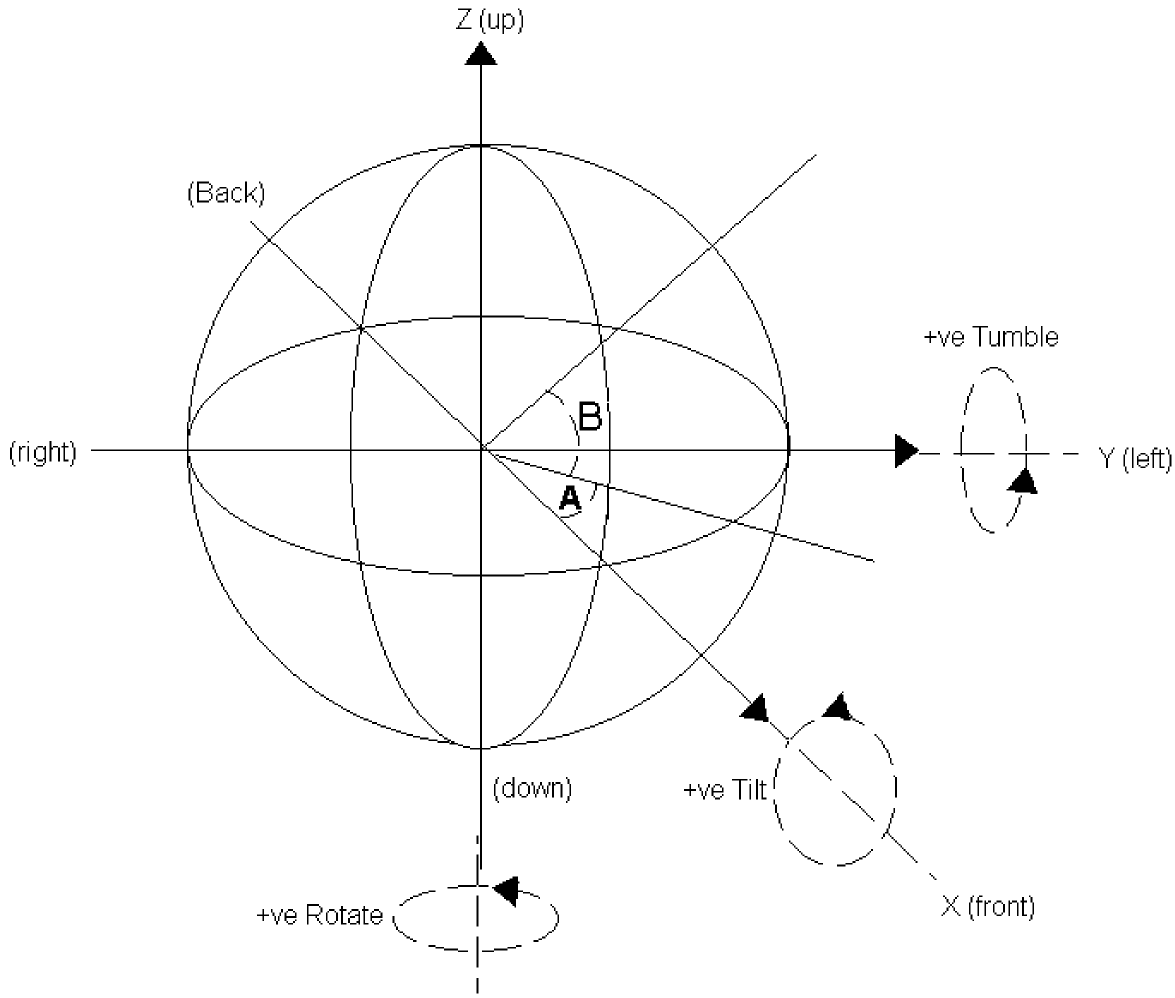
Maths equations for Ambisonic Processing

$$P_n = \frac{1}{N}(\sqrt{2}W + kX\cos\theta_n\cos\phi_n + kY\sin\theta_n\cos\phi_n + kZ\sin\phi_n)$$

This is the basic equation for defining the location of a sound. Phi is the angle above or below the horizontal plane and Theta is the angle location within the X-Y plane. N is the number of speakers in the array and the subscript n refers to a specific speaker.

