Theoretical Limitations of Allan Variance-based Regression for Time Series Model Estimation

Stéphane Guerrier, Roberto Molinari & Yannick Stebler

Abstract—**This paper formally proves the statistical inconsistency of the Allan variance-based estimation of latent (composite) model parameters. This issue has not been sufficiently investigated and highlighted since it is a technique that is still being widely used in practice, especially within the engineering domain. Indeed, among others, this method is frequently used for inertial sensor calibration which often deals with latent time series models and practitioners in these domains are often unaware of its limitations. To prove the inconsistency of this method we firstly provide a formal definition and subsequently deliver its theoretical properties, highlighting its limitations by comparing it with another statistically sound method.**

Index Terms—**Latent time series models, State Space Models, Sensor Calibration, Inertial Measurement Units, Error Modelling.**

I. INTRODUCTION

In the engineering domain mainly, but in other domains as well, researchers and practitioners often have to deal with the modelling of complex signals. These signals are often issued from latent or composite processes (i.e. a process made by the sum of different underlying processes) and their estimation has been dealt with using the Maximum-Likelihood Estimator (MLE) or linear regression based on the Allan variance (AVLR). The MLE is set up through a Kalman filter and an optimization typically based on an Expectation-Maximization algorithm making its implementation limited due to its complexity and its numerical instability as illustrated, for example, in [3, 8, 9].

To overcome the drawbacks of the MLE, the use of the AVLR is widespread especially in fields dealing with large amounts of data such as inertial sensor calibration procedures. A few examples of the latter can be found in [1] or [5] where this technique is used despite the fact that this method is not reliable from a statistical point of view in many cases. Therefore, practitioners keep making use of this approach either because they are unaware of these issues or because it is nevertheless a computationally feasible method which allows to easily estimate complex model parameters (albeit inconsistently). In this perspective, this paper intends to raise more awareness among researchers and practitioners about the inadequacy of the AVLR by formally showing the inconsistency of this estimator and underlining its relationship to a

S. **Guerrier** is Assistant Professor, Department of Statistics, University of Illinois at Urbana-Champaign, IL, 61820, USA (E-mail: stephane@illinois.edu).

R. **Molinari** is a Ph.D. student, Research Center for Statistics & Geneva School of Economics and Management, University of Geneva, 1211, Switzerland (E-mail: roberto.molinari@unige.ch).

Y. **Stebler** is senior R&D Engineer, u-blox, Thalwil, 8800, Switzerland (E-mail: yannick.stebler@gmail.com).

recently proposed estimator called the Generalized Method of Wavelet Moments (GMWM) (see [3]). Although the latter could be based on the Allan Variance (AV), the GMWM is based on the Wavelet Variance (WV) and is shown to have desirable statistical properties as well as being computationally straightforward. It therefore represents an ideal compromise between the optimal statistical properties of the MLE and the easy implementation of the AVLR, definitely making the use of the latter unreasonable.

II. STOCHASTIC PROCESSES PROPERTIES

In this section we define the processes that are used in this work to construct the latent process (Y_t). In App. A we give further details of these processes, especially concerning their implied Haar WV. The processes are the following:

(P1) *Gaussian White Noise* (WN) with parameter $\sigma^2 \in \mathbb{R}^+$. This process is defined as $X_t \sim \mathcal{N}\left(0, \sigma^2\right)$ and is sometimes referred to as *Angle (Velocity) Random Walk*.

(P2) *Quantization Noise* (QN) (see e.g. [6]) with parameter $Q^2 \in \mathbb{R}^+$. This process has a PSD of the form $S_X(f) = 4Q^2 \sin^2\left(\pi f (\Delta t)^{-1}\right) \Delta t, \; 2f < \Delta t$.

(P3) *Drift* with parameter $\omega \in \Omega$ where Ω is either $\mathbb{R}^+$ or $\mathbb{R}^-$. This process is defined as: $X_t = \omega t$ and is occasionally referred to as *Rate Ramp*.

(P4) *Random walk* (RW) with parameter $\gamma^2 \in \mathbb{R}^+$. This process is defined as $X_t = \sum_{t=1}^{T} \gamma Z_t$, where Z_t is a white noise. This process is often called *Rate Random Walk* in the engineering literature.

