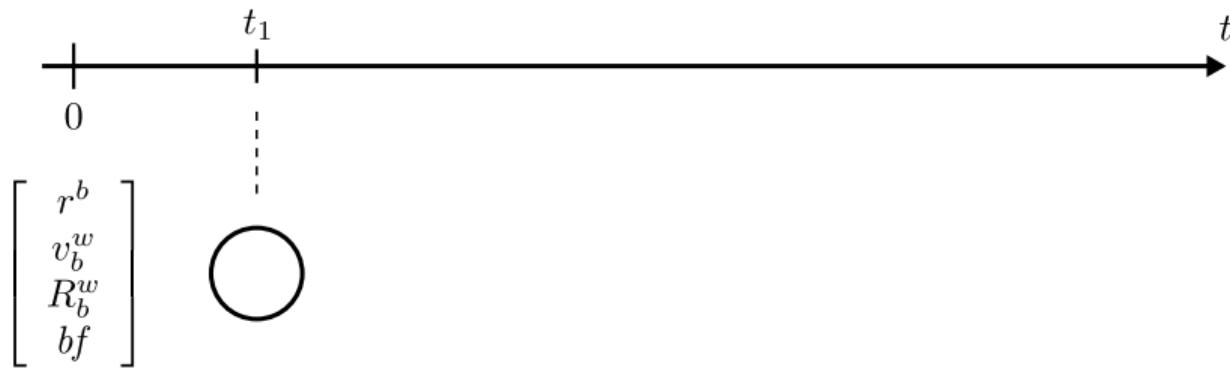
Quite useful: Hertzberg, C., et al. "Integrating generic sensor fusion algorithms with sound state representations through encapsulation of manifolds." *Information Fusion* 14.1 (2013): 57-77.

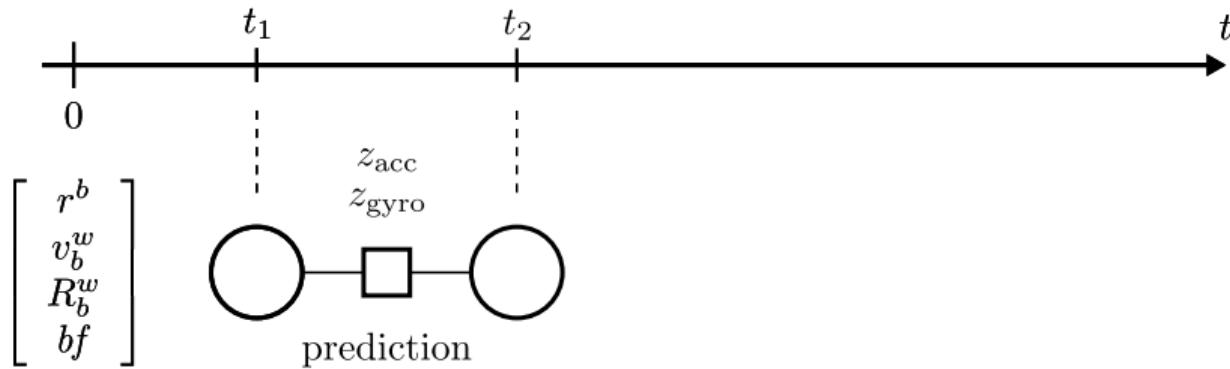
Measurements come one after the other, right?

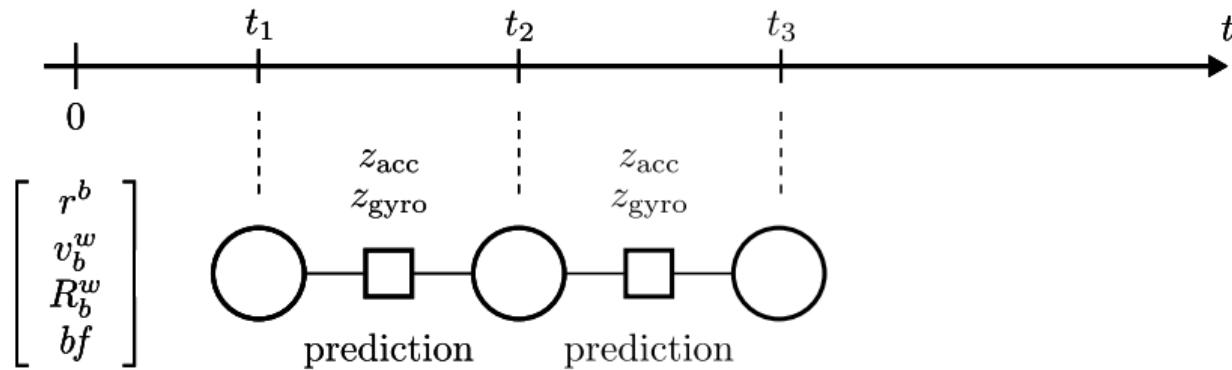
1. We remember the Markov assumption,
2. We have the state at time $t - 1$, \mathbb{X}_{t-1} ,
3. We have a notion of the distribution of \mathbb{X}_{t-1} , $P(\mathbb{X}_{t-1})$, a **belief** in robotics,
4. We have a control u_t (or other known forcing input, e.g. from IMU) [optional],
5. We have a way to tell $P(\mathbb{X}_t|u_t, \mathbb{X}_{t-1})$, (**prediction step**)
6. We can do Bayes: $P(\mathbb{X}_t) = \lambda P(z_t|\mathbb{X}_t) \int P(\mathbb{X}_t|u_t, \mathbb{X}_{t-1})P(\mathbb{X}_{t-1})d\mathbb{X}$ (**update step**)

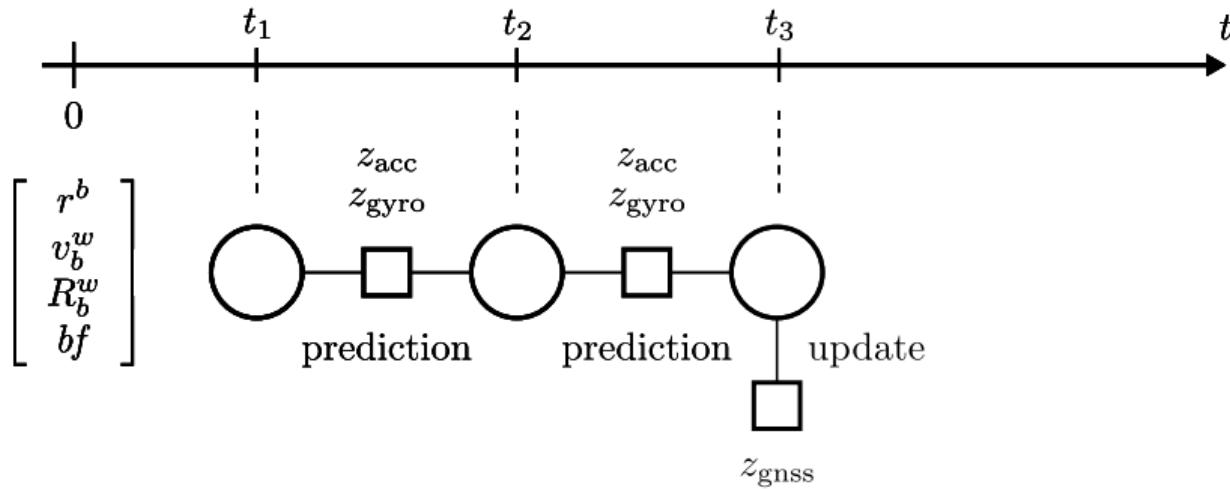
This leads to the **Kalman filter**, which is very good.

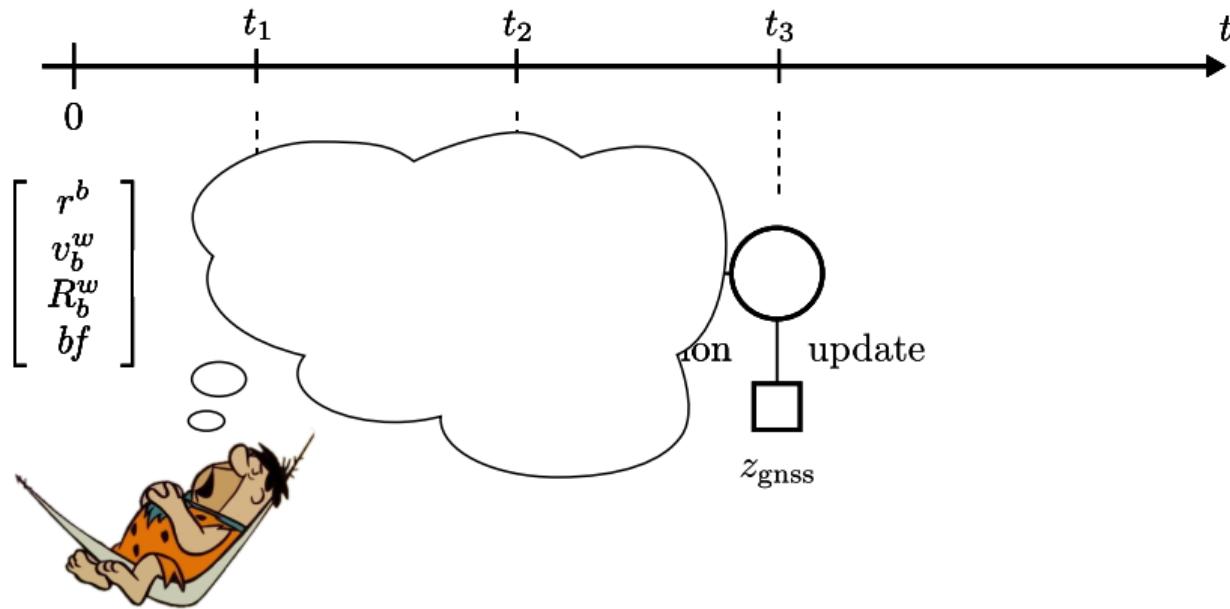
It is better to look at Bar-Shalom, Yaakov, et al. *Estimation with applications to tracking and navigation: theory algorithms and software*. John Wiley & Sons, 2004.

