

# Finite Automata Exercises

Course: Theory of Computation (CSE 2205)

*Prepared By*

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# Exercise 1

Construct the transition table for the following DFA:

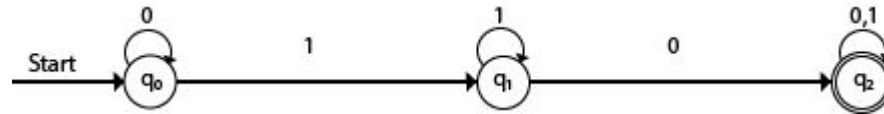
$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}$

$F = \{q_2\}$

Transition Diagram:



# Exercise 1 (Solution)

Construct the transition table for the following DFA:

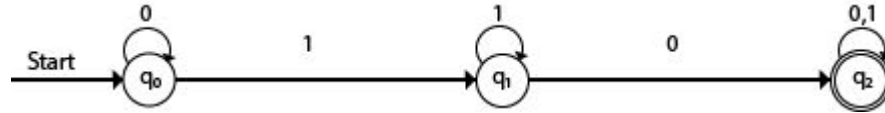
$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}$

$F = \{q_2\}$

Transition Diagram:



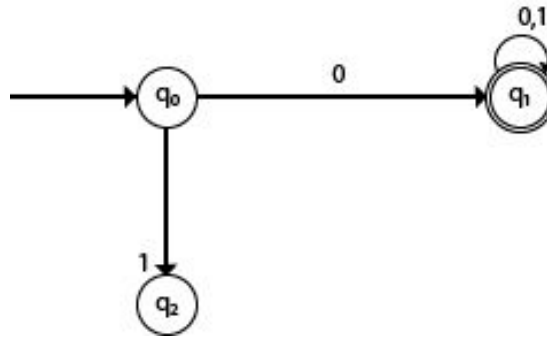
Present State	Next state for Input 0	Next State of Input 1
→q0	q0	q1
q1	q2	q1
*q2	q2	q2

## Exercise 2

Draw the state diagram and transition table for a DFA with  $\Sigma = \{0, 1\}$  accepts all starting with 0.

## Exercise 2 (Solution)

Draw the state diagram and transition table for a DFA with  $\Sigma = \{0, 1\}$  accepts all starting with 0.

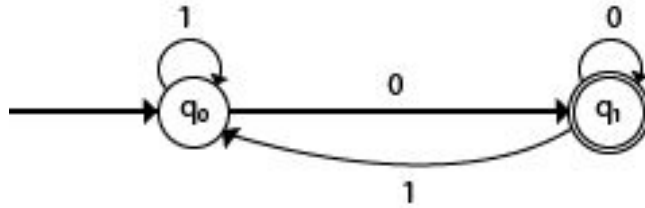


## Exercise 3

Draw the state diagram and transition table for a DFA with  $\Sigma = \{0, 1\}$  accepts all ending with 0.

## Exercise 3 (Solution)

Draw the state diagram and transition table for a DFA with  $\Sigma = \{0, 1\}$  accepts all ending with 0.



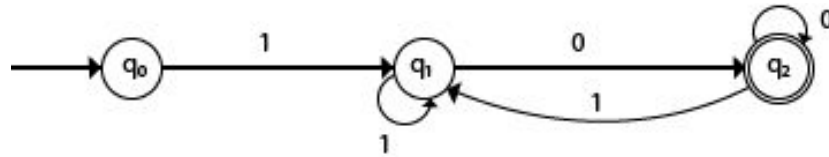
## Exercise 4

Design a FA with  $\Sigma = \{0, 1\}$  accepts those string which starts with 1 and ends with 0.



## Exercise 4 (Solution)

Design a FA with  $\Sigma = \{0, 1\}$  accepts those string which starts with 1 and ends with 0.



## Exercise 5

Design a FA with  $\Sigma = \{0, 1\}$  accepts the only input 101.

## Exercise 5 (Solution)

Design a FA with  $\Sigma = \{0, 1\}$  accepts the only input 101.

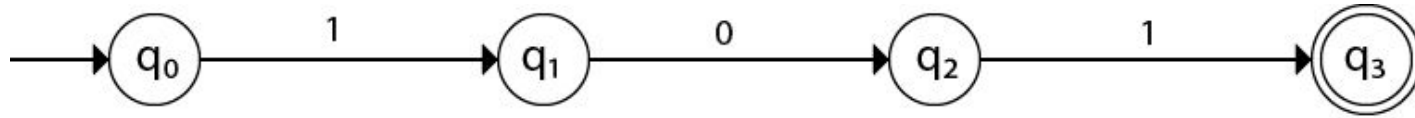


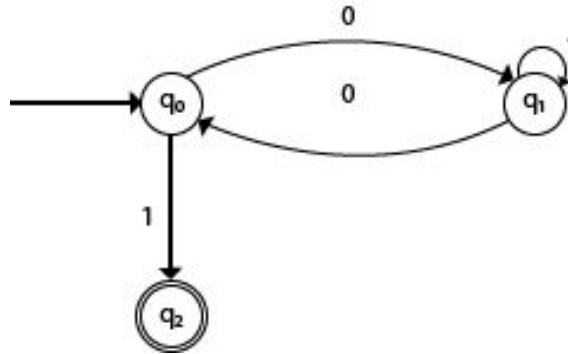
Fig: FA

## Exercise 6

Design a FA with  $\Sigma = \{0, 1\}$  accepts the strings with an even number of 0's followed by single 1.

## Exercise 6 (Solution)

Design a FA with  $\Sigma = \{0, 1\}$  accepts the strings with an even number of 0's followed by single 1.



# Exercise 7

Construct the transition table for the following NFA.

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}$

$F = \{q_2\}$

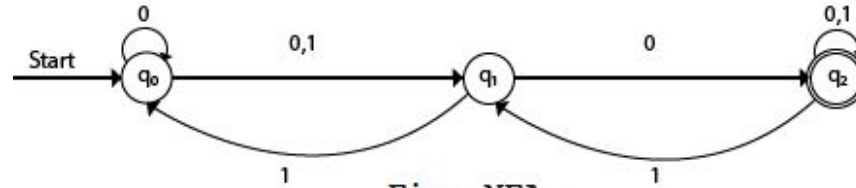


Fig: NFA

# Exercise 7 (Solution)

Construct the transition table for the following NFA.

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}$

$F = \{q_2\}$

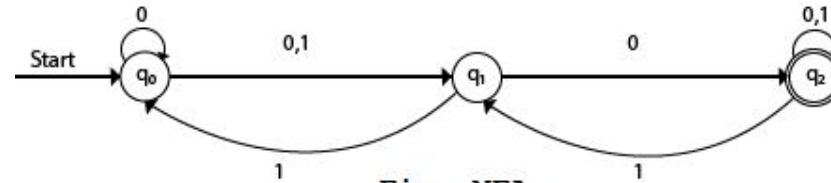


Fig: NFA

Present State	Next state for Input 0	Next State of Input 1
→q0	q0, q1	q1
q1	q2	q0
*q2	q2	q1, q2

## Exercise 8

Construct the NFA with  $\Sigma = \{0, 1\}$  that accepts all strings of length at least 2.



## Exercise 8 (Solution)

Construct the NFA with  $\Sigma = \{0, 1\}$  that accepts all strings of length at least 2.

