Université de technologie de Compiègne

Compiègne, France

Internship Project

Reliability and MTTF Comparison of Redundant System Architectures Under Weibull Aging

Dr. Mohamed Sallak

Galvan Camacho Sergio Marvin

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1 Abstract

Reliability analysis in engineering systems is often based on the assumption of constant failure rates, leading to lifetime models used to plan maintenance and component replacement. However, real components are subject to aging and wear phenomena, which result in time dependent failure rates and can be more accurately modeled using Weibull distributions.

This project investigates the impact of component aging on the reliability and mean time to failure (MTTF) of different redundant system architectures, including N parallel systems, 2 of 3 voting systems, and hybrid parallel series and series parallel configurations. For each architecture, the system reliability function and the corresponding MTTF are evaluated using numerical integration and computational simulation.

2 Introduction

Reliability is a key aspect in the design and operation of engineering systems, particularly in industrial, transportation, and safety critical applications. The ability of a system to perform its intended function over a specified period of time directly affects operational continuity, maintenance costs, and safety. As a result, reliability analysis plays an essential role in decision making during system design and life cycle management.

In classical reliability theory, component failures are often modeled using exponential lifetime distributions, which assume a constant failure rate. This assumption simplifies analytical calculations and is widely used in engineering practice. However, many real components experience degradation mechanisms such as wear, fatigue, corrosion, or thermal aging, leading to failure rates that vary over time. In these cases, the exponential model may not accurately represent the actual failure behavior.

Time dependent failure is a behavior commonly observed using the Weibull distribution, which provides flexibility through its shape parameter. Depending on its value, the model can describe early life failures, random failures or normal wear of a component, making it particularly suitable for reliability analysis of real engineering systems.

To improve system reliability, redundancy is widely employed in the design. Common redundant architectures include series and parallel systems, voting system such as 2 out of 3 configurations, and series parallel or parallel series structures. While the benefits of redundancy are understood under the assumption of a constant failure rate, their effectiveness may change significantly when components exhibit aging behavior.

The objective of this project is to analyze how component aging impacts system reliability and MTTF across different redundant architectures. Analytical modeling and computational simulations are used to compare the performance of these architectures to provide insights into the selection of appropriate redundancy strategies in systems subject to time dependent failure mechanisms.

3 Background

Reliability Theory Background

Reliability analysis provides a mathematical framework to quantify the ability of a system to perform its intended function over a specified period of time. In engineering applications, reliability metrics are used to support design decisions, compare alternative system architectures, and estimate system lifetime and maintenance requirements. In this project, reliability theory is used as the analytical foundation for comparing different redundant system architectures under both constant and time dependent failure mechanisms.

Let T denote the random variable representing the time to failure of a component or system. The reliability function is defined as the probability that the system remains operational beyond time T:

$$R(t) = P(T > t) \tag{1}$$

The reliability function is the central quantity used throughout this project. All system level reliability expressions are derived from the component reliability function R(t), and both analytical and simulation based results are ultimately expressed in terms of R(t).

The probability density function of failure is obtained as the negative derivative of the reliability function:

$$f(t) = -\frac{dR(t)}{dt} \tag{2}$$

From this definition, the hazard rate, also known as the failure rate, is defined as:

$$\lambda(t) = \frac{f(t)}{R(t)} \tag{3}$$

The hazard rate characterizes the instantaneous failure tendency of a component or system. In this project, the distinction between constant and time dependent hazard rates is essential to highlight the limitations of classical reliability models and motivate the use of aging models.

Mean Time to Failure (MTTF)

A key performance indicator used in this work is the mean time to failure, defined as the expected value of the system lifetime. For non repairable systems, the MTTF can be expressed directly in terms of the reliability function:

$$\text{MTTF} = \int_0^\infty R(t)\,dt \tag{4}$$

This formulation is particularly important for this project, as it allows the computation of MTTF for complex system architectures and non exponential lifetime distributions. When closed form solutions are not available, this integral is evaluated numerically or estimated using Monte Carlo simulations.