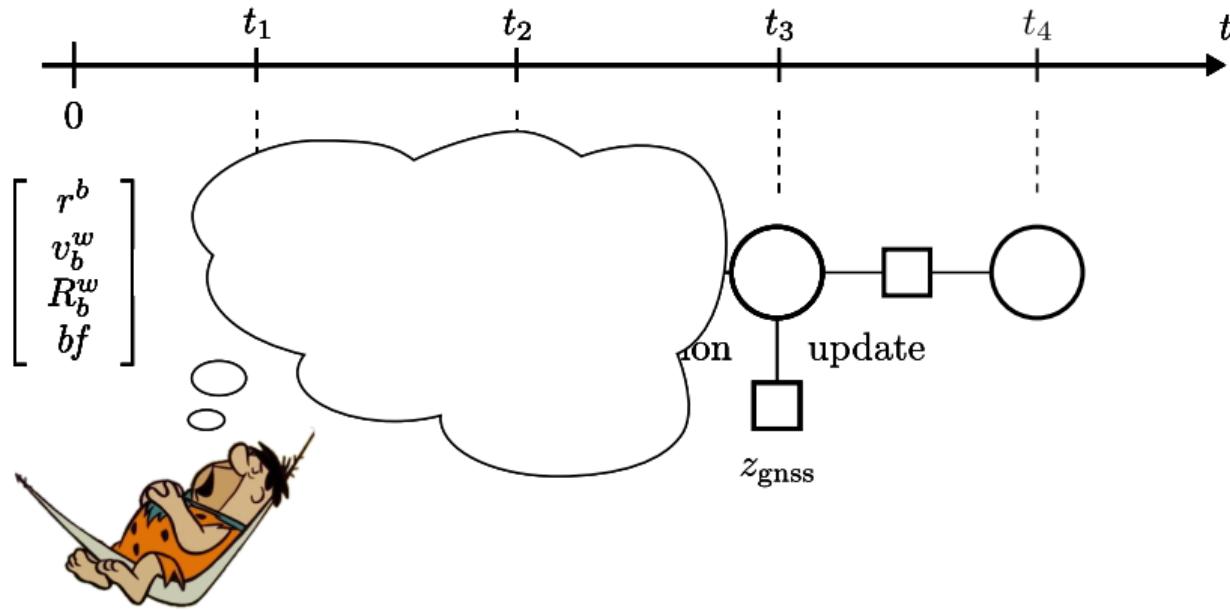


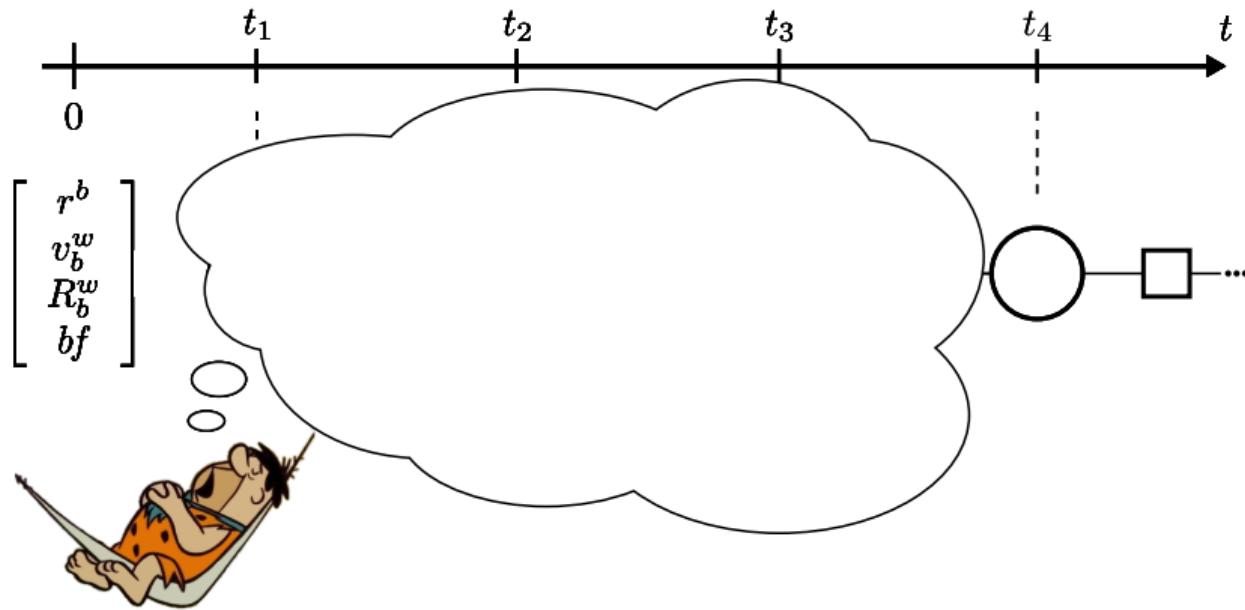












PROs:

1. Well established
2. Very low time-and-memory complexity

CONs:

1. New evidence can't correct past states (Markov!!)



2. High sensitivity to initial approximations (e.g., attitude initialization with low-cost IMUs)
3. Difficult rigorous formulation of spatial constraints
e.g., "*I was already here before*", as in airborne gravimetry (Skaloud et al, 2016)

We don't like *beliefs*.



Figure: The Nihilists (The Big Lebowski, 1998)

1. We consider all times and all measurements together
2. We know $P(z_j|\mathbb{X})$, then we define

$$\hat{\mathbb{X}} = \operatorname{argmax}_{\mathbb{X}} \prod_j \mathcal{L}(\mathbb{X}; z_j)$$

3. If ξ_j is additive and $\xi_j \sim \mathcal{N}(0, \Sigma_j)$:

$$\hat{\mathbb{X}} = \operatorname{argmin}_{\mathbb{X}} \sum_j \left(z_j - f_j(\mathbb{X}) \right) \Sigma_j^{-1} \left(z_j - f_j(\mathbb{X}) \right)^T \quad (\text{NL least-squares})$$

4. We can do robust estimation if you so wish.

Inertial navigation in DNs

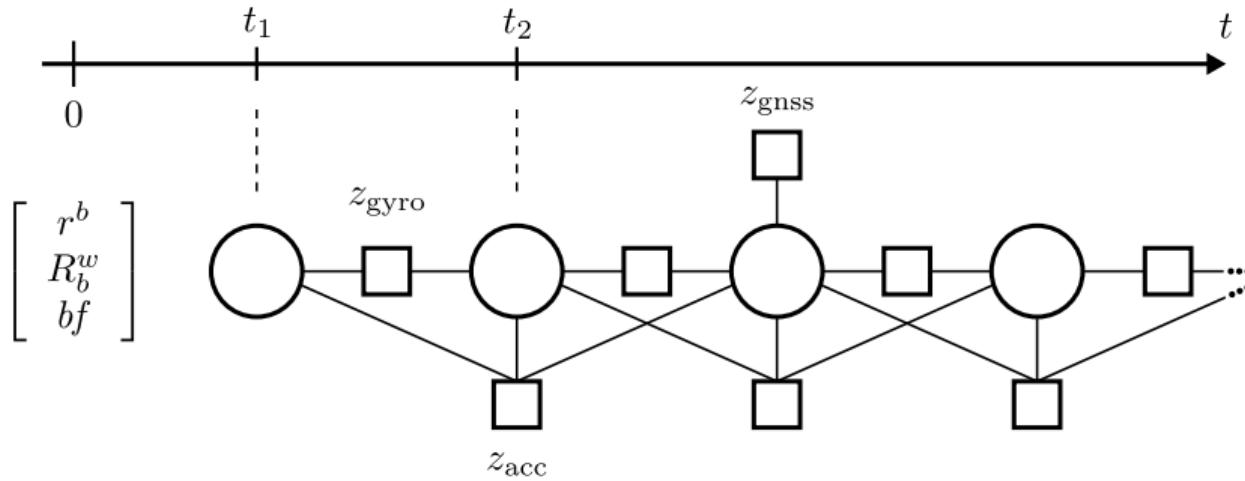


Figure: A simple DN for inertial navigation

Where did the velocities go?

1. v_b^w and ω_{wb}^b are not estimated explicitly
2. ... but approximated via finite differences of poses

Example: Gyroscope observation model

Earth rotation. Yes, it turns..

$$z_{\omega,t} = \dots + \underbrace{\frac{\log(R_b^w(t_{i-1})^T R_b^w(t_i))}{t_i - t_{i-1}}}_{\omega_{wb}^b(t)} + b\omega(t) + \xi_\omega(t).$$

For details see:

Cucci, D. A., Rehak, M., and Skaloud, J. "Bundle adjustment with raw inertial observations in UAV applications." *ISPRS Journal of Photogrammetry and Remote Sensing* 130 (2017): 1-12. [→ link]

1. Beliefs are hard to die, sometimes we need them.
2. Prior knowledge becomes zero-measurements, as if there was a sensor always measuring 0.
3. Example: auto-regressive 1st-order (AR1) process for the gyroscope bias

$$z_{\text{AR1},t} = 0 = \underbrace{b\omega(t) - \phi b\omega(t-1)}_{f(\mathbb{X})} + \xi$$

4. Many other types of prior knowledge can be formulated like this. ML becomes MAP.

For details see:

Cucci, D. A., et al. "A general approach to time-varying parameters in pose-graph optimization." *2017 European Navigation Conference (ENC). IEEE, 2017* [[→ link](#)]

PROs:

1. Joint & optimal estimation, no Forward/Backward/Smoothing needed
2. Lower sensitivity to initialization approximations (e.g., without large-heading models)
3. Direct integration of spatial constraints (e.g., optical observations)
4. Better de-correlation between parameters
5. As in geodetic networks, we can evaluate of redundancy, controllability, sensitivity, ...

CONs:

1. Severe time-and-memory complexity (but we can handle it!)
2. Ideal for post-processing, tricks needed for real-time.

Example 1 - INS/GNSS navigation

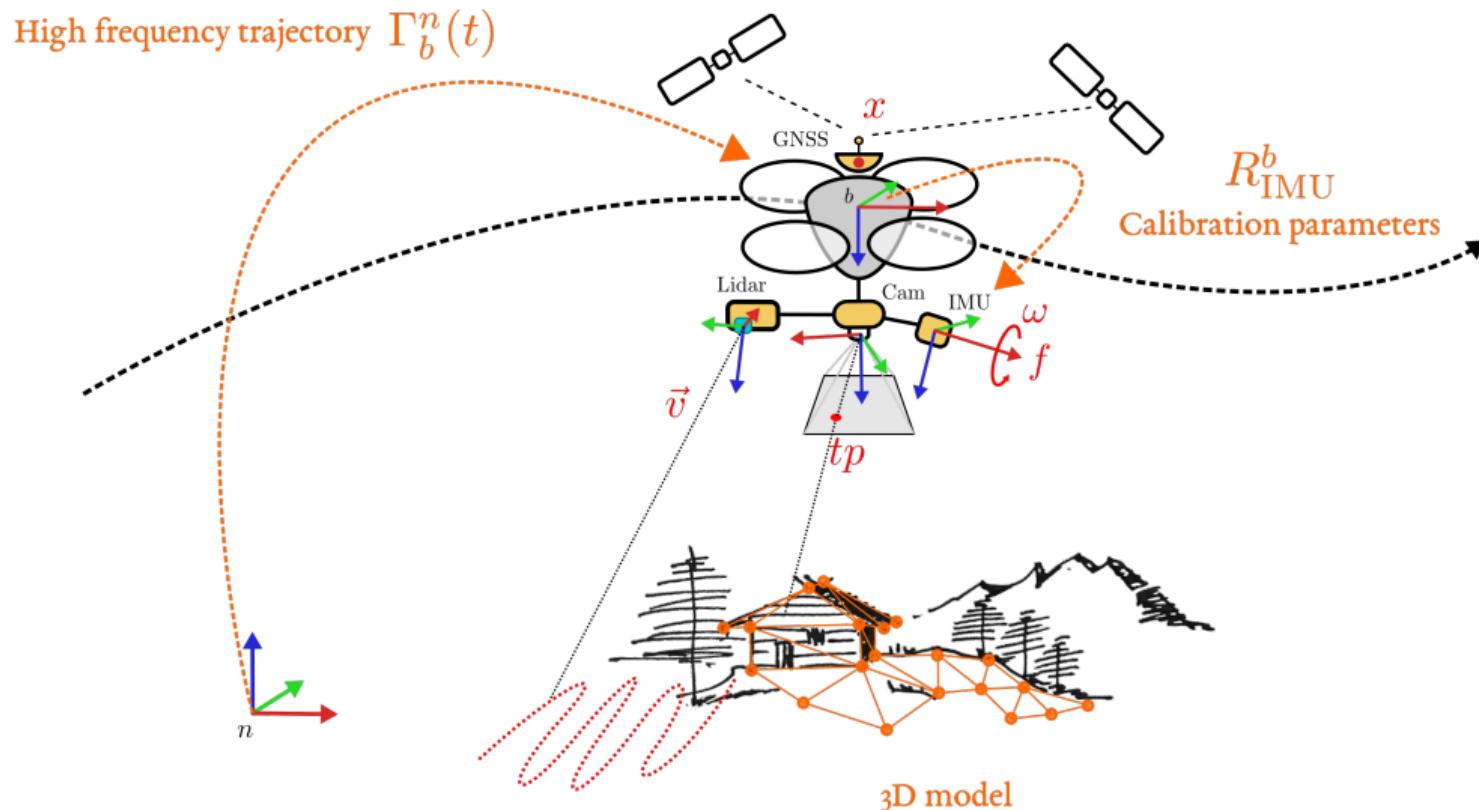
1. A solver for DN adjustment problems that run in the web browser