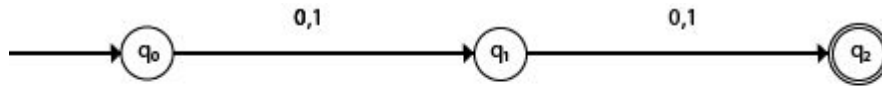


Fig: NFA

Present State	Next state for Input 0	Next State of Input 1
$\rightarrow q_0$	$q_1$	$q_1$
$q_1$	$q_2$	$q_2$
$*q_2$	$\epsilon$	$\epsilon$

## Exercise 9

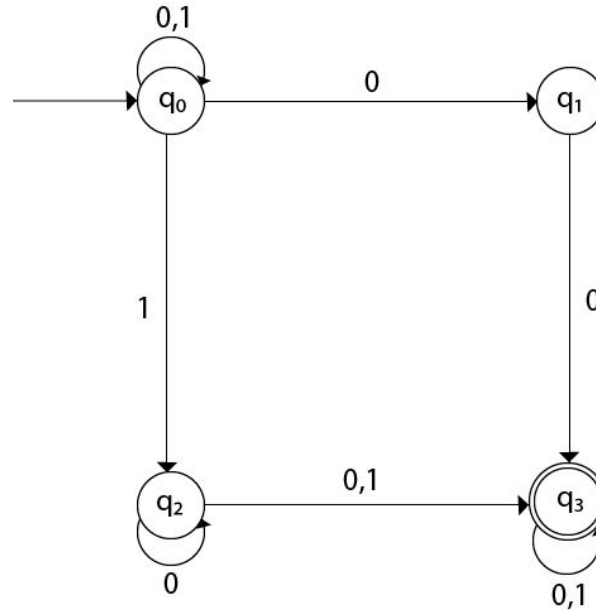
Design an NFA for the transition table as given below:

Present State	0	1
$\rightarrow q_0$	$q_0, q_1$	$q_0, q_2$
$q_1$	$q_3$	$\epsilon$
$q_2$	$q_2, q_3$	$q_3$
$\rightarrow q_3$	$q_3$	$q_3$

# Exercise 9 (Solution)

Design an NFA for the transition table as given below:

Present State	0	1
→q0	q0, q1	q0, q2
q1	q3	$\epsilon$
q2	q2, q3	q3
→q3	q3	q3

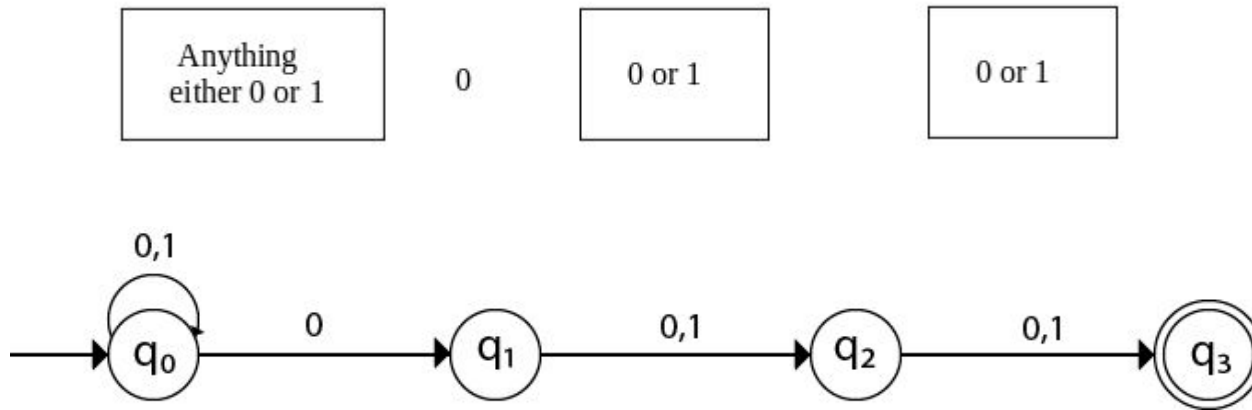


## Exercise 10

Design an NFA with  $\Sigma = \{0, 1\}$  accepts all strings in which the third symbol from the right end is always 0.

## Exercise 10 (Solution)

Design an NFA with  $\Sigma = \{0, 1\}$  accepts all strings in which the third symbol from the right end is always 0.

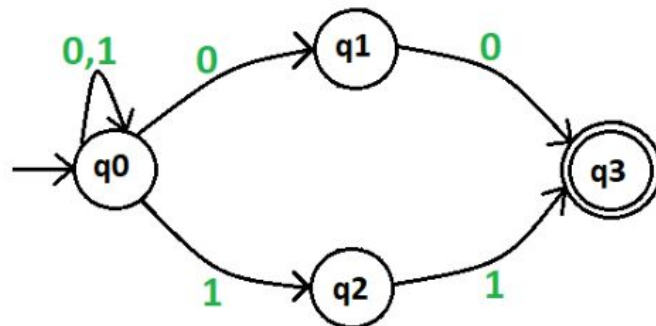


## Exercise 11

Draw a non-deterministic finite automaton that accepts 00 and 11 at the end of a string containing 0, 1 in it; e.g., it accepts 01010100 and 01010111, but not 000111010.

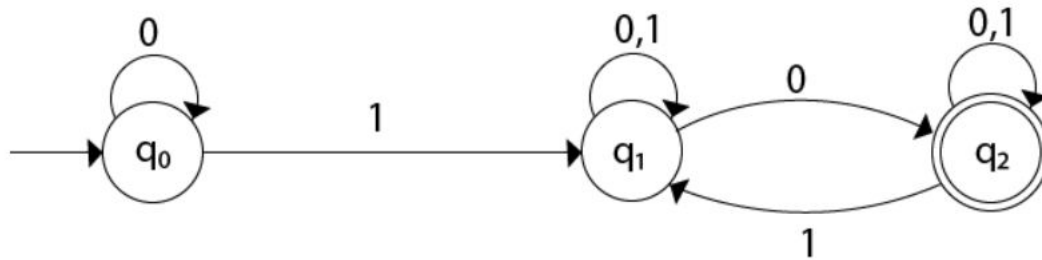
## Exercise 11 (Solution)

Draw a non-deterministic finite automaton that accepts 00 and 11 at the end of a string containing 0, 1 in it; e.g., it accepts 01010100 and 01010111, but not 000111010.



## Exercise 12

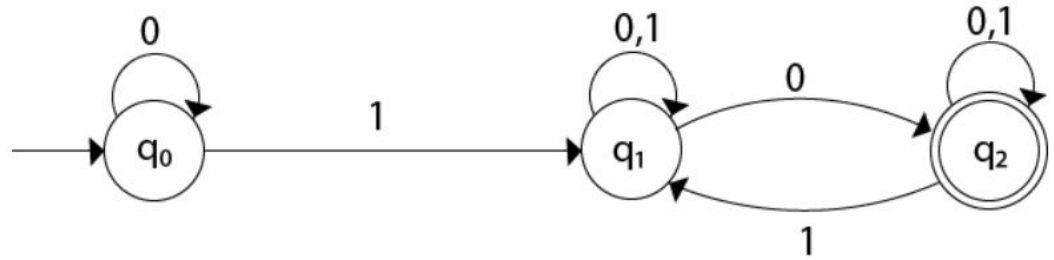
Convert the given NFA to DFA.





# Solution

State	0	1
$\rightarrow q_0$	$q_0$	$q_1$
$q_1$	$\{q_1, q_2\}$	$q_1$
$*q_2$	$q_2$	$\{q_1, q_2\}$



Step 1: For the given transition diagram, we will first construct the transition table.

# Solution

State	0	1
$\rightarrow q_0$	$q_0$	$q_1$
$q_1$	$\{q_1, q_2\}$	$q_1$
$*q_2$	$q_2$	$\{q_1, q_2\}$

Table: The transition table for the NFA

Step 2: Now we will obtain  $\delta'$  transition for all the states

The  $\delta'$  transition for state  $q_0$  is obtained as:

$$\delta'([q_0], 0) = [q_0]$$

$$\delta'([q_0], 1) = [q_1]$$

The  $\delta'$  transition for state  $q_1$  is obtained as:

$$\delta'([q_1], 0) = [q_1, q_2] \quad (\text{new state generated})$$

$$\delta'([q_1], 1) = [q_1]$$

# Solution

State	0	1
→q0	q0	q1
q1	{q1, q2}	q1
*q2	q2	{q1, q2}

Table: The transition table for the NFA

Step 2: Now we will obtain  $\delta'$  transition for all the states

The  $\delta'$  transition for state q0 is obtained as:

$$\delta'([q0], 0) = [q0]$$

$$\delta'([q0], 1) = [q1]$$

The  $\delta'$  transition for state q1 is obtained as:

$$\delta'([q1], 0) = [q1, q2] \quad (\text{new state generated})$$

$$\delta'([q1], 1) = [q1]$$

The  $\delta'$  transition for state q2 is obtained as:

$$\delta'([q2], 0) = [q2]$$

$$\delta'([q2], 1) = [q1, q2]$$

# Solution

State	0	1
$\rightarrow q_0$	$q_0$	$q_1$
$q_1$	$\{q_1, q_2\}$	$q_1$
$*q_2$	$q_2$	$\{q_1, q_2\}$