Exponential Lifetime Model

In classical reliability theory, component lifetimes are often modeled using the exponential distribution, which assumes a constant failure rate:

$$R(t) = e^{-\lambda t} \tag{5}$$

Under this assumption, the hazard rate is constant:

$$\lambda(t) = \lambda \tag{6}$$

and the mean time to failure is given by:

$$\text{MTTF} = \frac{1}{\lambda} \tag{7}$$

In this project, the exponential model serves as an analytical baseline and is used in conjunction with the fiabilipy library to compute reference reliability functions and MTTF values for different system configurations.

Weibull Lifetime Model

To model aging and wearout effects, the Weibull distribution is employed. The reliability function of a Weibull distributed component is given by:

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{8}$$

where β is the shape parameter and η is the scale parameter. The corresponding hazard rate is:

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \tag{9}$$

The Weibull model allows the representation of different failure regimes depending on the value of β. In particular, values of $\beta > 1$ correspond to aging behavior, which is the primary focus of this project. Weibull distributed lifetimes are used in the Monte Carlo simulations to assess how redundancy strategies perform under time dependent failure mechanisms.

4 Architectures

All configurations are composed of identical, non repairable components with independent failure behavior. This study considers these architectures because they represent the most commonly used redundancy strategies in engineering systems and allow a clear comparison between constant and time dependent failure models.

Series System

A series system is the simplest reliability architecture, in which the failure of any single component causes the failure of the entire system. Such systems are common in processes where all subsystems are required for correct operation.

For a series system composed of n identical components, the system reliability function is given by:

$$R_{\text{series}}(t) = \prod_{i=1}^{n} R_i(t) \tag{10}$$

For identical components, this expression simplifies to:

$$R_{\text{series}}(t) = [R(t)]^n \tag{11}$$

Series systems are highly sensitive to component aging, since the system reliability decreases rapidly as the number of components increases.

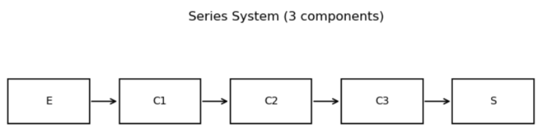


Figure 1: Series architecture (3 components) generated with `Matplotlib`.

Parallel System

In a parallel system, the system remains operational as long as at least one component is functioning. This configuration is widely used to increase reliability by introducing redundancy.

The reliability of a parallel system composed of n identical components is expressed as:

$$R_{\text{parallel}}(t) = 1 - \prod_{i=1}^{n}[1 - R_i(t)] \tag{12}$$

For identical components, this becomes:

$$R_{\text{parallel}}(t) = 1 - [1 - R(t)]^n \tag{13}$$

Parallel systems generally provide significant reliability improvements, although their effectiveness depends strongly on the failure behavior of individual components, particularly under aging conditions.

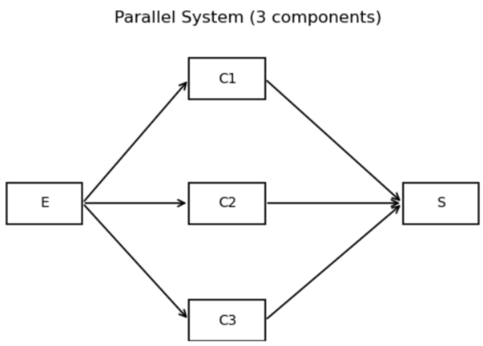


Figure 2: Parallel architecture (3 components) generated with `Matplotlib`.

Series Parallel System

A series parallel system consists of several stages connected in series, where each stage contains multiple components connected in parallel. The system fails if any stage fails completely, while redundancy within each stage provides local fault tolerance.

