DIGITAL LOGIC DESIGN EE(1005)

LECTURE-4,5,6

Chapter No:2
Number System, Operation and Codes

CHAPTER OUTLINE

- Decimal Numbers
- Binary Numbers
- Decimal-to-Binary Conversion
- Binary Arithmetic
- Complements of Binary Numbers
- Signed Numbers
- Arithmetic Operations with Signed Numbers
- Hexadecimal Numbers
- Octal Numbers
- Binary Coded Decimal (BCD)
- Digital Codes
- Error Codes

DECIMAL NUMBERS

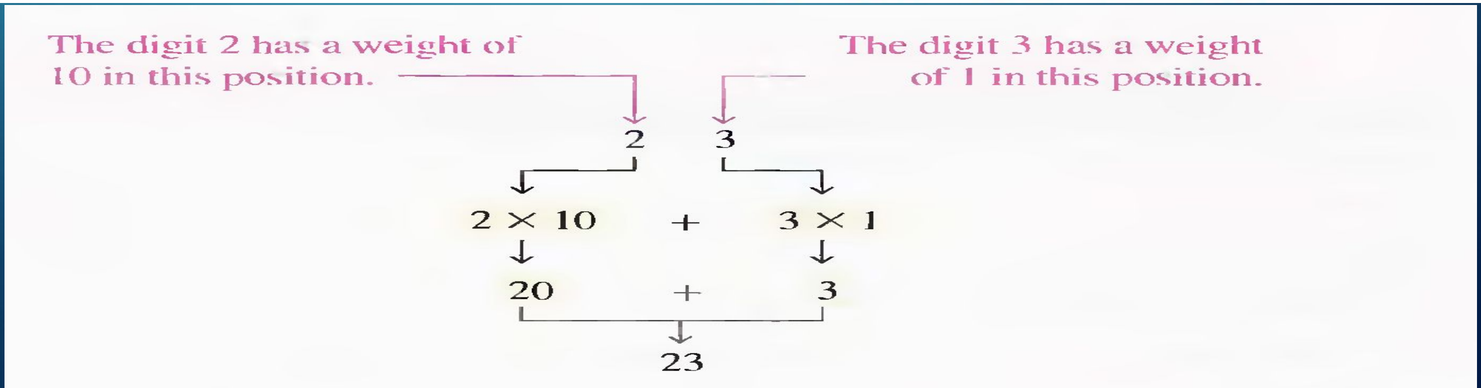
The position of each digit in a weighted number system is assigned a weight based on the **base** or **radix** of the system. The radix of decimal numbers is ten, because only ten symbols (0 through 9) are used to represent any number.

The column weights of decimal numbers are powers of ten that increase from right to left beginning with $10^0 = 1$:

$\ldots 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0.$

For fractional decimal numbers, the column weights are negative powers of ten that decrease from left to right:

$10^2\ 10^1\ 10^0.\ 10^{-1}\ 10^{-2}\ 10^{-3}\ 10^{-4}\ \ldots$



Decimal numbers can be expressed as the sum of the products of each digit times the column value for that digit. Thus, the number 9240 can be expressed as

$$(9 \times 10^3) + (2 \times 10^2) + (4 \times 10^1) + (0 \times 10^0)$$

or

$$9 \times 1{,}000 + 2 \times 100 + 4 \times 10 + 0 \times 1$$

Example Express the number 480.52 as the sum of values of each digit.

Solution

$$480.52 = (4 \times 10^2) + (8 \times 10^1) + (0 \times 10^0) + (5 \times 10^{-1}) + (2 \times 10^{-2})$$

BINARY NUMBERS

For digital systems, the binary number system is used. Binary has a radix of two and uses the digits 0 and 1 to represent quantities.

The column weights of binary numbers are powers of two that increase from right to left beginning with $2^0 = 1$:

$\ldots 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0.$

For fractional binary numbers, the column weights are negative powers of two that decrease from left to right:

$2^2\ 2^1\ 2^0.\ 2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}\ \ldots$

BINARY NUMBERS

A binary counting sequence for numbers from zero to fifteen is shown.

Notice the pattern of zeros and ones in each column.

Digital counters frequently have this same pattern of digits:

Decimal Number	Binary Number
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1