(P5) *Auto-Regressive* AR(1) process with autoregressive parameter $\rho \in (-1, +1)$ and innovation variance $\upsilon^2 \in \mathbb{R}^+$. This process is defined as $X_t = \rho X_{t-1} + \upsilon Z_t$, where Z_t is a white noise. AR(1) processes are sometimes used as an approximation for *Bias Instability* noises.

Considering these processes, we will now make the following assumptions:

(A1) A composite process (Y_t) is made of a sum which includes a subset or all processes from (P1) to (P5), where processes (P1) to (P4) are included only once and process (P5) can be included p times $(p < \infty)$ with $\rho^{(i)} < \rho^{(j)}$ for $1 \leq i < j \leq p$ (i.e. all (P5) processes have different ρ values).

(A2) If (Y_t) is a composite process then all sub-processes are independent.

(A3) The innovation process for processes (P1), (P4) and (P5) has a variance σ^2 such that $0 < \sigma^2 < \infty$ and processes (P2) and (P3) have $Q^2 < \infty$ and $|\omega| < \infty$ respectively.

We know that the AV and Haar-based WV are derived from an embedded first-order backward difference of their corresponding filters. Therefore, by denoting ∇ as the first-difference operator, the following lemma states the properties of the composite process (Y_t).

Lemma 1. *Under Assumptions (A1) to (A3), a stochastic process (Y_t) has the following properties:*

(i) $\mathbb{E}\left[|Y_t|^{4+2\delta}\right] < \infty$ *and/or* $\mathbb{E}\left[|\nabla Y_t|^{4+2\delta}\right] < \infty$ *for* $\delta > 0$,

(ii) *has mixing coefficient* $\alpha_{Y,h} = \mathcal{O}\left(\gamma^h\right)$ *and/or* $\alpha_{\nabla Y,h} = \mathcal{O}\left(\gamma^h\right)$ *as* $h \to \infty$ *for some* $0 < \gamma < 1$.

The proof of Lemma 1 can be found in App. B. This is a useful result since it states that the considered processes (or their first-order backward differences) are strongly mixing and allows to obtain the theoretical results necessary to prove the asymptotic properties of the estimators that are investigated and proposed in this work. These estimators are based respectively on the AV and WV whose definitions are given in App. C.

III. ALLAN VARIANCE LINEAR REGRESSION ESTIMATION

As highlighted earlier, the AVLR is, among others, a widely and commonly used approach in engineering, for example within sensor calibration procedures. This approach is possible since the AV $(\phi_{\tau_j}^2)$ can be expressed as a function $\phi_{\tau_j}^2(\boldsymbol{\theta})$ which depends on the scale and the parameter vector $\boldsymbol{\theta}$ of an arbitrary model $F_{\boldsymbol{\theta}}$. As mentioned earlier, the AVLR uses regressions on the linear parts of the $\log(\hat{\phi}_{\tau_j}) - \log(\tau_j)$ plot with slopes λ to estimate the parameter vector $\boldsymbol{\theta}$ of the associated τ_j^{λ} (or f^{α}) power-law process (see e.g. [4]).

To investigate the properties of this approach, we consider only processes producing power-law noises that have a linear representation in the $\log(\phi_{\tau_j}) - \log(\tau_j)$ plot such as processes (P1), (P2), (P3) and, under approximating conditions, (P4). Other processes with more complex Allan Deviation (AD) representation are difficult to estimate with this approach and therefore are not considered. Before studying the consistency of the AVLR estimator, the following corollary states the consistency of the AV estimator $\hat{\phi}_{\tau_j}^2$ proposed in [2].

Corollary 1. *Under the conditions of Lemma 1, $\hat{\phi}_{\tau_j}^2$ is a consistent estimator of $\phi_{\tau_j}^2(\boldsymbol{\theta})$.*

The proof of this corollary in App. D is a direct result of [7] for the WV estimator. Given this result, in the next sections we will define precisely the principle of AVLR for a single process as well as for latent time series models and study its properties in both cases. The formal definition of this method along with its theoretical limitations are therefore provided to make practitioners, who implicitly make use of it, aware of this problem.

