A Two-Step Computationally Efficient Procedure for IMU Classification and Calibration

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joint work with
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IMU Calibration

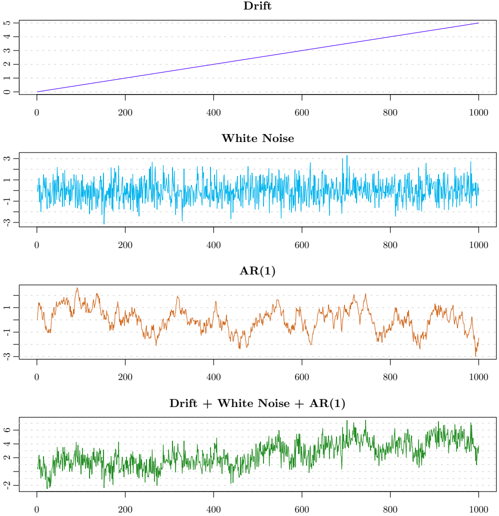
Errors in Inertial Sensors

- Possible causes:
  - Non-orthogonalities of the sensor axes
  - Environmental conditions (e.g. temperature)
  - Electronics
  - Dynamics
  - Others
- Error types:
  - Deterministic (calibration models, physical models, ...)
  - **Random error components** (typically latent time series models,...)

Correct stochastic sensor error modeling implies:

- Correct stochastic assumptions for inference
- **Better navigation or post-processing performance**

A latent time series model



Remarks:

- Not a linear regression model but a **state space model**.
- Computing the likelihood is not an easy task.
- **MLE (in fact EM-KF) fails.**

The GMWM estimator and its Model Selection Criterium

Definition: GMWM Estimator

The GMWM estimator is the solution of the following optimization problem

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\text{argmin}} \, \|\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}(\boldsymbol{\theta})\|_{\boldsymbol{\Omega}}^2,$$

in which $\boldsymbol{\Omega}$, a positive definite weighting matrix, is chosen in a suitable manner such that the above quadratic form is convex.

The Wavelet Variance Information Criterium

$$\text{WVIC} = \mathbb{E}\left[\mathbb{E}_0\left[\|\hat{\boldsymbol{\nu}}^0 - \boldsymbol{\nu}(\hat{\boldsymbol{\theta}})\|_{\boldsymbol{\Omega}}^2\right]\right],$$

where $\mathbb{E}[\cdot]$ and $\mathbb{E}_0[\cdot]$ represent specific probabilistic expectations which measure how well the WV implied by the estimated model fits the WV observed on a future replicate.

What we observe in reality

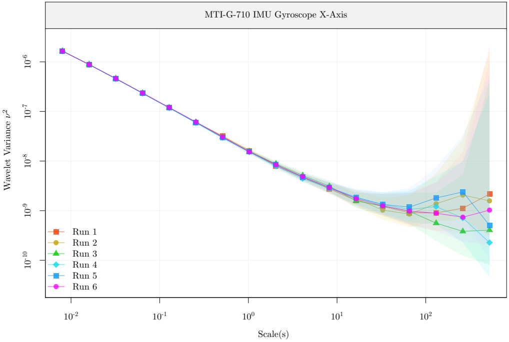
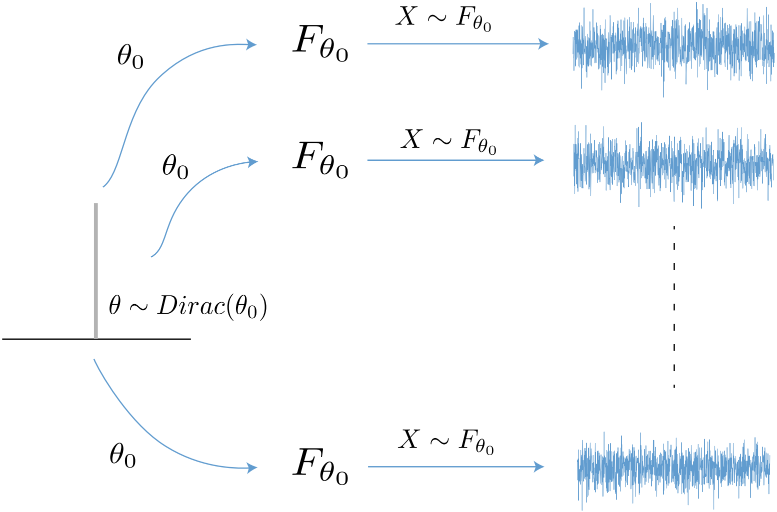


Figure: Data coming from an MTIG 710 IMU supplied by InvenSense.