Let the system be composed of m stages, where stage j contains n_j components in parallel. The reliability of stage j is given by:

$$R_{\text{stage},j}(t) = 1 - \prod_{k=1}^{n_j} [1 - R_{j,k}(t)] \tag{14}$$

The overall system reliability is then:

$$R_{\text{series parallel}}(t) = \prod_{j=1}^{m} R_{\text{stage},j}(t) \tag{15}$$

For identical components and identical stages, this expression reduces to:

$$R_{\text{series parallel}}(t) = [1 - (1 - R(t))^n]^m \tag{16}$$

Series parallel systems provide a compromise between structural simplicity and reliability improvement, offering increased robustness to aging compared to purely series architectures.

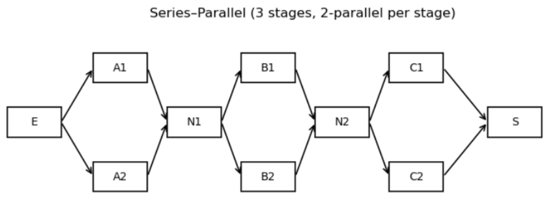


Figure 3: Series parallel architecture generated with `Matplotlib`.

Parallel Series System

A parallel series system is composed of multiple branches connected in parallel, where each branch consists of components connected in series. The system remains operational as long as at least one complete branch is functional.

Let the system contain b branches, where branch r includes s_r components connected in series. The reliability of branch r is given by:

$$R_{\text{branch},r}(t) = \prod_{\ell=1}^{s_r} R_{r,\ell}(t) \tag{17}$$

The system reliability is therefore:

$$R_{\text{parallel series}}(t) = 1 - \prod_{r=1}^{b} [1 - R_{\text{branch},r}(t)] \tag{18}$$

For identical branches with identical components, this expression simplifies to:

$$R_{\text{parallel series}}(t) = 1 - [1 - R(t)^s]^b \tag{19}$$

Although this architecture introduces redundancy at the system level, the series structure within each branch makes it sensitive to component aging, particularly for large values of s.

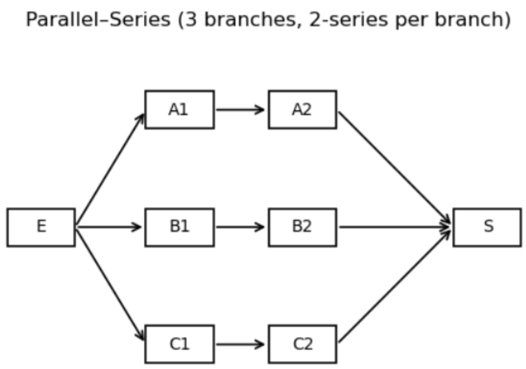


Figure 4: Parallel series architecture generated with `Matplotlib`.

Voting System (2 out of 3)

A voting system is a logical redundancy architecture in which the system operates successfully if a minimum number of components are functional. In a 2 out of 3 voting system, the system functions as long as at least two of the three components are operational.

For a general k-out-of-n system with identical and independent components, the system reliability is given by:

$$R_{k|n}(t) = \sum_{j=k}^{n} \binom{n}{j} [R(t)]^j [1 - R(t)]^{n-j} \quad (20)$$

For the specific case of a 2 out of 3 system, this expression becomes:

$$R_{2|3}(t) = 3R(t)^2 - 2R(t)^3 \quad (21)$$

Voting systems offer a balance between reliability improvement and structural complexity, making them attractive for applications where full parallel redundancy is impractical.

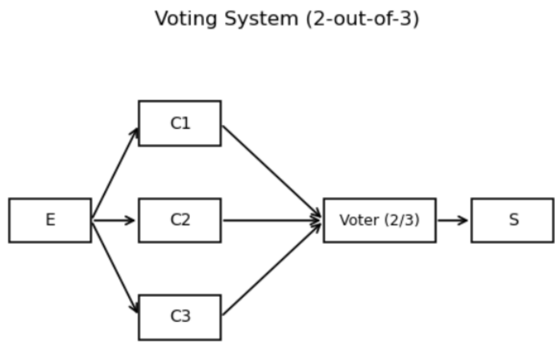


Figure 5: 2 out of 3 voting architecture generated with `Matplotlib`.

