

Copying answers and steps are strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. The calculations and answers should be written neatly on paper which is attached as a single pdf. Box your answers where appropriate. Thanks!

**Problem 1**

Solve the following homogeneous linear differential equations with constant coefficient.

- (a)  $(D^4 + 6D^3 + 15D^2 + 20D + 12)y = 0$   
 (b)  $(D^3 - 27)y = 0$

**Problem 2**

Solve the differential equations by method of Undetermine Coefficient-Superposition Approach.

- (a)  $(D^2 - 7D + 12)y = e^{2x}(x^3 - 5x^2)$   
 (b)  $y'' + y' - 2y = x^2 + 2\sin x - e^{3x}$

**Problem 3**

Solve differential using Variation of Parameters.

- (a)  $(D^2 + 1)y = \csc x$   
 (b)  $(D^2 - 1)y = \frac{2}{1 + e^x}$

**Problem 4**

Solve the differential equations by method of Undetermine Coefficient-Annihilator Approach.

- (a)  $y'' + y' + \frac{1}{4}y = e^x(\sin 3x + \cos 3x)$   
 (b)  $y'' + 2y' + y = x^2e^{-x}$

**Problem 5**

Solve Cauchy Euler equation  $x^2y'' + xy' - y = x^3e^x$

**Bonus Problem 6**

- (a) Determine whether the the given set of fuction is linearly independent or linearly dependent on  $(-\infty, \infty)$ .  
 i.  $y_1 = \cos 2x, y_2 = 1, y_3 = \cos^2 x$   
 ii.  $y_1 = x, y_2 = x^{-2}, y_3 = x^2 \ln x$   
 (b) Solve differential equation using reduction of order , then varify by formulation  $y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$ .  
 i.  $9y'' - 12y' + 4y = 0, y_1 = e^{2x/3}$   
 ii.  $y'' - 3y' + 2y = 5e^{3x}, y_1 = e^x$

**Bonus Problem 7**

A state game commission releases 40 elk into a game refuge. After 5 years, the elk population is 104. The commission believes that the environment can support no more than 4000 elk. The growth rate of the elk population  $p$  is

$$\frac{dp}{dt} = kp \left( 1 - \frac{p}{4000} \right), \quad 40 \leq p \leq 4000$$

where  $t$  is the number of years.

- Write a model for the elk population in terms of  $t$ .
- Graph the slope field for the differential equation and the solution that passes through the point  $(0, 40)$ .
- Use the model to estimate the elk population after 15 years.
- Find the limit of the model as  $t \rightarrow \infty$ .