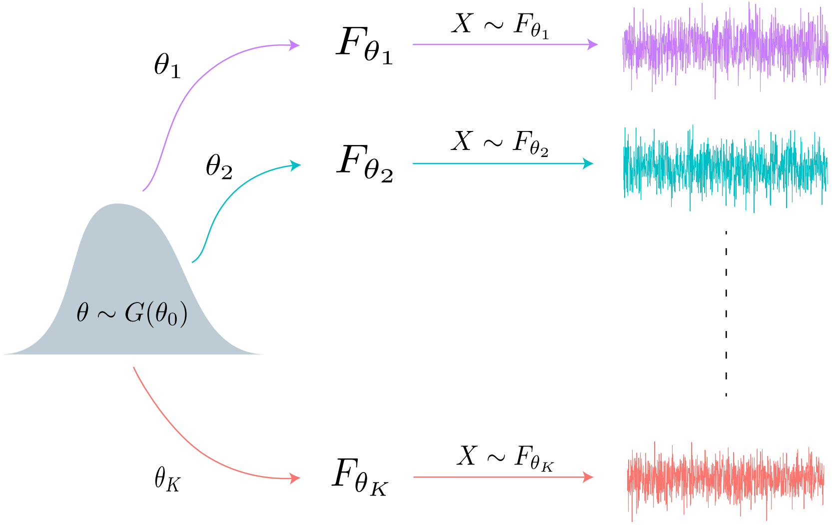
Remarks:

- Several sequences of the error signals issued from an IMU.

Current Assumption on IMU's Error Signal



What we observe in reality



Near stationarity

Context

- Lower grade IMUs.
- Several recording of IMU error signal.
- Static conditions.

Near stationarity

We define a nearly stationary time series, as one which exhibit the following properties:

1. Same model, but with **different parameter values** for each sequences.
2. The vector of parameter $\boldsymbol{\theta}$ has a **probability distribution $\boldsymbol{G}(\theta_0)$**, where $\mathbb{E}[\boldsymbol{\theta}] = \boldsymbol{\theta}_0$.
3. The distribution $\boldsymbol{G}(\theta_0)$ can be interpreted as **the internal sensor model**, which may account for unobserved factors (e.g. temperature).

Multisignal GMWM (MGMWM)

MGMWM

The MGMWM estimator can be defined as:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\text{argmin}} \, \|\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}(\boldsymbol{\theta})\|_{\boldsymbol{\Omega}}^2,$$

where $\hat{\boldsymbol{\nu}}$ represent the stack of WV computed and each sequences, and $\boldsymbol{\Omega}$ a block diagonal weighting matrix.

MGMWM considering independent signals

Considering $k = 1, \ldots, K$, the number replicates recorded from the same IMU in static condition, we define the **Multisignal GMWM** estimator as the solution of the following minimization problem:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\text{argmin}} \, \frac{1}{K} \sum_{k=1}^{K} \|\hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}(\boldsymbol{\theta})\|_{\boldsymbol{\Omega}}^2.$$

Let $\hat{\nu}_{jk}$, respectively $\nu_j\left(\boldsymbol{\theta}\right)$ be the j^{th} elements of the vectors $\hat{\boldsymbol{\nu}}_k$, the empirical WV computed on the k^{th} replicates of size $j = 1, \ldots, J_k$.