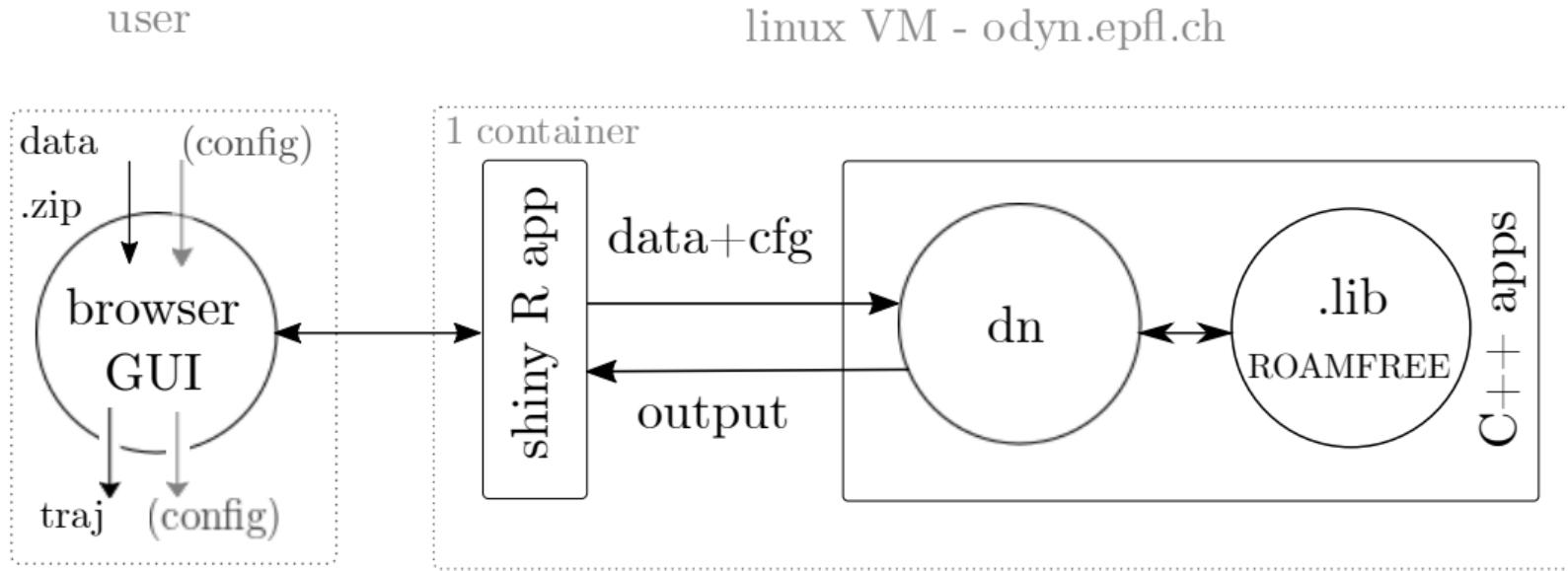


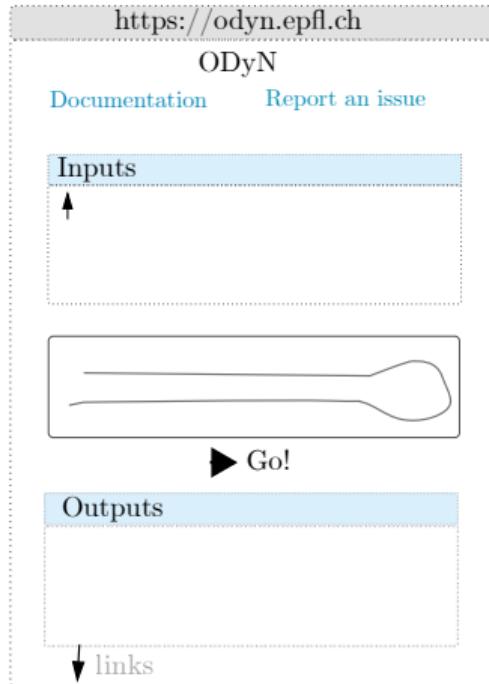
2. Based on the open-source **ROAMFREE** sensor fusion library
3. Available for free at [https://odyn.**EPFL**.ch](https://odyn.epfl.ch)
4. The processing happens on cloud servers
5. Introduced at this congress, for more info please see

Cucci, D.A., “ODyN: An Online Dynamic Network Solver For Photogrammetry And Lidar Geo-referencing”, *ISPRS Annals of Photogrammetry, Remote Sensing and Spatial Information Sciences*, V-1-2022, 153–159. [→ link]

ODyN: an Online Dynamic Network Solver







Documentation (github)

- readme
- data

Inputs

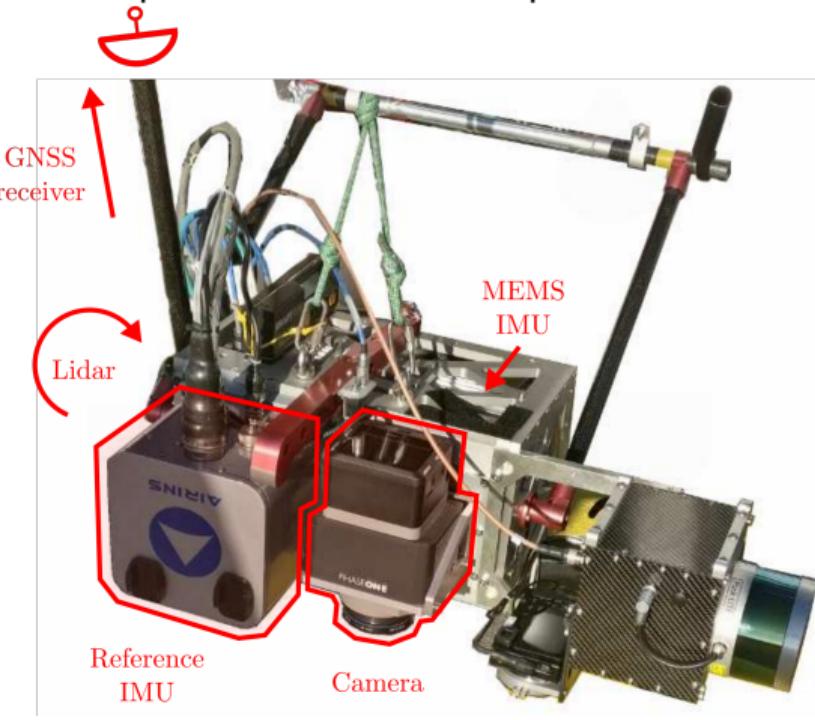
- data.zip (& config.)
- gui config (mounting, σ , etc.)

Outputs

- plots + stats
- estimated param.
- adjusted trajectory (link)

Example 1 - IMU + GNSS

Reference and UAV grade sensor mounted on the same platform on an helicopter.



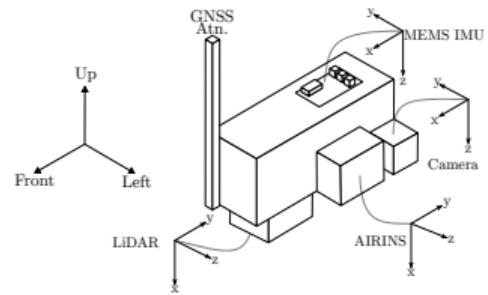
Example 1 - IMU + GNSS

Experiment (subset of):

Vallet et al. (2020), "Airborne And Mobile Lidar, Which Sensors For Which Application?" *ISPRS Archives* 43 (2020): 397-405.

DN - Processed:

1. Brun et al. (2022), "Lidar point-to-point correspondences for rigorous registration of kinematic scanning in dynamic networks", *J. ISPRS*, 89: 185-200 (open access).
2. Mouzakkidou et al. (2022), "On the benefit of concurrent adjustment of active and passive optical sensors with GNSS & raw inertial data", *ISPRS Annals* (Congress - Nice).



Sensors

- drone-like
- reference

Example 1 - IMU + GNSS

Goal:

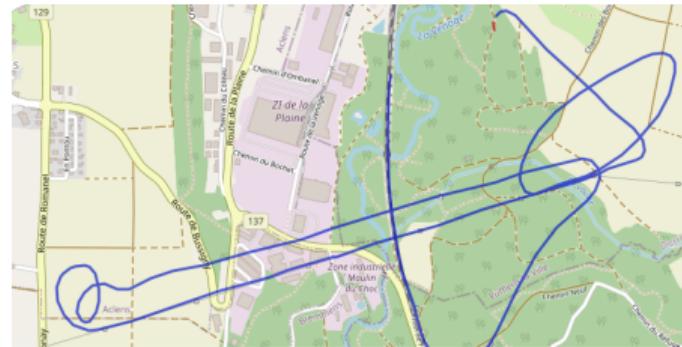
$$\text{IMU}(t, \Delta\theta, \Delta\nu) + \text{GNSS (PPK)} \rightarrow \text{PVA}(t)$$

Needed:

1. IMU.txt
2. GNSS.txt (or .cmb)

Optional:

3. initial_guess.txt (or .out)
4. reference.txt (or .out)
5. configuration.RData



Dynamic network (online App)
odyn.epfl.ch

Download data

- odyn.epfl.ch / Documentation (new TAB)
- /Examples/Configuration
- → data/configuration.RData
- /Examples/Inertial Navigation
- → data/INS.zip

Upload & process

- odyn.epfl.ch
- upload data [.zip]
- upload config [.Rdata]
- → Go!
- ... analyze

Example 1 - INS/GNSS navigation ODyN demo

More results on INS/GNSS with DNs

Cucci, D. A., et al. "A general approach to time-varying parameters in pose-graph optimization." *2017 European Navigation Conference (ENC)*. IEEE, 2017. [→ link]

1. ≈ 20 min of car driving, 9.5 km in Vuarrens(CH)
2. tested: MEMS IMU + standalone GNSS.
3. reference: nav. grade IMU + carrier-phase differential GNSS.

We compare

1. DN with respect to a Kalman smoother (Stebler, 2013)
2. ... different inertial sensor biases sampling rates

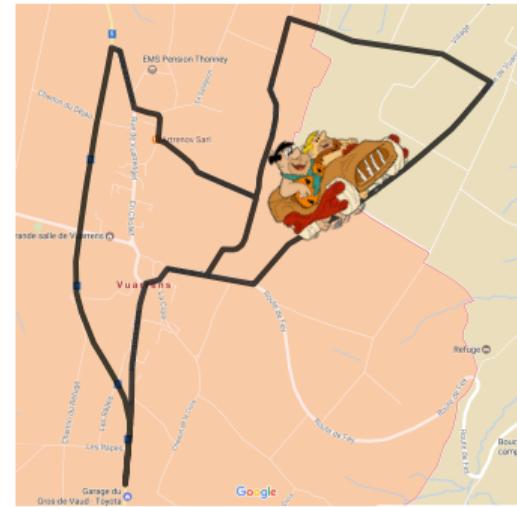


Figure: The trajectory.

	ROAMFREE			NAVPROC
	5s-2.5s	30s-10s	90s-30s	
ENU [m]	0.97 (-5%)	0.97 (-5%)	0.95 (-7%)	1.02
	1.54 (-1%)	1.54 (-1%)	1.51 (-1%)	1.53
	2.96 (-2%)	2.91 (-5%)	2.45 (-15%)	2.89
RPH [deg]	0.09 (-52%)	0.09 (-51%)	0.10 (-42%)	0.18
	0.12 (-47%)	0.12 (-47%)	0.12 (-45%)	0.23
	1.44 (-29%)	1.42 (-30%)	1.52 (-25%)	2.02

Table: East-North-Up (ENU) and Roll-Pitch-Heading (RPH) error RMS with respect to reference.

A very recent work

Wen, W., et al. "Factor graph optimization for GNSS/INS integration: A comparison with the extended Kalman filter." *NAVIGATION, Journal of the Institute of Navigation* 68.2 (2021): 315-331.

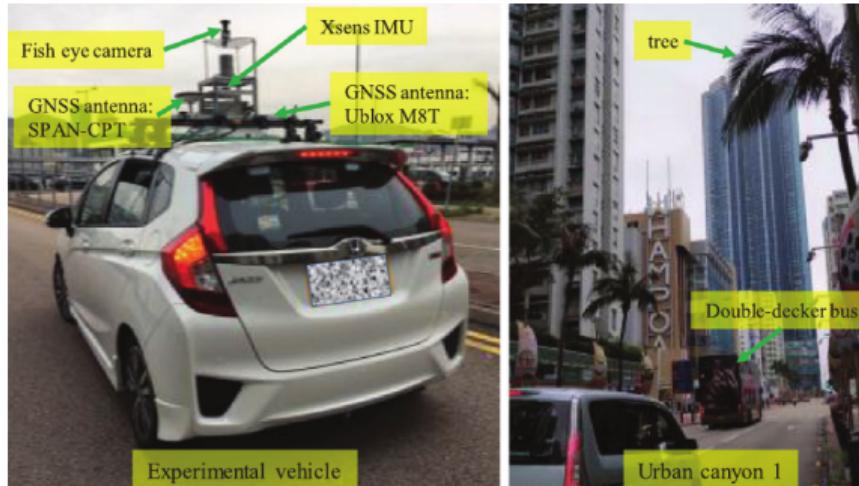


Figure: Experimental setup

Wen, W., et al. "Factor graph optimization for GNSS/INS integration: A comparison with the extended Kalman filter." *NAVIGATION, Journal of the Institute of Navigation* 68.2 (2021): 315-331.