1) AV-based Estimation for a Single Process: Suppose that the process (Y_t) is such that it has a linear representation in a $\log(\phi_{\tau_j}) - \log(\tau_j)$ plot for the set of scales $\boldsymbol{\eta} \in \mathcal{G}$ where

$$\mathcal{G} = \left\{\left\{\tau_k, ..., \tau_{k+h}\right\} | k, h \in \mathbb{N}^+, k + h \leq J\right\}$$

denotes all possible sets which contain adjacent scales having cardinality $|\boldsymbol{\eta}| > 0$. As a consequence of the linear relationship

that exists between $\log\left(\phi_{\tau_j}(\theta)\right)$ and $\log\left(\tau_j\right)$ we can write

$$\log\left(\phi_{\tau_j}(\theta)\right) = g(\theta) + \lambda \log\left(\tau_j\right), \ \ \forall \tau_j \in \boldsymbol{\eta} \tag{1}$$

where the function $g(\cdot)$ as well as the constant λ are known and depend on the model F_{θ}. The latter expressions are given explicitly further on for processes (P1) to (P3). However, when the estimated AD $(\hat{\phi}_{\tau_j})$ is computed from an observed finite sample (y_t), we obtain the relationship

$$\log\left(\hat{\phi}_{\tau_j}\right) = \log\left(\phi_{\tau_j}(\theta)\right) + \varepsilon_{\tau_j}, \ \ \forall \tau_j \in \boldsymbol{\eta}$$

where ε_{τ_j} are residuals. This linear relationship leads to the AVLR which consists in an AV-based "least-squares" estimator of θ, noted as $\hat{\theta}_{AV}$ and defined as

$$\hat{\theta}_{AV} = \underset{\theta \in \Theta}{\operatorname{argmin}} \sum_{\tau_j \in \boldsymbol{\eta}} \varepsilon_{\tau_j}^2. \tag{2}$$

Moreover, since $g(\cdot)$ and λ are known for a given model F_{θ} satisfying (1), $\hat{\theta}_{AV}$ can be defined explicitly as

$$\hat{\theta}_{AV} = g^{-1}\left\{\frac{1}{|\boldsymbol{\eta}|}\sum_{\tau_j \in \boldsymbol{\eta}}\left[\log\left(\hat{\phi}_{\tau_j}\right) - \lambda \log\left(\tau_j\right)\right]\right\}. \tag{3}$$

For example, let us consider that cases where (Y_t) follows (P1), (P2) or (P3), then Corollary 2 below states the consistency of $\hat{\theta}_{AV}$ in these cases and its proof is given in App. E which is a direct consequence of the exact linear representation of the AV for these processes.

Corollary 2. *For $\boldsymbol{\eta} \in \mathcal{G}$ and if (Y_t) follows either (P1), (P2) or (P3), the AVLR estimator $\hat{\theta}_{AV}$, as defined by (3), is a consistent estimator of θ under Corollary 1.*

When considering simple processes such as (P1), (P2) or (P3), using (3) is certainly not the best possible option for estimating θ as this estimator is far less efficient than other estimators such as, for example, the standard MLE or the GMWM (see for example Sec. IV).

Regarding process (P4), the engineering literature (see e.g. [1]) often approximates the AV with $\phi_{\tau_j}^2(\theta) \approx 6^{-1}\gamma^2\tau_j$. This approximation then satisfies (1) and the AVLR estimator for γ^2 can hence be defined as

$$\hat{\gamma}_{AV}^2 = 6\exp\left\{\frac{2}{|\boldsymbol{\eta}|}\sum_{\tau_j \in \boldsymbol{\eta}}\left[\log\left(\hat{\phi}_{\tau_j}\right) - \frac{1}{2}\log\left(\tau_j\right)\right]\right\}. \tag{4}$$

Although $\hat{\gamma}_{AV}^2$ may provide satisfactory results in practice and specially for large scales, the next corollary states that it does not converge to γ^2 (the true parameter value) but to $\tilde{\gamma}^2 > \gamma^2$.