Table: The transition table for the NFA

Step 2: Now we will obtain  $\delta'$  transition for all the states

Now we will obtain  $\delta'$  transition on  $[q_1, q_2]$ .

$$\delta'([q_1, q_2], 0) = \delta(q_1, 0) \cup \delta(q_2, 0)$$

$$= \{q_1, q_2\} \cup \{q_2\}$$

$$= [q_1, q_2]$$

$$\delta'([q_1, q_2], 1) = \delta(q_1, 1) \cup \delta(q_2, 1)$$

$$= \{q_1\} \cup \{q_1, q_2\}$$

$$= \{q_1, q_2\}$$

$$= [q_1, q_2]$$

# Solution

State	0	1
$\rightarrow q_0$	$q_0$	$q_1$
$q_1$	$\{q_1, q_2\}$	$q_1$
$*q_2$	$q_2$	$\{q_1, q_2\}$

Table: The transition table for the NFA

Step 3: Construct the transition table for the DFA

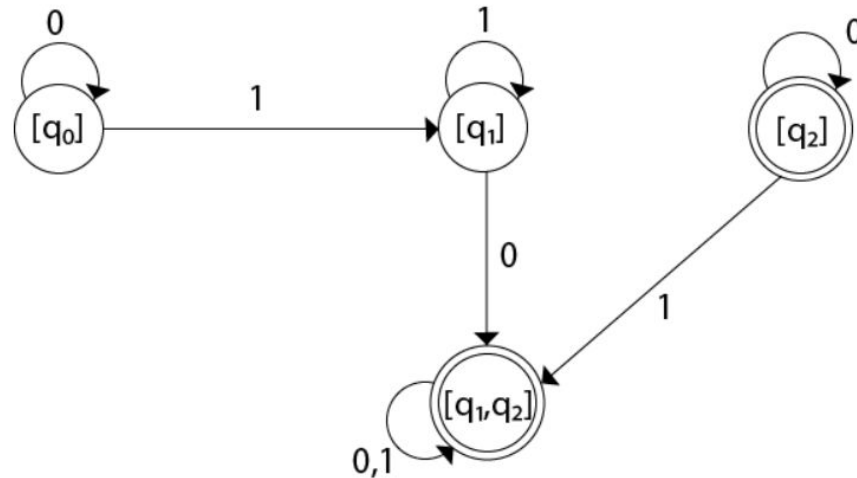
State	0	1
$\rightarrow [q_0]$	$[q_0]$	$[q_1]$
$[q_1]$	$[q_1, q_2]$	$[q_1]$
$*[q_2]$	$[q_2]$	$[q_1, q_2]$
$*[q_1, q_2]$	$[q_1, q_2]$	$[q_1, q_2]$

# Solution

State	0	1
→[q0]	[q0]	[q1]
[q1]	[q1, q2]	[q1]
*[q2]	[q2]	[q1, q2]
*[q1, q2]	[q1, q2]	[q1, q2]

Table: The transition table for the DFA

Step 4: Draw the transition diagram



The state  $q_2$  can be eliminated because  $q_2$  is an unreachable state.

# Solution

State	0	1
$\rightarrow [q_0]$	$[q_0]$	$[q_1]$
$[q_1]$	$[q_1, q_2]$	$[q_1]$
$*[q_2]$	$[q_2]$	$[q_1, q_2]$
$*[q_1, q_2]$	$[q_1, q_2]$	$[q_1, q_2]$

Table: The transition table for the DFA

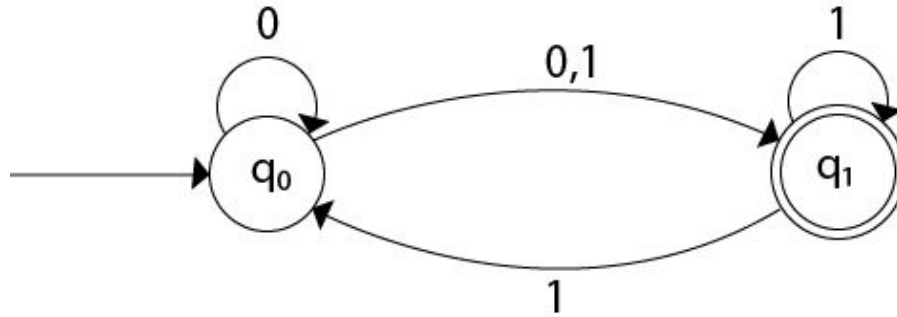
	0	1
$\emptyset$	$\emptyset$	$\emptyset$
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_1\}$	$\emptyset$	$\{q_2\}$
$*\{q_2\}$	$\emptyset$	$\emptyset$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$*\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$
$*\{q_1, q_2\}$	$\emptyset$	$\{q_2\}$
$*\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

Figure 2.12: The complete subset construction from Fig. 2.9

The state  $q_2$  can be eliminated because  $q_2$  is an unreachable state. You can also construct the whole table according to the complete subset construction approach. However, it will end up creating unreachable states that can be discarded.

## Exercise 13

Convert the given NFA to DFA.

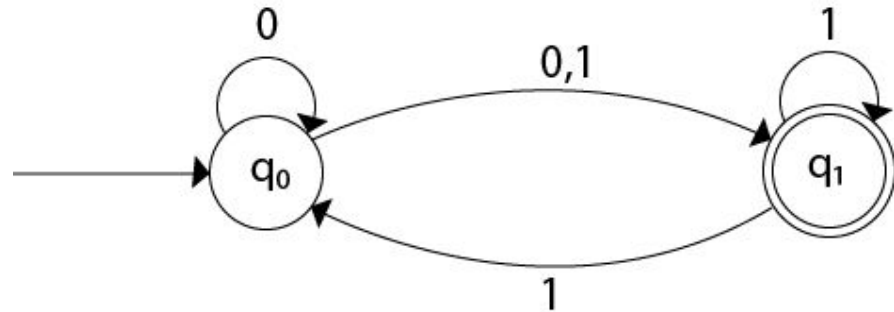




# Solution

Step 1: For the given transition diagram, we will first construct the transition table.

State	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	$\phi$	$\{q_0, q_1\}$



# Solution

Step 1: For the given transition diagram, we will first construct the transition table.

State	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	$\phi$	$\{q_0, q_1\}$

Step 2: Obtain transition states

Now we will obtain  $\delta'$  transition for state  $q_0$ .

$$\begin{aligned}\delta'([q_0], 0) &= \{q_0, q_1\} \\ &= [q_0, q_1] \quad (\text{new state generated})\end{aligned}$$

$$\delta'([q_0], 1) = \{q_1\} = [q_1]$$

The  $\delta'$  transition for state  $q_1$  is obtained as:

$$\delta'([q_1], 0) = \phi$$

$$\delta'([q_1], 1) = [q_0, q_1]$$

# Solution

Step 1: For the given transition diagram, we will first construct the transition table.

State	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	$\phi$	$\{q_0, q_1\}$

Step 2: Obtain transition states

Now we will obtain  $\delta'$  transition on  $[q_0, q_1]$ .

$$\delta'([q_0, q_1], 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$$

$$= \{q_0, q_1\} \cup \phi$$

$$= \{q_0, q_1\}$$

$$= [q_0, q_1]$$

Similarly,

$$\delta'([q_0, q_1], 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$$

$$= \{q_1\} \cup \{q_0, q_1\}$$

$$= \{q_0, q_1\}$$

$$= [q_0, q_1]$$

# Solution

Step 1: For the given transition diagram, we will first construct the transition table.

State	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	$\phi$	$\{q_0, q_1\}$

Step 3: Construct the transition table from the transition states

As in the given NFA,  $q_1$  is a final state, then in DFA wherever,  $q_1$  exists that state becomes a final state. Hence in the DFA, final states are  $[q_1]$  and  $[q_0, q_1]$ . Therefore set of final states  $F = \{[q_1], [q_0, q_1]\}$ .