n bits you can count up to a number equal to $2^n - 1$

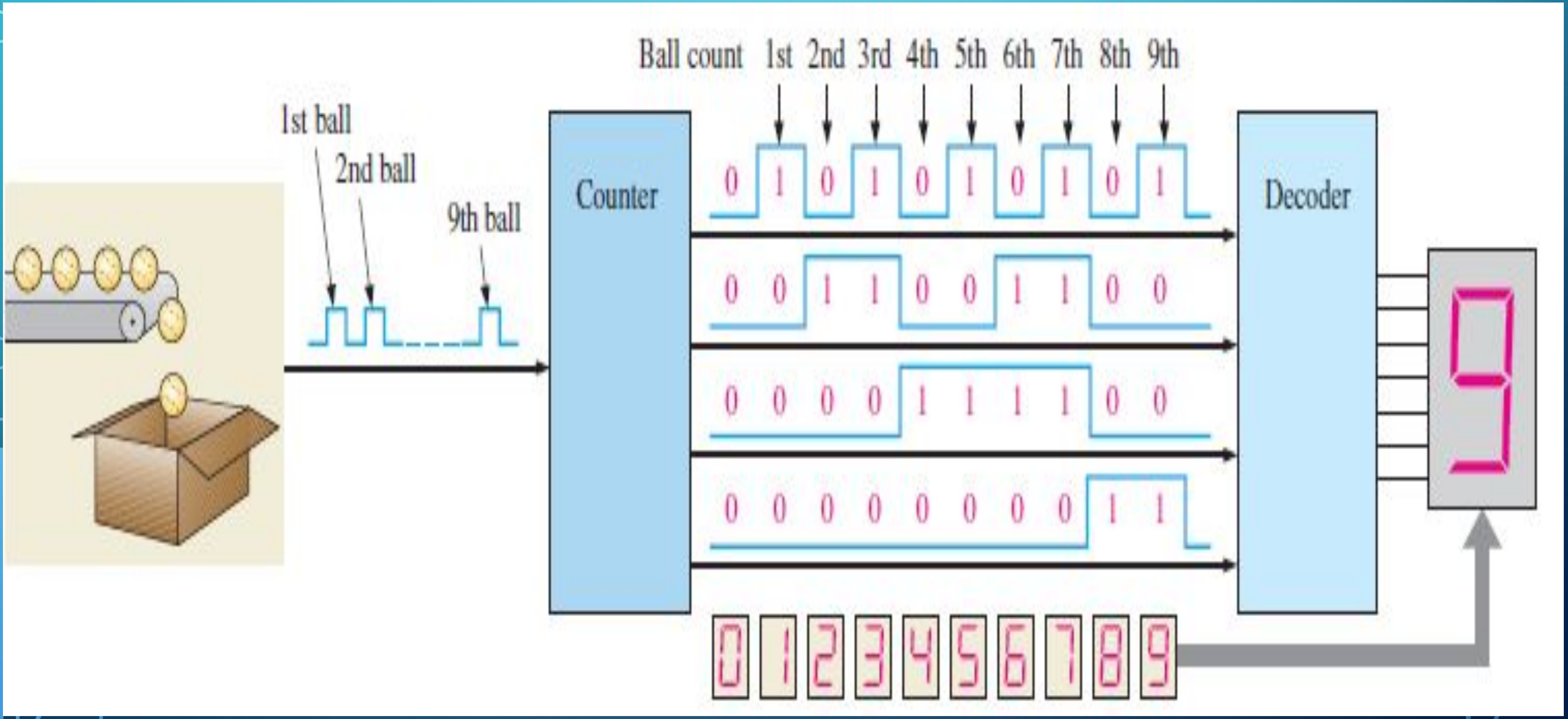
Largest decimal number $= 2^n - 1$

For example, with five bits ($n = 5$)

$2^5 - 1 = 32 - 1 = 31$

count from zero to thirty-one.

A SIMPLE BINARY COUNTING APPLICATION.



THE WEIGHTING STRUCTURE OF BINARY NUMBERS

The weight structure of a binary number is

$$2^{n-1} \ldots 2^3\, 2^2\, 2^1\, 2^0 \,.\, 2^{-1}\, 2^{-2} \ldots 2^{-n}$$

Binary point

POSITIVE POWERS OF TWO (WHOLE NUMBERS)									NEGATIVE POWERS OF TWO (FRACTIONAL NUMBER)					
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
256	128	64	32	16	8	4	2	1	1/2	1/4	1/8	1/16	1/32	1/64
									0.5	0.25	0.125	0.0625	0.03125	0.015625

$$1 \quad 1 \quad 0 \quad 1 \quad 1_2$$

$$2^4 + 2^3 + 0 + 2^1 + 2^0 = 16 + 8 + 2 + 1$$

$$= 27_{10}$$

Binary to Decimal Conversion

Weight: 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Binary number: 1 1 0 1 1 0 1

$$1101101 = 2^6 + 2^5 + 2^3 + 2^2 + 2^0$$
$$= 64 + 32 + 8 + 4 + 1 = \mathbf{109}$$

Convert the binary number 10010001 to decimal.

Weight: 2^{-1} 2^{-2} 2^{-3} 2^{-4}

Binary number: 0 . 1 0 1 1

$$0.1011 = 2^{-1} + 2^{-3} + 2^{-4}$$
$$= 0.5 + 0.125 + 0.0625 = \mathbf{0.6875}$$

Convert the binary number 10.111 to decimal.

DECIMAL TO BINARY CONVERSION

Sum-of-Weights Method

$$45_{10} = 32 + 8 + 4 + 1 = 2^5 + 0 + 2^3 + 2^2 + 0 + 2^0$$
$$= 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1_2$$

Repeated Division-by-2 Method

$$\frac{25}{2} = 12 + \text{remainder of } 1 \quad \text{LSB}$$
$$\frac{12}{2} = 6 + \text{remainder of } 0$$
$$\frac{6}{2} = 3 + \text{remainder of } 0$$
$$\frac{3}{2} = 1 + \text{remainder of } 1$$
$$\frac{1}{2} = 0 + \text{remainder of } 1 \quad \text{MSB}$$

$$25_{10} = 1 \quad 1 \quad 0 \quad 0 \quad 1_2$$

DECIMAL TO BINARY CONVERSION

You can convert a decimal fraction to binary by repeatedly multiplying the fractional results of successive multiplications by 2. The carries form the binary number.

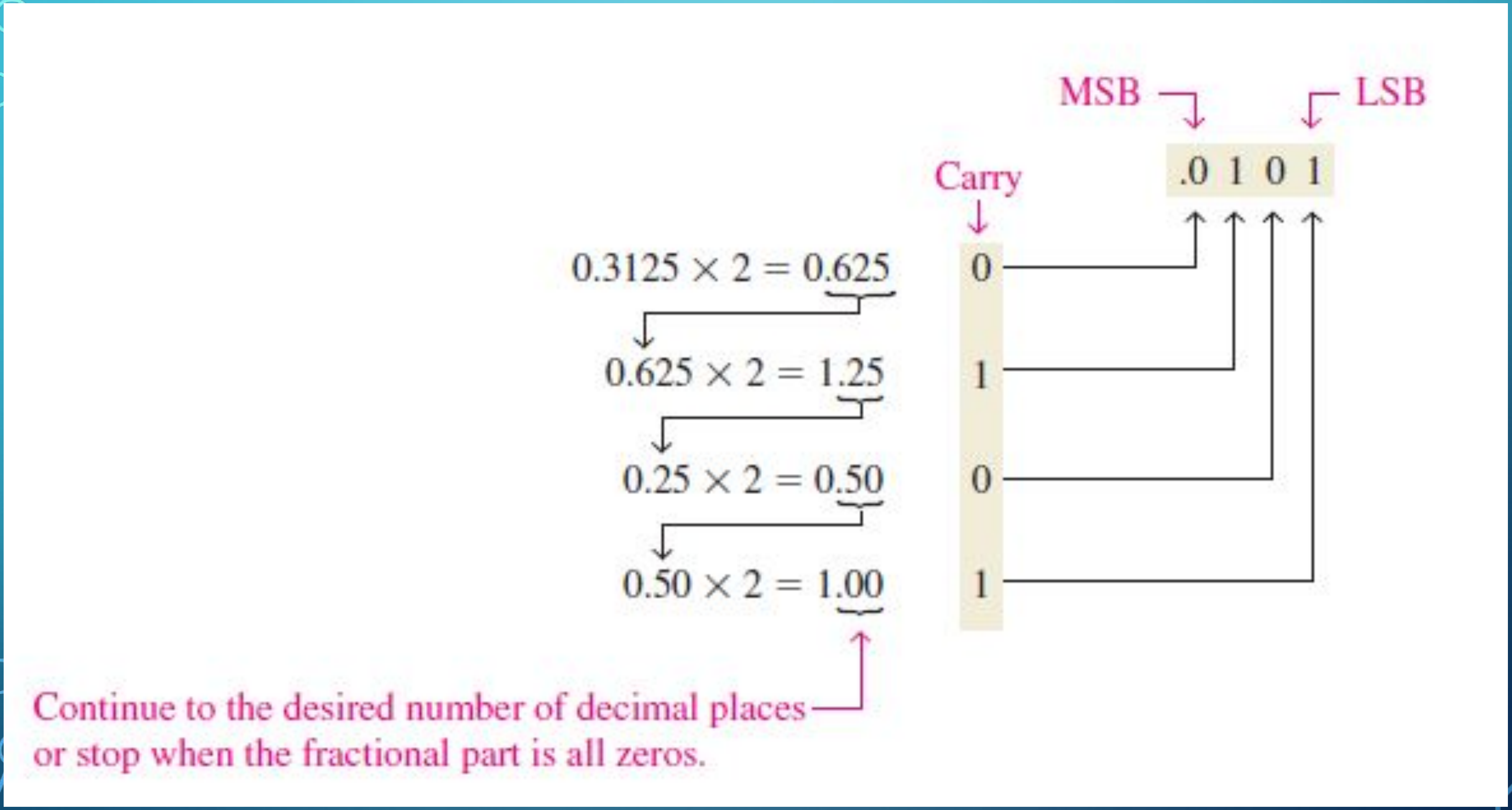
Example Convert the decimal fraction 0.188 to binary by repeatedly multiplying the fractional results by 2.

