Macro Prelim Session: Dean's Material[1]

Stefano Lord-Medrano

University of Wisconsin-Madison

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[1] **Disclaimer:** This slides are an attempt to summarized the material covered by Dean in Econ 712. They do not replace the lecture notes and additional material provided by him.

Outline

How to prepare and general info

- Definitely will be a **brand new model**. Practice to be fluent in setting up a model.
  - Usually, Dean makes a question based on recent research interests.
- Know all the different ingredients Dean may put into an OG model
  - Sources for these: problem sets, lecture notes, discussions
- Some examples: money, private info, production, idiosyncratic uncertainty, government policy
- The best way to prepare for Dean question is **practice**. Memorizing answers is worthless, better to understand the procedures.
- It is useful to be explicit in your procedure: Lagrangian, FOC, efficiency conditions, market clearing, and assumptions
- **Sanity check:** If there are any parametric assumptions explicitly made by Dean, your results should use them

OG Models

- The planner's problem for the baseline OG model is given by

$$\max_{\{c_t^{t-1}, c_t^t\}} u(c_t^{t-1}) + u(c_t^t),$$

$$s.t. \quad c_t^{t-1} + c_t^t \leq w_1 \quad \text{(Resource constraint)}$$

$$c_t^{t-1}, c_t^t \geq 0 \quad \text{(Non-negativity)}$$

- Under which conditions the resource constraint will be binding and there will be an interior solution?
  - LNS preferences: $u'(c) > 0$
  - Inada's conditions: $\lim_{c \to 0} u'(c) \to \infty$ and $\lim_{c \to \infty} u'(c) \to 0$
- With Dean, it is always a good idea to try to rewrite the optimization problem as an unconstrained one
  - However, be careful when collapsing constraints into one constraint

OG Models (Cont.)

- We can decentralize the planner's solution by incorporating fiat money to the agent's problem

$$\max_{\{c_t^t, c_{t+1}^t, M_{t+1}^t\} \in \mathbb{R}^3} u(c_t^t) + \beta u(c_{t+1}^t),$$

$$\begin{aligned} s.t. \quad p_t c_t^t + M_{t+1}^t &\leq p_t w_1 \quad \text{(Young budget const.)}, \\ p_{t+1} c_{t+1}^t &\leq M_{t+1}^t \quad \text{(Old budget const.)}, \\ c_t^t, c_{t+1}^t &\geq 0 \quad \text{(Non-negativity)}, \\ M_{t+1}^t &\geq 0 \quad \text{(Non-negativity)}. \end{aligned}$$

- The lagrangian associated to the problem is given by

$$\begin{aligned} \mathcal{L}(c_t^t, c_{t+1}^t, M_{t+1}^t; \lambda_1, \lambda_2) = u(c_t^t) + \beta u(c_{t+1}^t) + \lambda_1 \left[p_t w_1 - p_t c_t - M_{t+1}^t \right] \\ + \lambda_2 \left[M_{t+1}^t - p_{t+1} c_{t+1}^t \right] \end{aligned}$$

OG Models (Cont.)

- A **Competitive Equilibrium with Fiat Money** is a sequence $\{c_t^t, c_{t+1}^t, M_{t+1}^t\}_{\forall t}$ and a price system $\{p_t\}_{\forall t}$ such that
  1. Given prices p_t, the sequence $\{c_t^t, c_{t+1}^t, M_{t+1}^t\}$ solves the problem of the young agent at period t
  2. Markets clear:

$$\underline{\text{Good's market clearing}}: \quad c_t^{t-1} + c_t^t = w_1,$$
$$\underline{\text{Money market clearing}}: \quad M_{t+1}^t = \bar{M}.$$

- Combine the first order conditions with the market clearing conditions to solve for the allocations and the price level

- Instead of an endowment economy, we can add endogenous labor supply and demand in the model
  - In this more general framework, what is the firm problem?, who is the owner of the firm?, what are profits in equilibrium given technology?