Corollary 3. *Under the conditions of Corollary 1, we have that $\hat{\gamma}_{AV}^2$ as defined in (4) is such that $\hat{\gamma}_{AV}^2 \overset{\mathcal{P}}{\longmapsto} \tilde{\gamma}^2 > \gamma^2$ and therefore that $\hat{\gamma}_{AV}^2$ is not a consistent estimator of γ^2.*

Based on the proof of Corollary 3 in App. F, a consistent estimator of γ^2 can be obtained by introducing the following correction:

$$\hat{\gamma}_b^2 = c_{AV}^{-1}\,\hat{\gamma}_{AV}^2 \ \ \text{with} \ \ c_{AV} = \exp\left\{\frac{2}{|\boldsymbol{\eta}|}\sum_{\tau_j \in \boldsymbol{\eta}} b_{AV}(\tau_j)\right\} \tag{5}$$

and where

$$b_{AV}(\tau_j) = \log\left(\sqrt{1 + \frac{2}{\tau_j^2}}\right).$$

However, we must again point out that for such a simple process using (4) or (5) is certainly not the best option to estimate the innovations variance of a random walk.

2) AV-based Estimation for Composite Stochastic Processes: We consider now the situation in which we use (3) to estimate the parameters of a model F_{θ} made of *at least two* processes among (P1), (P2), (P3) and (P4). Let L denote the number of sub-processes such that $2 \leq L \leq 4$. Let also $\eta_l \in \mathcal{G}$, $l = 1, ..., L$ denote the set of scales on which the lth element of θ, say θ_l, will be estimated (i.e. apply (3) to the set η_l). Thus, we have $\hat{\theta}_{AV} = [\hat{\theta}_l]_{l=1,..,L}$ where

$$\hat{\theta}_l = g^{-1}\left\{\frac{1}{|\eta_l|}\sum_{\tau_j \in \eta_l}\left[\log\left(\hat{\phi}_{\tau_j}\right) - \lambda\log\left(\tau_j\right)\right]\right\}. \quad (6)$$

As stated in the following corollary, when (Y_t) is a composite stochastic process (with $L > 1$), $\hat{\theta}_{AV}$ is not consistent.

Corollary 4. *Let Assumptions (A1) and (A2) hold. Assume further that at least one of the sub-processes is either (P1), (P2), (P3) or (P4) with parameter θ and the other process(es) satisfy Assumption (A3). Then, the estimator $\hat{\theta}_l$ based on $\eta_l \in \mathcal{G}$ as defined in (6) is not a consistent estimator of θ and, consequently, $\hat{\theta}_{AV}$ is not a consistent estimator.*

The proof of this corollary can be found in App. G. AVLR estimators are therefore generally inconsistent when applied to latent processes and their use is not justified for the estimation of single processes (as they are clearly outperformed by standard statistical methods such as the MLE). Although the properties of the AVLR formally shown in this section are supposedly known, their widespread usage in the engineering literature and others highlight a lack of awareness towards these issues which, despite the good statistical properties of the AVLR for specific simple processes, indicate that these methods are not optimal and should be avoided in practice.

IV. COMPARISONS BETWEEN THE AVLR AND THE GMWM

In this section we compare the AVLR and GMWM estimators when applied to a single process or a sum of them. To do so, let us define the GMWM estimator which is the result of the following generalized least-squares problem

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmin}} ||\hat{\nu} - \nu(\theta)||_{\Omega}^2 \quad (7)$$

where the expression $||\mathbf{x}||_{\Omega}^2$ denotes the quadratic form $\mathbf{x}^T\Omega\mathbf{x}$, $\hat{\nu}$ is the estimated WV proposed by [7], $\nu(\theta)$ is the WV implied by the model F_{θ} and Ω is a positive definite weighting matrix chosen in a suitable manner (see [3] for details). Based on the properties of the WV estimator $\hat{\nu}$, [3] prove the statistical consistency and asymptotic normality of this estimator.

To start the comparison, let us take the case where the signal (Y_t) is composed of a single process among (P1), (P2) or (P3).

In this case, the AVLR estimator $\hat{\theta}_{AV}$ defined by Eq. (3) can be seen as a special case of the GMWM estimator developed in the previous section which, to distinguish it from the AVLR estimator, we denote as $\hat{\theta}_{\nu}$. This statement is formally given in the next lemma whose proof can be found in App. H.