1. DN (factor-graph) vs Kalman Filtering (not smoothing, sadly)
2. Loosely and **tightly** coupled GNSS
3. Urban canyons (they look only at position)

All data	Loosely coupled EKF	Tightly coupled EKF	Loosely coupled FGO	Tightly coupled FGO
Mean error	9.14 m	8.03 m	7.01 m	3.64 m
Std	7.60 m	7.15 m	6.41 m	2.84 m
Used Time	0.053 s	0.071 s	15.41 s	75.30 s

Figure: Positioning performance and computational load (time)

Optical constraints

1. **Photogrammetry**: Image observations / tie-points / 2D - 2D correspondences
2. **Laser scanning**: LiDAR observations / 3D - 3D correspondences

Photogrammetry

1. Automatic identification of feature points in images
2. Bundle Adjustment → camera poses, features 3D position, calibration (optionally)
3. Absolute scale and geo-referencing via Ground Control Points (GCPs)

Issue: GCPs are time/cost intensive

Poorly distributed / insufficient GCPs lead to large-scale deformations.

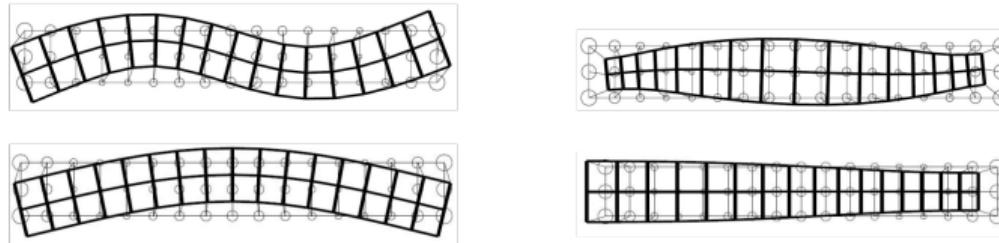


Figure: Typical corridor *quasi-systematic* errors.
From W. Förstner, UAV-g, Sept, 4th 2017.

GNSS/Inertial fusion gives priors for the camera position and orientation.

1. Global accuracy with less/no GCPs
2. Complex pipeline

Issues: initial alignment in KF, bias/bore sight determination, prior uncertainties, etc.

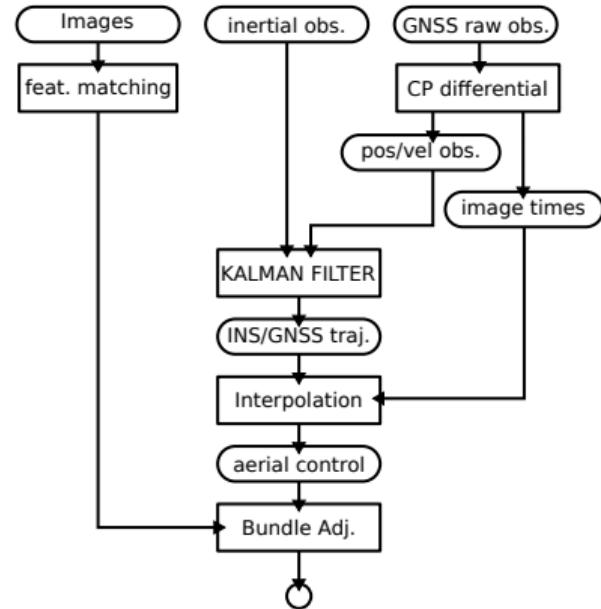


Figure: AAT pipeline

A single step, joint-adjustment solution.

Higher redundancy → stronger conditions on parameter estimation.

Issues: less to none :)

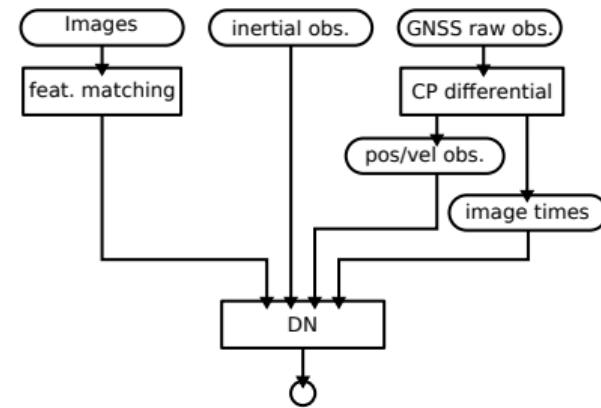
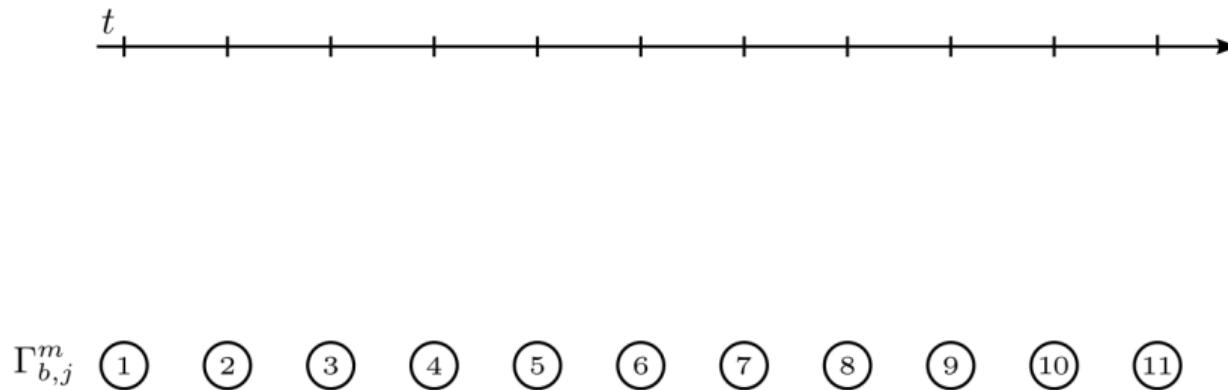
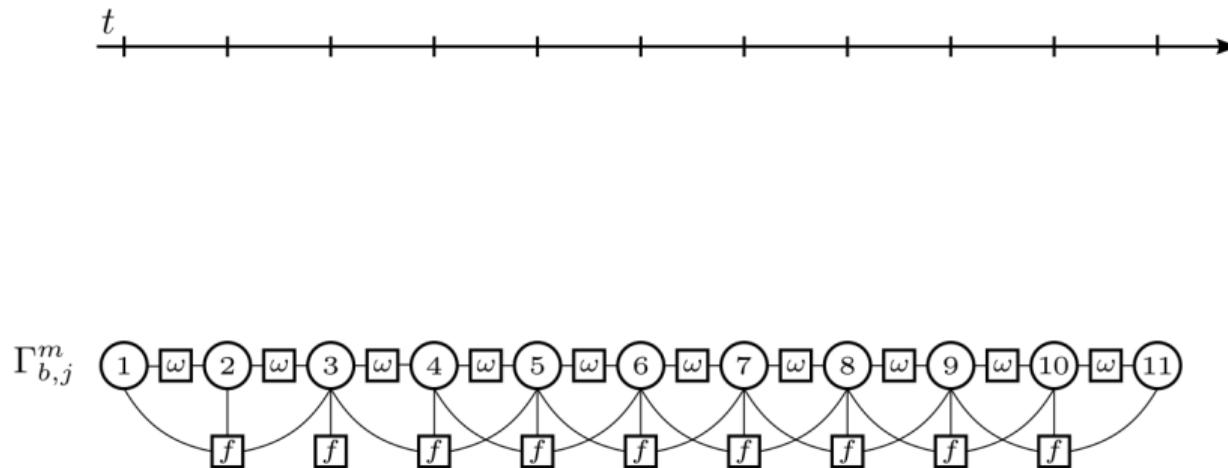
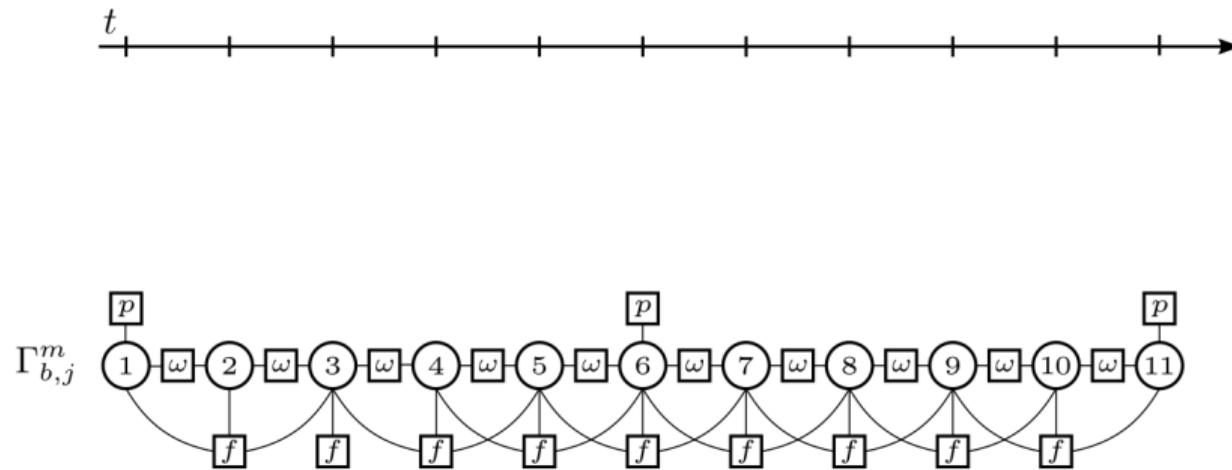