The transition table for the constructed DFA will be:

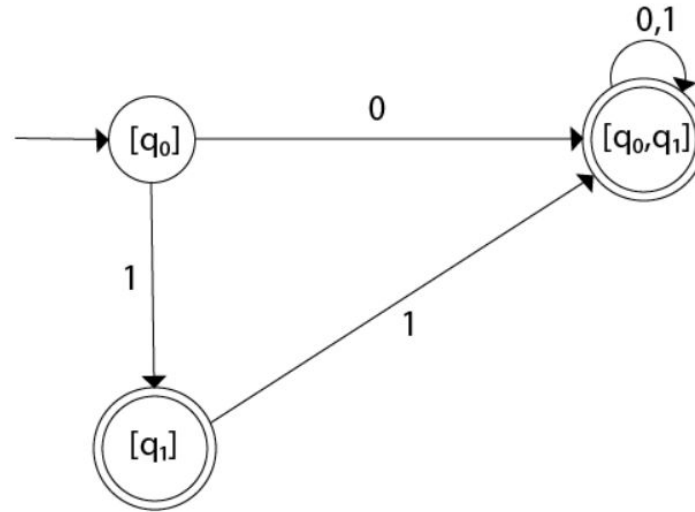
State	0	1
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_1]$
$*[q_1]$	$\phi$	$[q_0, q_1]$
$*[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_1]$

# Solution

The transition table for the constructed DFA will be:

State	0	1
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_1]$
$*[q_1]$	$\phi$	$[q_0, q_1]$
$*[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_1]$

Step 4: Construct the transition diagram from the transition table



# Solution

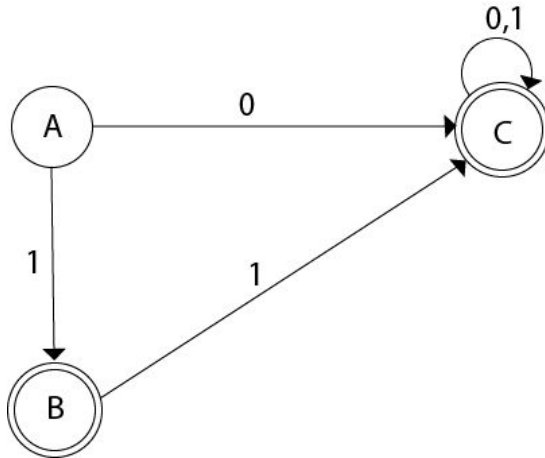
Even we can change the name of the states of DFA.

$A = [q_0]$

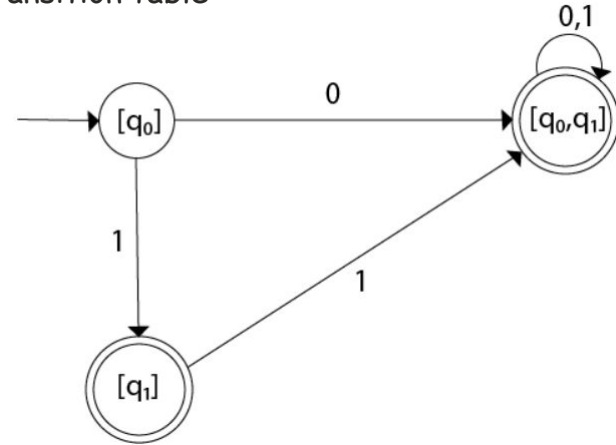
$B = [q_1]$

$C = [q_0, q_1]$

With these new names, the DFA will be as follows:



Step 4: Construct the transition diagram from the transition table

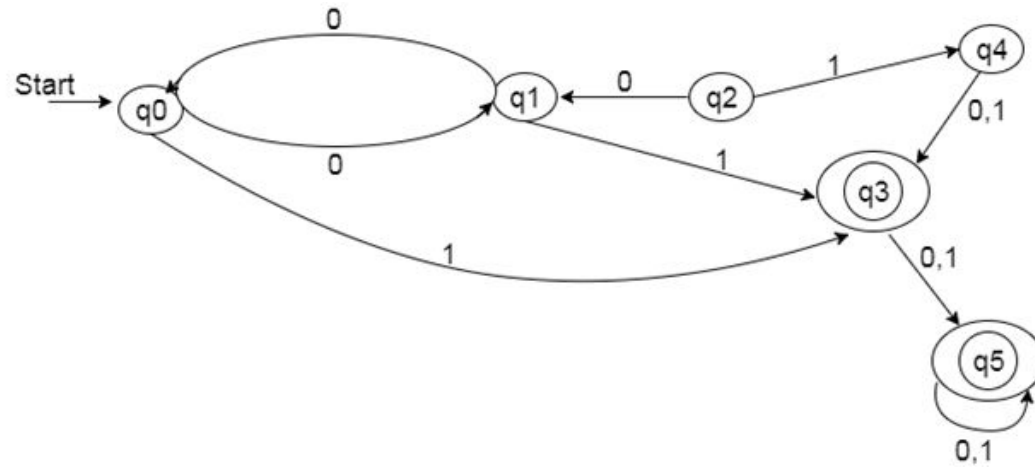


# Minimization of DFA

- Minimization of DFA means reducing the number of states from a given FA. Thus, we get the FSM (finite state machine) with redundant states after minimizing the FSM.
- We have to follow the various steps to minimize the DFA. These are as follows:
- Step 1: Remove all the states that are unreachable from the initial state via any set of the transition of DFA.
- Step 2: Draw the transition table for all pairs of states.

# Exercise 14

Minimize the following DFA.





# Solution

Minimize the following DFA.

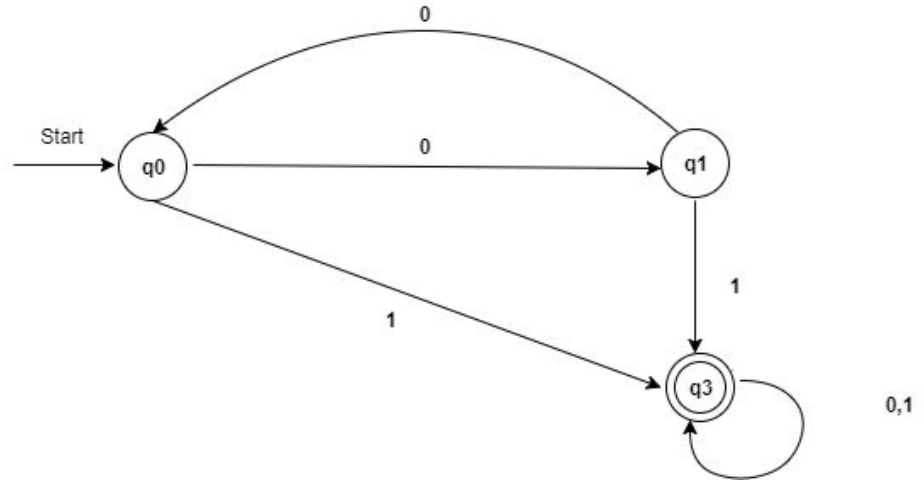
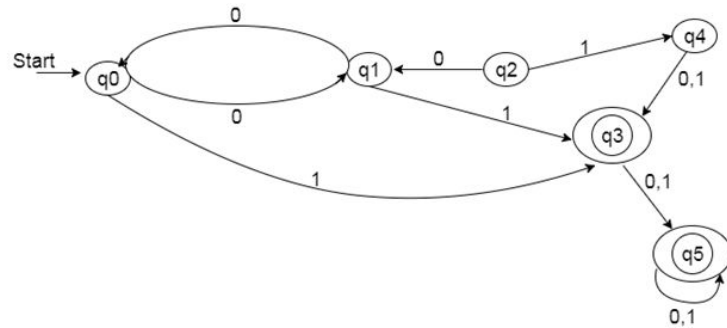
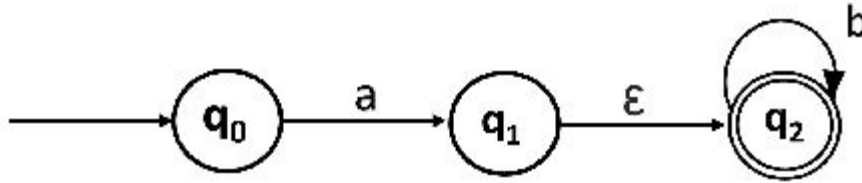


Fig. Minimized DFA

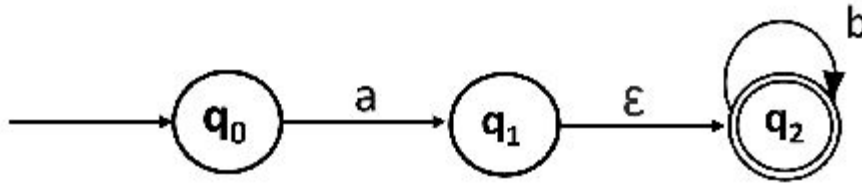
## Exercise 16: Eliminating $\epsilon$ Transitions

Convert the following NFA with  $\epsilon$  to NFA without  $\epsilon$ .