Solution

$0.188 \times 2 = 0.376$	carry = 0	MSB
$0.376 \times 2 = 0.752$	carry = 0	
$0.752 \times 2 = 1.504$	carry = 1	
$0.504 \times 2 = 1.008$	carry = 1	
$0.008 \times 2 = 0.016$	carry = 0	

Answer = .00110 (for five significant digits)

DECIMAL TO BINARY CONVERSION



Binary to Decimal Conversion

1. Convert 100011011011_2 to its decimal equivalent.
2. What is the weight of the MSB of a 16-bit number?

1. 2267 2. 32768

Decimal to Binary Conversion

1. Convert 83_{10} to binary
2. Convert 729_{10} to binary
3. How many bits are required to count up to decimal 1 million?

1. 1010011 2. 1011011001 3. 20bits

Convert the binary number 10010001 to decimal.

$10010001 = 145$

Convert the binary number 10.111 to decimal.

$10.111 = 2.875$

RULES OF BINARY ADDITION

The four basic rules for adding binary digits (bits) are as follows:

$0 + 0 = 0$	Sum of 0 with a carry of 0
$0 + 1 = 1$	Sum of 1 with a carry of 0
$1 + 0 = 1$	Sum of 1 with a carry of 0
$1 + 1 = 10$	Sum of 0 with a carry of 1

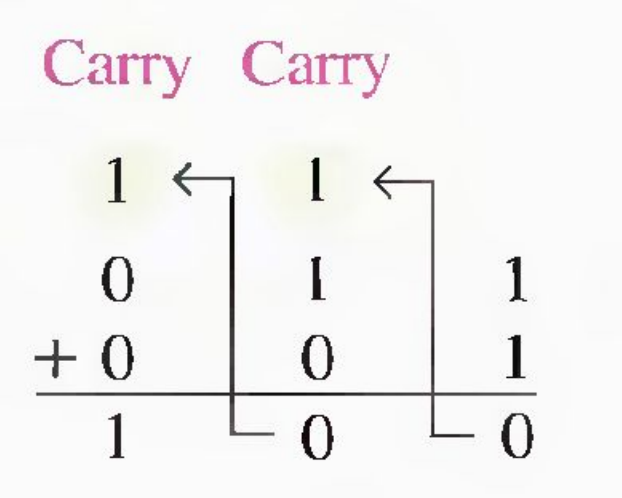
11 + 11 = 110 (3+3=6)

100+ 10 = 110 (4+2=6)

111+11 = 1010 (7+3=10)

110+100=1010 (6+4=10)

Note: (1+1+1 = 11)



RULES OF BINARY SUBTRACTION

The four basic rules for subtracting bits are as follows:

$$0 - 0 = 0$$
$$1 - 1 = 0$$
$$1 - 0 = 1$$
$$10 - 1 = 1 \qquad 0 - 1 \text{ with a borrow of } 1$$

11-01 = 10 (3-1=2)

11-10 = 01 (3-2=1)

101 -011 = 010 (5-3=2)

$$\begin{array}{r} 11 \\ -\,01 \\ \hline 10 \end{array} \qquad \begin{array}{r} 3 \\ -\,1 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 101 \\ -\,011 \\ \hline 010 \end{array} \qquad \begin{array}{r} 5 \\ -\,3 \\ \hline 2 \end{array}$$

RULES OF BINARY MULTIPLICATION

The four basic rules for multiplying bits are as follows:

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

(a)

$$\begin{array}{r} 11 \\ \times\ 11 \\ \hline 11 \\ +11 \\ \hline 1001 \end{array} \qquad \begin{array}{r} 3 \\ \times\ 3 \\ \hline 9 \end{array}$$

Partial products

(b)

$$\begin{array}{r} 111 \\ \times\ 101 \\ \hline 111 \\ 000\ \\ +111\ \ \\ \hline 100011 \end{array} \qquad \begin{array}{r} 7 \\ \times\ 5 \\ \hline 35 \end{array}$$

Partial products

Rules of binary Division

Division in binary follows the same procedure as division in decimal. The equivalent decimal divisions are also given.

(a)
$$\begin{array}{r} 10 \\ 11\overline{)110} \\ \underline{11} \\ 000 \end{array} \qquad \begin{array}{r} 2 \\ 3\overline{)6} \\ \underline{6} \\ 0 \end{array}$$

(b)
$$\begin{array}{r} 11 \\ 10\overline{)110} \\ \underline{10} \\ 10 \\ \underline{10} \\ 00 \end{array} \qquad \begin{array}{r} 3 \\ 2\overline{)6} \\ \underline{6} \\ 0 \end{array}$$

Examples

Perform the following binary additions:

(a) $1101 + 1010$ (b) $10111 + 01101$

(a) $1101 + 1010 = 10111$ (b) $10111 + 01101 = 100100$

Perform the following binary subtractions:

(a) $1101 - 0100$ (b) $1001 - 0111$

(a) $1101 - 0100 = 1001$ (b) $1001 - 0111 = 0010$

Perform the indicated binary operations:

(a) 110×111 (b) $1100 \div 011$

(a) $110 \times 111 = 101010$ (b) $1100 \div 011 = 100$

1' s and 2's Complements of Binary Numbers

Subtraction of a number from another can be accomplished by adding the complement of the subtrahend to the minuend.