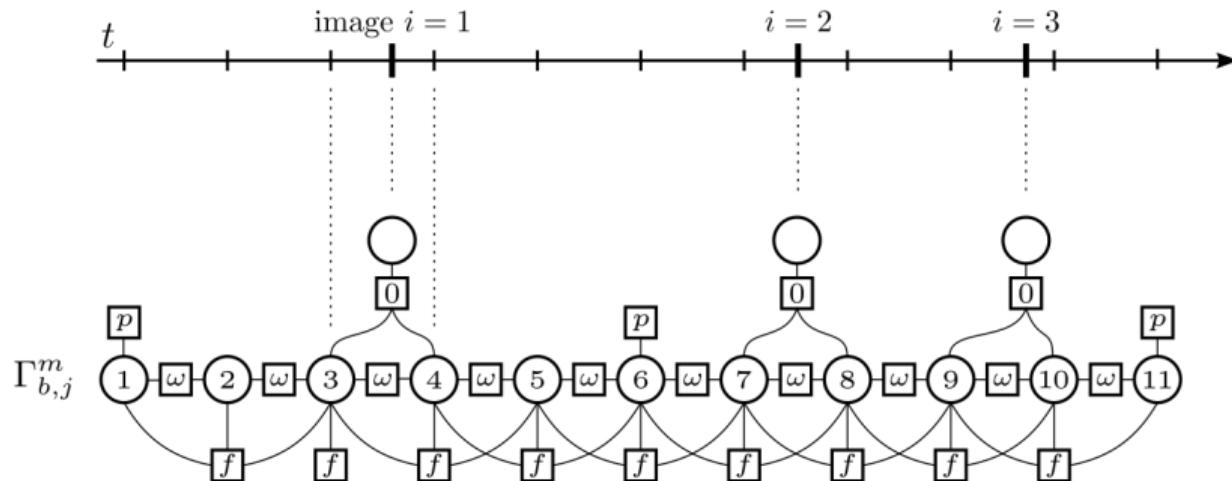
Figure: DN pipeline



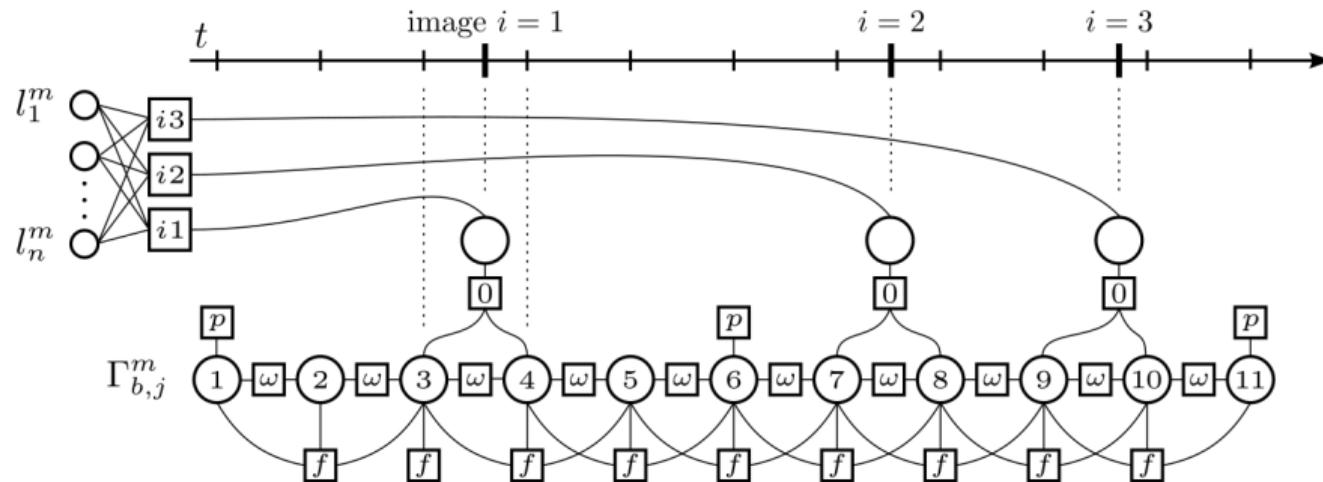




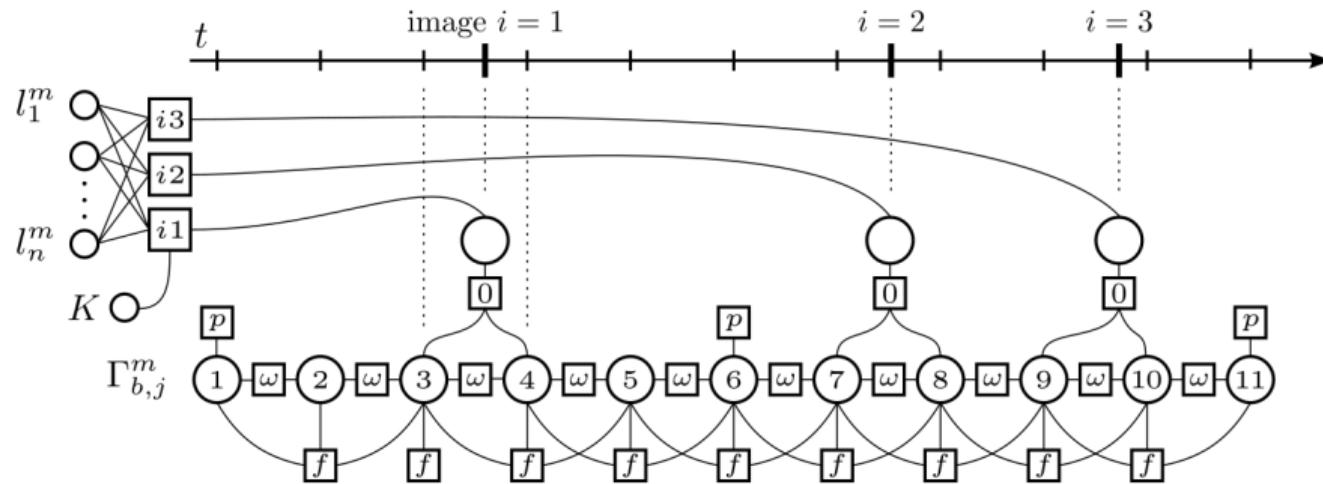
Dynamic Networks for Navigation and Mapping



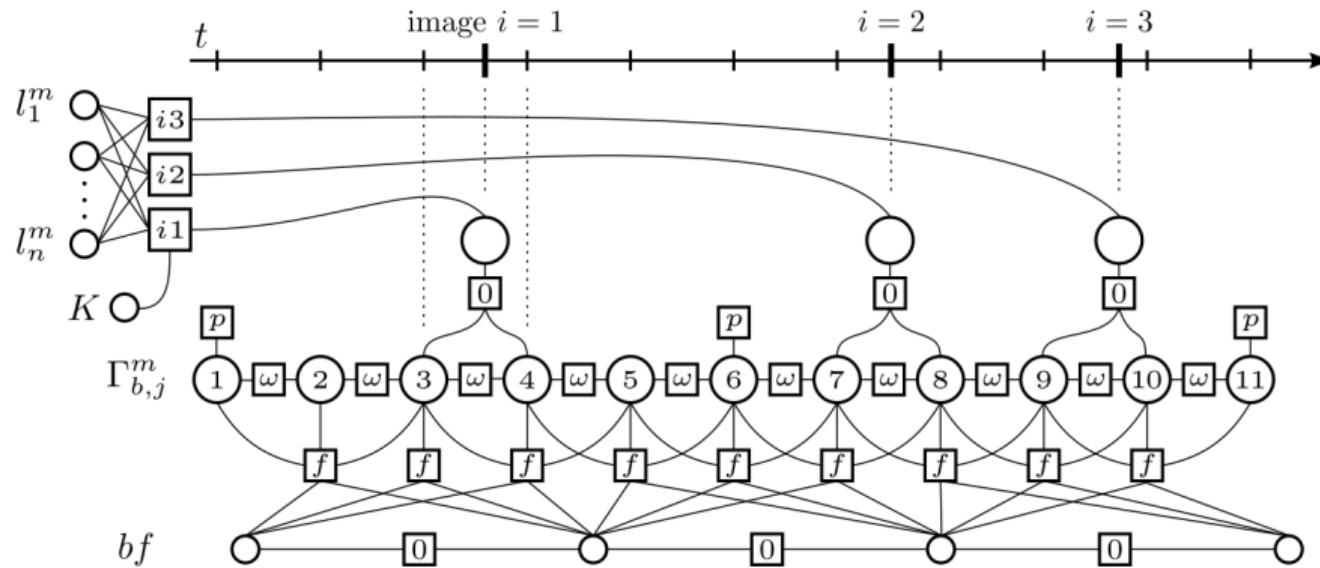
Dynamic Networks for Navigation and Mapping

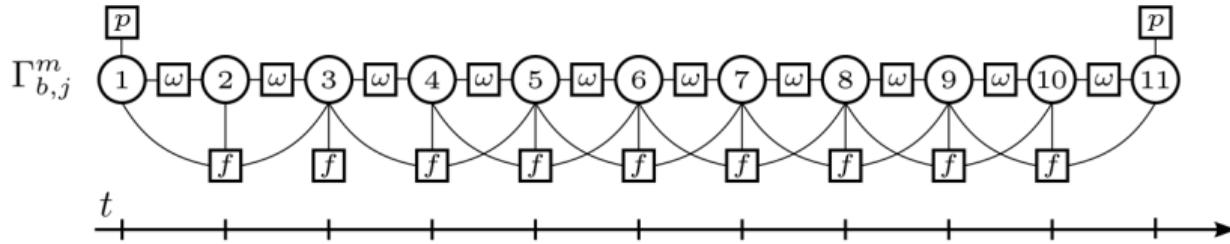


Dynamic Networks for Navigation and Mapping

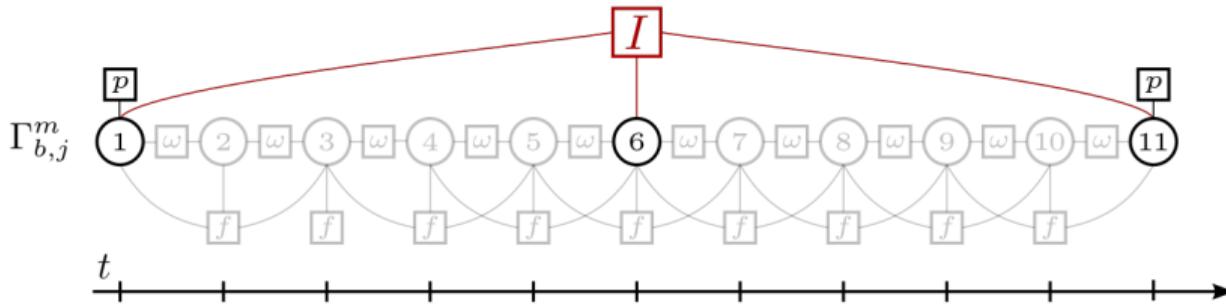


Dynamic Networks for Navigation and Mapping





1. Bloating number of unknowns (e.g.: 1 KHz IMU, minimum 6×1000 unknowns/s)
2. Ill-conditioning of the normal equations



1. Reduces the number of measurement **without** loss of information (no downsampling)
2. Complications with IMU stochastic processes (not entirely understood)

1. Original publication:

Lupton, T., and Sukkarieh, S. "Efficient integration of inertial observations into visual SLAM without initialization." *2009 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE*, 2009.

2. All-in with Lie Algebra

Barrau, A., and Bonnabel, S. "A mathematical framework for IMU error propagation with applications to preintegration." *2020 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2020.

3. Our contribution: **gravity models** and **Earth rotation** in IMU-preintegration:

Cucci, D. A., and Skaloud, J. "On Raw Inertial Measurements in Dynamic Networks." *ISPRS Annals of Photogrammetry, Remote Sensing & Spatial Information Sciences* 4 (2019). [[→ link](#)]

Visual-inertial Odometry

This is very, very similar to what is called visual-inertial odometry today, e.g.,

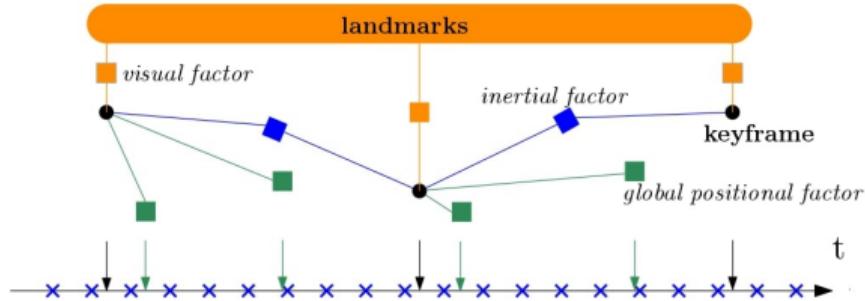
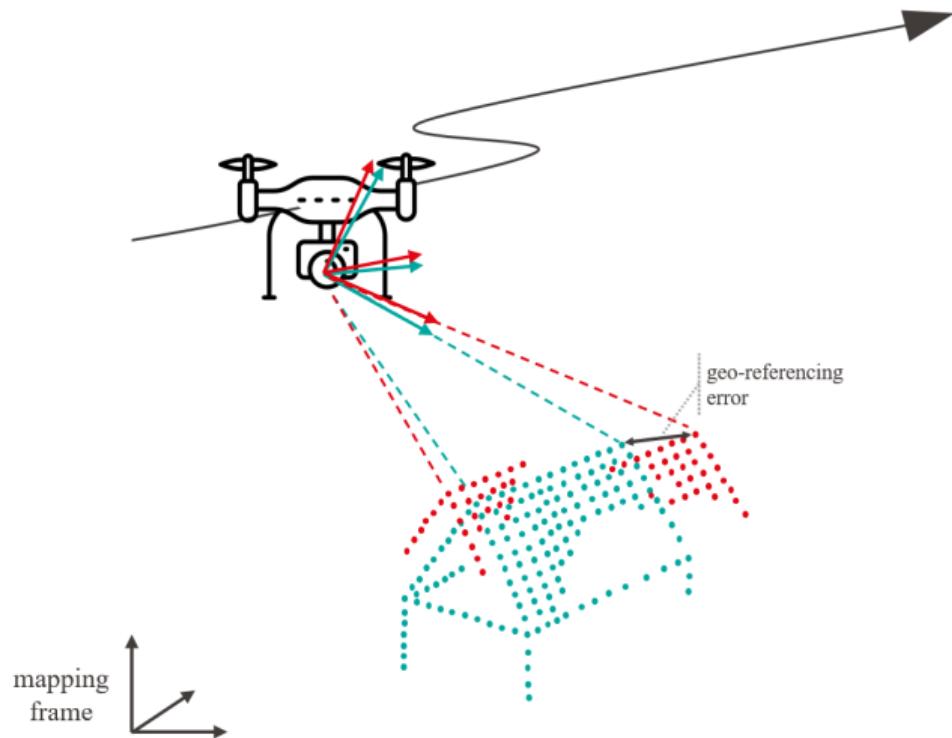


Figure: Representation of the proposed optimization-based multi-sensor fusion in [1]

- [1] Cioffi, G., and Scaramuzza, D.. “Tightly-coupled fusion of global positional measurements in optimization-based visual-inertial odometry.” *2020 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2020.