# Solution

Convert the following NFA with  $\epsilon$  to NFA without  $\epsilon$ .



We will first obtain  $\epsilon$ -closures of  $q_0$ ,  $q_1$  and  $q_2$  as follows:

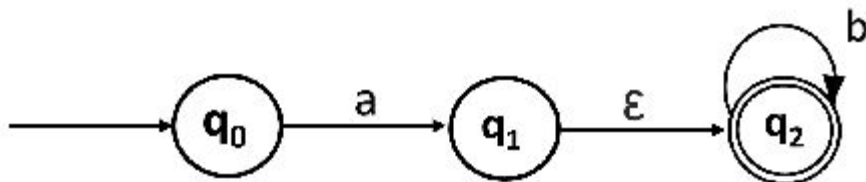
$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

# Solution

Convert the following NFA with  $\epsilon$  to NFA without  $\epsilon$ .



We will first obtain  $\epsilon$ -closures of  $q_0$ ,  $q_1$  and  $q_2$  as follows:

$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

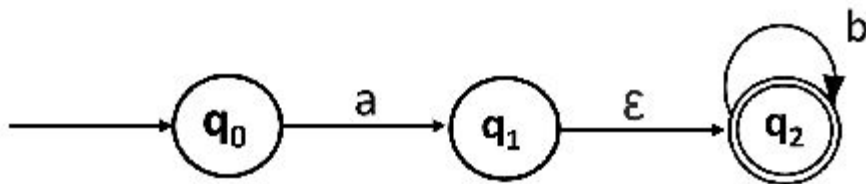
Now the  $\delta'$  transition on each input symbol is obtained as:

$$\begin{aligned}\delta'(q_0, a) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_0, \epsilon), a)) \\ &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), a)) \\ &= \epsilon\text{-closure}(\delta(q_0, a)) \\ &= \epsilon\text{-closure}(q_1) \\ &= \{q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\delta'(q_0, b) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_0, \epsilon), b)) \\ &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), b)) \\ &= \epsilon\text{-closure}(\delta(q_0, b)) \\ &= \emptyset\end{aligned}$$

# Solution

Convert the following NFA with  $\epsilon$  to NFA without  $\epsilon$ .



We will first obtain  $\epsilon$ -closures of  $q_0$ ,  $q_1$  and  $q_2$  as follows:

$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

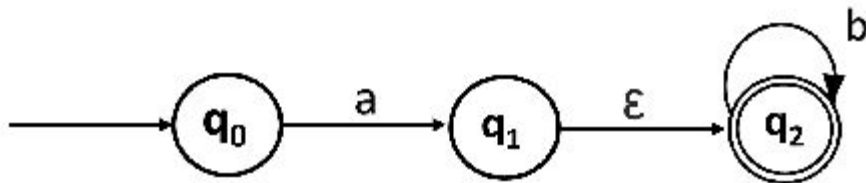
Now the  $\delta'$  transition on  $q_1$  is obtained as:

$$\begin{aligned}\delta'(q_1, a) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_1, \epsilon), a)) \\ &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), a)) \\ &= \epsilon\text{-closure}(\delta(q_1, q_2), a) \\ &= \epsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a)) \\ &= \epsilon\text{-closure}(\Phi \cup \Phi) \\ &= \Phi\end{aligned}$$

$$\begin{aligned}\delta'(q_1, b) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_1, \epsilon), b)) \\ &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), b)) \\ &= \epsilon\text{-closure}(\delta(q_1, q_2), b) \\ &= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b)) \\ &= \epsilon\text{-closure}(\Phi \cup q_2) \\ &= \{q_2\}\end{aligned}$$

# Solution

Convert the following NFA with  $\epsilon$  to NFA without  $\epsilon$ .



We will first obtain  $\epsilon$ -closures of  $q_0$ ,  $q_1$  and  $q_2$  as follows:

$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

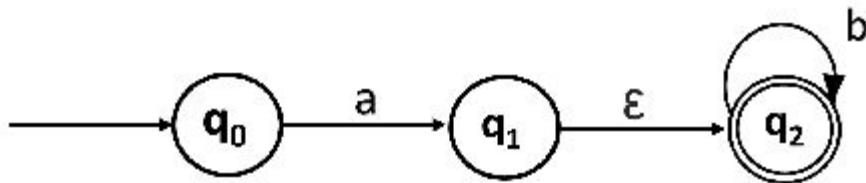
The  $\delta'$  transition on  $q_2$  is obtained as:

$$\begin{aligned}\delta'(q_2, a) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_2, \epsilon), a)) \\ &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), a)) \\ &= \epsilon\text{-closure}(\delta(q_2, a)) \\ &= \epsilon\text{-closure}(\Phi) \\ &= \Phi\end{aligned}$$

$$\begin{aligned}\delta'(q_2, b) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_2, \epsilon), b)) \\ &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), b)) \\ &= \epsilon\text{-closure}(\delta(q_2, b)) \\ &= \epsilon\text{-closure}(q_2) \\ &= \{q_2\}\end{aligned}$$

# Solution

Convert the following NFA with  $\epsilon$  to NFA without  $\epsilon$ .



We will first obtain  $\epsilon$ -closures of  $q_0$ ,  $q_1$  and  $q_2$  as follows:

$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

Now we will summarize all the computed  $\delta'$  transitions:

$$\delta'(q_0, a) = \{q_1, q_2\}$$

$$\delta'(q_0, b) = \Phi$$

$$\delta'(q_1, a) = \Phi$$

$$\delta'(q_1, b) = \{q_2\}$$

$$\delta'(q_2, a) = \Phi$$

$$\delta'(q_2, b) = \{q_2\}$$

# Solution

$\epsilon$ -closures of  $q_0$ ,  $q_1$  and  $q_2$ :

$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

All the computed  $\delta'$  transitions:

$$\delta'(q_0, a) = \{q_1, q_2\}$$

$$\delta'(q_0, b) = \Phi$$

$$\delta'(q_1, a) = \Phi$$

$$\delta'(q_1, b) = \{q_2\}$$

$$\delta'(q_2, a) = \Phi$$

$$\delta'(q_2, b) = \{q_2\}$$

States	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	$\Phi$
$*q_1$	$\Phi$	$\{q_2\}$
$*q_2$	$\Phi$	$\{q_2\}$

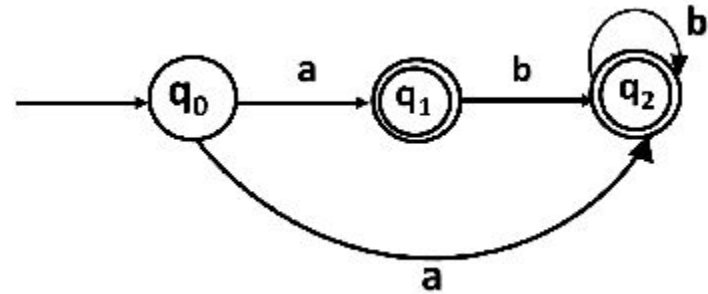
State  $q_1$  and  $q_2$  become the final state as  $\epsilon$ -closure of  $q_1$  and  $q_2$  contain the final state  $q_2$ .



