Sliding Mode Control With Integral Action

UNAM

January 28, 2022

SMC with integral action: an extension of the super-twisting controller for a arbitrary relative degree. When the relative degree $\rho = 1$, the controller by super-twisting for the plant

$$\Sigma_{DI} : \begin{cases} \dot{x}_i = x_{i+1}, & i = 1, \dots, \rho - 1, \\ \dot{x}_\rho \in [-C, C] + [K_m, K_M]u, \end{cases} \tag{1}$$

is given by

$$u = -k_1 \sqrt{|\sigma|}\mathrm{sign}(\sigma) - k_2 \int_0^t \mathrm{sign}(\sigma(\tau))d\tau, \tag{2}$$

which produces a continuous control signal u.

For $\rho > 1$ a natural extension of the super-twisting for (1) consists of the following integral action controller

$$u = \wp_1(\mathbf{x}) + \int_0^t \wp_2(\mathbf{x})d\tau, \tag{3}$$

where $\wp_1(\mathbf{x})$ is a continuous state feedback and $\wp(\mathbf{x})_2$, is the integral action, is a *discontinuous* function of the $\mathbf{x}$ state.

Sliding Mode Control With Integral Action

This controller extends the case of the arbitrary relative degree of the classic super-twisting:

- It can compensates *exactly* and in finite time the matched perturbations and/or uncertainties of the, which are *Lipschitz* functions, which derivative is bounded.
- Makes the signals $\sigma = \dot{\sigma} = \cdots = \sigma^{(\rho-1)} = \sigma^{(\rho)} \equiv 0$ cancel simultaneously after a finite time.
- For its implementation only is required to have the value of $\mathbf{x}(t) = (\sigma(t), \ldots, \sigma^{(\rho-1)}(t))^T$. This signals can be calculated with the robust exact differentiator.
- The control signal u is continuous.
- The precision is better than the sliding mode control of order ρ

Continuous Singular Terminal Sliding Mode Algorithm (CSTSMA)

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + f(x,t) \\ \sigma &= x_1 \end{cases}$$

$$u = -k_1 \lfloor \phi \rceil^{1/2} - k_3 \int_0^t \lfloor \phi \rceil^0 d\tau, \tag{4}$$

where $\phi = \left(x_2 + k_2 \lfloor x_1 \rceil^{2/3}\right)$, and k_1, k_2, k_3 are appropriate positive gains.

Continuous Nonsingular Terminal Sliding Mode Algorithm (CNTSMA)

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + f(x,t) \\ \sigma &= x_1 \end{cases}$$

$$u = -k_1\lfloor\phi_N\rceil^{1/3} - k_3\int_0^t \lfloor\phi_N\rceil^0 d\tau, \quad (5)$$

where $\phi_N = \left(x_1 + k_2\lfloor x_2\rceil^{3/2}\right)$, and k_1, k_2, k_3 are appropriate positive gains.

$$\begin{cases} \dot{x}_1 & = x_2 \\ \dot{x}_2 & = u + f(t) \\ \sigma & = x_1 \end{cases}$$

$$u = -k_1\lfloor x_1\rceil^{1/3} - k_2\lfloor x_2\rceil^{1/2} - \int_0^t (k_3\lfloor x_1(\tau)\rceil^0)d\tau, \tag{6}$$

where k_1, k_2, k_3 are appropriate positive gains.
NONLINEAR PID!!!
New Notation: $\lfloor z\rceil^p = |z|^p sgn(z)$

Continuous Twisting Algorithm (CTA), Moreno, Fridman et al , 2013-18

$$\begin{cases} \dot{x}_1 & = x_2 \\ \dot{x}_2 & = u + f(t) \\ \sigma & = x_1 \end{cases}$$

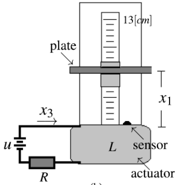
$$u = -k_1\lfloor x_1 \rceil^{1/3} - k_2\lfloor x_2 \rceil^{1/2} - \int_0^t (k_3\lfloor x_1(\tau) \rceil^0 + k_4\lfloor x_2(\tau) \rceil^0) d\tau, \qquad (7)$$

where k_1, k_2, k_3, k_4 are appropriate positive gains.
NONLINEAR PID!!!

Integral control: experimental validation

The sliding mode control with integral action was implemented in a system of magnetic suspension (Model 730) using a magnetic disk instead of a ball.

(a)



(b)

Example of the SMCIA

The mathematical model is the following

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{k}{m}x_2 - \frac{aL_0}{2m}\frac{x_3^2}{(a+x_1)^2} + g \\
\dot{x}_3 &= \frac{1}{L(x_1)}\left(-R_3 + aL_0\frac{x_2x_3}{(a+x_1)^2} + u\right)
\end{aligned}
\tag{8}
$$

Example of the SMCIA

Parameters and states of the magnetic suspension.

- $x_1 = y \in \mathbb{R}_+$ (vertical distance)
- $x_2 = \dot{y}$ (velocity)
- m (mass of the ball)
- g (gravity)
- k (viscous friction)
- $L(x_1) = L_1 + \frac{aL_0}{a+x_1}$ (inductance that depend of the distance)
- $x_3 = i \in \mathbb{R}_+$ (electric Current)
- R (electrical resistance)
- u (control)

$k, m, a, L_0, L_1, g, R > 0$, and the only measure signal is x_1.

Example of the SMCIA

Values of the parameters of the magnetic suspension.

- $L_0 = 0.245[H]$
- $L_1 = 0.1[H]$
- $R = 1.75[\Omega]$
- $m = 0.12[kg]$
- $g = 9.81[\frac{m}{s^2}]$
- $a = 8.8[mm]$
- $k = 0.1[\frac{Ns}{m}]$

The control objective is: x_1 must follow a trajectory of a non-harmonic reference signal $r(t)$, which is constant $(0.025[mm])$ when $0 \leq t \leq 30$ and its fourth temporal derivative (d^4r/dt^4) is a square signal, which is discontinuous bounded by $\Delta_2 = \frac{0.02}{\pi}$

Example of the SMCIA

Due to the relative degree $\rho = 3$ match the order of the system and the system (1) can be transform to the normal form.

The experiments where made for the discontinuous integral control $(d = -1)$ and two approximations of the homogeneous degrees $d = -\frac{1}{2}$ and $d = 0$ being the last a linear controller. The results of the experiment are shown in the following figure. First Position, second the error ans last the control signal.