Laser scanning

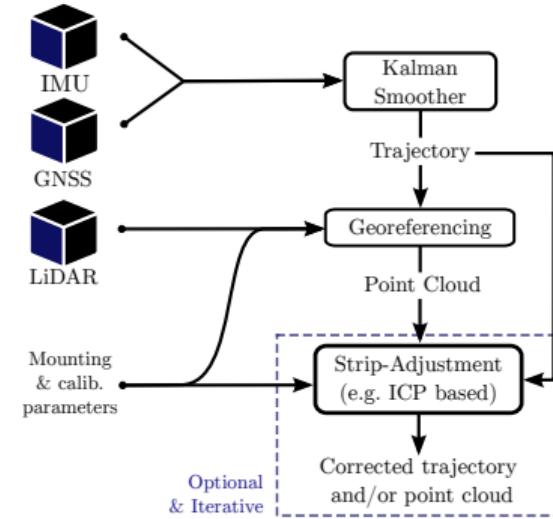
1. In rotating scanners the pose of the sensor is different for every acquired point
2. We need a **high-frequency** trajectory to correctly geo-reference lidar observations
3. Today, this is obtainable only with **inertial sensors** → DNs :)



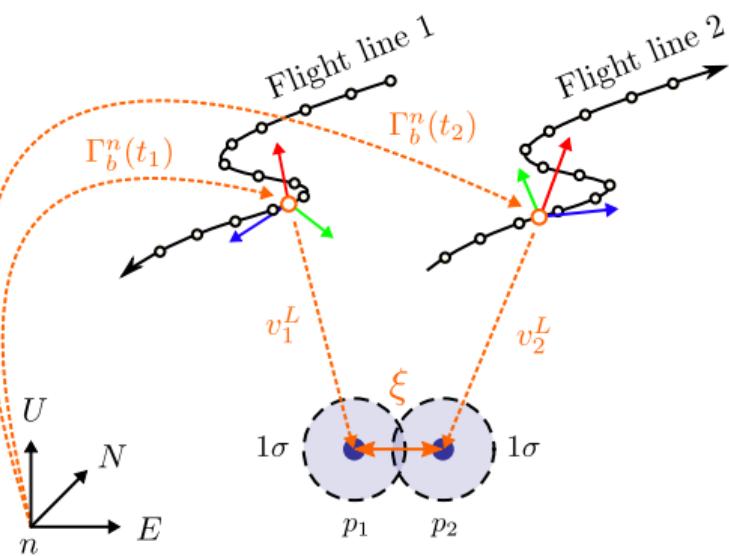
Laser scanning

Conventional processing methodology:

1. INS/GNSS navigation to get a **high-frequency** trajectory
2. Direct geo-referencing of laser observations
3. (Optional) Corrections at **point-cloud level**



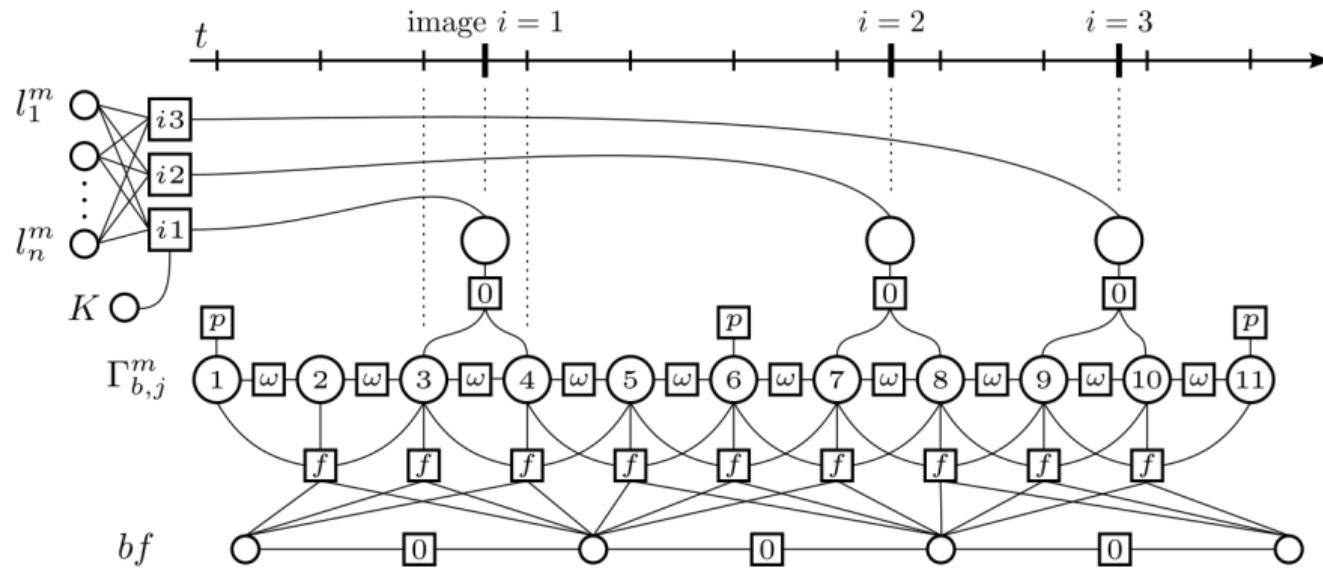
Matching lidar observations corresponding to the same point on ground (up to the GSD).



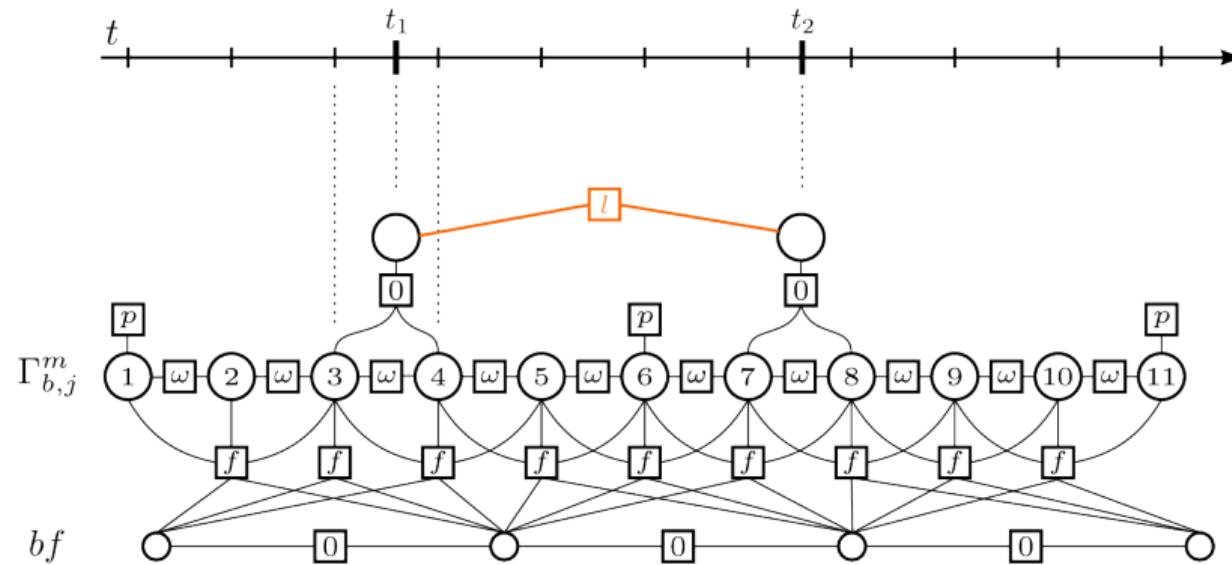
Which can be expressed as:

$$\begin{aligned} \tilde{\Gamma}_{b,t_1}^n R_L^b v_1^L - \tilde{\Gamma}_{b,t_2}^n R_L^b v_2^L &= \\ R_L^b v_1^L - \underbrace{\left[\tilde{\Gamma}_{b,t_1}^n \right]^{-1} \tilde{\Gamma}_{b,t_2}^n R_L^b v_2^L}_{\Gamma_{b,t_2}^{b,t_1}} &= 0 + \xi, \end{aligned}$$

DN structure (photogrammetry)

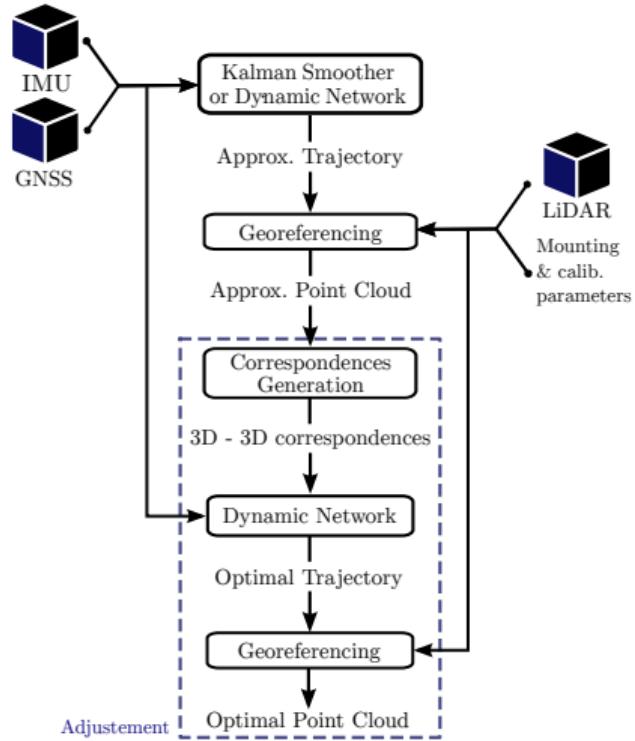


DN structure (laser scanning)



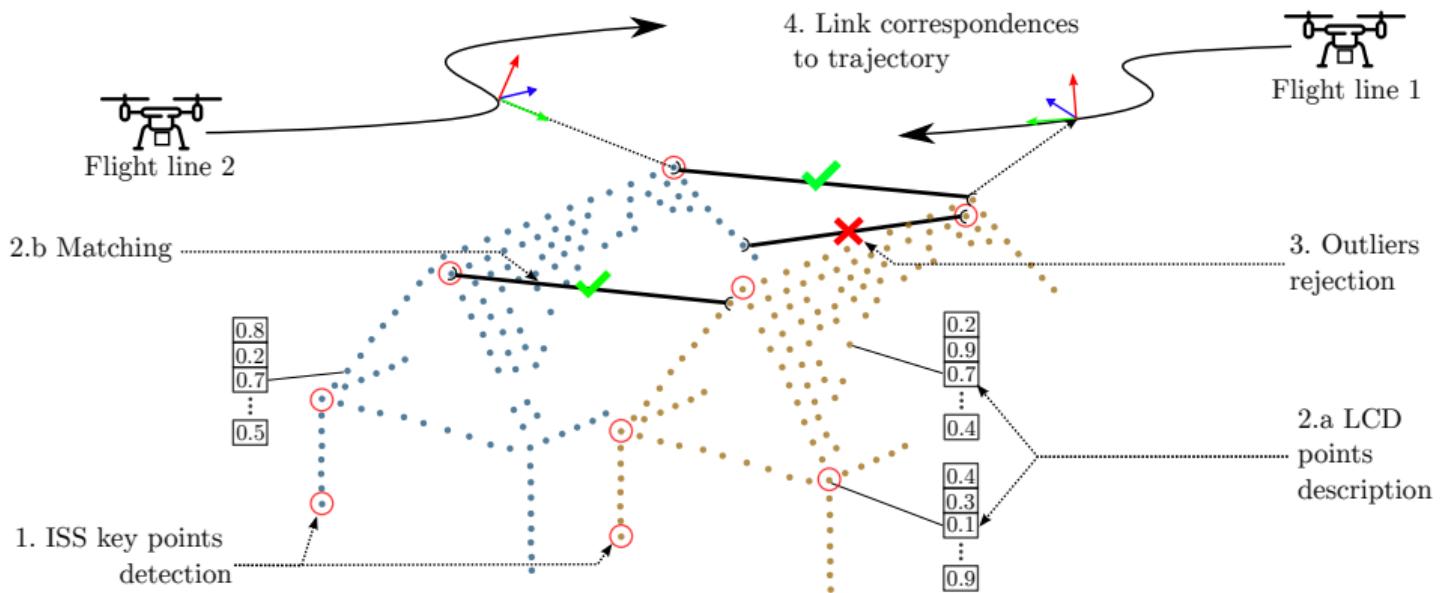
with DNs:

1. INS/GNSS navigation (possibly with DNs) to get an **approximated** trajectory
2. Identify 3D - 3D correspondences
3. Fuse everything with DNs to get a **corrected** trajectory
4. Direct geo-referencing of laser observations



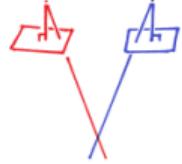
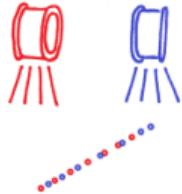
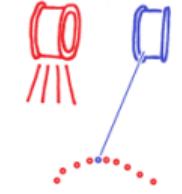
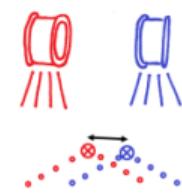
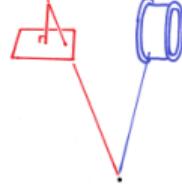
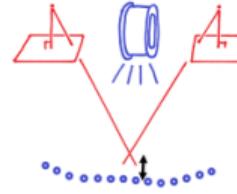
How do I obtain 3D - 3D correspondences?

