GMWM Package - Part II

Automatic and Computationally Efficient Method For Model Selection In Inertial Sensor Calibration

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September 16, 2015

The GMWM package in action

Outline

1. **Visualizing the signal(s)**:
   - Is the signal disturbed (contaminated)?
   - Which models could best describe the visualized WV?
2. **Estimating the model(s)**
   - The `gmwm.imu()` function for parameter estimation
   - The options for estimation
3. **Inference**
   - Confidence intervals for the parameters
   - Goodness-of-fit of the model to the signal
4. **Model Selection**
   - The efficient computation of the WVIC
   - Model selection from a set of user-specified models
   - Automatic model selection from a set of all sub-models of a single user-specified model

The GMWM package in action

Getting to know the package

The package comes with an example dataset called `imu6`

- The data comes from the calibration of an XSens MTi-G sensor
- To use the data within the `R` session:
  1. Load the `gmwmdata` package (separate from the `gmwm` package)
  2. Make the dataset available in the session by typing `data(imu)`
- Load external data (e.g. binary data) with `read.imu()`

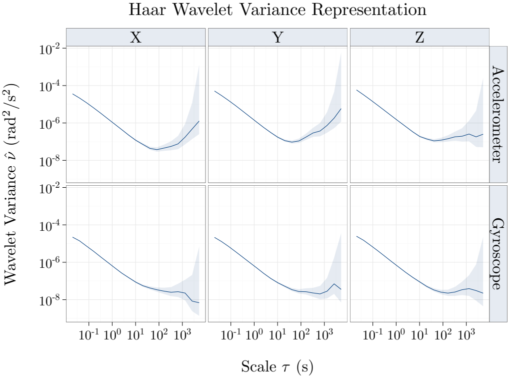
```
install.packages("gmwmdata")
library(gmwmdata)
data(imu6)
head(imu6)
```

	Gyro. X	Gyro. Y	Gyro. Z	Accel. X	Accel. Y	Accel. Z
1	−0.011138	−0.017646	−0.009531	1.083731	0.023897	9.638113
2	−0.006765	−0.013657	−0.021974	1.059858	0.042061	9.646901
3	...	...	...	...	...	...

Visualizing the data

The function `wvar.imu()`

Once the calibration data has been entered, it is possible to plot the observed WV vs τ on a log-log scale



R code:

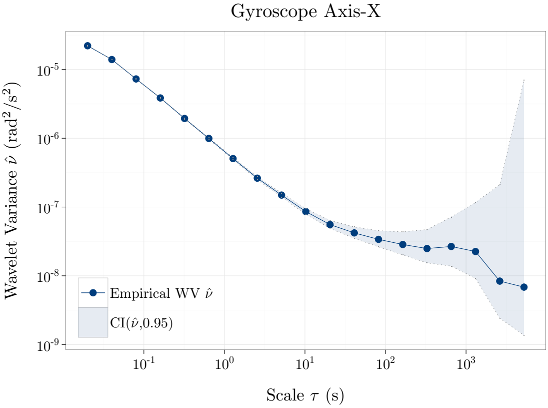
```
imu.obj = imu(imu6,
gyroscope = 1:3,
accelerometer = 4:6)
wv = wvar.imu(imu.obj)
plot(wv)
```

(Run time: 3.70 [sec])

Visualizing the data

The function `wvar()`

It is obviously possible to plot the WV from the axis of an accelerometer or gyroscope individually



R code:

```
WV.gx = wvar(imu6[,1])
plot(WV.gx)
```

(Run time: 0.65 [sec])

The Allan variance can also be computed through the function `avar()`

GMWM estimation

The `gmwm.imu()` function

The function tailor-made for IMU error modeling is `gmwm.imu()`

Main arguments of the `gmwm.imu()` function

- `model`: The structure of the model is specified through this argument
- `data`: The signal to be modelled
- `compute.v`: The method for computing the weighting matrix $\boldsymbol{\Omega}$
  - `fast`: Estimated diagonal matrix
  - `diag`: Use a diagonal matrix with the asymptotic variance of $\hat{\boldsymbol{\nu}}$
  - `bootstrap`: Estimate $\boldsymbol{\Omega}$ through parametric bootsrap