Testing for Near Stationarity

This frameworks allows to use the MGMWM objective function as a test statistics, which is defined below:

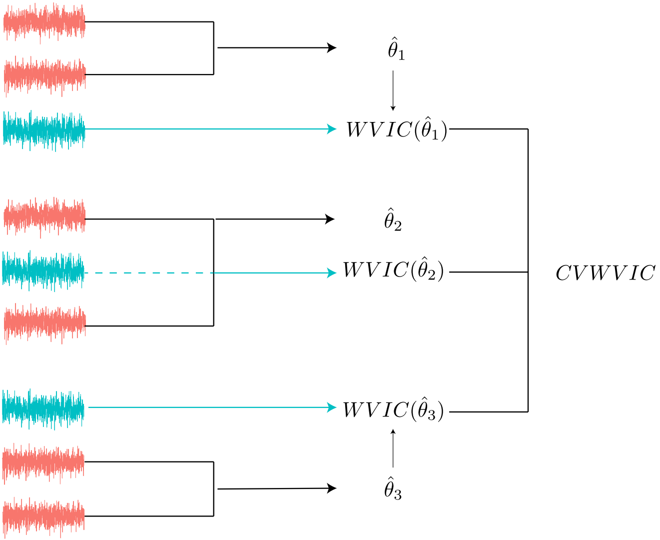
$$g(\hat{\boldsymbol{\theta}}) = \|\hat{\boldsymbol{\theta}} - \boldsymbol{\nu}(\hat{\boldsymbol{\theta}})\|^2_{\hat{\boldsymbol{\Omega}}}.$$

Using this statistic we would therefore like to test the following null and alternative hypotheses

$$\mathcal{H}_0 : \boldsymbol{\theta}_k = \boldsymbol{\theta}_0, \ \forall\, k,$$
$$\mathcal{H}_a : \mathcal{H}_0 \text{ is false.}$$

Under $\mathcal{H}_0$, we fall in the case where the distribution G is a Dirac function and $\boldsymbol{\theta}_k = \boldsymbol{\theta}_0, \forall k$.

WVIC with Multiple Signal Replicates



Thank you very much for your attention!

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Any questions?

More info...

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Two Estimators: Average GMWM vs MGMWM

Average GMWM vs MGMWM

We define $\hat{\boldsymbol{\theta}}^{\circ}$ as the Average GMWM (AGMWM) defined as:

$$\hat{\boldsymbol{\theta}}^{\circ} = \frac{1}{K} \sum_{k=1}^{K} \tilde{\boldsymbol{\theta}}_k \, ,$$

with $\tilde{\boldsymbol{\theta}}_k = \text{argmin}_{\boldsymbol{\theta}_k \in \boldsymbol{\Theta}} \|\hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}(\boldsymbol{\theta}_k)\|^2_{\boldsymbol{\Omega}_k}$. Remeber that we define the Multisignal GMWM (MGMWM) as

$$\hat{\boldsymbol{\theta}}^{\star} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\text{argmin}} \, \frac{1}{K} \sum_{k=1}^{K} \|\hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}(\boldsymbol{\theta})\|^2_{\boldsymbol{\Omega}_k}.$$

Two Estimators: Average GMWM vs MGMWM

Properties

It turns out that the MGMWM appears far more appropriate than the AGMWM for two main reasons:

- The MGMWM is more efficient than the AGMWM, i.e.

$$\frac{\mathrm{tr}\left(\min_{\Omega_i} \mathrm{var}\left[\hat{\boldsymbol{\theta}}^{\circ}\right]\right)}{\mathrm{tr}\left(\min_{\Omega_i} \mathrm{var}\left[\hat{\boldsymbol{\theta}}^{\star}\right]\right)} \xmapsto{\mathcal{P}} c > 1.$$

- From Jensen inequality implies that
  - If $\boldsymbol{\nu}(\boldsymbol{\theta})$ is linear, i.e for stochastic processes WN, DR and QN, or if $G(\boldsymbol{\theta}_0)$ is a Dirac function,

$$\hat{\boldsymbol{\theta}}^{\star} - \hat{\boldsymbol{\theta}}^{\circ} \xmapsto{\mathcal{P}} 0.$$

  - If $\boldsymbol{\nu}(\boldsymbol{\theta})$ is not linear i.e for stochastic processes RW and AR1, than

$$\hat{\boldsymbol{\theta}}^{\star} - \hat{\boldsymbol{\theta}}^{\circ} \xmapsto{\mathcal{P}} \delta \neq 0.$$