Brun, A. et us, "LiDAR Point-to-point Correspondences for Rigorous Registration of Kinematic Scanning in Dynamic Networks." *ISPRS Journal of Photogrammetry and Remote Sensing* 189 (2022): 185-200. [→ link]

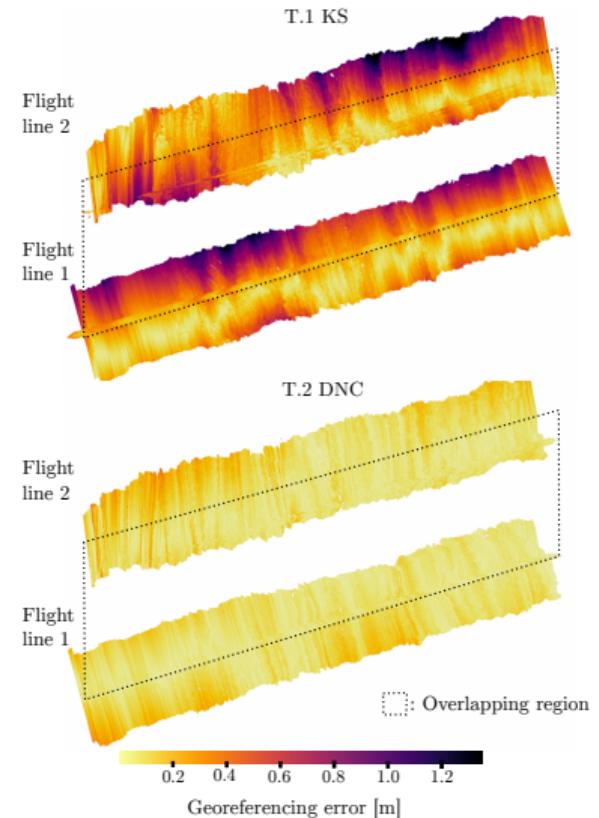


More correspondences?

Cledat, E., and J. Skaloud. "Fusion Of Photo With Airborne Laser Scanning" *ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences* 1 (2020): 173-180.

2.1 Image-Image	2.2 Plane-Plane	2.3 Point-Patch	2.4 Point-Point	2.5 2D Point Point-3D	2.6 Tie-Point Point-cloud
					
SIFT, SURF, ORB, KAZE, etc.	(robust) plane-fit	Point to closest surface	0. Manual point selection 1. rasterize LIDAR point-cloud and apply SIFT, SURF, etc. 2. target 3. ICP 4. Neural Network	3. Plane intersection	Dense matching

Better trajectory → Better point cloud.



Example 2 - INS/GNSS with 3D - 3D correspondences
[ODyN demo](#)

Input: LiDAR_p2p.txt see documentation

396765.649713,396660.533676,271.420000,-2.570000,18.060000,263.280000,-2.520000,-135.520000

396767.558842,396659.053466,263.810000,-2.520000,-0.480000,259.870000,-2.470000,-126.380000

Other files: as in INS.zip

Configuration

- 3 lever-arms (1 new)
- 2 boresight (1 new)

What we want?

- adjusted trajectory
- estimated parameters
(e.g., boresight, IMU biases)

What we do after it?

- DiSO

Compare different processing pipelines in a challenging corridor mapping scenario

- AAT with absolute **position control** from GNSS [PC]
- AAT with absolute **orientation control** from previous INS/GNSS integration [AC]
- Dynamic Networks [DN]

A fixed wing UAV, 1.5m wingspan, 600 g payload including:

1. Custom 20 Mpx camera developed by IGN, France with Zeiss Biogon 35 mm lenses,
2. Gecko4Nav redundant IMU board with two Intersense NavChip MEMs IMUs,
3. Topcon B110 GPS/GLONASS L1/L2 receiver.

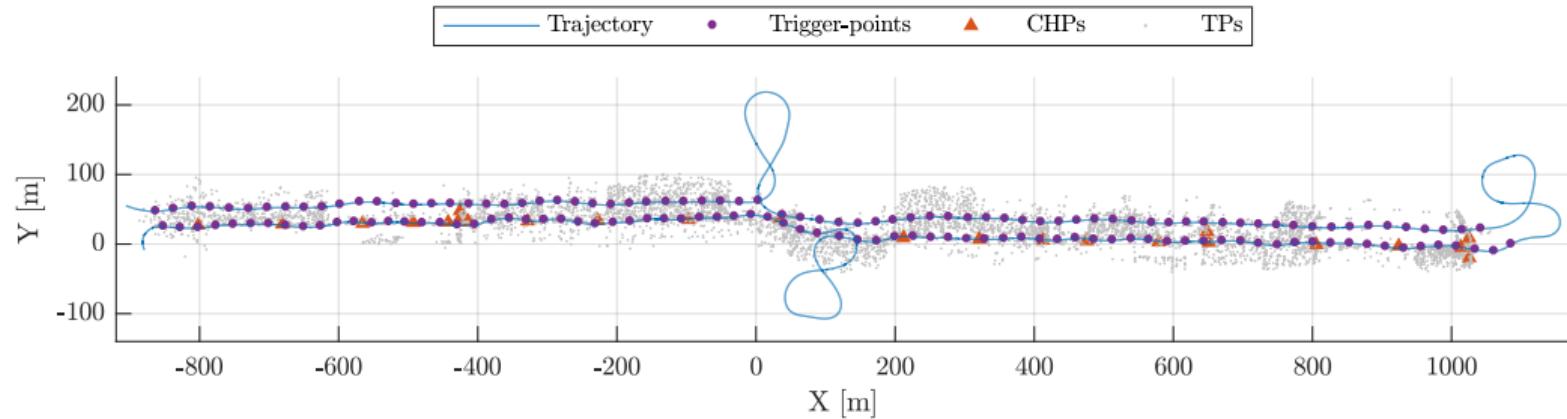


Figure: The payload

1. [PC] GNSS processing with Novatel Grafnav, then Agisoft Photoscan
2. [AC] GNS/INS with Applanix Posproc (custom), then Agisoft Photoscan
3. [DN] Tiepoints from Photoscan, then joint DN adjustment with ROAMFREE

Two Km corridor mapping flight (released as open-data, see later on)

1. two flight lines at 100 m elevation (GSD \sim 2 cm)
2. Alignment maneuvers
3. 4 GCPs, 20 checkpoints



	mean						std						RMS					
	E		N		U		E		N		U		E		N		U	
	mm	px	mm	px	mm	px	mm	px	mm	px	mm	px	mm	px	mm	px	mm	px
DN	-1	-0.03	-3	-0.15	0	0.01	10	0.51	13	0.64	28	1.41	10	0.51	13	0.66	28	1.41
AC	-13	-0.67	-8	-0.39	16	0.82	15	0.77	7	0.35	16	0.79	20	1.02	10	0.52	23	1.14
PC	-10	-0.48	-4	-0.18	11	0.53	12	0.60	6	0.30	13	0.67	15	0.76	7	0.35	17	0.85

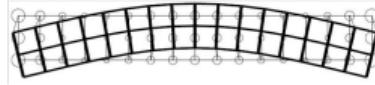
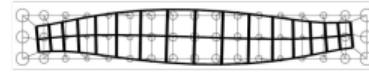
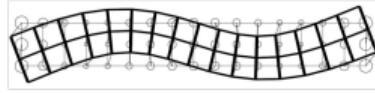
Table: Statistic of the error in the estimated checkpoint position.

Interpretation:

1. DN is the best in terms of mean error
2. **BUT** it's more noisy (higher standard deviation)

1. The camera is too good for the IMU
2. The IMU introduces noise but helps fighting large-scale systematic errors.

Overall statistics hide spatially correlated errors, i.e., these effects:



Let's imagine the error on the ground is a function of the position, e.g.

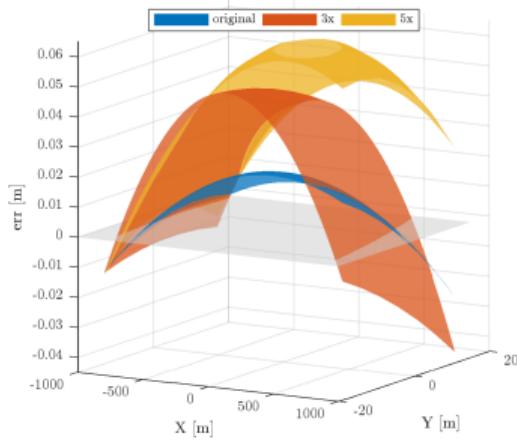
$$err = f(x, y, z) + \xi, \xi \sim \mathcal{N}(0, \Sigma) \quad (1)$$

The terrain is almost flat, we restrict to a second order polynomial function in x and y :

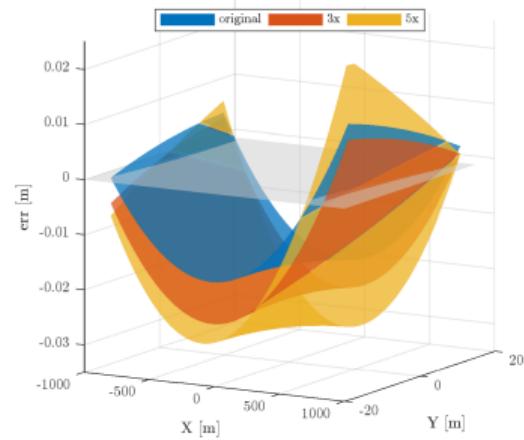
$$err = ax^2 + by^2 + cxy + dx + ey + f. \quad (2)$$

1. We downsample the images by a factor of 3x and 5x
2. We fit such model to the obtained residuals for [PC] and [DN]

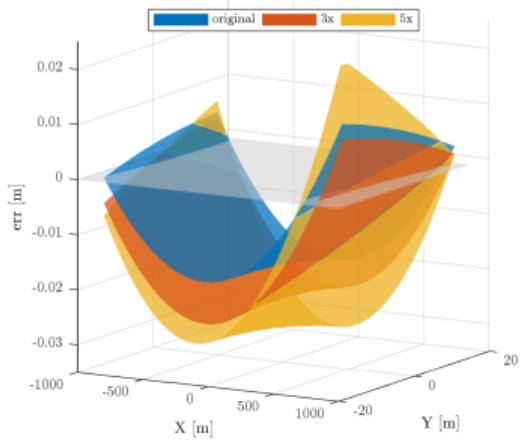
Global deformations [PC]



(a) err_x

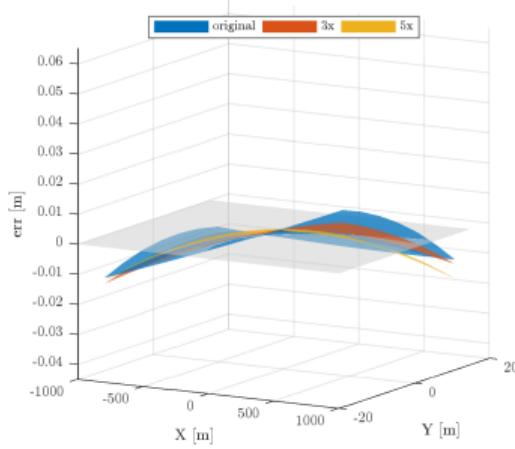


(b) err_y

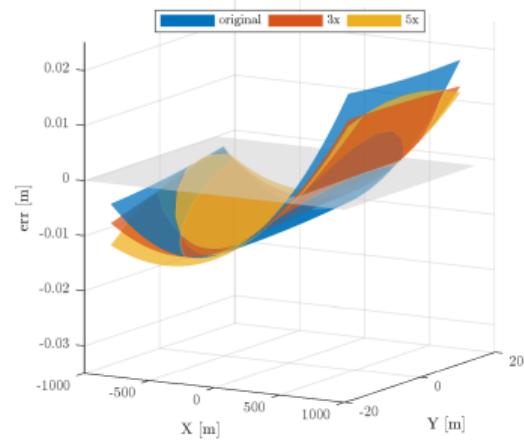


(c) err_z

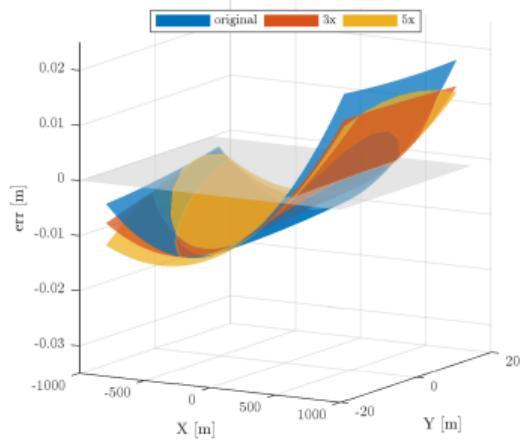
Global deformations [DN]