Multiple equilibria - Kehoe

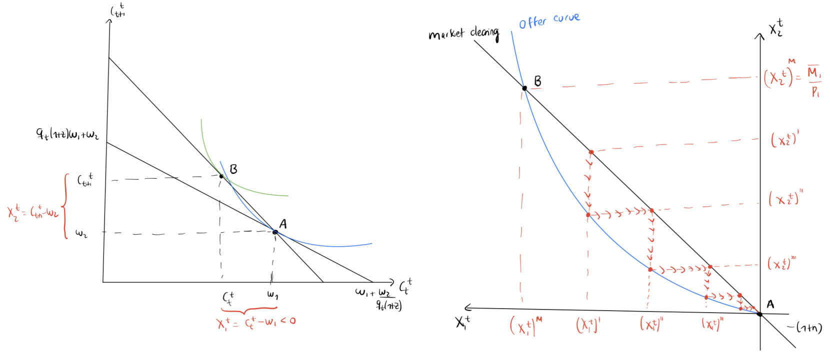
- A little bit more general framework as the decentralized economy: old agents have an endowment
- We get the intertemporal budget constraint by consolidating the sequential constraints through the holdings of nominal balances:

$$\begin{aligned} p_t c_t^t + p_{t+1} c_{t+1}^t &= p_t w_1 + p_{t+1} w_2, \\ \iff c_t^t + \frac{p_{t+1}}{p_t} c_{t+1}^t &= w_1 + \frac{p_{t+1}}{p_t} w_2, \\ \iff p_t \underbrace{\left(c_t^t - w_1\right)}_{\equiv x_1^t} + p_{t+1} \underbrace{\left(c_{t+1}^t - w_2\right)}_{\equiv x_2^t} &= 0, \end{aligned}$$

- From the intertemporal budget constraint and Euler's equation, we can trace the offer curve by allowing some changes in the relative prices $\frac{p_t}{p_{t+1}}$

Multiple equilibria - Kehoe (Cont.)

Figure: Offer curve



(a) Budget line (b) Offer curve and transitory dynamics

- The key result from Kehoe OG model is that we have a continuum of Pareto ranked equilibrium which are Pareto dominated by the monetary equilibrium (under the assumption that $w_1 > w_2$)

Multiple equilibria - Kehoe (Cont.)

- When defining the social planner's problem, we need to impose the overtaking criterion which allows us to have a well-defined objective function (Why?)
- The planners problem is given by

$$\max_{c_1^0, \{c_t^{t-1}, c_t^t\}_{t\geq 1}} u(c_1^0) + \sum_{t=1}^{\infty} u(c_t^t, c_{t+1}^t),$$
$$s.t. \quad c_t^{t-1} + c_t^t = w_1 + w_2, \quad \forall t.$$

- A good exercise for this type of problem is PS3 of last year

Investment

- A CE is a set of sequences for quantities $\{c_t^t, c_{t+1}^t, k_{t+1}^t\}_{t=0}^{\infty}, \{K_t, L_t, Y_t, \Pi_t\}_{t=0}^{\infty}$ and prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that:
  1. Given prices w_t, r_t, the sequences $\{c_t^t, c_{t+1}^t, k_{t+1}^t\}_{t=0}^{\infty}$ solve the problem of the young in generation t defined as:
  $$\max_{\{c_t^t, c_{t+1}^t, k_{t+1}^t\}} u(c_t^t) + \beta u(c_{t+1}^t)$$
  $$\text{s.t. } c_t^t + k_{t+1}^t = w_t$$
  $$c_{t+1}^t = (1 + r_{t+1})k_{t+1}^t + \Pi_{t+1}$$
  2. Given prices w_t, r_t, the sequences $\{K_t, L_t, Y_t\}_{t=0}^{\infty}$ (noting that $L_t = 1$ for all t) solve the static profit maximization problem of the firm defined as:
  $$\max_{\{K_t\}} F(K_t) - w_t - r_t K_t$$
  $$\text{s.t. } F(K_t) = k_t^{\alpha}$$
  3. Markets clear:
     1. Goods market: $c_t^t + c_t^{t-1} + K_{t+1} - (1 - \delta)K_t = F(K_t, 1) = Y_t$
     2. Capital market: $k_{t+1}^t = K_{t+1}$
     3. Labor market: $1 = L_t^s = L_t^d$
- The planners problem is to choose sequences for c_t^t, c_{t+1}^t and K_{t+1} such that it maximizes $\log(c_1^0) + \sum_{t=1}^{\infty}[\log(c_t^t) + \beta \log(c_{t+1}^t)]$ subject to the resource constraint

