A Two-Step Computationally Efficient Procedure for IMU Classification and Calibration

Gaetan Bakalli[1]

joint work with
Ahmed Radi[2], Prof. Stéphane Guerrier[3],
Yuming Zhang[3], Dr. Roberto Molinari[3]
& Prof. Sameh Nassar[2]

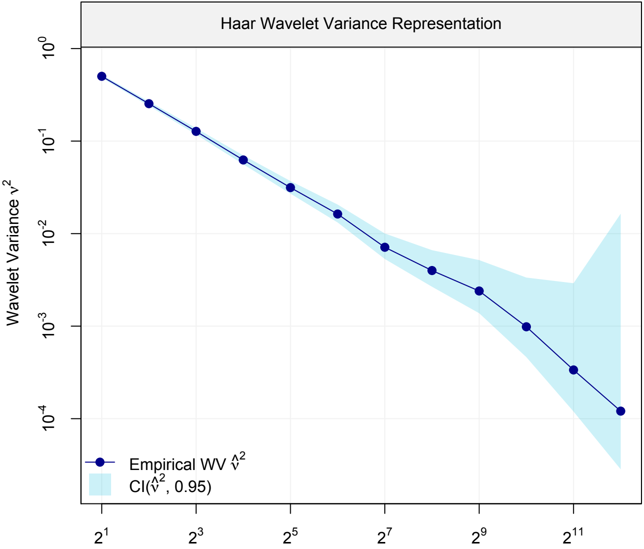
[1] University of Geneva
[2] University of Calgary
[3] Pennsylvania State University

IEEE/ION PLANS , Monterey 2018

April 25, 2018

Outline

Wavelet Variance (Allan Variance) log-log plot



The GMWM estimator and its Model Selection Criterium

Definition: GMWM Estimator

The GMWM estimator is the solution of the following optimization problem

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\text{argmin}} \, \|\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}(\boldsymbol{\theta})\|_{\boldsymbol{\Omega}}^2,$$

in which $\boldsymbol{\Omega}$, a positive definite weighting matrix, is chosen in a suitable manner such that the above quadratic form is convex.

The Wavelet Variance Information Criterium

$$\text{WVIC} = \mathbb{E}\left[\mathbb{E}_0\left[\|\hat{\boldsymbol{\nu}}^0 - \boldsymbol{\nu}(\hat{\boldsymbol{\theta}})\|_{\boldsymbol{\Omega}}^2\right]\right],$$

where $\mathbb{E}[\cdot]$ and $\mathbb{E}_0[\cdot]$ represent specific probabilistic expectations which measure how well the WV implied by the estimated model fits the WV observed on a future replicate.

What we observe in reality

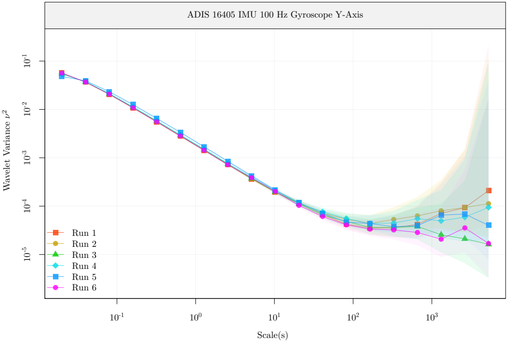
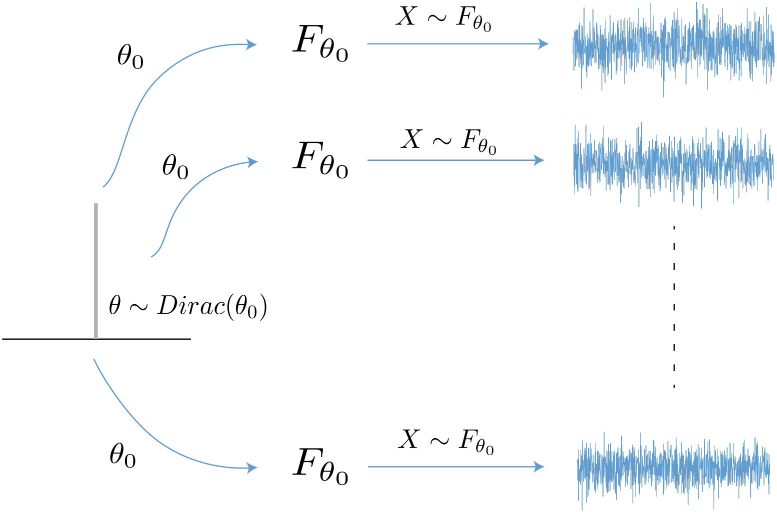


Figure: Data coming from an MTIG 710 IMU supplied by InvenSense.

