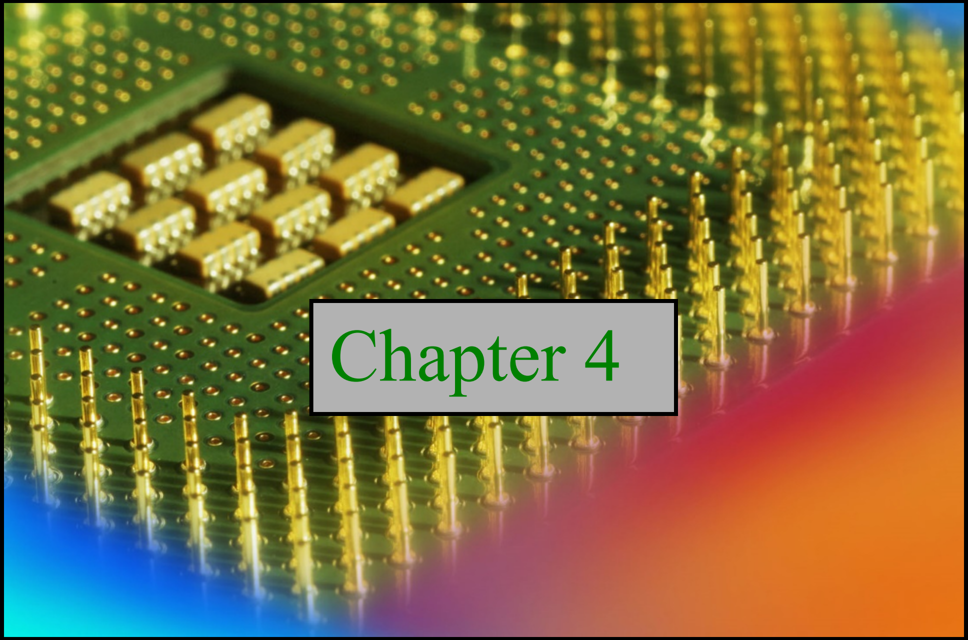
Digital Fundamentals

Tenth Edition

Floyd



Summary

Boolean Addition

In Boolean algebra, a **variable** is a symbol used to represent an action, a condition, or data. A single variable can only have a value of 1 or 0.

The **complement** represents the inverse of a variable and is indicated with an overbar. Thus, the complement of A is $\overline{A}$.

A **literal** is a variable or its complement.

Addition is equivalent to the OR operation. The sum term is 1 if one or more if the literals are 1. The sum term is zero only if each literal is 0.

Example Determine the values of A, B, and C that make the sum term of the expression $\overline{A} + B + \overline{C} = 0$?

Solution Each literal must = 0; therefore $A = 1$, $B = 0$ and $C = 1$.

Summary

Boolean Multiplication

In Boolean algebra, multiplication is equivalent to the AND operation. The product of literals forms a product term. The product term will be 1 only if all of the literals are 1.

Example What are the values of the A, B and C if the product term of $A \cdot \overline{B} \cdot \overline{C} = 1$?

Solution Each literal must = 1; therefore $A = 1$, $B = 0$ and $C = 0$.

Summary

Commutative Laws

The **commutative laws** are applied to addition and multiplication. For addition, the commutative law states

In terms of the result, the order in which variables are ORed makes no difference.

$$A + B = B + A$$

For multiplication, the commutative law states

In terms of the result, the order in which variables are ANDed makes no difference.

$$AB = BA$$

Summary

Associative Laws

The **associative laws** are also applied to addition and multiplication. For addition, the associative law states

When ORing more than two variables, the result is the same regardless of the grouping of the variables.

$$A + (B + C) = (A + B) + C$$

For multiplication, the associative law states

When ANDing more than two variables, the result is the same regardless of the grouping of the variables.

$$A(BC) = (AB)C$$

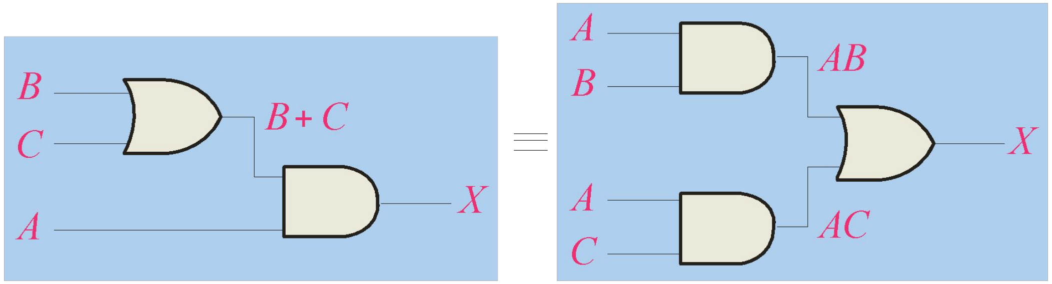
Summary

Distributive Law

The **distributive law** is the factoring law. A common variable can be factored from an expression just as in ordinary algebra. That is

$$\boxed{AB + AC = A(B + C)}$$

The distributive law can be illustrated with equivalent circuits:



$A(B + C)$ $\qquad$ $AB + AC$

Summary

Rules of Boolean Algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = \mathrm{A}$
5. $A + A = A$
6. $A + \overline{A} = 1$

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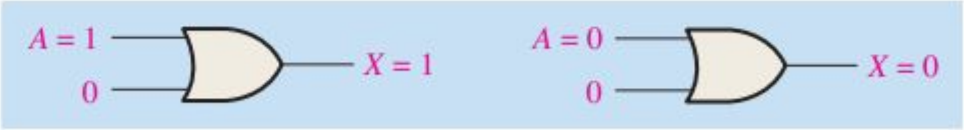
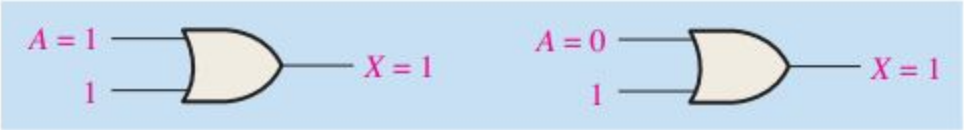
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Rule 1: $A + 0 = A$ A variable ORed with 0 is always equal to the variable. If the input variable A is 1, the output variable X is 1, which is equal to A. If A is 0, the output is 0, which is also equal to A. This rule is illustrated in Figure 4–8, where the lower input is fixed at 0.