GMWM estimation

The `model` argument

The GMWM package can estimate each specific model or any latent model made by a combination of all or a subset of the following models

- `AR1()`: a first-order autoregressive process (reparametrization of Gauss-Markov process)
- `WN()`: white noise process
- `QN()`: quantization noise (rounding error)
- `RW()`: random walk process
- `DR()`: drift
- `AR()`: p-order autoregressive process
- `MA()`: q-order moving average process
- `ARMA()`: autoregressive-moving average processes

GMWM estimation

Latent model syntax

To specify a latent model we use the sign "+" between the available models while the `AR1()` model can be included more than once (say k times) and is can be specified as `k*AR1()`, for example

- `ARMA()+WN()+RW()`
- `3*AR1()+DR()`

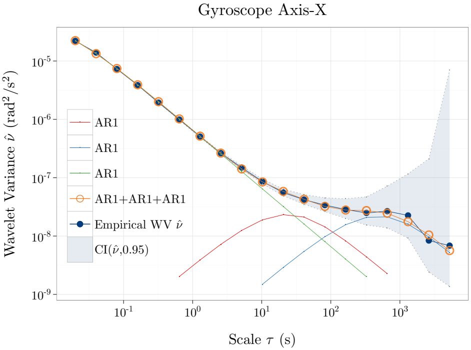
Convergence

If the `gmwm.imu()` function has problems of convergence, one can specify **starting values** for the parameters using the brackets in the syntax of each model (e.g. `WN(sigma2=0.5)`)

GMWM estimation

Visually assessing the fit

The function `plot()` applied to the object of a GMWM estimation allows to see how well the WV implied by the estimated model fits the observed WV



R code:

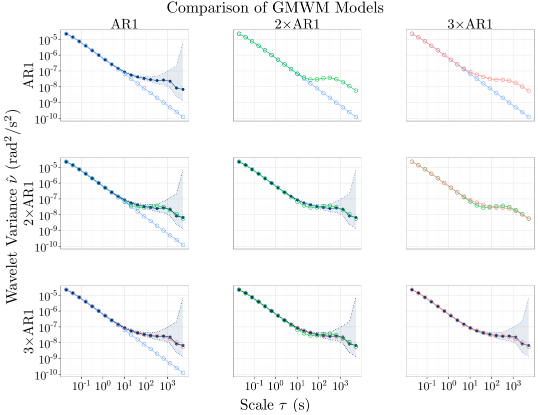
```
mod = gmwm.imu(3*AR1(),
imu6[,1]) plot(mod,
process.decomp = T)
```

(Run time: 0.56 [sec])

Model Comparison

Suppose we want to compare three models...

AR1(), 2*AR1() and 3*AR1()



R code:

```
Xt = imu[,1]
m1 = gmwm.imu(AR1(),Xt)
m2 = gmwm.imu(2*AR1(),Xt)
m3 = gmwm.imu(3*AR1(),Xt)
compare.models(m1, m2, m3)
```

(Run time: 5.20 [sec])

Inference

Output of estimation

Aside from the value of the estimated parameters, the output of the function `gmwm.imu` provides other information for inference which include **confidence intervals** and **goodness of fit** test (GoF)

```
mod = gmwm.imu(2*AR1(), imu6[,1])
summary(mod, inference = T)
```

```
Model Information:
          Estimates        CI Low        CI High            SE
AR1    9.998700e-01 9.998336e-01 9.999064e-01 2.211547e-05
SIGMA2 5.223319e-11 4.343878e-11 6.102759e-11 5.346620e-12
AR1    1.265324e-01 1.265324e-01 1.265324e-01 1.765050e-08
SIGMA2 5.031185e-05 5.021566e-05 5.040805e-05 5.848209e-08

* The initial values of the parameters used in the minimization
of the GMWM objective function were  generated by the program
underneath seed: 1337.

Objective Function: 20.9297

Asymptotic Goodness of Fit:
Test Statistic: 903.8 on 15 degrees of freedom
The resulting p-value is: 0
```