Investment (Cont.)

- From the previous problem and under the assumption that $u(\cdot) = \log(\cdot)$, we get the following solution:

$$c_t^t = \frac{w_t + \frac{\Pi_{t+1}}{1+r_{t+1}-\delta}}{1+\beta} \quad \text{and} \quad k_{t+1}^t = \frac{\beta w_t - \frac{\Pi_{t+1}}{1+r_{t+1}-\delta}}{1+\beta}$$

- Using our solutions to household and firm problems, along with market clearing, we find:

$$K_{t+1} = \frac{\beta(1-\alpha)(K_t)^\alpha}{1+\beta} \Rightarrow K_{SS}^{CE} = \left(\frac{\beta(1-\alpha)}{1+\beta}\right)^{\frac{1}{1-\alpha}}$$

- From the planner problem we get a 2-order difference equation for capital which yields the following capital in steady-state:

$$K_{SS}^{SP} = \left(\frac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}$$

- Why CE and SP are different?, are there any other ways of decentralizing the economy?

Idiosyncratic uncertainty

- In this setting, there is uncertainty about the type (i.e., employed or unemployed) of the agent when old
- When young, they can choose to work n_t^t or invest in human capital H_t^t
- The planners problem is given by:

$$\max_{\{c_t^{e,t-1}, c_t^{u,t-1}, n_t^t, H_t^t\}} \sum_{t=1}^{\infty} (1-v)\log(c_t^{e,t-1}) + v\log(c_t^{u,t-1})$$
$$\text{s.t. } (1-v)c_t^{e,t-1} + vc_t^{u,t-1} = n_t^t + (1-v)AH_{t-1}^{t-1}$$
$$n_t^t + H_t^t = 1$$

- For decentralizing the economy, we introduce **Arrow-Debreu securities**: claims to consumption in period $t+1$ for both states of nature
  - With AD securities, we have a "complete market" problem (Why?)
  - With fiat money there is an "incomplete markets" problem (Why?)

Idiosyncratic uncertainty (Cont.)

- The household faces prices q_t^e and q_t^u and solves:

$$
\begin{aligned}
\max_{\{a_{t+1}^{e,t}, a_{t+1}^{u,t}, H_t^t\}} \; & (1-v)\log(c_{t+1}^{e,t}) + v\log(c_{t+1}^{u,t}) = \mathbb{E}_t\left[\log(c_{t+1}^{s,t})\right] \\
\text{s.t. } \; & q_t^e a_{t+1}^{e,t} + q_t^u a_{t+1}^{u,t} = n_t^t \\
& c_{t+1}^{e,t} = AH_t^t + a_{t+1}^{e,t} \\
& c_{t+1}^{u,t} = a_{t+1}^{u,t} \\
& n_t^t + H_t^t = 1, \;\; c_{t+1}^{s,t} \geq 0, \; H_t^t \in [0,1]
\end{aligned}
$$

- A competitive Arrow-Debreu (Complete Markets) equilibrium is given by sequences $\{q_t^s\}_{t=1}^{\infty}$ and $\{c_t^{s,t-1}, n_t^t, H_t^t\}_{t=1}^{\infty}$ for $s \in \{e, u\}$ such that:
  1. Given prices $\{q_t^s\}_{t=1}^{\infty}$, the allocation solves the expected utility maximization problem faced by each generation
  2. Markets clear in every period i.e.
     1. $(1-v)c_t^{e,t-1} + vc_t^{u,t-1} = n_t^t + (1-v)AH_{t-1}^{t-1}$
     2. $(1-v)a_t^{e,t-1} + va_t^{u,t-1} = 0$
- How does the decentralized equilibrium with fiat money looks like?