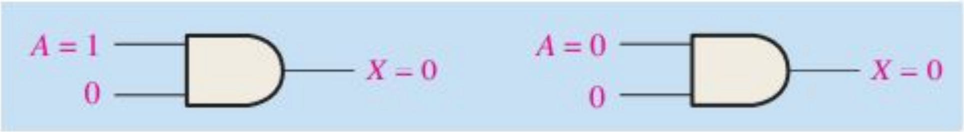
$X = A + 0 = A$

Rule 2: $A + 1 = 1$ A variable ORed with 1 is always equal to 1. A 1 on an input to an OR gate produces a 1 on the output, regardless of the value of the variable on the other input. This rule is illustrated in Figure 4–9, where the lower input is fixed at 1.



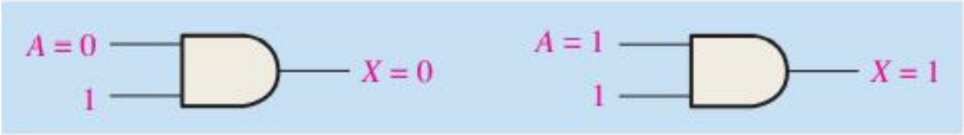
$X = A + 1 = 1$

Rule 3: $A \cdot 0 = 0$ A variable ANDed with 0 is always equal to 0. Any time one input to an AND gate is 0, the output is 0, regardless of the value of the variable on the other input. This rule is illustrated in Figure 4–10, where the lower input is fixed at 0.



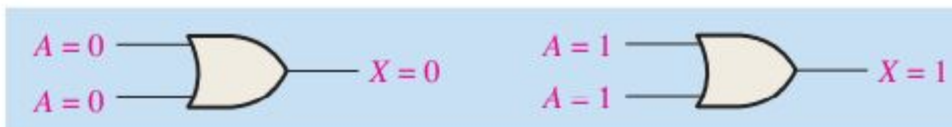
$X = A \cdot 0 = 0$

Rule 4: $A \cdot 1 = A$ A variable ANDed with 1 is always equal to the variable. If A is 0, the output of the AND gate is 0. If A is 1, the output of the AND gate is 1 because both inputs are now 1s. This rule is shown in Figure 4–11, where the lower input is fixed at 1.



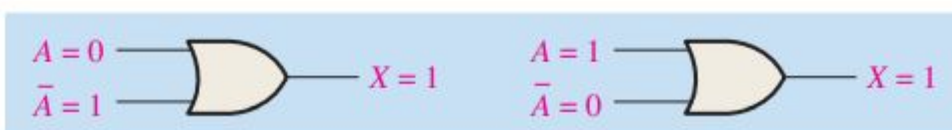
$X = A \cdot 1 = A$

Rule 5: $A + A = A$ A variable ORed with itself is always equal to the variable. If A is 0, then $0 + 0 = 0$; and if A is 1, then $1 + 1 = 1$. This is shown in Figure 4–12, where both inputs are the same variable.



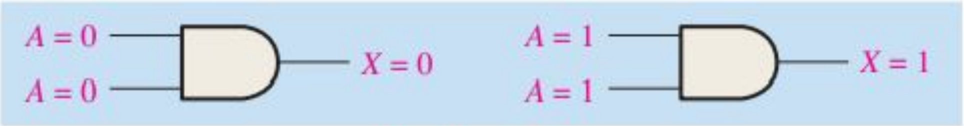
$X = A + A = A$

Rule 6: $A + \bar{A} = 1$ A variable ORed with its complement is always equal to 1. If A is 0, then $0 + \bar{0} = 0 + 1 = 1$. If A is 1, then $1 + \bar{1} = 1 + 0 = 1$. See Figure 4–13, where one input is the complement of the other.



$X = A + \bar{A} = 1$

Rule 7: $A \cdot A = A$ A variable ANDed with itself is always equal to the variable. If $A = 0$, then $0 \cdot 0 = 0$; and if $A = 1$, then $1 \cdot 1 = 1$. Figure 4–14 illustrates this rule.

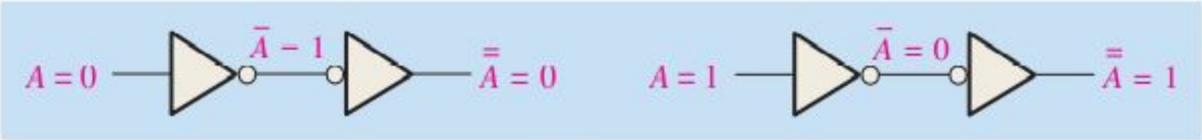


$X = A \bullet A = A$

Rule 8: $A \cdot \overline{A} = 0$ A variable ANDed with its complement is always equal to 0. Either A or $\overline{A}$ will always be 0; and when a 0 is applied to the input of an AND gate, the output will be 0 also. Figure 4–15 illustrates this rule.

$X = A \bullet \overline{A} = 0$

Rule 9: $\overline{\overline{A}} = A$ The double complement of a variable is always equal to the variable. If you start with the variable A and complement (invert) it once, you get $\overline{A}$. If you then take $\overline{A}$ and complement (invert) it, you get A, which is the original variable. This rule is shown in Figure 4–16 using inverters.