Lemma 2. *Let the process (Y_t) follow either (P1), (P2) or (P3) for scale vector $\tau \subset \mathcal{G}$. Let also $\hat{\theta}_{AV}$ and $\hat{\theta}_{\nu}$ denote the AVLR and GMWM estimators defined by Eq. (2) and (7), respectively. Then we have that $\hat{\theta}_{AV} = \hat{\theta}_{\nu}$ when the matrix*

$$\Omega^{\star} = \left[\omega_{i,j}^{\star}\right]_{i,j=1,...,|\tau|}$$

is diagonal with elements

$$\omega_{i,j}^{\star} = \begin{cases} \left(\frac{\log(\hat{\phi}_{\tau_i}) - \log(\phi_{\tau_i}(\theta))}{\hat{\nu}_{\tau_i} - \nu_{\tau_i}(\theta)}\right)^2 & \text{if } i = j \\ 0 & \text{if } i \neq j. \end{cases} \quad (8)$$

Hence, when (Y_t) follows either (P1), (P2) or (P3) (or (P4) using the unbiased estimator in (5)), $\hat{\theta}_{AV}$ (or $\hat{\gamma}_b^2$) are in fact GMWM estimators of θ. Therefore, they inherit their asymptotic properties, i.e. consistency and asymptotic normality. However, depending on the matrix Ω on which the GMWM estimator is based, it can be more or less asymptotically efficient. Corollary 5 states that there always exists a GMWM estimator that is asymptotically more efficient than the AVLR estimator given in Eq. (3). The proof is given in App. I.

Corollary 5. *Consider the same setting as in Lemma 2. Then we have that there always exists at least one matrix Ω° such that*

$$\lim_{T \to \infty} \frac{\operatorname{var}\left(\hat{\theta}_{\nu}^{\circ}\right)}{\operatorname{var}\left(\hat{\theta}_{AV}\right)} < 1, \quad \tau \in \mathcal{G}$$

where $\hat{\theta}_{\nu}^{\circ}$ denotes the GMWM estimator of θ based on the matrix Ω°.

The matrix Ω° is defined as V^{-1} where V denotes the covariance matrix of $\hat{\nu}$ and can be estimated, for example, via bootstrap methods (see App. J). To conclude this comparison, it must be underlined that the GMWM and AVLR are equivalent in terms of computational efficiency with the main bottleneck being the computation of the WV and AV whose operations are of the order $T\log_2(T)$ (the same order as the widely used fast Fourier transform algorithm).

V. AN EXAMPLE

Let us now provide an example illustrating the conclusions of Sec. III-2 concerning the inconsistency of the AVLR estimator on composite processes. For this purpose, assume that we observe a process (Y_t) which is issued from the following model F_{θ}:

$$Y_t = X_t^{(1)} + X_t^{(2)} \quad (9)$$

where $X_t^{(1)}$ and $X_t^{(2)}$ are as defined in (P1) and (P4), respectively. This process is particularly relevant in the field of sensor calibration as underlined by [10] who use a modified version of the AVLR to estimate this process for rate gyroscopes. Considering this model, we therefore have the true parameter

vector $\boldsymbol{\theta}_0 = \left(\sigma_0^2, \gamma_0^2\right)$ for which the theoretical AV of this system (under Assumption (A2)) is given by

$$\phi_{\tau_j}^2(\boldsymbol{\theta}_0) = \frac{12\sigma_0^2 + \left(\tau_j^2 + 2\right)\gamma_0^2}{6\tau_j}. \tag{10}$$

The true $\log(\phi_{\tau_j}^2(\boldsymbol{\theta}_0))$ value is unknown in practice and only its estimation, i.e. $\log(\hat{\phi}_{\tau_j}^2)$ is available. Suppose we are interested in firstly estimating σ_0^2. In this case, the AVLR would consist in applying a linear regression on the first j^* scales in order to estimate σ_0^2. This concept is illustrated in Fig. 1 showing the AV sequence (lower panel) of a simulated signal (upper panel) (y_t) with $T = 10^4$ issued from (9). In this case, we chose to perform the linear regression on the $j^* = 5$ first scales ("full" black dots in the lower panel) for estimating σ_0^2. Since we are interested in estimating σ_0^2, we
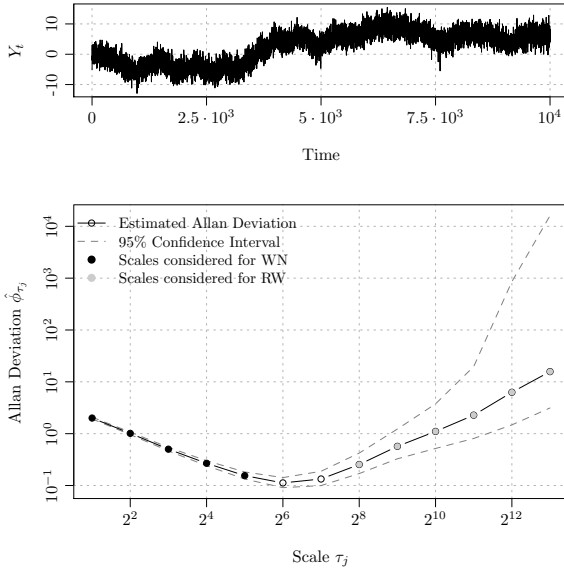