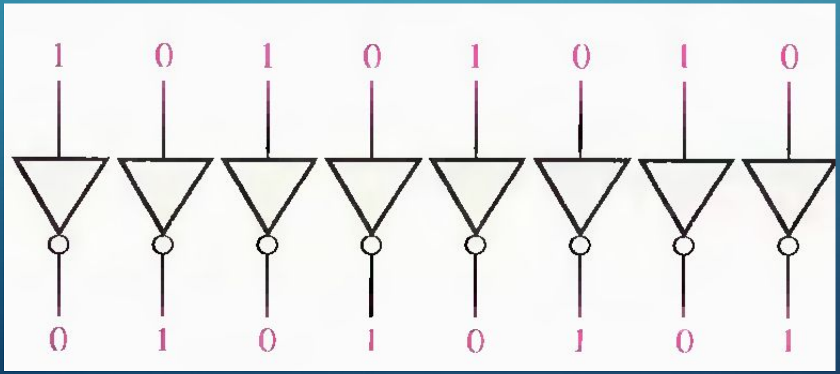
Subtraction of binary numbers using the 1's and 2's complements method allows subtraction only by addition.

1' s Complements of Binary Numbers

Finding the 1's Complement

The 1's **complement** of a binary number is found by changing all 1s to 0s and all 0s to 1s, as illustrated below:

1 0 1 1 0 0 1 0	Binary number
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	
0 1 0 0 1 1 0 1	1's complement



2's Complements of Binary Numbers

Finding the 2's Complement

The 2's complement of a binary number is found by adding 1 to the LSB of the 1's complement.

$$\text{2's complement} = (\text{1's complement}) + 1$$

10110010	Binary number
01001101	1's complement
+ 1	Add 1
01001110	2's complement

An alternative method of finding the 2's complement of a binary number is as follows:

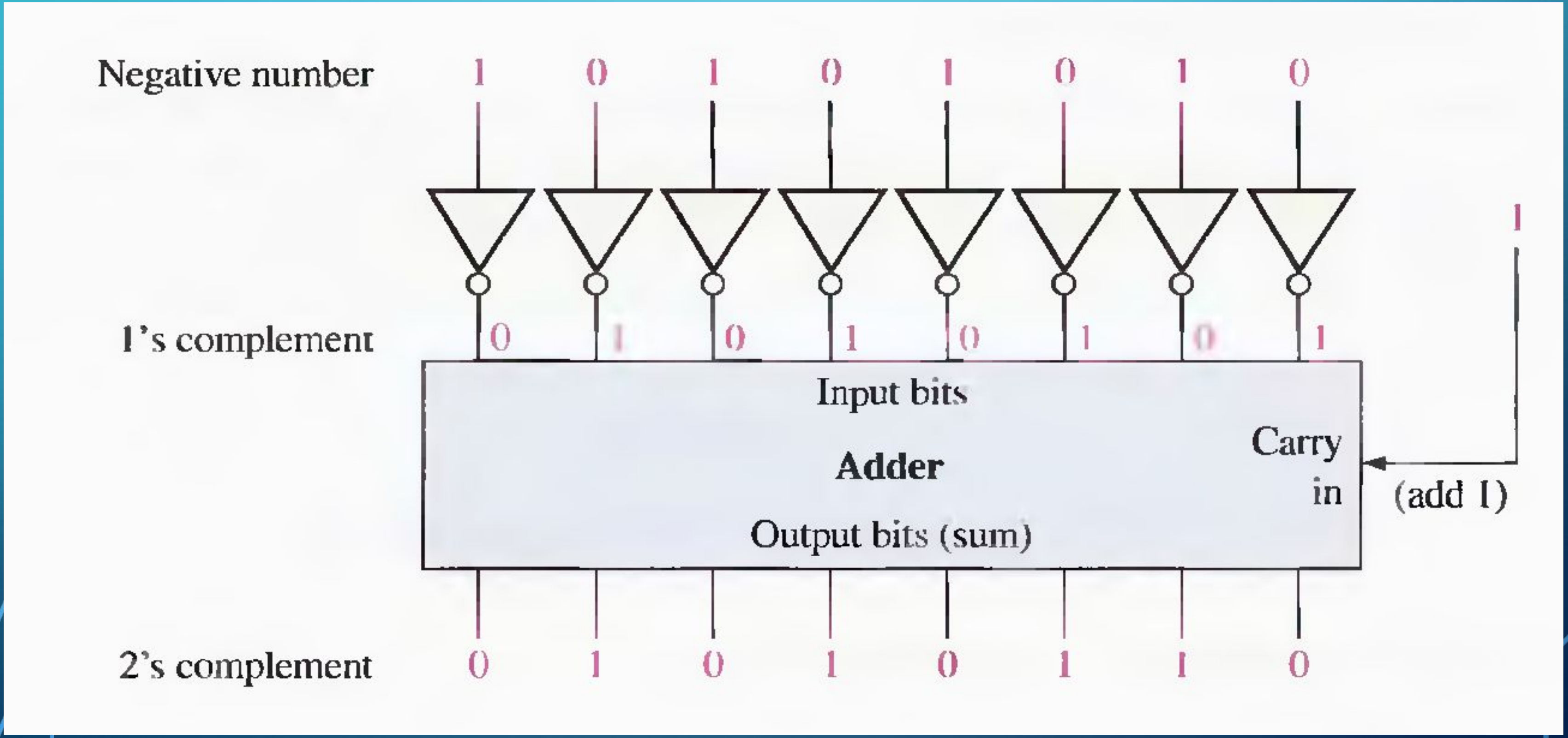
1. Start at the right with the LSB and write the bits as they are up to and including the first 1.
2. Take the 1's complements of the remaining bits.