WIC model selection

Two options for model selection

- **Manual**: the function `rank.models()` allows the user to specify the set of models from which to select
- **Automatic**: the function `auto.imu()` allows to specify one single model from which all sub-models are generated to create a set from which to select

R code:

```
rank.models(3*AR1()+WN(),
2*AR1()+QN(), imu6[,1], nested =
F, bootstrap = F, model.type="imu",
robust = F)
```

```
mod.res = auto.imu(imu6, model
= 3*AR1()+WN()+RW()+QN()+DR(),
bootstrap = F, robust = F)
```

WIC model selection

Automatic IMU model selection

Suppose we want to apply the `auto.imu()` function to the X-axis gyroscope in the `imu` dataset and consider all model combinations within the `4*AR1() + WN() + RW()` model

```
Xt = imu6[,1]
mod = 4*AR1()+WN()+RW()
mod.sel = auto.imu(Xt, model = mod)
summary(mod.sel)
```

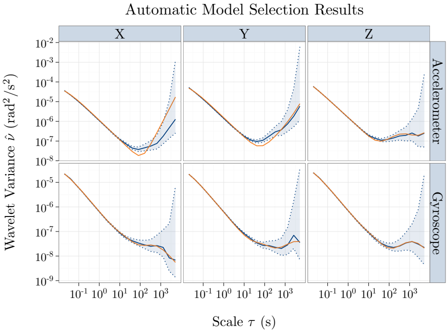
```
The model ranking for data column 1:
                              Obj Fun  Optimism   Criterion  GoF P-Value
1. AR1 AR1 AR1                17.2769    0.2974     17.5743            0
2. AR1 AR1 AR1 RW             17.2679    0.3139     17.5818            0
3. AR1 AR1 AR1 AR1            17.6022    0.4330     18.0351            0
4. AR1 AR1 RW                 20.1044    0.4871     20.5915            0
5. AR1 AR1                    20.9864    0.2070     21.1933            0
...
19. RW                     28420.4972    0.0160  28420.5132            0
```

(Run time: 50.25 [sec] ≈ 14x faster than the bootstrap option 685.4 [sec])

WIC Model Selection

Visualizing the Automatic IMU Model Selection

We can observe the results of the `auto.imu()` function by requesting a plot. The plot will contain the empirical wavelet variance in addition to the best implied wavelet variance that we find. In this particular case, we used the defaults of the `auto.imu()` (63 models $\times$ 6 columns).



R code:

```
imu.obj = imu(imu6, gyroscope = 1:3,
accelerometer = 4:6, axis = c('X',
'Y', 'Z'))
auto.mod = auto.imu(imu.obj)
plot(auto.mod)
```

(Run time: 377.77 [sec])

Upcoming features

Aside from **fine-tuning existing features** of the package (additional options, function optimization, support documentation, etc.), the GMWM package will be updated with the following features

- Sensor calibration under **dynamic conditions**

$$Y_t = \boldsymbol{\rho_t} Y_{t-1} + \epsilon_t$$

- **Multivariate latent models**

$$\begin{aligned} Y_t^{(x)} &= V_t + W_t^{(x)} \\ Y_t^{(y)} &= V_t + W_t^{(y)} \\ Y_t^{(z)} &= V_t + W_t^{(z)} \end{aligned}$$

- Others (e.g. additional moments for the GMWM (AGMWM), other wavelet filters, unit-root and independence testing)

Thank you very much for your attention!

A special thanks to

- Dr. Yannick Stebler (EPFL/u-blox)
- Prof. Maria-Pia Victoria-Feser (U. Geneva)
- Wenchao Yang (UIUC)

Any questions?

More info...

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