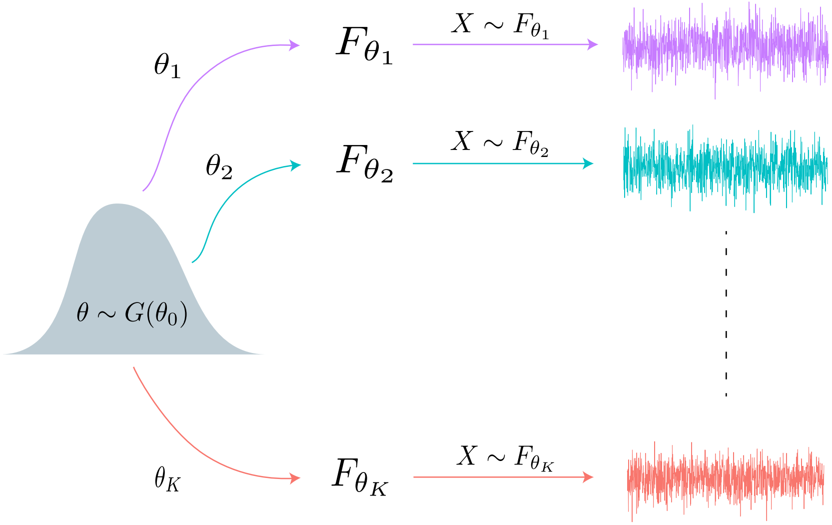
Remarks:

- Several sequences of the error signals issued from an IMU.

Current Assumption on IMU's Error Signal



What we observe in reality



Near stationarity

Context

- Lower grade IMUs.
- Several recording of IMU error signal.
- Static conditions.

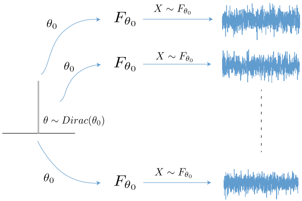
Near stationarity

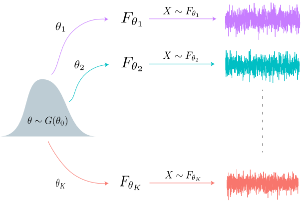
We define a nearly stationary time series, as one which exhibit the following properties:

1. Same model, but with **different parameter values** for each sequences.
2. The vector of parameter $\boldsymbol{\theta}$ has a **probability distribution $\boldsymbol{G}(\boldsymbol{\theta}_0)$**, where $\mathbb{E}[\boldsymbol{\theta}] = \boldsymbol{\theta}_0$.
3. The distribution $\boldsymbol{G}(\boldsymbol{\theta}_0)$ can be interpreted as **the internal sensor model**, which may account for unobserved factors (e.g. temperature).

Near-Stationarity test

$$\mathcal{H}_0 : \hat{\theta}_k = \theta_0 \text{ for all } k \qquad \mathcal{H}_a : \mathcal{H}_0 \text{ is false}$$





Multisignal GMWM (MGMWM)

MGMWM

Considering $k = 1, \dots, K$, the number replicates recorded from the same IMU in static condition, we define the **Multisignal GMWM** estimator as the solution of the following minimization problem:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\operatorname{argmin}} \, \|\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}(\boldsymbol{\theta})\|^2_{\boldsymbol{\Omega}},$$

where $\hat{\boldsymbol{\nu}}$ represent the stack of WV computed and each sequences, and $\boldsymbol{\Omega}$ a block diagonal weighting matrix.

MGMWM considering independent signals

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\operatorname{argmin}} \, \frac{1}{K} \sum_{k=1}^{K} \|\hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}(\boldsymbol{\theta})\|^2_{\boldsymbol{\Omega}_k}.$$

Let $\hat{\nu}_{jk}$, respectively $\nu_j(\boldsymbol{\theta})$ be the j^{th} elements of the vectors $\hat{\boldsymbol{\nu}}_k$, the empirical WV computed on the k^{th} replicates of size $j = 1, \dots, J_k$.