5 Modeling

Analytical Reliability Modeling

For each system architecture, the system reliability function $R_{\text{sys}}(t)$ is derived analytically from the component reliability function $R(t)$ using classical reliability block diagram (RBD) rules.

Closed form expressions are obtained for simple architectures such as series, parallel, and voting systems. These expressions are used to compute reference reliability curves and serve as a baseline for comparison with simulation results.

Numerical Evaluation of MTTF

The mean time to failure of each system is computed using the integral definition:

$$\text{MTTF} = \int_0^\infty R_{\text{sys}}(t)\, dt \quad (22)$$

For exponential lifetime models, analytical solutions exist for many architectures. However, for Weibull distributed lifetimes and complex architectures, closed form solutions are generally not available. In these cases, numerical integration techniques are employed to estimate the MTTF.

Monte Carlo Simulation Procedure

For each system architecture, a Monte Carlo simulation procedure was implemented following these steps:

1. Generate random failure times for each component using a Weibull distribution.
2. Determine the system failure time according to the system architecture.
3. Repeat the process for a large number of realizations.
4. Estimate the system reliability function and MTTF from the simulated failure times.

The number of simulation runs was selected to ensure statistical convergence of the estimated reliability metrics.

6 Simulation and Software Framework

Fiabilipy

The reliability analysis presented in this work is partially based on the open source Python library fiabilipy, originally developed more than ten years ago for the modeling of reliability block diagrams (RBDs). The original library was designed to compute system reliability metrics, such as reliability functions and mean time to failure, under the assumption of constant failure rates using exponential lifetime distributions.

Fiabilipy provides a structured framework to define series, parallel, and voting system architectures, making it suitable for analytical reliability studies. However, due to its age, the original implementation presents limitations regarding compatibility with modern Python versions and lacks native support for time dependent failure models, such as Weibull aging.

Software Modifications and Updates

As part of this project, the original fiabilipy codebase was updated to ensure compatibility with modern Python environments. Several deprecated imports and outdated dependencies were corrected to allow the library to run reliably under Python 3.9. These updates were necessary to restore functionality and reproducibility in current scientific computing workflows.

In addition to compatibility fixes, minor refactoring was performed to improve code clarity, maintainability, and execution stability. The analytical logic of the original library was preserved, while internal structures were adjusted to facilitate integration with external simulation methods.

New Version and Weibull Integration

A modified version of the original fiabilipy library was developed to support time dependent failure models. While the analytical structure of the original library was preserved, Weibull distributed lifetimes were integrated through external simulation routines.

Monte Carlo simulation was used to generate random component failure times following a Weibull distribution. System failure times were then determined based on the logical structure of each architecture. This approach allows the estimation of reliability functions and MTTF values without relying on closed form analytical expressions.

7 Results

Comparison of MTTF across architectures

Figure 6 presents the Monte Carlo estimation of the Mean Time to Failure for each architecture under Weibull aging ($\beta = 2.0$, $\lambda = 10^{-4}$).

The results clearly show that redundancy significantly improves system lifetime compared to a pure series configuration. The series system exhibits the lowest MTTF, as failure of any single component causes system failure. In contrast, the parallel architecture provides the highest MTTF, since the system only fails when all components fail.

Hybrid architectures (series parallel and parallel series) exhibit intermediate performance, confirming that their reliability behavior lies between pure series and pure parallel systems. The 2 out of 3 voting architecture also provides substantial improvement over the series system, though slightly below full parallel redundancy.

These results validate classical reliability theory: redundancy increases lifetime, but the improvement depends strongly on the structural configuration of components.