$\overline{\overline{A}} = A$

Rule 10: $A + AB = A$ This rule can be proved by applying the distributive law, rule 2, and rule 4 as follows:

$$
\begin{aligned}
A + AB &= A \cdot 1 + AB = A(1 + B) && \text{Factoring (distributive law)} \\
&= A \cdot 1 && \text{Rule 2: } (1 + B) = 1 \\
&= A && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

Rule 11: $A + \overline{A}B = A + B$ This rule can be proved as follows:

$$
\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B && \text{Rule 10: } A = A + AB \\
&= (AA + AB) + \overline{A}B && \text{Rule 7: } A = AA \\
&= AA + AB + A\overline{A} + \overline{A}B && \text{Rule 8: adding } A\overline{A} = 0 \\
&= (A + \overline{A})(A + B) && \text{Factoring} \\
&= 1 \cdot (A + B) && \text{Rule 6: } A + \overline{A} = 1 \\
&= A + B && \text{Rule 4: drop the 1}
\end{aligned}
$$

Rule 12: $(A + B)(A + C) = A + BC$ This rule can be proved as follows:

$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= A + AC + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\
&= A(1 + B) + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\
&= A + BC && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

Summary

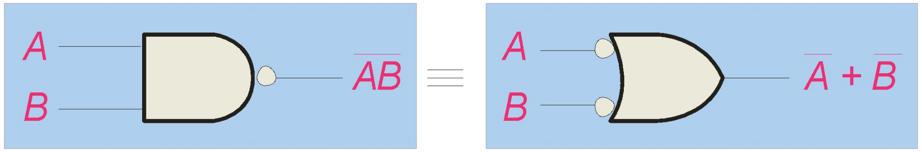
DeMorgan's Theorem

DeMorgan's 1st Theorem

The complement of a product of variables is equal to the sum of the complemented variables.

$$\overline{AB} = \overline{A} + \overline{B}$$

Applying DeMorgan's first theorem to gates:



NAND Negative-OR

Inputs		Output	
A	B	$\overline{AB}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

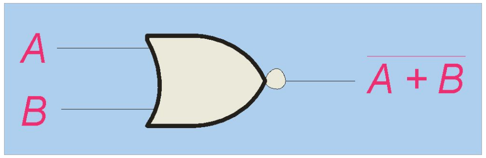
DeMorgan's Theorem

DeMorgan's 2nd Theorem

The complement of a sum of variables is equal to the product of the complemented variables.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

Applying DeMorgan's second theorem to gates:



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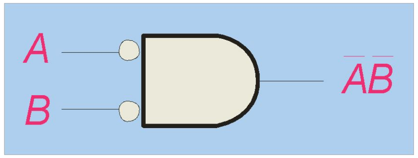


Negative-AND

Inputs		Output	
A	B	$\overline{A + B}$	$\overline{A}\,\overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Summary

DeMorgan's Theorem

Example Apply DeMorgan's theorem to remove the overbar covering both terms from the expression $X = \overline{\overline{C} + D}$.

Solution To apply DeMorgan's theorem to the expression, you can break the overbar covering both terms and change the sign between the terms. This results in $X = \overline{\overline{C}} \cdot \overline{D}$. Deleting the double bar gives $X = C \cdot \overline{D}$.

EXERCISE

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorem to the expression $\overline{\overline{X} + \overline{Y} + \overline{Z}}$.

Apply DeMorgan's theorem to the expression $\overline{\overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}}$.

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + B\overline{C}} + D(\overline{E + \overline{F}})}$$

Step 1. Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable. Let $\overline{A + B\overline{C}} = X$ and $D(\overline{E + \overline{F}}) = Y$.

Step 2. Since $\overline{X + Y} = \overline{X}\,\overline{Y}$,

$$\overline{(\overline{A + B\overline{C}}) + (D(\overline{E + \overline{F}}))} = (\overline{\overline{A + B\overline{C}}})(\overline{D(\overline{E + \overline{F}})})$$

Step 3. Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$(\overline{\overline{A + B\overline{C}}})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D(\overline{E + \overline{F}})})$$

Step 4. Applying DeMorgan's theorem to the second term,

$$(A + B\overline{C})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$$

Step 5. Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the $E + \overline{F}$ part of the term.

$$(A + B\overline{C})(\overline{D} + \overline{\overline{E + \overline{F}}}) = (A + B\overline{C})(\overline{D} + E + \overline{F})$$

The following three examples will further illustrate how to use DeMorgan's theorems.

EXAMPLE 4–5

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(\overline{A + B + C})D}$ **(b)** $\overline{ABC + DEF}$ **(c)** $\overline{A\overline{B} + \overline{C}D + EF}$

Solution

(a) Let $A + B + C = X$ and $D = Y$. The expression $\overline{(\overline{A + B + C})D}$ is of the form $\overline{XY} = \overline{X} + \overline{Y}$ and can be rewritten as

$$\overline{(\overline{A + B + C})D} = \overline{\overline{A + B + C}} + \overline{D}$$

Next, apply DeMorgan's theorem to the term $\overline{\overline{A + B + C}}$.

$$\overline{\overline{A + B + C}} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

(b) Let $ABC = X$ and $DEF = Y$. The expression $\overline{ABC + DEF}$ is of the form $\overline{X + Y} = \overline{X}\,\overline{Y}$ and can be rewritten as

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{ABC}$ and $\overline{DEF}$.

$$(\overline{ABC})(\overline{DEF}) = (\overline{A} + \overline{B} + \overline{C})(\overline{D} + \overline{E} + \overline{F})$$

(c) Let $A\overline{B} = X$, $\overline{C}D = Y$, and $EF = Z$. The expression $\overline{A\overline{B} + \overline{C}D + EF}$ is of the form $\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$ and can be rewritten as

$$\overline{A\overline{B} + \overline{C}D + EF} = (\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{A\overline{B}}$, $\overline{\overline{C}D}$, and $\overline{EF}$.

$$(\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF}) = (\overline{A} + B)(C + \overline{D})(\overline{E} + \overline{F})$$

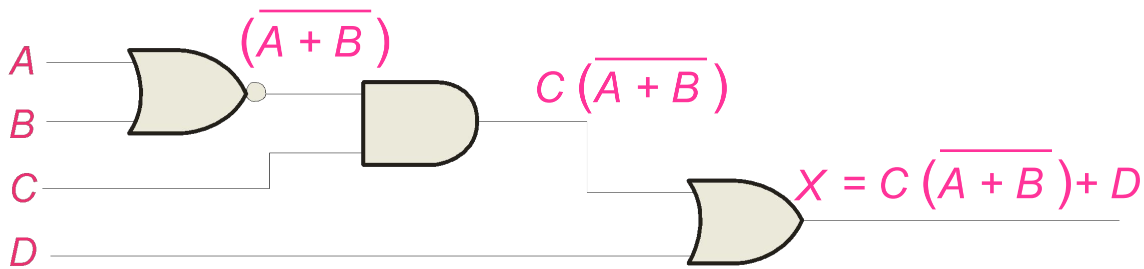
Summary

Boolean Analysis of Logic Circuits

Combinational logic circuits can be analyzed by writing the expression for each gate and combining the expressions according to the rules for Boolean algebra.