10111000	Binary number
01001000	2's complement

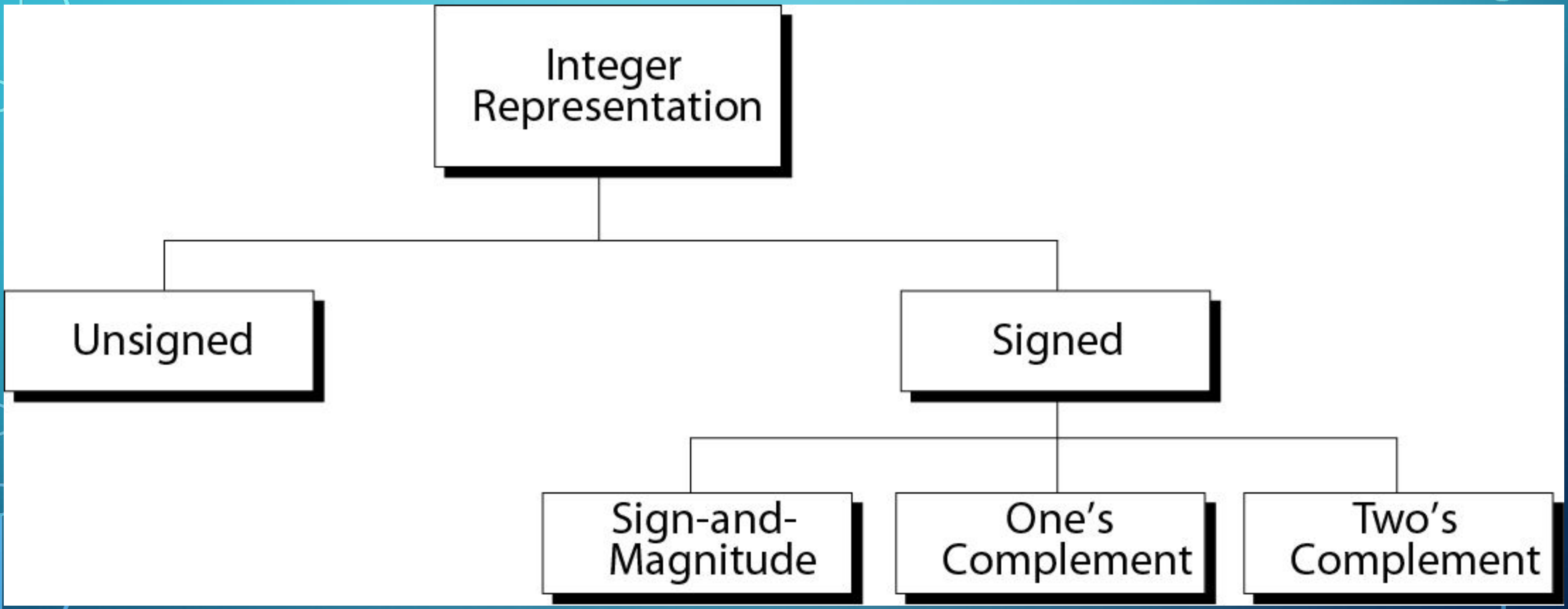
1's complements of original bits

These bits stay the same.

Example of obtaining the 2's complement of a negative binary number.



Taxonomy of integers



Signed Numbers

Digital systems, such as the computer, must be able to handle both positive and negative numbers. A signed binary number consists of both sign and magnitude information. The sign indicates whether a number is positive or negative, and the magnitude is the value of the number. There are three forms in which signed integer (whole) numbers can be represented in binary:

- *Sign-magnitude,*
- *1's complement, and*
- *2' complement.*

The Sign Bit

The left-most bit in a signed binary number is the **sign bit,** which tells you whether the number is positive or negative.

A 0 sign bit indicates a positive number, and a 1 sign bit indicates a negative number.

Signed Numbers

Sign-Magnitude Form

When a signed binary number is represented in sign-magnitude, the left-most bit is the sign bit and the remaining bits are the magnitude bits. The magnitude bits are in true (uncomplemented) binary for both positive and negative numbers. For example, the decimal number +25 is expressed as an 8-bit signed binary number using the sign-magnitude form as

$$\underset{\uparrow\ \text{Sign bit}}{0}\underbrace{0011001}_{\text{Magnitude bits}}$$

The decimal number −25 is expressed as

10011001

In the sign-magnitude form, a negative number has the same magnitude bits as the corresponding positive number but the sign bit is a 1 rather than a zero.

Signed Numbers
1’s and 2’s Complement forms

Positive numbers in I's and 2’s complement forms are represented the same way as the positive sign- magnitude numbers. Negative numbers, however, are the l's and 2’s complements of the corresponding positive numbers.

1’s Complement :

The decimal number -25 is expressed as the I' s complement of + 25 (0001100 I) as 11100110

In the 1’s complement form, a negative number is the 1’s complement of the corresponding positive number.

2’s Complement :

The decimal number -25 is expressed as the 2' s complement of + 25 (0001100 I) as 11100111

In the 2’s complement form, a negative number is the 2’s complement of the corresponding positive number.

Examples (Signed Numbers)

Express the decimal number −39 as an 8-bit number in the sign-magnitude, 1's complement, and 2's complement forms.

First, write the 8-bit number for +39.

00100111

In the *sign-magnitude form*, −39 is produced by changing the sign bit to a 1 and leaving the magnitude bits as they are. The number is

10100111

In the *1's complement form*, −39 is produced by taking the 1's complement of +39 (00100111).

11011000

In the *2's complement form*, −39 is produced by taking the 2's complement of +39 (00100111) as follows:

11011000	1's complement
+ 1	
11011001	2's complement

Examples (Signed Numbers)

Express + 19 and - 19 in sign-magnitude, I's complement, and 2's complement.

	SIGN-MAGNITUDE	1'S COMP	2'S COMP
+19	00010011	00010011	00010011
−19	10010011	11101100	11101101

The Decimal Value of Signed Numbers

Determine the decimal value of this signed binary number expressed in sign-magnitude: 10010101.

The seven magnitude bits and their powers-of-two weights are as follows:

2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	0	1	0	1

$$16 + 4 + 1 = 21$$

Summing the weights where there are 1s,

The sign bit is 1; therefore, the decimal number is -21

Determine the decimal value of the sign-magnitude number 01110111

$$01110111 = +119_{10}$$

The Decimal Value of Signed Numbers

1's Complement

Determine the decimal values of the signed binary numbers expressed in 1's complement: 00010111

The bits and their powers-of-two weights for the positive number are as follows:

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	0	1	0	1	1	1

Summing the weights where there are 1s,

$$16 + 4 + 2 + 1 = \mathbf{+23}$$

The Decimal Value of Signed Numbers

1's Complement

Determine the decimal values of the signed binary numbers expressed in 1's complement: 11101000

The bits and their powers-of-two weights for the negative number are as follows. Notice that the negative sign bit has a weight of -2^7 or -128.