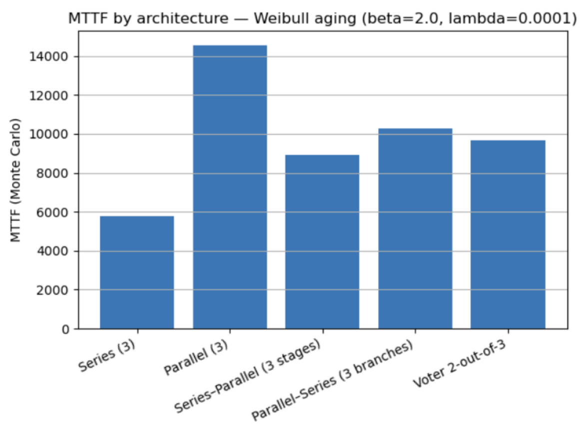


Figure 6: Comparison of mean time to failure (MTTF) across system architectures under Weibull aging.

Reliability evolution under Weibull aging

Figure 7 shows the reliability functions $R(t)$ for all architectures under Weibull aging.

The series system displays the fastest reliability decay, reflecting its vulnerability to single component failures. The parallel system maintains high reliability over a much longer time horizon, illustrating the robustness introduced by redundancy.

Hybrid configurations demonstrate reliability curves that gradually transition between series like and parallel like behavior. The voting architecture initially behaves closer to parallel, but its long term degradation reflects the requirement of maintaining at least two functioning components.

The non exponential shape of the curves confirms the aging behavior induced by the Weibull distribution.

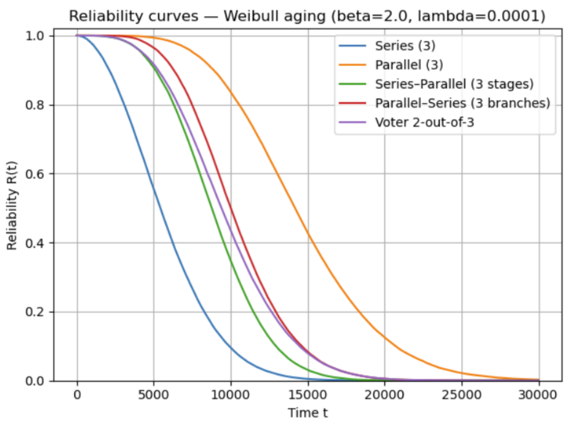


Figure 7: System reliability curves $R(t)$ under Weibull aging for different architectures.

Sensitivity of MTTF to aging intensity

Figure 8 illustrates the sensitivity of system MTTF to variations in the Weibull scale parameter λ, which controls the aging intensity.

As expected, increasing λ produces a monotonic decrease in MTTF for all architectures. However, the rate of degradation differs significantly among architectures.

The series configuration is the most sensitive to aging intensity, exhibiting the steepest decline. In contrast, the parallel architecture shows the slowest relative degradation, indicating that redundancy mitigates the impact of accelerated aging.

Hybrid and voting systems again display intermediate sensitivity, reinforcing the structural dependence of reliability performance.

This analysis highlights that redundancy not only increases nominal lifetime, but also reduces sensitivity to parameter uncertainty in aging models.

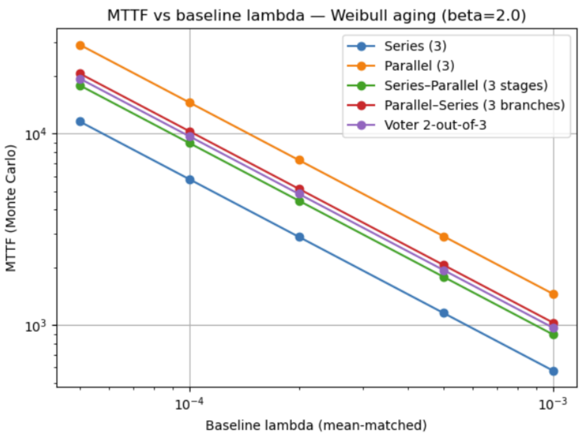


Figure 8: Sensitivity of system MTTF to the baseline Weibull scale parameter λ.

Lifetime Expectations

Figure 9 compares reliability curves under different lifetime scenarios:

- Normal Weibull aging,
- Infant mortality (early life failures),
- Heterogeneous components,
- Mixture distributions.