Fig. 1. Top: Plot of a process (Y_t) made by the sum of a WN and a RW process; Bottom: estimated WV of the process (Y_t) shown in the top part of the figure.

can use the estimator based on (3), i.e.

$$\hat{\sigma}_{AV}^2 = \frac{1}{2} \exp\left\{ \frac{2}{|\boldsymbol{\tau}|} \sum_{\tau_j \in \boldsymbol{\tau}} \left[\log\left(\hat{\phi}_{\tau_j}\right) + \frac{1}{2}\log\left(\tau_j\right) \right] \right\}$$
$$= \frac{1}{2} \exp\left\{ \frac{1}{|\boldsymbol{\tau}|} \sum_{\tau_j \in \boldsymbol{\tau}} \left[\log\left(\hat{\phi}_{\tau_j}^2\right) + \log\left(\tau_j\right) \right] \right\}. \tag{11}$$

Substituting Eq. (10) into Eq. (11), by the continuous mapping theorem we have

$$\hat{\sigma}_{AV}^2 \xmapsto{\mathcal{P}} \frac{1}{2} \exp\left[\frac{1}{j^*} \sum_{i=1}^{j^*} \log\left(\frac{12\sigma_0^2 + \left(\tau_i^2 + 2\right)\gamma_0^2}{6\tau_i} \right) + \log(\tau_i) \right] \tag{12}$$

Now two cases can be investigated:

1) If $\gamma^2 = 0$, we have from (12) that

$$\hat{\sigma}_{AV}^2 \xmapsto{\mathcal{P}} \sigma_0^2$$

which demonstrates that in our example $\hat{\sigma}_{AV}^2$ is a consistent estimator of σ_0^2 if and only if $\gamma_0^2 = 0$.

2) If $\gamma_0^2 > 0$, then $\frac{(\tau_i^2 + 2)\gamma_0^2}{6\tau_i} > 0$ in (12) and we have

$$\hat{\sigma}_{AV}^2 \xmapsto{\mathcal{P}} \sigma_0^2 + c, \quad \text{with} \quad c > 0.$$

This last inequality confirms that the commonly used AVLR method does not provide a consistent estimator for σ_0^2 when $\gamma_0^2 > 0$. In other words, $\hat{\sigma}_{AV}^2$ is a consistent estimator of σ_0^2 if and only if $\gamma_0^2 = 0$. The simulation study illustrated in Fig. 2 supports these results and confirms the statements of Theorem 2 and of Corollary 4. The simulation was made using the process given in (9) with parameters $\sigma_0^2 = 4$ and $\gamma_0^2 = 0.01$. We compare the Mean Squared Error (MSE) of the AVLR and GMWM estimators as the sample size increases (starting from a sample size of $n = 5,000$). Each time, the parameter γ_0^2 was estimated using the estimator in (4) and the scales $j^* = 8, \ldots, J$. As it can be seen, the MSE of the
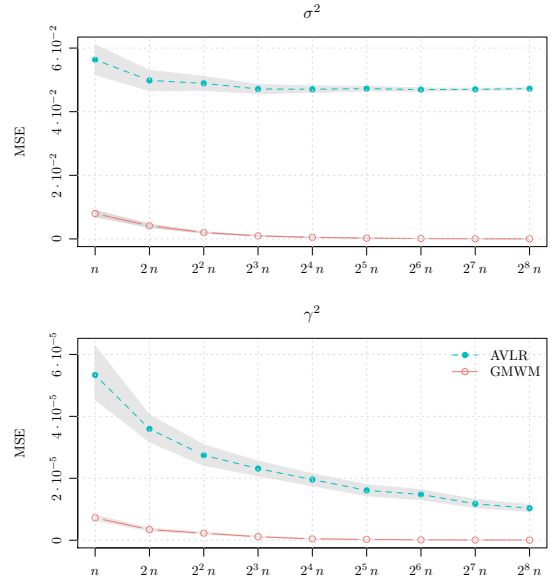