The following is a set of formulas from David Malham's article linked above as well as an image of the coordinate plane in which we work, and the relevant shifts in it due to tilting etc.



ENCODING EQUATIONS
The position of a sound within a three dimensional soundfield is encoded in the four signals which make up the B format thus;
X = cosA.cosB (front-back)
Y = sinA.cosB (left-right)
Z = sinB (up-down)
W = 0.707 (pressure signal)
where A is the anti-clockwise angle from centre front and B is the elevation.. If you limit the positions of sounds to

within the unit sphere by ensuring that
$(x + y + z)$
is always less than or equal to one then the equations can be more simply written as;
$X = x$
$Y = y$
$Z = z$
$W = 0.707$
where x,y,z are the coordinates of the sound source. The value of W is given as 0.707 rather than 1 since this allows for a more even distribution of levels within the four channels. There is a catch in this simplicity, however, since if you attempt to move off the surface of the notional unit sphere and in towards the centre, the dropping levels in the X,Y,Z channels will reduce the overall sound level, rather than there being the expected increase as the apparent position of the sound source moves nearer the centre.
One fix that will keep the overall level reasonably constant is to make W vary thus;
$W = 1 - 0.293(x + y + z)$
Malham further notes that there are ways to bring the perceived sound off of the surface of a sphere envisioned around the listener as well although these methods lack mathematical precision as the perception of distance appears to be heavily dependent on features other than just amplitude.
In addition, Malham determines the construction of matrices containing localization data sound which can be manipulated to appear to rotate, tilt and tumble.

ROTATING A POINT ABOUT THE Z-AXIS
If A is the positive angle of rotation and C is the angle between the X-axis and the untransformed position, (x,y), we have;
$x = r*\cos C$, $y = r*\sin C$
$x' = r*\cos (A+C)$, $y'= r*\sin (A+C)$
simplifying;
$x' = r*\cos C * \cos A - r*\sin C * \sin A$
$y' = r*\cos C * \sin A + r*\sin C * \cos A$
and substituting for x and y
$x' = x * \cos A - y * \sin A$ $y' = x * \sin A + y * \cos A$
w and z remain unchanged since the rotation is about the Z-axis, for points on the surface of the unit sphere w = 0.707.
If the same procedure is applied to to the tilt and rotate equations this gives the following;

TILT.
$x' = x$
$w' = w$
$y' = y * \cos B - z * \sin B$
$z' = y * \sin B + z * \cos B$

TUMBLE.
$x' = x * \cos B - z * \sin B$
$w' = w$
$y' = y$
$z' = x * \sin B + z * \cos B$
These equations can now be combined to perform transformations such as rotate-tilt which give an angular rotation of the whole input soundfield to the left by an angle of A from the centre front. Then it tilts the B-format soundfield by an angle B from the horizontal.

ROTATE-TILT.
$x' = x * \cos A - y * \sin A$
$w' = w$
$y' = x * \sin A * \cos B + y * \cos A * \cos B - z * \sin B$
$z' = x * \sin A * \sin B + y * \cos A * \sin B + z * \cos B$
Any combination of the many possible soundfield manipulations can be realised by using one matrix of scaling coefficients thus;
$X' = K1.X + K2.W + K3.Y + K4.Z$
$W' = K5.X + K6.W + K7.Y + K8.Z$
$Y' = K9.X + K10.W + K11.Y + K12.Z$
$Z' = K13.X + K14.W + K15.Y + K16.Z$
where K1 - K16 are the scaling coefficients formed by the soundfield manipulations applied to the incoming signals X, W, Y, and Z. X', W', Y' and Z' are the resultant B-format output signals.