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	1	0	1	0	0	0

Summing the weights where there are 1s,

$$-128 + 64 + 32 + 8 = -24$$

Adding 1 to the result, the final decimal number is

$$-24 + 1 = -23$$

Determine the decimal value of the 1's complement number 11101011.

The Decimal Value of Signed Numbers

2's Complement

Determine the decimal values of the signed binary numbers expressed in 2's complement:

(a) The bits and their powers-of-two weights for the positive number are as follows:

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	0	1	1	0

Summing the weights where there are 1s,

$$64 + 16 + 4 + 2 = \mathbf{+86}$$

(b) The bits and their powers-of-two weights for the negative number are as follows. Notice that the negative sign bit has a weight of $-2^7 = -128$.

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	0	1	0	1	0

Summing the weights where there are 1s,

$$-128 + 32 + 8 + 2 = \mathbf{-86}$$

Determine the decimal value of the 2's complement number 11010111.

Summary of integer representation

Contents of Memory	Unsigned	Sign-and-Magnitude	One's Complement	Two's Complement
------------	------------	----------	----------	---------
0000	**0**	**+0**	**+0**	**+0**
0001	**1**	**+1**	**+1**	**+1**
0010	**2**	**+2**	**+2**	**+2**
0011	**3**	**+3**	**+3**	**+3**
0100	**4**	**+4**	**+4**	**+4**
0101	**5**	**+5**	**+5**	**+5**
0110	**6**	**+6**	**+6**	**+6**
0111	**7**	**+7**	**+7**	**+7**
1000	**8**	**−0**	**−7**	**−8**
1001	**9**	**−1**	**−6**	**−7**
1010	**10**	**−2**	**−5**	**−6**
1011	**11**	**−3**	**−4**	**−5**
1100	**12**	**−4**	**−3**	**−4**
1101	**13**	**−5**	**−2**	**−3**
1110	**14**	**−6**	**−1**	**−2**
1111	**15**	**−7**	**−0**	**−1**

Range of unsigned integers

# of Bits	Range	
8	0	255
16	0	65,535

Range of sign-and-magnitude integers

# *of Bits*	*Range*			
8	−127	−0	+0	+127
16	−32767	−0	+0	+32767
32	−2,147,483,647	−0	+0	+2,147,483,647

Note:

There are two 0s in one's complement representation: positive and negative.

In an 8-bit allocation:

+0 □ 00000000

-0 □ 11111111

Range of two's complement integers

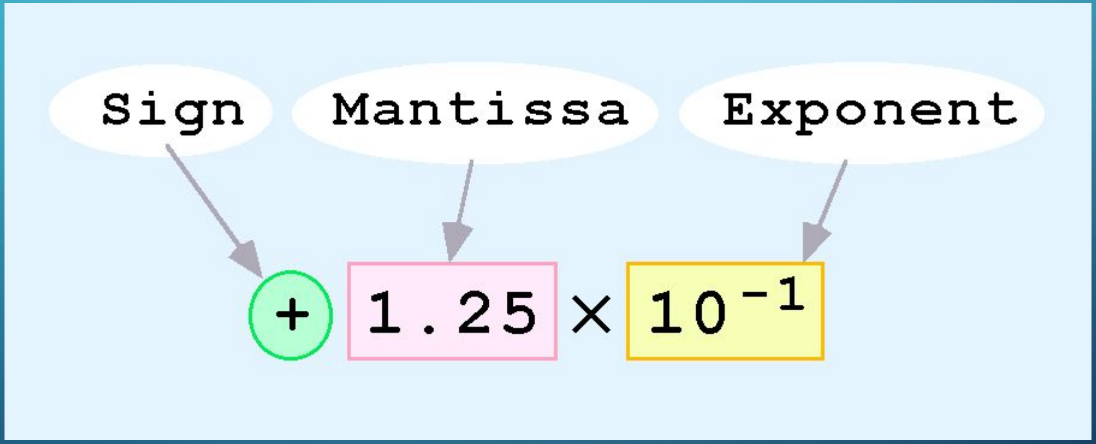
# of Bits	**Range**		
8	−128	0	+127
16	−32,768	0	+32,767
32	−2,147,483,648	0	+2,147,483,647

The Decimal Value of Signed Numbers
1's and 2's Complement

From these examples, you can see why the 2's complement form is preferred for representing signed integer numbers: To convert to decimal, it simply requires a summation of weights regardless of whether the number is positive or negative. The 1's complement system requires adding 1 to the summation of weights for negative numbers but not for positive numbers. Also, the 1's complement form is generally not used because two representations of zero (00000000 or 11111111) are possible.

FLOATING-POINT REPRESENTATION

- Computers use a form of scientific notation for floating-point representation
- Numbers written in scientific notation have three components:



SIGNED NUMBERS

- Floating-point numbers
  - Can represent very large or very small numbers based on scientific notation. Binary point “floats”.
- Two Parts
  - **Mantissa** represents magnitude of number
  - **Exponent** represents number of places that binary point is to be moved
- Three forms
  - Single-precision (32 bits) float
  - Double-precision (64 bits) double
  - Extended-precision (80 bits) long double
  - Also have Quadruple and Quadruple extended!