Example Apply Boolean algebra to derive the expression for *X*.

Solution Write the expression for each gate:



Applying DeMorgan's theorem and the distribution law:

$$X = C(\overline{A}\,\overline{B}) + D = \overline{A}\,\overline{B}\,C + D$$

EXAMPLE 4–8

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

Solution The following is not necessarily the only approach.

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 ($AB + AB = AB$) to the first two terms.

$$AB + AC + B + BC$$

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B$$

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

$$B + AC$$

Related Problem Simplify the Boolean expression $A\overline{B} + A(\overline{B + C}) + B(\overline{B + C})$.

EXAMPLE 4–9

Simplify the following Boolean expression:

$$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$$

Note that brackets and parentheses mean the same thing: the term inside is multiplied (ANDed) with the term outside.

Solution

Step 1: Apply the distributive law to the terms within the brackets.

$$(A\overline{B}C + A\overline{B}BD + \overline{A}\,\overline{B})C$$

Step 2: Apply rule 8 ($\overline{B}B = 0$) to the second term within the parentheses.

$$(A\overline{B}C + A \cdot 0 \cdot D + \overline{A}\,\overline{B})C$$

Step 3: Apply rule 3 ($A \cdot 0 \cdot D = 0$) to the second term within the parentheses.

$$(A\overline{B}C + 0 + \overline{A}\,\overline{B})C$$

Step 4: Apply rule 1 (drop the 0) within the parentheses.

$$(A\overline{B}C + \overline{A}\,\overline{B})C$$

Step 5: Apply the distributive law.

$$A\overline{B}CC + \overline{A}\,\overline{B}C$$

Step 6: Apply rule 7 ($CC = C$) to the first term.

$$A\overline{B}C + \overline{A}\,\overline{B}C$$

Step 7: Factor out $\overline{B}C$.

$$\overline{B}C(A + \overline{A})$$

Step 8: Apply rule 6 ($A + \overline{A} = 1$).

$$\overline{B}C \cdot 1$$

Step 9: Apply rule 4 (drop the 1).

$$\overline{B}C$$

EXAMPLE 4–11

Simplify the following Boolean expression:

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

Solution **Step 1:** Apply DeMorgan's theorem to the first term.

$$(\overline{AB})(\overline{AC}) + \overline{A}\,\overline{B}C$$

Step 2: Apply DeMorgan's theorem to each term in parentheses.

$$(\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C$$

Step 3: Apply the distributive law to the two terms in parentheses.

$$\overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

Step 4: Apply rule 7 ($\overline{A}\,\overline{A} = \overline{A}$) to the first term, and apply rule 10 [$\overline{A}\,\overline{B} + \overline{A}\,\overline{B}C = \overline{A}\,\overline{B}(1 + C) = \overline{A}\,\overline{B}$] to the third and last terms.

$$\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Step 5: Apply rule 10 [$\overline{A} + \overline{A}\,\overline{C} = \overline{A}(1 + \overline{C}) = \overline{A}$] to the first and second terms.

$$\overline{A} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Step 6: Apply rule 10 [$\overline{A} + \overline{A}\,\overline{B} = \overline{A}(1 + \overline{B}) = \overline{A}$] to the first and second terms.

$$\overline{A} + \overline{B}\,\overline{C}$$

SOP and POS forms

Boolean expressions can be written in the **sum-of-products** form (**SOP**) or in the **product-of-sums** form (**POS**). These forms can simplify the implementation of combinational logic, particularly with PLDs. In both forms, an overbar cannot extend over more than one variable.

An expression is in SOP form when two or more product terms are summed as in the following examples:

$\overline{A}\,\overline{B}\,\overline{C} + A\,B$ $\quad$ $A\,B\,C + \overline{C}\,D$ $\quad$ $C\,D + E$

An expression is in POS form when two or more sum terms are multiplied as in the following examples:

$(A + B)(\overline{A} + C)$ $\quad$ $(A + B + C)(\overline{B} + D)$ $\quad$ $(A + B)C$

EXAMPLE 4–12

Convert each of the following Boolean expressions to SOP form:

(a) $AB + B(CD + EF)$ **(b)** $(A + B)(B + C + D)$ **(c)** $\overline{(A + B) + C}$

Solution

(a) $AB + B(CD + EF) = AB + BCD + BEF$

(b) $(A + B)(B + C + D) = AB + AC + AD + BB + BC + BD$

(c) $\overline{(\overline{A + B}) + C} = (\overline{\overline{A + B}})\overline{C} = (A + B)\overline{C} = A\overline{C} + B\overline{C}$

Summary

SOP Standard form

In **SOP standard form**, every variable in the domain must appear in each term. This form is useful for constructing truth tables or for implementing logic in PLDs.

You can expand a nonstandard term to standard form by multiplying the term by a term consisting of the sum of the missing variable and its complement.

Example

Convert $X = \overline{A}\,\overline{B} + A\,B\,C$ to standard form.

Solution

The first term does not include the variable C. Therefore, multiply it by the $(C + \overline{C})$, which = 1:

$$X = \overline{A}\,\overline{B}\,(C + \overline{C}) + A\,B\,C$$
$$= \overline{A}\,\overline{B}\,C + \overline{A}\,\overline{B}\,\overline{C} + A\,B\,C$$

EXAMPLE 4–13

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Solution The domain of this SOP expression is A, B, C, D. Take one term at a time. The first term, $A\overline{B}C$, is missing variable D or $\overline{D}$, so multiply the first term by $D + \overline{D}$ as follows:

$$A\overline{B}C = A\overline{B}C(D + \overline{D}) = A\overline{B}CD + A\overline{B}C\overline{D}$$

In this case, two standard product terms are the result.

