# Assignment # 2
## Differential Equations (MT-224)
### Date of Submission: 5th May, 2021
### Total marks: 6 (CLO-3)

Find the general solution of the following:                     **[marks: 16; weightage: 4]**

1.  $y'' + 4y' + 3y = 0$
    **Answer:** $y = c_1 e^{-x} + c_2 e^{-3x}$

2.  $y''' - y'' + y' - y = 0$
    **Answer:** $y = c_1 e^x + c_2 \sin x + c_3 \cos x$

3.  $2x^2 y'' + 3xy' - 15y = 0$,
    **Answer:** $y = C_1 x^{\frac{5}{2}} + C_2 x^{-3}$

4.  $y'' - 3y' + 2y = x^2 e^x$
    **Answer:** $y = c_1 e^x + c_2 e^{2x} - \frac{1}{3} x^3 e^x - x^2 e^x$

5.  $y'' + 4y = xe^x + x Sin 2x$
    **Answer:** $y = c_1 \sin 2x + c_2 \cos 2x + \frac{1}{5} xe^x - \frac{2}{25} e^x + \frac{1}{16} x Sin 2x - \frac{1}{8} x^2 \cos 2x$

6.  $y'' - 2y' + y = xe^x \ln x$
    **Answer:** $y = c_1 e^x + c_2 xe^x + e^x(\frac{-5}{36} x^3 + \frac{x^3}{6} \ln x)$

7.  $x^2 y'' - x(x + 2)y' + (x + 2)y = x^3$
    **Answer:** $y = c_1 x + c_2 xe^x - x - x^2$

8.  $y'' - 4y' - 12y = 2t^3 - t + 3$
    **Answer:** $y = C_1 e^{-2t} + C_2 e^{6t} - \frac{1}{6} t^3 + \frac{1}{6} t^2 - \frac{1}{9} t - \frac{5}{27}$

9.  $y'' + 5y' + 6y = 2x$
    **Answer:** $y = A e^{-2x} + B e^{-3x} + \frac{x}{3} - \frac{5}{18}$

10. $y'' + 5y' - 9y = e^{-2x} + 2 - x$
    **Answer:** $y = A e^{1.40x} + B e^{-6.40x} - \frac{1}{15} e^{-2x} + \frac{1}{9} x - \frac{13}{81}$

11. $y'' - 100y = 9t^2 e^{10t} + \cos t - t \sin t$
    **Answer:** $y = C_1 e^{10t} + C_2 e^{-10t} + \frac{t \sin t}{101} + \frac{3t^3}{20} e^{10t} - \frac{9t^2}{400} e^{10t} + \frac{9t}{4000} e^{10t} - \frac{99 \cos t}{10201}$

12. $y'' - 2y' + 2y = e^x \tan x$
    **Answer:** $y = e^x(C_1 \cos x + C_2 \sin x) - e^x \cos x \ln(\sec x + \tan x)$

13. $x^2 y'' - 4xy' + 6y = 2x^4 + x^2$
    **Answer:** $y = C_1 x^2 + C_2 x^3 + x^4 = x^2 \ln x$

14. $x^2 y'' + 10xy' + 8y = x^2$
    **Answer:** $y = C_1 x^{-1} + C_2 x^{-8} + \frac{1}{50} x^2$

15. $x^2 y'' - 3xy' + 13y = 4 + 3x$
    **Answer:** $y = x^2[C_1 \cos(3 \ln x) + C_2 \sin(3 \ln x)] + \frac{4}{13} + \frac{3}{10} x$

16. $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 3 + \ln x^3$
    **Answer:** $y = C_1 x + C_2 x^2 + C_3 x^3 - \frac{\ln x}{2} - \frac{17}{12}$

**Solve the following initial value problems** [marks: 4; weightage: 2]

17. $y'' - 2y' + y = \frac{1}{x}e^x$, $\qquad\qquad\qquad$ $y(1) = 0, y'(1) = 1$

**Answer:** $y = e^{x-1}(e-1)(1-x) + xe^x \ln x$

18. $y'' + 4y = Sin^2 2x$ $\qquad\qquad\qquad\qquad$ $y(1) = 0, y'(0) = 0$

**Answer:** $y = -\frac{1}{6}cos2x + \frac{1}{6}Cos^2 2x + \frac{1}{12}Sin^2 2x$

19. $y'' - 6y' - 7y = -9e^{-2x}$ $\qquad\qquad\qquad$ $y(0) = -2, y'(0) = -13$

**Answer:** $y = e^{-x} - 2e^{7x} - e^{-2x}$

20. $y'' - 4y' + 4y = 2e^{2x} - 12cos3x - 5sin3x$ $\quad$ $y(0) = -2, y'(0) = 4$

**Answer:** $y = -2e^{2x} + 5xe^{2x} + x2 e^{2x} + sin 3x$