The series architecture is highly sensitive to early life failures, showing dramatic reliability reduction when infant mortality dominates. This occurs because a single early failure immediately collapses the system.

In contrast, the parallel system benefits from redundancy during early life degradation: even if some components fail prematurely, remaining components maintain functionality. This explains why parallel MTTF can even increase under certain heterogeneity scenarios.

For hybrid systems (Figures 11a and 11b), the impact of early life mortality is moderated but still visible. The structural arrangement determines how localized failures propagate to system level collapse.

The voting architecture (Figure 12) demonstrates robust performance against moderate heterogeneity but remains sensitive to systematic infant mortality affecting multiple components simultaneously.

Overall, these results confirm that architecture strongly influences not only average lifetime, but also robustness to different physical failure mechanisms.

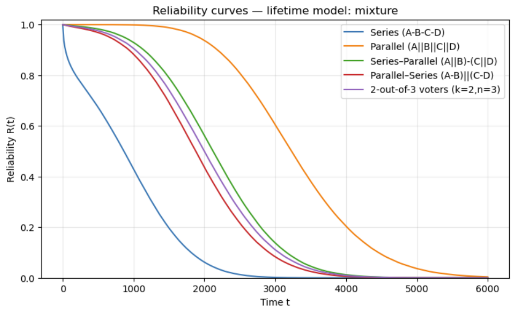
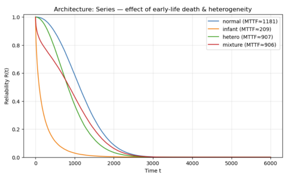
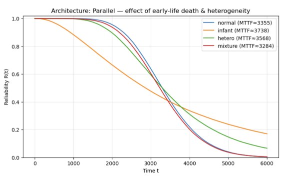


Figure 9: Comparison of reliability under mixed lifetime models (mixture).

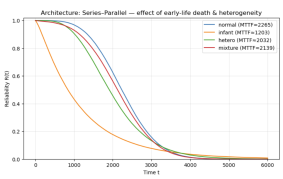


(a) Series

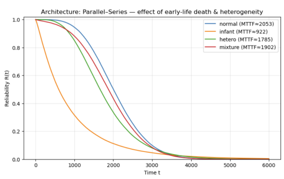


(b) Parallel

Figure 10: Effect of early life failures and heterogeneity on lifetime expectations (basic architectures).



(a) Series Parallel



(b) Parallel Series

Figure 11: Effect of early life failures and heterogeneity on lifetime expectations (hybrid architectures).

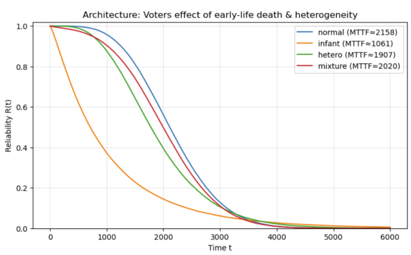


Figure 12: Effect of early life failures and heterogeneity on a 2 out of 3 voting architecture.

8 Conclusions

This project investigated the impact of Weibull aging on the reliability and mean time to failure (MTTF) of several redundant system architectures using both analytical formulations and Monte Carlo simulations.

The results demonstrate that:

- Redundancy significantly increases MTTF compared to pure series configurations.
- Parallel architectures provide the highest lifetime and lowest sensitivity to aging intensity.
- Hybrid and voting architectures offer intermediate trade offs between robustness and structural complexity.
- Systems with redundancy are less sensitive to variations in aging parameters.
- Early life failures disproportionately affect series systems, while parallel systems exhibit improved resilience.

The Monte Carlo results are consistent with classical reliability theory, validating the implementation and confirming the correctness of the modified `fiabilipym` framework.

Furthermore, the integration of aging models beyond exponential assumptions provides a more realistic representation of physical degradation processes. The Weibull model successfully captures increasing hazard rates and their system level consequences.

Overall, this work demonstrates that reliability performance is fundamentally architecture dependent and that aging effects must be explicitly considered in system design.

9 References

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