The second term, $\overline{A}\,\overline{B}$, is missing variables C or $\overline{C}$ and D or $\overline{D}$, so first multiply the second term by $C + \overline{C}$ as follows:

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C + \overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

The two resulting terms are missing variable D or $\overline{D}$, so multiply both terms by $D + \overline{D}$ as follows:

$$\begin{aligned}\overline{A}\,\overline{B} &= \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} = \overline{A}\,\overline{B}C(D + \overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D + \overline{D})\\ &= \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\end{aligned}$$

In this case, four standard product terms are the result.

The third term, $AB\overline{C}D$, is already in standard form. The complete standard SOP form of the original expression is as follows:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

Summary

POS Standard form

In **POS standard form**, every variable in the domain must appear in each sum term of the expression.

You can expand a nonstandard POS expression to standard form by adding the product of the missing variable and its complement and applying rule 12, which states that $(A + B)(A + C) = A + BC$.

Example Convert $X = (\overline{A} + \overline{B})(A + B + C)$ to standard form.

Solution The first sum term does not include the variable C. Therefore, add $C\overline{C}$ and expand the result by rule 12.

$$X = (\overline{A} + \overline{B} + C\overline{C})(A + B + C)$$

$$= (\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})(A + B + C)$$

EXAMPLE 4-15

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Solution The domain of this POS expression is A, B, C, D. Take one term at a time. The first term, $A + \overline{B} + C$, is missing variable D or $\overline{D}$, so add $D\overline{D}$ and apply rule 12 as follows:

$$A + \overline{B} + C = A + \overline{B} + C + D\overline{D} = (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})$$

The second term, $\overline{B} + C + \overline{D}$, is missing variable A or $\overline{A}$, so add $A\overline{A}$ and apply rule 12 as follows:

$$\overline{B} + C + \overline{D} = \overline{B} + C + \overline{D} + A\overline{A} = (A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})$$

The third term, $A + \overline{B} + \overline{C} + D$, is already in standard form. The standard POS form of the original expression is as follows:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D) =$$
$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Summary

Converting SOP to POS

Step 1. Evaluate each product term in the SOP expression. That is, determine the binary numbers that represent the product terms.

Step 2. Determine all of the binary numbers not included in the evaluation in Step 1.

Step 3. Write the equivalent sum term for each binary number from Step 2 and express in POS form.

EXAMPLE 4–17

Convert the following SOP expression to an equivalent POS expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

Solution The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

Remember, these are the binary values that make the sum term 0. The equivalent POS expression is

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

Summary

TABLE 4–8

INPUTS			OUTPUT
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Solution There are four 1s in the output column and the corresponding binary values are 011, 100, 110, and 111. Convert these binary values to product terms as follows:

$$011 \longrightarrow \overline{A}BC$$
$$100 \longrightarrow A\overline{B}\,\overline{C}$$
$$110 \longrightarrow AB\overline{C}$$
$$111 \longrightarrow ABC$$

The resulting standard SOP expression for the output X is

$$X = \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

For the POS expression, the output is 0 for binary values 000, 001, 010, and 101. Convert these binary values to sum terms as follows:

$$000 \longrightarrow A + B + C$$
$$001 \longrightarrow A + B + \overline{C}$$
$$010 \longrightarrow A + \overline{B} + C$$
$$101 \longrightarrow \overline{A} + B + \overline{C}$$

The resulting standard POS expression for the output X is

$$X = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + \overline{C})$$

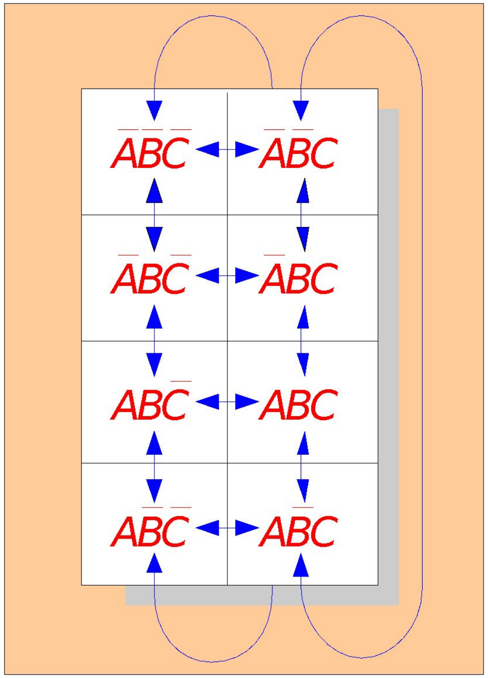
Summary

Karnaugh maps

The Karnaugh map (K-map) is a tool for simplifying combinational logic with 3 or 4 variables. For 3 variables, 8 cells are required (2^3).

The map shown is for three variables labeled *A, B,* and *C*. Each cell represents one possible product term.

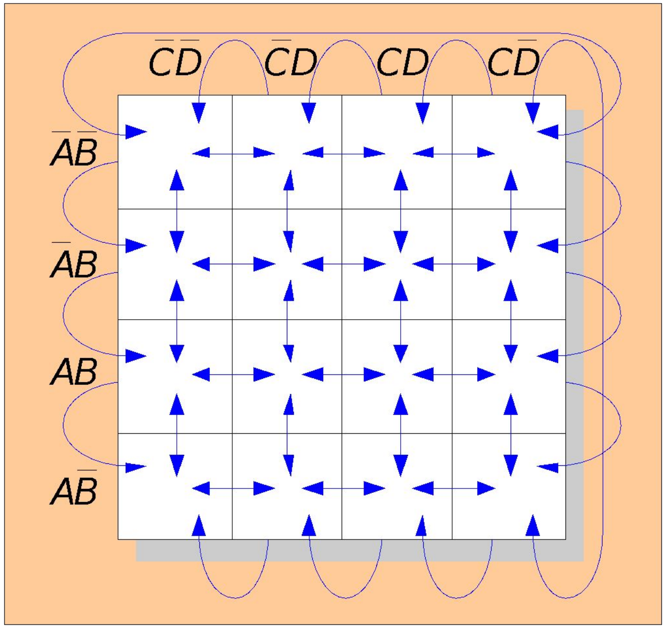
Each cell differs from an adjacent cell by only one variable.



