

Greedy 1:

a. Assume T and T' are distinct minimum spanning trees. Then some edge $(u; v)$ is in T but not in T' . Since all edge weights are distinct the edges on the unique path from u to v in T' must then be strictly lighter than $(u; v)$ contradicting the fact that T is a minimum spanning tree. It can easily be seen by example that the second-best minimum spanning trees is not unique.

b. Let T be a minimum spanning tree. We wish to find a tree that has the smallest possible weight that is larger than the weight of T . We can insert an edge $(u; v) \notin T$ by removing some other edge on the unique path between u and v . By the above property such a replacement must increase the weight of the tree. By carefully considering the cases it is seen that replacing two or more edges will not produce a tree better than the second best minimum spanning tree.

c. To compute this in $O(V^2)$ time for all nodes in the tree do the following: For each node v perform a traversal of the tree. In order to compute $\max(v; k)$ for all $k \in V$ simply maintain the largest weight encountered so far for the path being investigated. Doing this yields a linear time algorithm for each node and we therefore obtain a total of $O(V^2)$ time.

d. Using the idea of the second subexercise and the provided algorithm in the third subexercise, we can now compute T_2 from T in the following way:

Compute $\max(u; v)$ for all vertices in T . Compute for any edge $(u; v)$ not in T the difference $w(u; v) - \max(u; v)$. The two edges yielding the smallest positive difference should be replaced.

Greedy 2:

a. Suppose a permutation of S is $\langle r_1, r_2, \dots, r_n \rangle$, the total completion time is $\sum_{i=1}^n (n-i+1) \cdot p_{r_i}$. The optimal solution is to sort p_i into increasing order.

b. Preemption will not yield a better solution if there is no new task. Each time there is a new task, assume that the current running task is preempted, let the current condition be a new scheduling task without preemption.