ADD/SUB : 4 COMBINATIONS

Positive / Positive
Positive Answer

$$\begin{array}{r} 9 \\ +\ \ 5 \\ \hline 14 \end{array}$$

Negative / Positive
Negative Answer

$$\begin{array}{r} (-9) \\ +\ \ 5 \\ \hline -\ 4 \end{array}$$

Positive / Negative
Positive Answer

$$\begin{array}{r} 9 \\ +\ (-5) \\ \hline 4 \end{array}$$

Negative / Negative
Negative Answer

$$\begin{array}{r} (-9) \\ +\ (-5) \\ \hline -\ 14 \end{array}$$

POSITIVE / POSITIVE COMBINATION

Positive / Positive
Positive Answer

$$\begin{array}{rr} & 9 \\ + & 5 \\ \hline & 14 \end{array} \quad \begin{array}{c} \longrightarrow \\ \longrightarrow \\ \\ \longleftarrow \end{array} \quad \begin{array}{rr} & 00001001 \\ + & 00000101 \\ \hline & 00001110 \end{array}$$

Both Positive Numbers
Use Straight Binary Addition

POSITIVE / NEGATIVE COMBINATION

Positive / Negative
Positive Answer

$$\begin{array}{r} 9 \\ + \ (-5) \\ \hline 4 \end{array} \qquad \begin{array}{r} 00001001 \\ + \ 11111011 \\ \hline \cancel{1}]00000100 \end{array}$$

8^{th} Bit = 0 : Answer is Positive
Disregard 9^{th} Bit

1-Positive / 1-Negative
Take 2's Complement
Of Negative Number (-5)

$$\begin{array}{r} 00000101 \\ \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ 11111010 \\ +1 \\ \hline 11111011 \end{array}$$

2's
Complement
Process

NEGATIVE / POSITIVE COMBINATION

Positive / Negative
Negative Answer

$$\begin{array}{r} (-9) \\ +\ \ 5 \\ \hline -\ \ 4 \end{array} \longrightarrow \begin{array}{r} 11110111 \\ +\ \ 00000101 \\ \hline 11111100 \end{array}$$

8th Bit = 1 : Answer is Negative
Take 2's Complement to Check Answer

1-Positive / 1-Negative
Take 2's Complement
Of Negative Number (-9)

2's Complement Process

$$\begin{array}{r} 11111100 \\ \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ 00000011 \\ +1 \\ \hline 00000100 \end{array}$$

$$\begin{array}{r} 00001001 \\ \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ 11110110 \\ +1 \\ \hline 11110111 \end{array}$$

2's Complement Process

NEGATIVE / NEGATIVE COMBINATION

Negative / Negative
Negative Answer

(-9)	→		11110111
+ (-5)	→	+	11111011
- 14			~~1~~]11110010

2's Complement Numbers, See Conversion Process In Previous Slides

8^{th} Bit = 1 : Answer is Negative
Disregard 9^{th} Bit
Take 2's Complement to Check Answer

2-Negative
Take 2's Complement Of Both Negative Numbers

2's Complement Process

11110010
↓↓↓↓↓↓↓↓
00001101
+1
00001110

Overflow Condition When two numbers are added and the number of bits required to represent the sum exceeds the number of bits in the two numbers, an **overflow** results as indicated by an incorrect sign bit. An overflow can occur only when both numbers are positive or both numbers are negative. The following 8-bit example will illustrate this condition.

01111101	125
+ 00111010	+ 58
10110111	183

Sign incorrect

Magnitude incorrect

Add the signed numbers: 01000100, 00011011, 00001110, and 00010010.

The equivalent decimal additions are given for reference.

68	01000100	
+ 27	+ 00011011	Add 1st two numbers
95	01011111	1st sum
+ 14	+ 00001110	Add 3rd number
109	01101101	2nd sum
+ 18	+ 00010010	Add 4th number
127	**01111111**	Final sum

Subtraction (Examples)

Perform each of the following subtractions of the signed numbers:

(a) 00001000 − 00000011 (b) 00001100 − 11110111

Like in other examples, the equivalent decimal subtractions are given for reference.

(a) In this case, $8 - 3 = 8 + (-3) = 5$.

	00001000	Minuend (+8)
	+ 11111101	2's complement of subtrahend (−3)
Discard carry →	**1 00000101**	Difference (+5)

(b) In this case, $12 - (-9) = 12 + 9 = 21$.

00001100	Minuend (+12)
+ 00001001	2's complement of subtrahend (+9)
00010101	Difference (+21)

HEXADECIMAL NUMBER SYSTEM

The **hexadecimal number system** uses base 16. Thus, it has 16 possible digit symbols. It uses the digits 0 through 9 plus the letters A, B, C, D, E, and F as

16^4	16^3	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}	16^{-4}

Hexadecimal point

0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Binary-to-Hexadecimal Conversion

Converting a binary number to hexadecimal is a straightforward procedure. Simply break the binary number into 4-bit groups, starting at the right-most bit and replace each 4-bit group with the equivalent hexadecimal symbol.

(a) $\underbrace{1100}_{C}\underbrace{1010}_{A}\underbrace{0101}_{5}\underbrace{0111}_{7} = CA57_{16}$

(b) $\underbrace{0011}_{3}\underbrace{1111}_{F}\underbrace{0001}_{1}\underbrace{0110}_{6}\underbrace{1001}_{9} = 3F169_{16}$

Two zeros have been added in part (b) to complete a 4-bit group at the left.

Convert the binary number 1001111011110011100 to hexadecimal.

$4F79C_{16}$

Hexadecimal-to-Binary Conversion

(a) 1 0 A 4

1000010100100

(b) C F 8 E

1100111110001110

(c) 9 7 4 2

1001011101000010

Convert the hexadecimal number 6BD3 to binary.

0110101111010011_2

Hexadecimal-to-Decimal Conversion

(a)

$$\overbrace{0001}^{\text{1}}\overbrace{1100}^{\text{C}} = 2^4 + 2^3 + 2^2 = 16 + 8 + 4 = \mathbf{28_{10}}$$

(b)

$$\overbrace{1010}^{\text{A}}\overbrace{1000}^{\text{8}}\overbrace{0101}^{\text{5}} = 2^{11} + 2^9 + 2^7 + 2^2 + 2^0 = 2048 + 512 + 128 + 4 + 1 = \mathbf{2693_{10}}$$

Another way to convert a hexadecimal number to its decimal equivalent is

16^3	16^2	16^1	16^0
4096	256	16	1

$$\begin{aligned} 356_{16} &= 3 \times 16^2 + 5 \times 16^1 + 6 \times 16^0 \\ &= 768 + 80 + 6 \\ &= 854_{10} \end{aligned}$$

$$\begin{aligned} 2AF_{16} &= 2 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 \\ &= 512 + 160 + 15 \\ &= 687_{10} \end{aligned}$$

Decimal-to-Hexadecimal Conversion

Hexadecimal remainder

$$\frac{650}{16} = 40.625 \rightarrow 0.625 \times 16 = 10 = \quad A$$

$$\frac{40}{16} = 2.5 \longrightarrow 0.5 \times 16 = 8 = \quad 8$$

$$\frac{2}{16} = 0.125 \longrightarrow 0.125 \times 16 = 2 = \quad 2$$

Stop when whole number quotient is zero.