# Solution

States	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	$\emptyset$
$*q_1$	$\emptyset$	$\{q_2\}$
$*q_2$	$\emptyset$	$\{q_2\}$

The NFA can be shown by the following transition diagram:



## Conversion from NFA with $\epsilon$ to DFA

**Step 1:** We will take the  $\epsilon$ -closure for the starting state of NFA as a starting state of DFA.

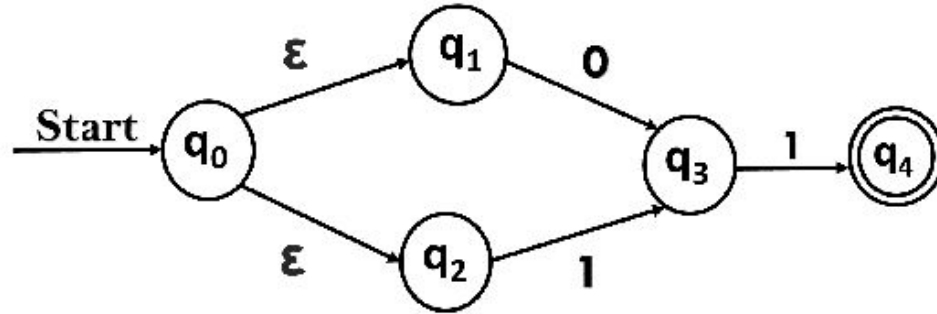
**Step 2:** Find the states for each input symbol that can be traversed from the present. That means the union of transition value and their closures for each state of NFA present in the current state of DFA.

**Step 3:** If we found a new state, take it as current state and repeat step 2.

**Step 4:** Repeat Step 2 and Step 3 until there is no new state present in the transition table of DFA.

**Step 5:** Mark the states of DFA as a final state which contains the final state of NFA.

## Conversion from NFA with $\epsilon$ to DFA



$\epsilon$ -closure  $\{q_0\} = \{q_0, q_1, q_2\}$

$\epsilon$ -closure  $\{q_1\} = \{q_1\}$

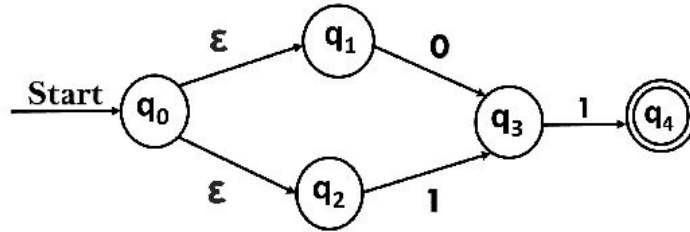
$\epsilon$ -closure  $\{q_2\} = \{q_2\}$

$\epsilon$ -closure  $\{q_3\} = \{q_3\}$

$\epsilon$ -closure  $\{q_4\} = \{q_4\}$

Now, let  $\epsilon$ -closure  $\{q_0\} = \{q_0, q_1, q_2\}$  be state A.

# Conversion from NFA with $\epsilon$ to DFA



$\epsilon$ -closure  $\{q_0\} = \{q_0, q_1, q_2\}$

$\epsilon$ -closure  $\{q_1\} = \{q_1\}$

$\epsilon$ -closure  $\{q_2\} = \{q_2\}$

$\epsilon$ -closure  $\{q_3\} = \{q_3\}$

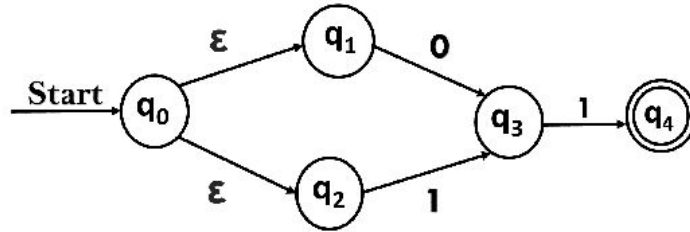
$\epsilon$ -closure  $\{q_4\} = \{q_4\}$

$$\begin{aligned}\delta'(A, 0) &= \epsilon\text{-closure } \{\delta((q_0, q_1, q_2), 0)\} \\ &= \epsilon\text{-closure } \{\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)\} \\ &= \epsilon\text{-closure } \{q_3\} \\ &= \{q_3\} \quad \text{call it as state B.}\end{aligned}$$

$$\begin{aligned}\delta'(A, 1) &= \epsilon\text{-closure } \{\delta((q_0, q_1, q_2), 1)\} \\ &= \epsilon\text{-closure } \{\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)\} \\ &= \epsilon\text{-closure } \{q_3\} \\ &= \{q_3\} = B.\end{aligned}$$

Now, let  $\epsilon$ -closure  $\{q_0\} = \{q_0, q_1, q_2\}$  be state A.

# Conversion from NFA with $\epsilon$ to DFA



$\epsilon$ -closure  $\{q_0\} = \{q_0, q_1, q_2\}$

$\epsilon$ -closure  $\{q_1\} = \{q_1\}$

$\epsilon$ -closure  $\{q_2\} = \{q_2\}$

$\epsilon$ -closure  $\{q_3\} = \{q_3\}$

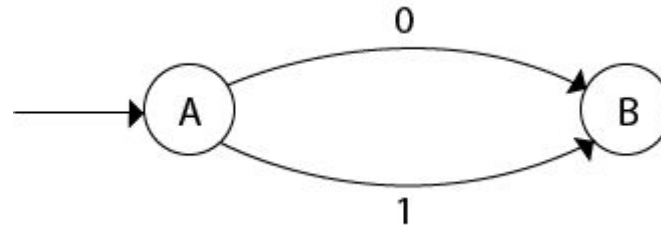
$\epsilon$ -closure  $\{q_4\} = \{q_4\}$

$\delta'(A, 0) = \epsilon\text{-closure } \{\delta((q_0, q_1, q_2), 0)\}$   
 $= \epsilon\text{-closure } \{\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)\}$   
 $= \epsilon\text{-closure } \{q_3\}$   
 $= \{q_3\}$  **call it as state B.**

$\delta'(A, 1) = \epsilon\text{-closure } \{\delta((q_0, q_1, q_2), 1)\}$   
 $= \epsilon\text{-closure } \{\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)\}$   
 $= \epsilon\text{-closure } \{q_3\}$   
 $= \{q_3\} = B.$

Now, let  $\epsilon$ -closure  $\{q_0\} = \{q_0, q_1, q_2\}$  be state A.

The partial DFA will be



# Conversion from NFA with $\epsilon$ to DFA

$$\epsilon\text{-closure } \{q_0\} = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure } \{q_1\} = \{q_1\}$$

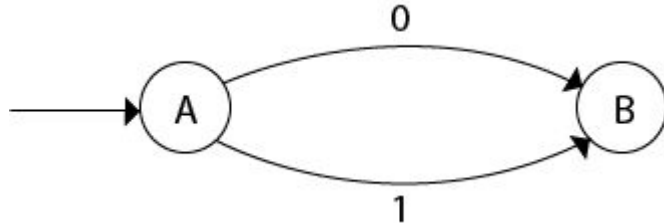
$$\epsilon\text{-closure } \{q_2\} = \{q_2\}$$

$$\epsilon\text{-closure } \{q_3\} = \{q_3\}$$

$$\epsilon\text{-closure } \{q_4\} = \{q_4\}$$

Now, let  $\epsilon\text{-closure } \{q_0\} = \{q_0, q_1, q_2\}$  be state A.