Summary

Karnaugh maps

A 4-variable map has an adjacent cell on each of its four boundaries as shown.



Each cell is different only by one variable from an adjacent cell.

Grouping follows the rules given in the text.

The following slide shows an example of reading a four variable map using binary numbers for the variables…

Summary

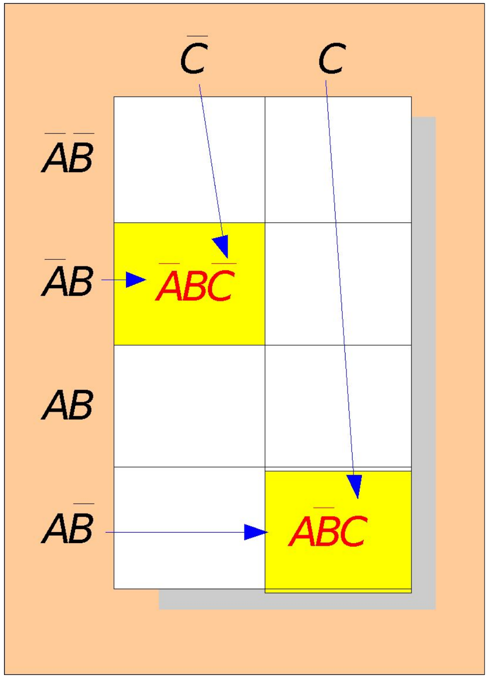
Karnaugh maps

Alternatively, cells can be labeled with the variable letters. This makes it simple to read, but it takes more time preparing the map.

Example Read the terms for the yellow cells.

Solution

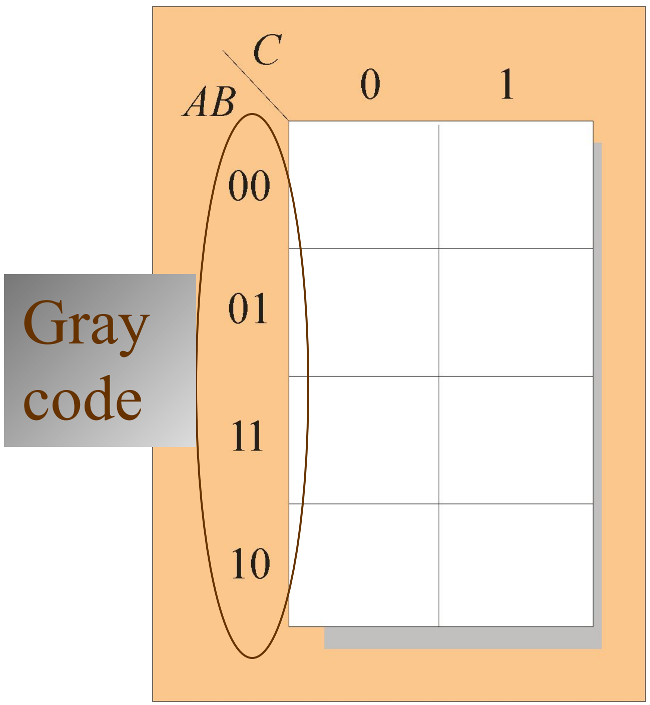
The cells are $\overline{A}B\overline{C}$ and $A\overline{B}C$.



Summary

Karnaugh maps

Cells are usually labeled using 0's and 1's to represent the variable and its complement.



The numbers are entered in gray code, to force adjacent cells to be different by only one variable.

Ones are read as the true variable and zeros are read as the complemented variable.

Summary

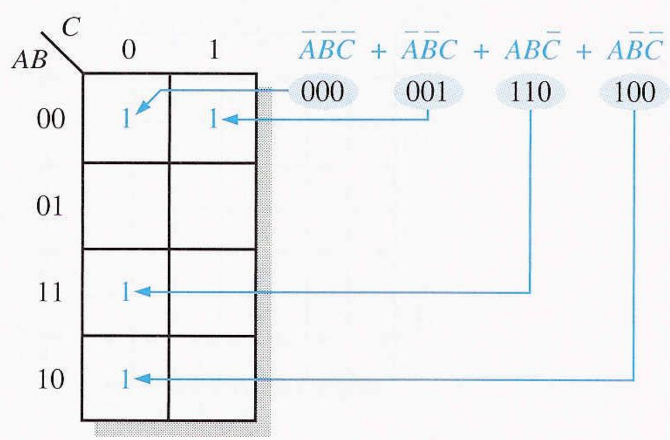
Mapping an SOP expression in K-Map

Step 1. Determine the binary value of each product term in the standard SOP expression. After some practice, you can usually do the evaluation of terms mentally.

Step 2. As each product term is evaluated, place a 1 on the Karnaugh map in the cell having the same value as the product term.

FIGURE 4–24

Example of mapping a standard SOP expression.



Summary

EXAMPLE 4–22

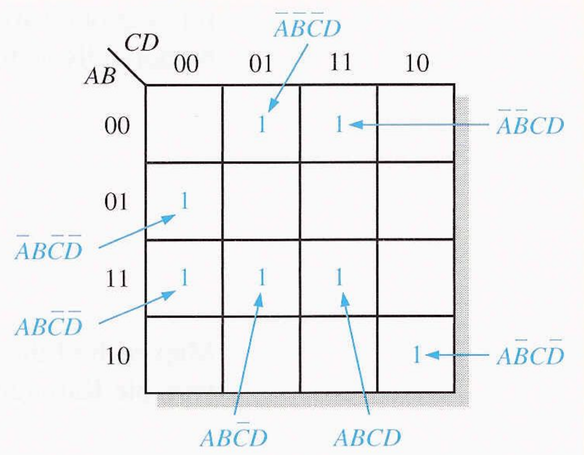
Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

Solution Evaluate the expression as shown below. Place a 1 on the 4-variable Karnaugh map in Figure 4–26 for each standard product term in the expression.