Fig. 2. Top: MSE and corresponding confidence intervals of the AVLR and GMWM estimators for the WN parameter $\sigma_0^2 = 4$ of the composite process in (9); Bottom: MSE and corresponding confidence intervals of the AVLR and GMWM estimators for the RW parameter $\gamma_0^2 = 0.01$ of the composite process in (9).

AVLR does not converge to zero as a consistent estimator would be expected to do whereas the MSE of the GMWM does exactly this and also at a faster rate. The confidence intervals of the MSE for the estimators show how the MSE of the two estimators is significantly different and also underline how the GMWM, apart from being consistent, can also be seen as being more efficient as stated in Section IV.

VI. CONCLUSIONS

In many fields dealing with large time series, among which engineering, a widely and frequently used alternative to more

complex methods (e.g. MLE) is the AVLR which considerably simplifies the task of estimation. However, the latter method does not lead to consistent and reliable estimates in most cases thereby not justifying its widespread usage. To stress the latter aspect, in this paper we described and proved the statistical properties of this estimator and showed that, although it is consistent in some simple cases, it is in general an inconsistent method and should therefore be avoided in practice in favor of other consistent estimators. An easy-to-implement alternative is the recently proposed GMWM estimator (see [3]) which provides a statistically sound and computationally efficient approach to estimate composite models as well as simple processes. Considering the latter method, we proved that, even when the AVLR is consistent, the GMWM nevertheless provides more efficient estimators and is to be generally preferred to the AVLR. Finally, these conclusions are supported by an example which highlights the drawbacks of the AVLR and the advantages of the GMWM in these contexts.

REFERENCES

[1] El-Sheimy, N., Hou, H., Niu, X.: Analysis and Modeling of Inertial Sensors using Allan Variance. IEEE Transactions on Instrumentation and Measurement **57**(1), 140–149 (2008)

[2] Greenhall, C.A.: Recipes for Degrees of Freedom of Frequency Stability Estimators. IEEE Transactions on Instrumentation and Measurement **40**(6), 994–999 (1991)

[3] Guerrier, S., Skaloud, J., Stebler, Y., Victoria-Feser, M.P.: Wavelet-variance-based estimation for composite stochastic processes. Journal of the American Statistical Association **108**(503), 1021–1030 (2013)

[4] IEEE Standard 1293: Specification Format Guide and Test Procedure for Linear, Single-Axis, Non-gyroscopic Accelerometers (1998)

[5] Jain, S., Nandy, S., Chakraborty, G., Kumar, C., Ray, R., Shome, S.: Error modeling of laser range finder for robotic application using time domain technique. In: Signal Processing, Communications and Computing (ICSPCC), 2011 IEEE International Conference on, pp. 1–5. IEEE (2011)

[6] Papoulis, A.: Probability, Random Variables, and Stochastic Process, third edition edn. McGraw-Hill, Inc. (1991)

[7] Percival, D.B.: On Estimation of the Wavelet Variance. Biometrika **82**(3), 619–631 (1995)

[8] Stebler, Y.: Modeling and Processing Approaches for Integrated Inertial Navigation. Ph.D. thesis, École Polytechnique Fédérale de Lausanne (2012)

[9] Stebler, Y., Guerrier, S., Skaloud, J., Victoria-Feser, M.P.: Constrained Expectation-Maximization Algorithm for Stochastic Inertial Error Modeling: Study of Feasibility. Measurement Science and Technology **22**(8), 085,204 (2012)

[10] Vaccaro, R.J., Zaki, A.S.: Statistical modeling of rate gyros. Instrumentation and Measurement, IEEE Transactions on **61**(3), 673–684 (2012)