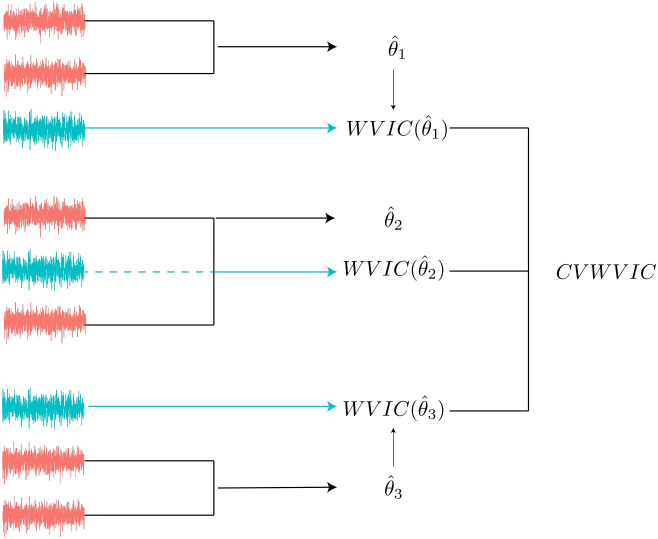
Back to the WVIC

The Wavelet Variance Information Criterium

$$\text{WVIC} = \mathbb{E}\left[\mathbb{E}_0\left[\|\hat{\boldsymbol{\nu}}^0 - \boldsymbol{\nu}(\hat{\boldsymbol{\theta}})\|^2_{\boldsymbol{\Omega}}\right]\right],$$

where $\mathbb{E}[\cdot]$ and $\mathbb{E}_0[\cdot]$ represent specific probabilistic expectations which measure how well the WV implied by the estimated model fits the WV observed on a future replicate.

WVIC with Multiple Signal Replicates



mgmwm.smac-group.com

GitHub, Inc.

README.md

build unknown | repo status Active | licence CC BY-NC-SA 4.0 | R>= 3.4.0 | CRAN not published | Package version 0.1.0 | last change 2018-01-15

`mgmwm` Overview

Multisignal GMWM (`mgmwm`) R package estimates parameters from multiple replicates coming from an IMU error signal, apply the near-stationarity test and select the nost approriate model

To see what `mgmwm` is capable of, please refer to the "Vignettes" tabs above.

IEEE/ION PLANS Monterey 2018 presentation

You'll find hereunder the presentation of the mgmwm package and its related theory:

A Two-Step Computationally Efficient Procedure for IMU Classification and Calibration.

Install Instructions

Installing the package through GitHub

For users who are interested in having the latest developments, this option is ideal. Though, more dependancies are required to run a stable version of the package. Most importantly, users **must** have a compiler installed on their machine that is compatible with R (e.g. Clang).

The setup to obtain the development version of `mgmwm` *is platform dependent.*

Requirements and Dependencies

OS X

Some users report the need to use X11 to suppress shared library errors. To install X11, visit xquartz.org.

Linux

Both curl and libxml are required.

For **Debian** systems, enter the following in terminal:

Empirical WV

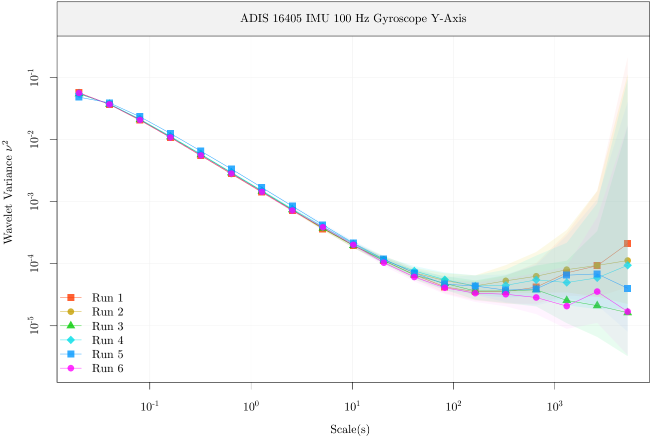


Figure: Empirical WV of 6 replicates coming from an ADIS 16405 IMU gyroscopes Y-axis recorded a 100hrz