$\overline{A}\,\overline{B}CD$	$\overline{A}B\overline{C}\,\overline{D}$	$AB\overline{C}D$	$ABCD$	$AB\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}D$	$A\overline{B}C\overline{D}$
0011	0100	1101	1111	1100	0001	1010

▶ **FIGURE 4–26**

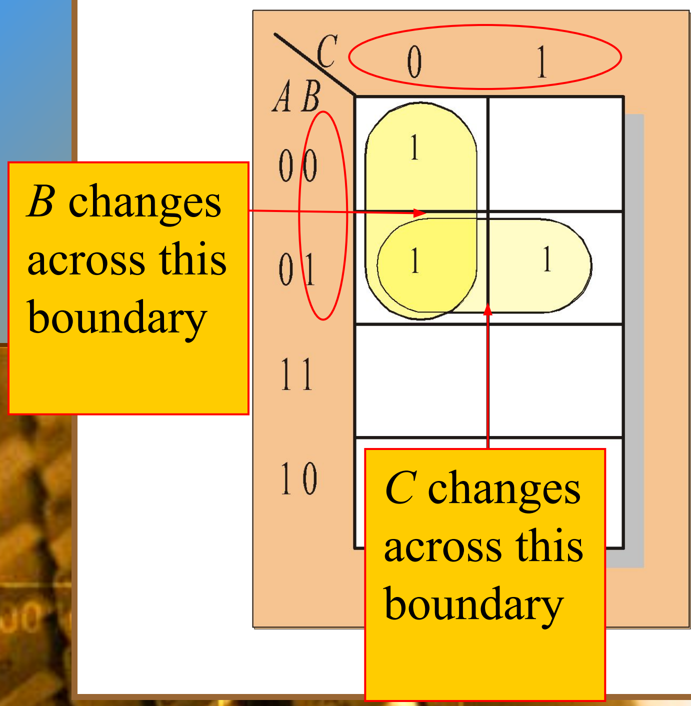


Summary

Karnaugh maps

K-maps can simplify combinational logic by grouping cells and eliminating variables that change.

Example Group the 1's on the map and read the minimum logic.



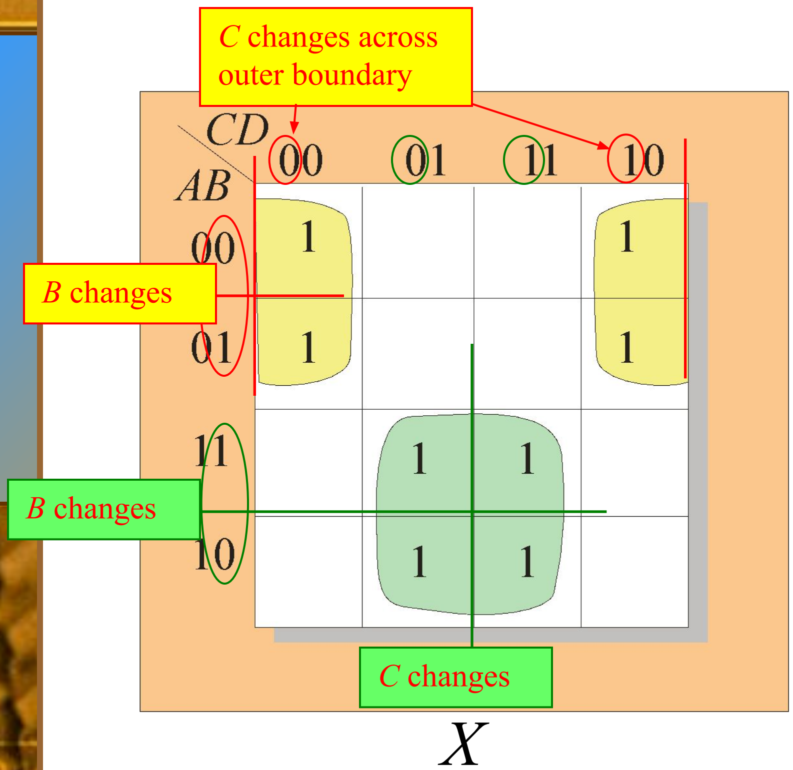
Solution

1. Group the 1's into two overlapping groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The vertical group is read $\overline{A}\overline{C}$.
4. The horizontal group is read $\overline{A}B$.

$$X = \overline{A}\overline{C} + \overline{A}B$$

Karnaugh maps

Example Group the 1's on the map and read the minimum logic.



Solution

1. Group the 1's into two separate groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The upper (yellow) group is read as $\overline{A}\overline{D}$.
4. The lower (green) group is read as AD.

$$X = \overline{A}\overline{D} + AD$$

EXAMPLE 4–27

Determine the product terms for each of the Karnaugh maps in Figure 4–32 and write the resulting minimum SOP expression.

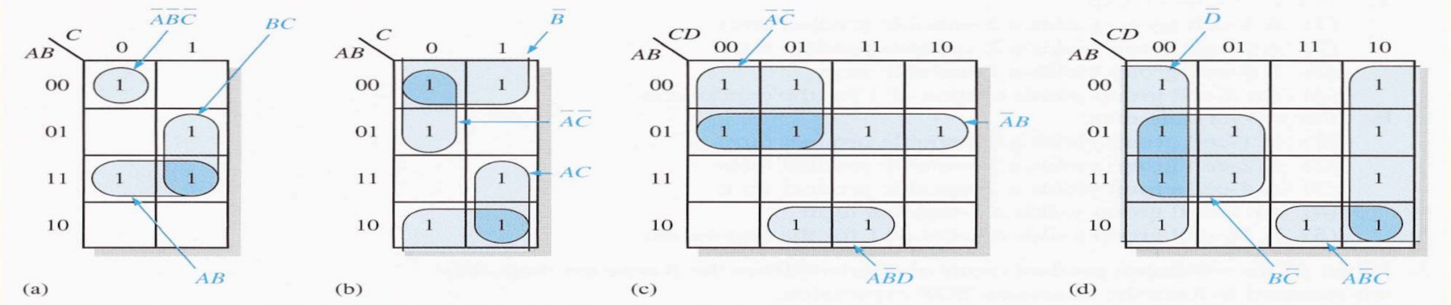


FIGURE 4–32

Solution The resulting minimum product term for each group is shown in Figure 4–32. The minimum SOP expressions for each of the Karnaugh maps in the figure are