Ricardian equivalence

- We introduce a government that needs to finance a stream of expenditures that the household might not value
- **Problem:** The government can tax current generations or issue debt and tax future generations
- David Ricardo argue that both options will yield the same outcome in equilibrium if the tax structure is not distortionary
- The government budget constraint is the following

$$\underbrace{G_t + B_t}_{\text{expenditures}} = \underbrace{\Upsilon_t + q_t B_{t+1}}_{\text{income}}, \quad \forall t,$$

$$\Longrightarrow \sum_{j=1}^{\infty} \left(\prod_{s=1}^{j-1} q_s \right) G_j + B_1 = \sum_{j=1}^{\infty} \left(\prod_{s=1}^{j-1} q_s \right) \Upsilon_j$$

Ricardian equivalence (Cont.)

- Any two arbitrary sequences of taxes $\{\Upsilon_t\}_{t=1}^{\infty}$ and $\{\Upsilon'_t\}_{t=1}^{\infty}$ with $\Upsilon_t \neq \Upsilon'_t$ that have the same present value:

$$\sum_{j=1}^{\infty} \left(\prod_{s=1}^{j-1} q_s \right) \Upsilon_j = \sum_{j=1}^{\infty} \left(\prod_{s=1}^{j-1} q_s \right) \Upsilon'_j$$

- Induces the same allocation in equilibrium iff the tax structure is lump-sum or does not induce any wedge in efficiency conditions

- **Remember**: when we add a government to the baseline model, we need to add an extra component in the definition of competitive equilibrium (i.e., budget constraint) and modify the market clearing condition

Government commitment problems

- **Main problem:** time inconsistency (i.e., it depends who is taking the decision first, the household or the government)
- **Framework:** Static model with distortionary taxation, productive and unproductive storage technologies and a benevolent government
- The household problem is given by

$$\max_{\{c,g,x,m\}} u(c,g),$$
$$s.t. \quad x + m = w,$$
$$c = m + (1-\tau)Rx, \quad \text{with } R > 1,$$

where c denotes consumption, g is the public good, x is the amount of resources invested in the productive tech., m is the amount stored in the pillow tech., w is the endowment, and τ is the proportional tax on returns

Government commitment problems (Cont.)

- **Commitment equilibrium (Ramsey):** government first commits to a specific tax τ and then the household takes decisions about investment and consumption
- **No commitment equilibrium:** households move first and then the government chooses how much to tax the productive technology return
- We solve both games using backward induction
- **Intuition:** When households move first, investment is perfectly inelastic $\Rightarrow$ the government will over-tax to provide more public goods
- Can we support a Ramsey equilibrium in an infinitely repeated game? **Yes**, using grim-trigger strategies!
  - Any deviation from a particular strategy (i.e., tax schedule) will be punished by playing any other action (i.e., $x = 0$)

Private information frictions

- We will look at the planners problem under two types of information settings: observable and unobservable types $\theta \in \Theta$
- The key to solving the planners problem with unobservable types is to introduce **incentive compatibility constraints**
    - The planner designs a mechanism for truthful telling (i.e., no late types will mask themselves as early types)
- We can decentralize the planners problem using banks
- The Diamond and Dybvig model introduces idiosyncratic uncertainty with a unit measure of agents in a simple dynamic model with private information about agents' preferences

Private commitment frictions

- We analyze the housing market under commitment to repay a mortgage and with no commitment
- Under commitment, the solution to the household problem is straightforward
- When there is no commitment, we can introduce a collateral constraint (Kiyotaki-Moore) to enforce repayment in which lenders can seize a fraction of the collateral value of the house
- In an infinitely repeated version of this problem, we can implement the commitment equilibrium/allocation by introducing exclusion as a grim trigger strategy