The partial DFA will be



$$\delta'(B, 0) = \epsilon\text{-closure } \{\delta(q_3, 0)\}$$

$$= \phi$$

$$\delta'(B, 1) = \epsilon\text{-closure } \{\delta(q_3, 1)\}$$

$$= \epsilon\text{-closure } \{q_4\}$$

$$= \{q_4\} \quad \text{i.e. state C}$$

$$\delta'(C, 0) = \epsilon\text{-closure } \{\delta(q_4, 0)\}$$

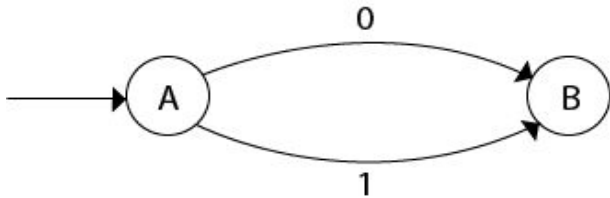
$$= \phi$$

$$\delta'(C, 1) = \epsilon\text{-closure } \{\delta(q_4, 1)\}$$

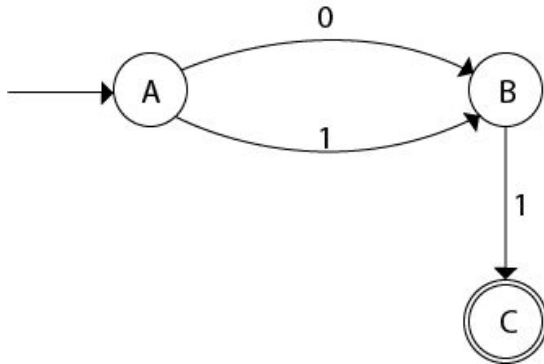
$$= \phi$$

# Conversion from NFA with $\epsilon$ to DFA

The partial DFA will be



The final DFA will be



$$\delta'(B, 0) = \epsilon\text{-closure} \{ \delta(q3, 0) \}$$

$$= \phi$$

$$\delta'(B, 1) = \epsilon\text{-closure} \{ \delta(q3, 1) \}$$

$$= \epsilon\text{-closure} \{ q4 \}$$

$$= \{ q4 \} \quad \text{i.e. state C}$$

$$\delta'(C, 0) = \epsilon\text{-closure} \{ \delta(q4, 0) \}$$

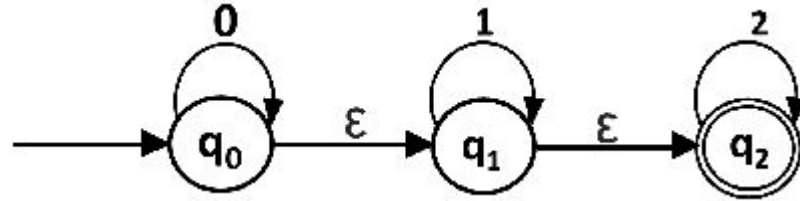
$$= \phi$$

$$\delta'(C, 1) = \epsilon\text{-closure} \{ \delta(q4, 1) \}$$

$$= \phi$$

# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.



$\epsilon$ -closure( $q_0$ ) = { $q_0$ ,  $q_1$ ,  $q_2$ }

$\epsilon$ -closure( $q_1$ ) = { $q_1$ ,  $q_2$ }

$\epsilon$ -closure( $q_2$ ) = { $q_2$ }

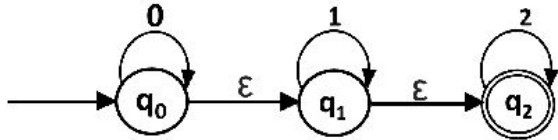
Now we will obtain  $\delta'$  transition.

Let  $\epsilon$ -closure( $q_0$ ) = { $q_0$ ,  $q_1$ ,  $q_2$ } call it as state A.



# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.



$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

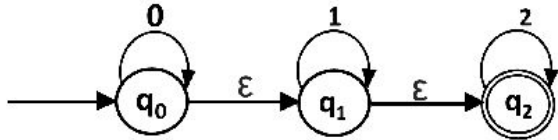
$$\begin{aligned}\delta'(A, 0) &= \epsilon\text{-closure}\{\delta((q_0, q_1, q_2), 0)\} \\ &= \epsilon\text{-closure}\{\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)\} \\ &= \epsilon\text{-closure}\{q_0\} \\ &= \{q_0, q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\delta'(A, 1) &= \epsilon\text{-closure}\{\delta((q_0, q_1, q_2), 1)\} \\ &= \epsilon\text{-closure}\{\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)\} \\ &= \epsilon\text{-closure}\{q_1\} \\ &= \{q_1, q_2\} \quad \text{call it as state B}\end{aligned}$$

$$\begin{aligned}\delta'(A, 2) &= \epsilon\text{-closure}\{\delta((q_0, q_1, q_2), 2)\} \\ &= \epsilon\text{-closure}\{\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2)\} \\ &= \epsilon\text{-closure}\{q_2\} \\ &= \{q_2\} \quad \text{call it state C}\end{aligned}$$

# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.



$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

Thus we have obtained

$$\delta'(A, 0) = A$$

$$\delta'(A, 1) = B$$

$$\delta'(A, 2) = C$$

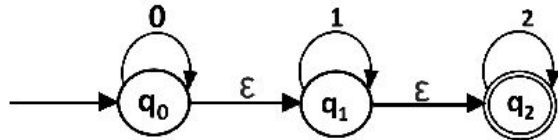
$$\begin{aligned}\delta'(A, 0) &= \epsilon\text{-closure}\{\delta((q_0, q_1, q_2), 0)\} \\ &= \epsilon\text{-closure}\{\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)\} \\ &= \epsilon\text{-closure}\{q_0\} \\ &= \{q_0, q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\delta'(A, 1) &= \epsilon\text{-closure}\{\delta((q_0, q_1, q_2), 1)\} \\ &= \epsilon\text{-closure}\{\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)\} \\ &= \epsilon\text{-closure}\{q_1\} \\ &= \{q_1, q_2\} \quad \text{call it as state B}\end{aligned}$$

$$\begin{aligned}\delta'(A, 2) &= \epsilon\text{-closure}\{\delta((q_0, q_1, q_2), 2)\} \\ &= \epsilon\text{-closure}\{\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2)\} \\ &= \epsilon\text{-closure}\{q_2\} \\ &= \{q_2\} \quad \text{call it state C}\end{aligned}$$

# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.



$\epsilon$ -closure( $q_0$ ) = { $q_0, q_1, q_2$ }

$\epsilon$ -closure( $q_1$ ) = { $q_1, q_2$ }

$\epsilon$ -closure( $q_2$ ) = { $q_2$ }

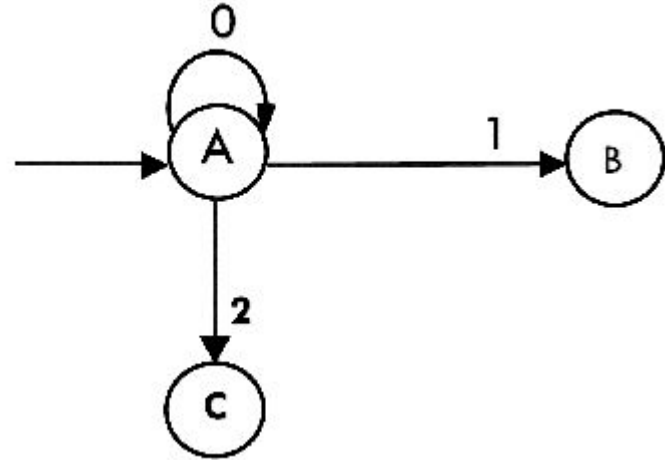
Thus we have obtained

$\delta'(A, 0) = A$

$\delta'(A, 1) = B$

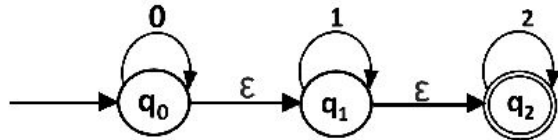
$\delta'(A, 2) = C$

The partial DFA will be:

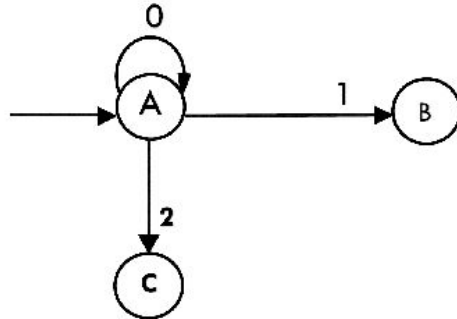


# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.



The partial DFA will be:



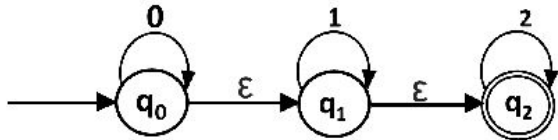
$$\begin{aligned}\delta'(B, 0) &= \epsilon\text{-closure}\{\delta((q1, q2), 0)\} \\ &= \epsilon\text{-closure}\{\delta(q1, 0) \cup \delta(q2, 0)\} \\ &= \epsilon\text{-closure}\{\emptyset\} \\ &= \emptyset\end{aligned}$$

$$\begin{aligned}\delta'(B, 1) &= \epsilon\text{-closure}\{\delta((q1, q2), 1)\} \\ &= \epsilon\text{-closure}\{\delta(q1, 1) \cup \delta(q2, 1)\} \\ &= \epsilon\text{-closure}\{q1\} \\ &= \{q1, q2\} \quad \text{i.e. state B itself}\end{aligned}$$

$$\begin{aligned}\delta'(B, 2) &= \epsilon\text{-closure}\{\delta((q1, q2), 2)\} \\ &= \epsilon\text{-closure}\{\delta(q1, 2) \cup \delta(q2, 2)\} \\ &= \epsilon\text{-closure}\{q2\} \\ &= \{q2\} \quad \text{i.e. state C itself}\end{aligned}$$

# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.