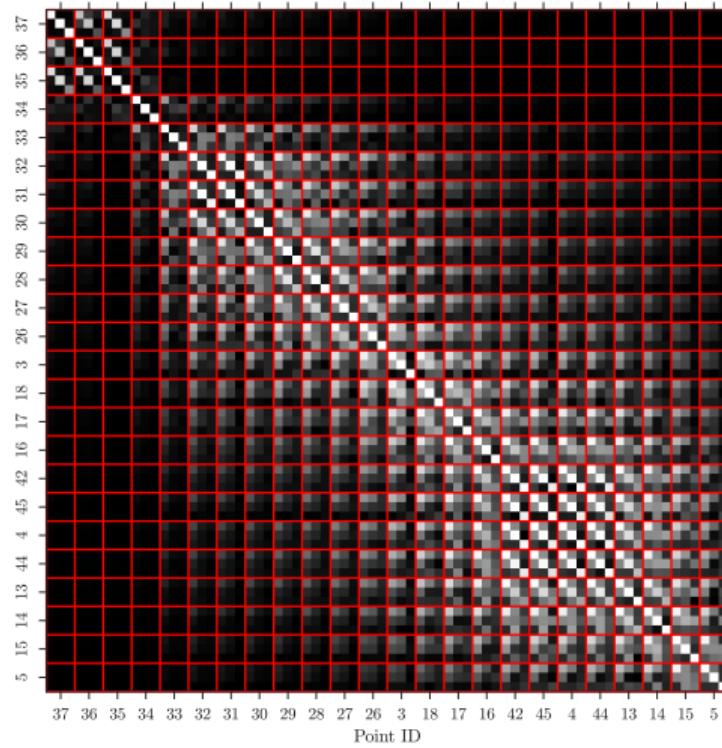
(d) err_x

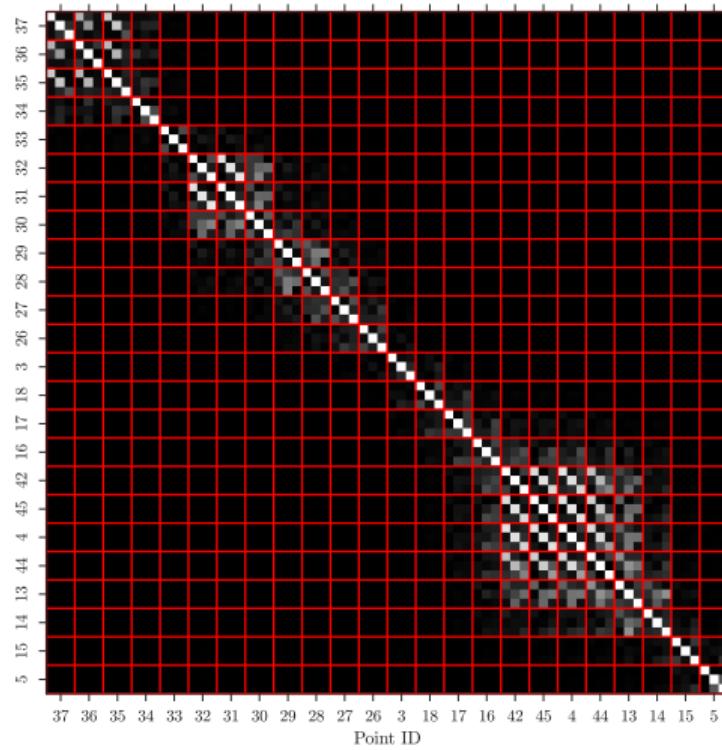


(e) err_y



(f) err_z





DN vs PC:

1. will have better decorrelation of the residuals.
2. will improve provided that the IMU is sufficiently good

DN vs AC:

1. substantially simpler pipeline.
2. less experience-intensive process, easier to get better results
(no alignment problems, better bias determination, boresight calibration)

The data for this flight (and similar ones) has been released as **open-data** (at last? ISPRS Congress).

1. Why: access to high quality data **with reference** is not obvious.
2. Goal: benchmarking traditional approaches and faster testing of new concepts.

Skaloud, J., Cucci, D. A., and Joseph Paul, K. "Fixed-wing Micro UAV Open Data With Diccam And Raw INS/GNSS" *ISPRS Annals of Photogrammetry, Remote Sensing & Spatial Information Sciences* (2021). [[→ link](#)]



Example 3 - INS/GNSS with image observations
ODyN demo

- **Task.** Combine Example 2 with Example 3 into a common adjustment of all optical (lidar/photo) and navigation (GNSS/raw-IMU) data.
- **Prerequisite.** Create a new .zip and configuration (via GUI).
- **Analysis.** Is there an improvement?
- **Hint.** Look at the details, e.g., →

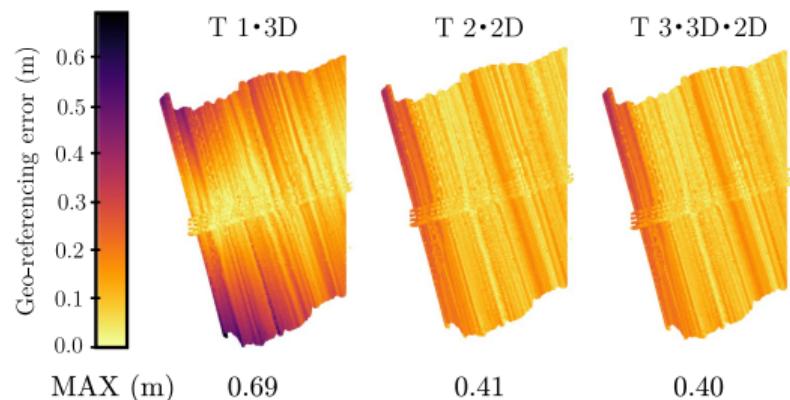


Figure: Extreme west part of the point cloud.

Mouzakkidou, K., Cucci, D.A., and Skaloud, J., "On the benefit of concurrent adjustment of active and passive optical sensors with GNSS & raw inertial data", *ISPRS Annals*, V-1-2022: 161-168. [→ link]

Another example

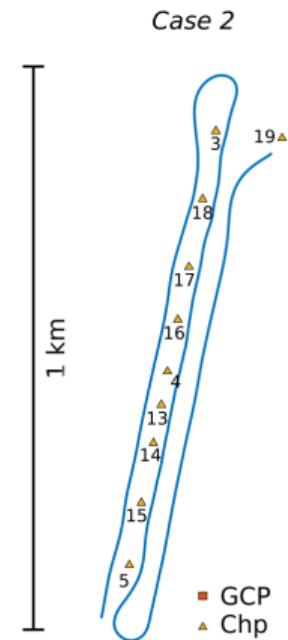
1. 1 Km corridor
2. Same IMU as before
3. Worse camera (Sony NEX-5R)
4. NO GCPs

We compare

1. AAT (Kalman smoother and then Bundle-adjustment)
2. DN

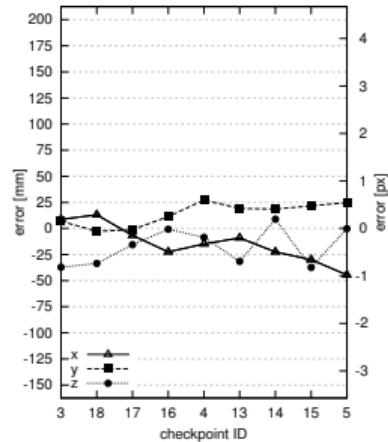
More cases in:

Cucci, D. A., Rehak, M., and Skaloud, J. "Bundle adjustment with raw inertial observations in UAV applications." *ISPRS J. of Photogrammetry and Remote Sensing* 130 (2017): 1-12. [→ link]

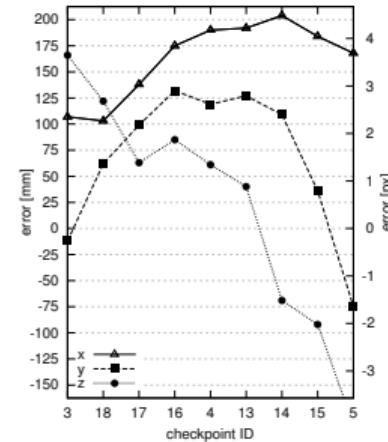


Results

DN



AAT



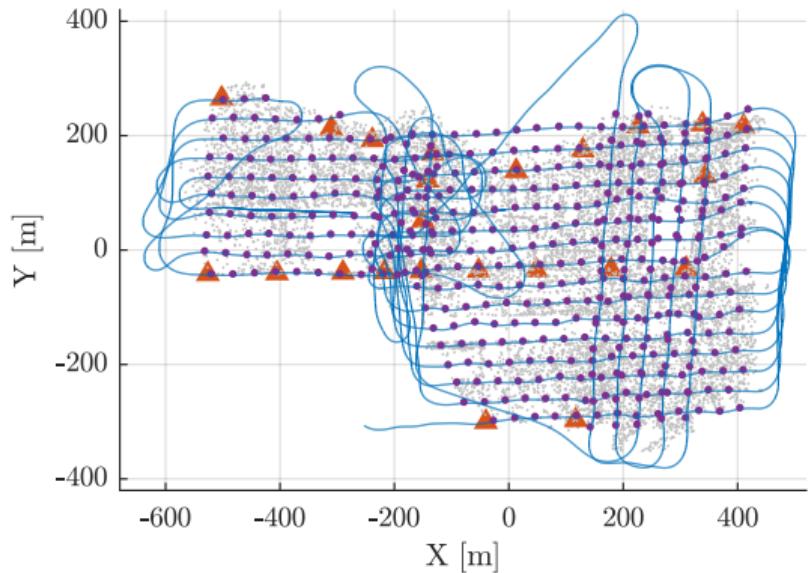
E N U

mean (mm)	-14	+14	-17
RMS (mm)	22	17	24

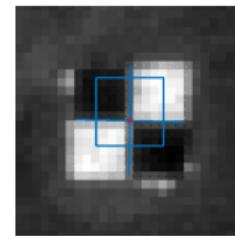
E N U

mean (mm)	+160	+66	+12
RMS (mm)	164	95	108

Example 4 - INS/GNSS with image observations (UAV)
ODyN demo



- 440 images, IGN-digicam/Biogon 35mm
- 26 lines, 2 levels, ~35 min
- 65/45 % overlap, 1/5000 s
- 2× raw INS 400 Hz, GNSS 10 Hz
- 2-4 cm GSD, 21 GCPs at 0.1 pix



Skaloud, J., Cucci, D.A., and Paul, K.J. "Fixed-wing micro UAV open data with digicam and raw INS/GNSS". *ISPRS Annals* 51 (2021): 105-111. [[→ link](#)]

1. DNs are a very general sensor fusion framework. Many things can be added:
 - 1.1 more navigation sensors: magnetometers, barometers, ...
 - 1.2 other exteroceptive sensors, e.g., LiDARs,
 - 1.3 *robust* fusion, outlier rejection, fault detection and isolation, ...
2. The one-shot, all-together, tight/joint adjustment is better than multi-stage or cascade processing methods
3. Redundancy can be *quantified*
 - 3.1 e.g., redundancy numbers, parameters cross-correlation, ...
4. Redundancy allows for *calibration*
 - 4.1 lever arms, boresights, scale factors, magnetic distortion, camera calibration, etc.

1. *General.* DN is the rigorous problem formulation for the common adjustment of navigation/optical data. It improves modeling, error propagation & parameter estimation.
2. *On INS/GNSS redundancy.* Observability of (IMU time-correlated) errors is limited by trajectory dynamics. Inclusion of optical constraints mitigates this weakness.
3. *Weak geometry (e.g., corridor, intermittent GNSS, point/line scanners).* One-step (DN), rather than sequential (KF + AT), adjustment is preferable for geometry weak scenarios as it limits the deformations in object space.
4. *Example drones – nominal GNSS / low-cost IMU / lidar.* The inclusion of (a subset of) point-to-point correspondences into DN effectively mitigated the “wavy” patterns in the point cloud registration caused by the attitude errors in the drone trajectory.

Questions?

EPFL