First Model estimated

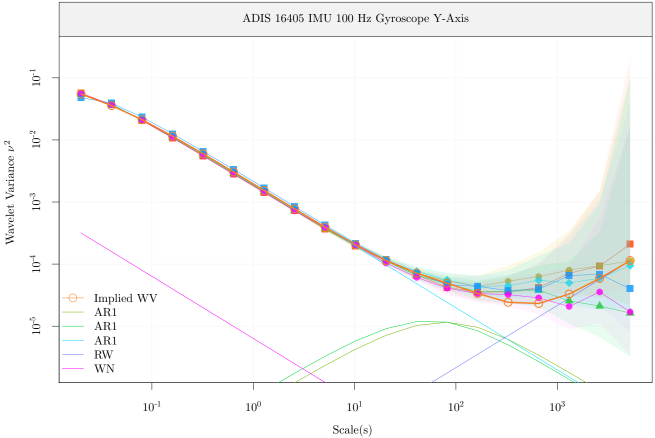


Figure: Empirical and Implied WV (in orange) of 6 replicates coming from an ADIS 16405 IMU gyroscopes Y-axis recorded a 100hrz. Model fitted is composed of 3 AR1 + RW + WN. Plain lines represent the contribution of each individual process.

Selected model trough WVIC

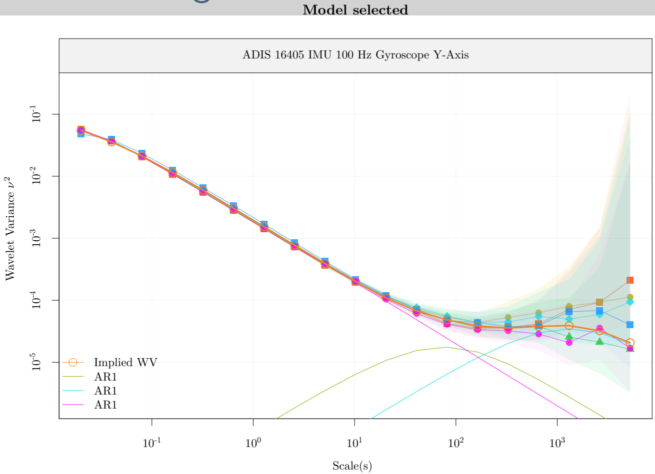


Figure: Empirical and Implied WV (in orange) of 6 replicates coming from an ADIS 16405 IMU gyroscopes Y-axis recorded a 100hrz. Model selected is composed of 3 AR1. Plain lines represent the contribution of each individual process.

WVIC comparison for all nested models.

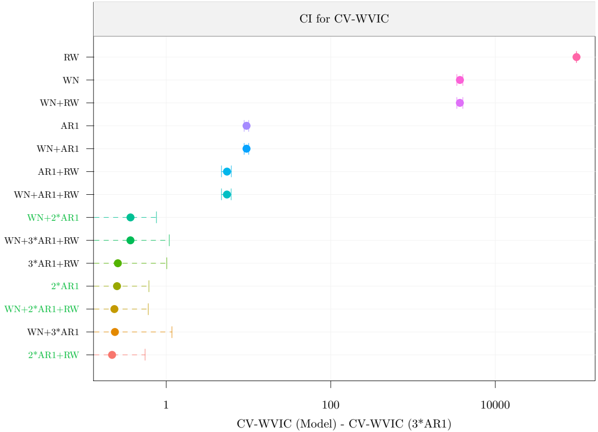


Figure: Comparison of the WVIC criteria for every model nested in 3 AR1 + RW + WN with the selected model (log scale). The dots represent the value of the WVIC, the lines alongside its respective confidence interval. Model in green represent "equivalent" models with less or equivalent model complexity.