Thus we have obtained

$$\delta'(B, 0) = \phi$$

$$\delta'(B, 1) = B$$

$$\delta'(B, 2) = C$$

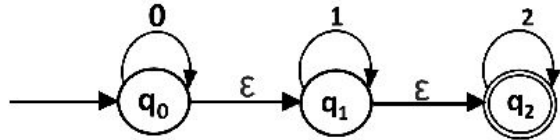
$$\begin{aligned}\delta'(B, 0) &= \epsilon\text{-closure}\{\delta((q1, q2), 0)\} \\ &= \epsilon\text{-closure}\{\delta(q1, 0) \cup \delta(q2, 0)\} \\ &= \epsilon\text{-closure}\{\phi\} \\ &= \phi\end{aligned}$$

$$\begin{aligned}\delta'(B, 1) &= \epsilon\text{-closure}\{\delta((q1, q2), 1)\} \\ &= \epsilon\text{-closure}\{\delta(q1, 1) \cup \delta(q2, 1)\} \\ &= \epsilon\text{-closure}\{q1\} \\ &= \{q1, q2\} \quad \text{i.e. state B itself}\end{aligned}$$

$$\begin{aligned}\delta'(B, 2) &= \epsilon\text{-closure}\{\delta((q1, q2), 2)\} \\ &= \epsilon\text{-closure}\{\delta(q1, 2) \cup \delta(q2, 2)\} \\ &= \epsilon\text{-closure}\{q2\} \\ &= \{q2\} \quad \text{i.e. state C itself}\end{aligned}$$

# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.



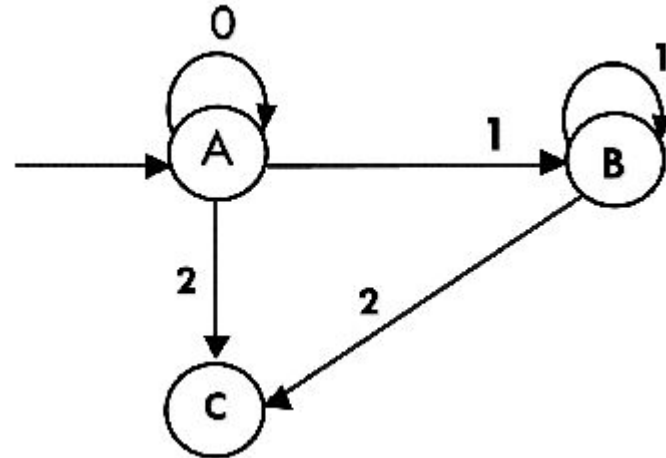
Thus we have obtained

$$\delta'(B, 0) = \phi$$

$$\delta'(B, 1) = B$$

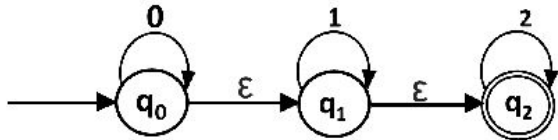
$$\delta'(B, 2) = C$$

The partial transition diagram will be



# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.



$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

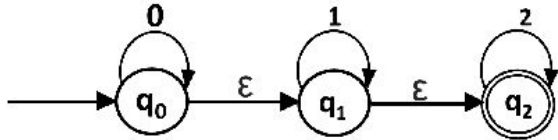
$$\begin{aligned}\delta'(C, 0) &= \epsilon\text{-closure}\{\delta(q_2, 0)\} \\ &= \epsilon\text{-closure}\{\phi\} \\ &= \phi\end{aligned}$$

$$\begin{aligned}\delta'(C, 1) &= \epsilon\text{-closure}\{\delta(q_2, 1)\} \\ &= \epsilon\text{-closure}\{\phi\} \\ &= \phi\end{aligned}$$

$$\begin{aligned}\delta'(C, 2) &= \epsilon\text{-closure}\{\delta(q_2, 2)\} \\ &= \{q_2\}\end{aligned}$$

# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.

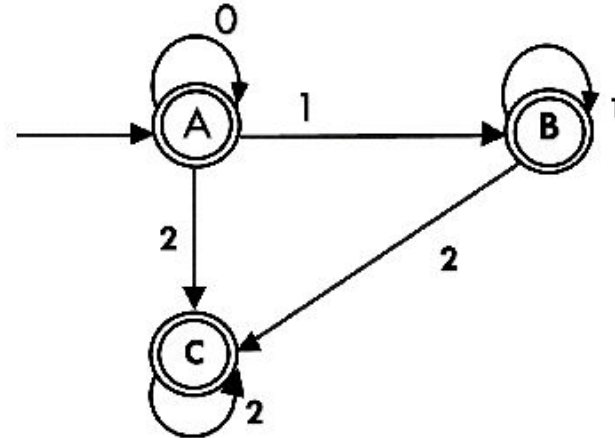


$$\begin{aligned}\delta'(C, 0) &= \epsilon\text{-closure}\{\delta(q_2, 0)\} \\ &= \epsilon\text{-closure}\{\phi\} \\ &= \phi\end{aligned}$$

$$\begin{aligned}\delta'(C, 1) &= \epsilon\text{-closure}\{\delta(q_2, 1)\} \\ &= \epsilon\text{-closure}\{\phi\} \\ &= \phi\end{aligned}$$

$$\begin{aligned}\delta'(C, 2) &= \epsilon\text{-closure}\{\delta(q_2, 2)\} \\ &= \{q_2\}\end{aligned}$$

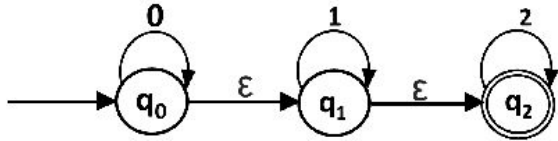
Hence the final DFA is:





# Conversion from NFA with $\epsilon$ to DFA

Convert the given NFA into its equivalent DFA.

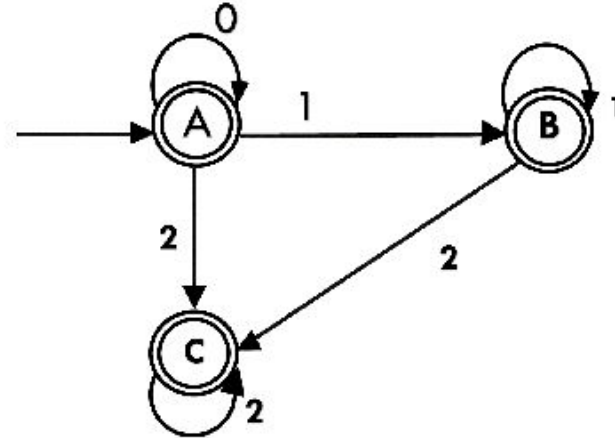


As  $A = \{q_0, q_1, q_2\}$  in which final state  $q_2$  lies, hence A is a final state.

$B = \{q_1, q_2\}$  in which the state  $q_2$  lies, hence B is also a final state.

$C = \{q_2\}$  in which the state  $q_2$  lies, hence C is also a final state.

Hence the final DFA is:



# Resource List

1. [DFA | Deterministic Finite Automata - Javatpoint](#)
2. [Examples of DFA](#)
3. [NFA | Non-Deterministic Finite Automata - Javatpoint](#)
4. [Examples of NFA - Javatpoint](#)
5. [Automata Conversion from NFA to DFA - Javatpoint](#)
6. [Minimization of DFA - Javatpoint](#)
7. Neso Academy Playlist:  
<https://youtu.be/40i4PKpM0cI?si=1ddTHOJYCJGMpKLG>