(a) $AB + BC + \overline{A}\,\overline{B}\,\overline{C}$ (b) $\overline{B} + \overline{A}\,\overline{C} + AC$

(c) $\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$ (d) $\overline{D} + A\overline{B}C + B\overline{C}$

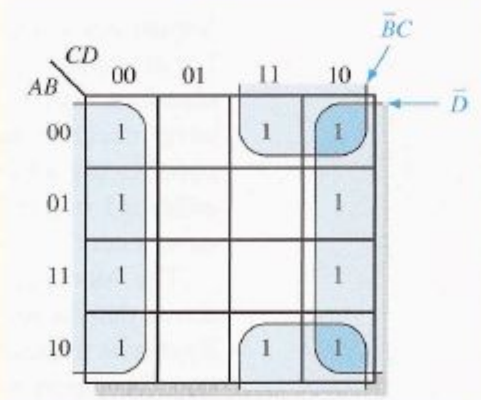
EXAMPLE 4–29

Use a Karnaugh map to minimize the following SOP expression:

$$\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D}$$

Solution The first term $\bar{B}\bar{C}\bar{D}$ must be expanded into $A\bar{B}\bar{C}\bar{D}$ and $\bar{A}\bar{B}\bar{C}\bar{D}$ to get the standard SOP expression, which is then mapped; and the cells are grouped as shown in Figure 4–34.

▶ FIGURE 4–34



Notice that both groups exhibit "wrap around" adjacency. The group of eight is formed because the cells in the outer columns are adjacent. The group of four is formed to pick up the remaining two 1s because the top and bottom cells are adjacent. The product term for each group is shown. The resulting minimum SOP expression is

$$\bar{D} + \bar{B}C$$

Summary

Example

Use K-Map To simplify the given Boolean Expression

$$F\ (A, B, C, D) = \Sigma\ (0, 1,2\ 4,5,7,11,15)$$

Summary

Mapping a Standard POS Expression

EXAMPLE 4–30

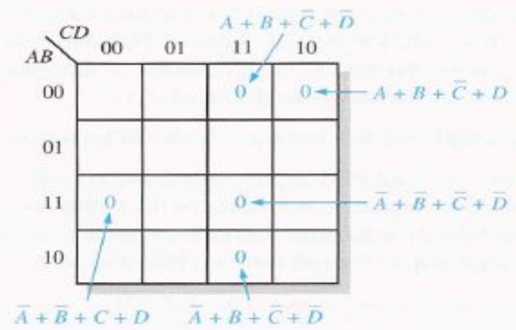
Map the following standard POS expression on a Karnaugh map:

$$(\overline{A} + \overline{B} + C + D)(\overline{A} + B + \overline{C} + \overline{D})(A + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + \overline{C} + \overline{D})$$

Solution Evaluate the expression as shown below and place a 0 on the 4-variable Karnaugh map in Figure 4–38 for each standard sum term in the expression.

$$\underset{1100}{(\overline{A} + \overline{B} + C + D)}\underset{1011}{(\overline{A} + B + \overline{C} + \overline{D})}\underset{0010}{(A + B + \overline{C} + D)}\underset{1111}{(\overline{A} + \overline{B} + \overline{C} + \overline{D})}\underset{0011}{(A + B + \overline{C} + \overline{D})}$$

FIGURE 4–38

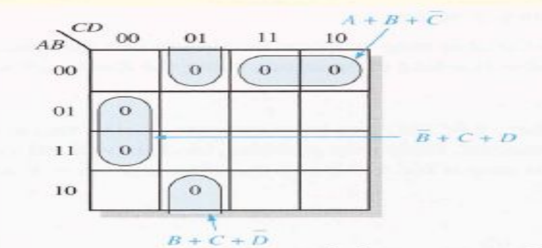


Convert POS to SOP Using K-Map

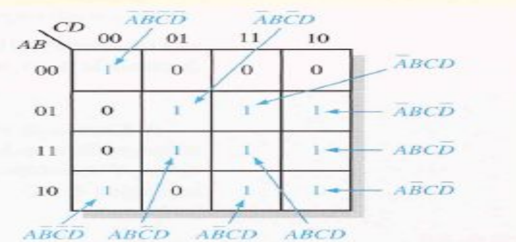
EXAMPLE 4–33

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

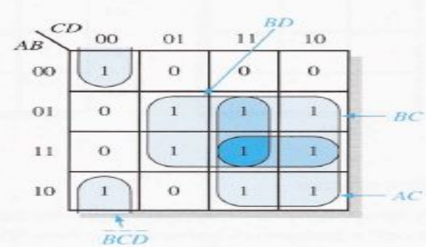
$$(\bar{A} + \bar{B} + C + D)(A + \bar{B} + C + D)(A + B + C + \bar{D})(A + B + \bar{C} + \bar{D})(\bar{A} + B + C + \bar{D})(A + B + \bar{C} + D)$$



(a) Minimum POS: $(A + B + C)(\bar{B} + C + D)(B + C + \bar{D})$



(b) Standard SOP:
$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}CD + ABCD$



(c) Minimum SOP: $AC + BC + BD + \bar{B}\bar{C}\bar{D}$

Selected Key Terms

Variable A symbol used to represent a logical quantity that can have a value of 1 or 0, usually designated by an italic letter.

Complement The inverse or opposite of a number. In Boolean algebra, the inverse function, expressed with a bar over the variable.

Sum term The Boolean sum of two or more literals equivalent to an OR operation.

Product term The Boolean product of two or more literals equivalent to an AND operation.

Selected Key Terms

Sum-of-products (SOP) A form of Boolean expression that is basically the ORing of ANDed terms.

Product of sums (POS) A form of Boolean expression that is basically the ANDing of ORed terms.

Karnaugh map An arrangement of cells representing combinations of literals in a Boolean expression and used for systematic simplification of the expression.

VHDL A standard hardware description language. IEEE Std. 1076-1993.