Implied WV for equivalent models

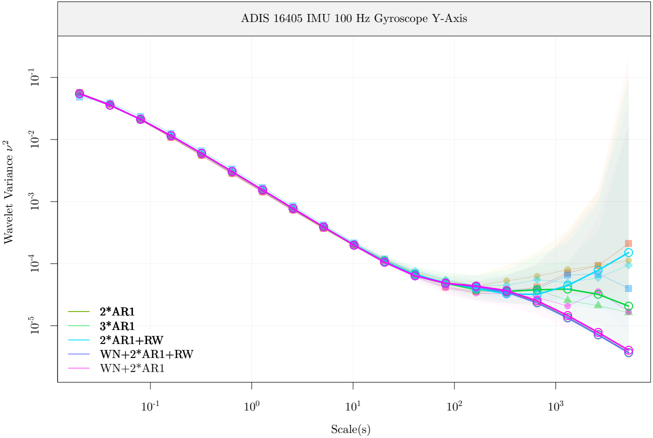


Figure: Fit comparison for "equivalent" models.

WVIC comparison for equivalent nested models.

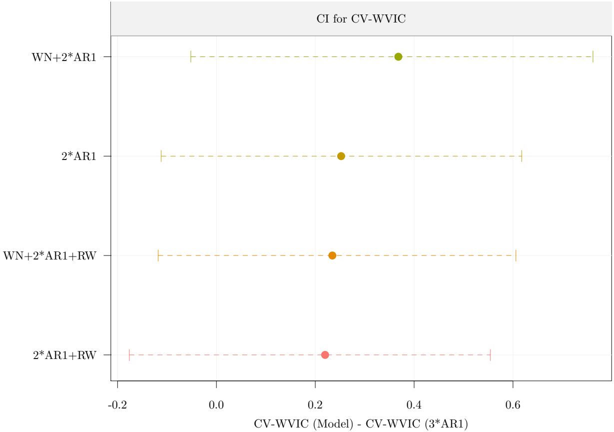


Figure: Comparison of the WVIC criteria for "equivalent" model nested in 3 AR1 + RW + WN with the selected model.

Thank you very much for your attention!

A special thanks to

- Prof. Nasser El-Sheimy (U. Geneva)
- Justin Lee (Penn State University)

Any questions?

More info...

SMAC-group.com
github.com/SMAC-Group
gaetan.bakalli@unige.ch
@SMAC_Group

Two Estimators: Average GMWM vs MGMWM

Average GMWM vs MGMWM

We define $\hat{\boldsymbol{\theta}}^{\circ}$ as the Average GMWM (AGMWM) defined as:

$$\hat{\boldsymbol{\theta}}^{\circ} = \frac{1}{K} \sum_{k=1}^{K} \tilde{\boldsymbol{\theta}}_k \, ,$$

with $\tilde{\boldsymbol{\theta}}_k = \text{argmin}_{\boldsymbol{\theta}_k \in \boldsymbol{\Theta}} \|\hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}(\boldsymbol{\theta}_k)\|^2_{\boldsymbol{\Omega}_k}$. Remeber that we define the Multisignal GMWM (MGMWM) as

$$\hat{\boldsymbol{\theta}}^{\star} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\text{argmin}} \, \frac{1}{K} \sum_{k=1}^{K} \|\hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}(\boldsymbol{\theta})\|^2_{\boldsymbol{\Omega}_k}.$$

Two Estimators: Average GMWM vs MGMWM

Properties

It turns out that the MGMWM appears far more appropriate than the AGMWM for two main reasons:

- The MGMWM is more efficient than the AGMWM, i.e.

$$\frac{\operatorname{tr}\left(\min_{\Omega_i} \operatorname{var}\left[\hat{\boldsymbol{\theta}}^{\circ}\right]\right)}{\operatorname{tr}\left(\min_{\Omega_i} \operatorname{var}\left[\hat{\boldsymbol{\theta}}^{\star}\right]\right)} \xmapsto{\mathcal{P}} c > 1.$$

- From Jensen inequality implies that
  - If $\boldsymbol{\nu}(\boldsymbol{\theta})$ is linear, i.e for stochastic processes WN, DR and QN, or if $G(\boldsymbol{\theta}_0)$ is a Dirac function,

$$\hat{\boldsymbol{\theta}}^{\star} - \hat{\boldsymbol{\theta}}^{\circ} \xmapsto{\mathcal{P}} 0.$$

  - If $\boldsymbol{\nu}(\boldsymbol{\theta})$ is not linear i.e for stochastic processes RW and AR1, than

$$\hat{\boldsymbol{\theta}}^{\star} - \hat{\boldsymbol{\theta}}^{\circ} \xmapsto{\mathcal{P}} \delta \neq 0.$$