2 8 A Hexadecimal number

MSD LSD

USEFULNESS OF HEX

Hex is often used in a digital system as sort of a "Shorthand" way to represent string of bits.

When dealing with the large numbers of bits , it is more convenient and less error to write the binary in hex.

Would you rather check 50 numbers like this one 0110111001100111

or 50 numbers like this one 6E67 ?

Digital circuit work in binary.

Hex is simply used as a convenience for the

Octal Numbers

Octal uses eight characters the numbers 0 through 7 to represent numbers. There is no 8 or 9 character in octal.

Binary number can easily be converted to octal by grouping bits 3 at a time and writing the equivalent octal character for each group.

Example

Express 1 001 011 000 001 110_2 in octal:

Solution

Group the binary number by 3-bits starting from the right. Thus, 113016_8

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Octal Numbers

Octal is also a weighted number system. The column weights are powers of 8, which increase from right to left.

$$\text{Column weights} \begin{cases} 8^3 & 8^2 & 8^1 & 8^0 \\ 512 & 64 & 8 & 1 \end{cases}$$

Example Express 3702_8 in decimal.

Solution Start by writing the column weights:

512 64 8 1

3 7 0 2_8

$3(512) + 7(64) + 0(8) + 2(1) = 1986_{10}$

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Code:

When the numbers , letters or word are represented by a special group of symbols, we say they are being encoded, and the group of symbols is called a Code.

Straight binary Code:

When a decimal number is represented by its equivalent binary number, called Straight binary Code.

BCD Code:

If each digit of a decimal number is represented by its binary equivalent, the result is a code called Binary –Coded –Decimal (BCD)

Since a decimal digit can be as large as 9, four bits are required to code each digit .

Decimal/BCD conversion.

Decimal Digit	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

BINARY CODED DECIMAL

8	7	4	(decimal)
↓	↓	↓	
1000	0111	0100	(BCD)

$$\underbrace{0110}_{6}\ \underbrace{1000}_{8}\ \underbrace{0011}_{3}\ \underbrace{1001}_{9}$$

If any of the “forbidden” four bit numbers ever occurs in a machine using the BCD code, it is usually an indication that an error has occurred.

$$\underbrace{0111}_{7}\ \underset{\downarrow}{1100}\ \underbrace{0001}_{1}$$

The forbidden code group indicates an error in the BCD number

COMPARISON OF BCD AND BINARY

BCD is not another number system like binary , octal and hexadecimal .

It is the decimal system with each digit encoded in its binary equivalent.

$$137_{10} = 10001001_2 \quad \text{(binary)}$$
$$137_{10} = 0001\ 0011\ 0111 \quad \text{(BCD)}$$

Advantage:

Easy conversion

Disadvantage:

BCD required more bits

Example:	Decimal	Binary	BCD
	11	***1011***	***0001 0001***

ADDITION OF BCD NUMBERS

Add the following BCD numbers:

(a) 0011 + 0100

(b) 00100011 + 00010101

(c) 10000110 + 00010011

(d) 010001010000 + 010000010111

Solution

The decimal number additions are shown for comparison.

(a)
$$\begin{array}{r} 0011 \\ +\,0100 \\ \hline \mathbf{0111} \end{array} \qquad \begin{array}{r} 3 \\ +\,4 \\ \hline 7 \end{array}$$

(b)
$$\begin{array}{rr} 0010 & 0011 \\ +\,0001 & 0101 \\ \hline \mathbf{0011} & \mathbf{1000} \end{array} \qquad \begin{array}{r} 23 \\ +\,15 \\ \hline 38 \end{array}$$

(c)
$$\begin{array}{rr} 1000 & 0110 \\ +\,0001 & 0011 \\ \hline \mathbf{1001} & \mathbf{1001} \end{array} \qquad \begin{array}{r} 86 \\ +\,13 \\ \hline 99 \end{array}$$

(d)
$$\begin{array}{rrr} 0100 & 0101 & 0000 \\ +\,0100 & 0001 & 0111 \\ \hline \mathbf{1000} & \mathbf{0110} & \mathbf{0111} \end{array} \qquad \begin{array}{r} 450 \\ +\,417 \\ \hline 867 \end{array}$$

Note that in each case the sum in any 4-bit column does not exceed 9, and the results are valid BCD numbers.

Related Problem

Add the BCD numbers: 1001000001000011 + 0000100100100101.

Addition of BCD numbers

NOTE THAT IN EACH CASE THE SUM IN ANY 4 BIT COLUMN DOES NOT EXCEED 9, AND THE RESULT ARE INVALID BCD NUMBERS.

Add the following BCD numbers:

(a) 1001 + 0100

(b) 1001 + 1001

(c) 00010110 + 00010101

(d) 01100111 + 01010011

Addition of BCD numbers

The decimal number additions are shown for comparison.

(a)

$$
\begin{array}{rrlr}
 & 1001 & & 9 \\
 & \underline{+\,0100} & & \underline{+4} \\
 & 1101 & \text{Invalid BCD number } (>9) & 13 \\
 & +\,0110 & \text{Add 6} & \\
\cline{1-2}
\underbrace{0001}_{\substack{\downarrow \\ 1}} & \underbrace{0011}_{\substack{\downarrow \\ 3}} & \text{Valid BCD number} &
\end{array}
$$

(b)

$$
\begin{array}{rrlr}
 & 1001 & & 9 \\
 & \underline{+\,1001} & & \underline{+\,9} \\
1 & 0010 & \text{Invalid because of carry} & 18 \\
 & +\,0110 & \text{Add 6} & \\
\cline{1-2}
\underbrace{0001}_{\substack{\downarrow \\ 1}} & \underbrace{1000}_{\substack{\downarrow \\ 8}} & \text{Valid BCD number} &
\end{array}
$$

Addition of BCD numbers

(c)	0001	0110		16
	+ 0001	0101		+ 15
	0010	1011	Right group is invalid (>9), left group is valid.	31
		+ 0110	Add 6 to invalid code. Add carry, 0001, to next group.	
	0011	0001	Valid BCD number	
	↓	↓		
	3	1		

(d)		0110	0111		67
		+ 0101	0011		+ 53
		1011	1010	Both groups are invalid (>9)	120
		+ 0110	+ 0110	Add 6 to both groups	
	0001	0010	0000	Valid BCD number	
	↓	↓	↓		
	1	2	0		

Related Problem

Add the BCD numbers: 01001000 + 00110100.