Relay Systems

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The following relay system is proposed

$$\begin{aligned} \dot{y}_1 &= y_2, \ldots, \dot{y}_{l-1} = y_l, \\ \dot{y}_l &= \sum_{j=1}^{n} a_{l,j} y_j + k \mathsf{sign} y_1, \\ \dot{y}_i &= \sum_{j=1}^{n} a_{i,j} y_j, \qquad i = l+1, \ldots, n \end{aligned} \tag{1}$$

where

$a_{i,j} = const$,

$k \neq 0$

$y_1 = y_2 = \ldots = y_l = 0$ is the lth order sliding set.

The main results is as follows:

- for stability of equilibrium point of relay control system (1) with second order sliding $(l = 2)$ three main cases are singled out: exponentially stable, stable and unstable;
- it is shown that the equilibrium point of the system (1) is always unstable with $l \geq 3$. Consequently, all higher order sliding modes arriving in the relay control systems are unstable with order of sliding more than 2

2-sliding stability in relay systems

Consider a simple example of a second order dynamic system

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= a y_1 + b y_2 + k \,\mathsf{sign}\, y_1 \end{aligned} \tag{2}$$

The 2-sliding set is given here by $y_1 = y_2 = 0$. At first, let $k < 0$. Consider the Lyapunov function

$$E = \frac{y_2^2}{2} - a\frac{y_1^2}{2} + |k||y_1| - \frac{b}{2} y_1 y_2 \tag{3}$$

Function E is an energy integral of system (2).

Computing the derivative of function E achieve

$$\dot{E} = \frac{b}{2}y_2^2 + \frac{b}{2}|y_1|(|k| - a|y_1| - by_2 \mathsf{sign} y_1)$$

for some positive $\alpha_1 \le \alpha_2, \beta_1 \le \beta_2$

$$\alpha_1|y_1| + \beta_1 y_2^2 \le E \le \alpha_2|y_1| + \beta_2 y_2^2.$$

- $-\gamma_2 E \le \dot{E} \le -\gamma_1 E$ or $\gamma_1 E \le \dot{E} \le \gamma_2 E$ hold for $b < 0$ or $b > 0$ respectively, in a small vicinity of the origin with some $\gamma_2 \ge \gamma_1 > 0$
- Let now $k > 0$. The trajectories cannot leave the set $y_1 > 0, y_2 = \dot{y}_1 > 0$ if $a \ge 0$, and $y_1 < k/|a|$ if $a < 0$
- Starting with infinitisimally small $y_1 > 0, y_2 > 0$, any trajectory inevitably leaves some fixed origin vicinity.

2-sliding stability in relay systems

It allows three main cases of stability of system (2)

- *Exponentially stable case.* Under the conditions

$$b < 0, \quad k < 0 \tag{4}$$

  the equilibrium point $y_1 = y_2 = 0$ is exponentially stable.

- *Unstable case.* Under the condition

$$k > 0 \ \ or \ \ b > 0$$

  the equilibrium point $y_1 = y_2 = 0$ is unstable.

- *Critical case.*

$$k \leq 0, \quad b \leq 0, \quad bk = 0$$

  With $b = 0, k < 0$ the equilibrium point $y_1 = y_2 = 0$ is stable.

Let us illustrate the idea of the proof on an example of a simple third order system

$$\begin{aligned}
\dot{y}_1 &= y_2, \\
\dot{y}_2 &= y_3, \\
\dot{y}_3 &= a_{31}y_1 + a_{32}y_2 + a_{33}y_3 - k\,\mathsf{sign}\,y_1, \quad k > 0.
\end{aligned} \tag{5}$$

Consider the Lyapunov function.

$$V = y_1 y_3 - \frac{1}{2} y_2^2$$

Thus,

$$\dot{V} = -k|y_1| + y_1(a_{31}y_1 + a_{32}y_2 + a_{33}y_3)$$

and $\dot{V}$ is negative at least in a small neighbourhood of origin $(0, 0, 0)$