Macro Prelim Session: Rishabh's Material[1]

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[1] **Disclaimer:** This slides are an attempt to summarize the material covered by Rishabh in Econ 712. They do not replace the lecture notes and additional material provided by him. Any errors are my own.

Outline

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How to prepare and general info

- Rishabh's section has two questions: one **completely new** and the other one is "similar" to **previous finals/prelims** (not a rule but there is a high probability)
  - New question has more weight (usually 35 pts out of 50)
  - Dominant strategy: try to solve as many as possible past questions from Rishabh
- It is useful to be explicit in your procedure: Lagrangian, FOC, efficiency conditions, market clearing, and assumptions
- Learn how to define properly what a competitive equilibrium is under different frameworks (i.e., AD and sequential markets)
- His questions are oriented to test if you know the **tools** and **techniques** to solve a particular problem
- When working with private info problems, try to identify which type of agent has an incentive to lie and develop some intuition to provide an answer

Complete Markets

- Consider the following setup:
  1. Agents: $i \in \{1, ..., I\}$
  2. Endowment: stochastic $\{e_{it}(s^t)\}$
  3. Preferences: $u(c_{it}(s^t))$
  4. Probability: $\pi_t(s^t)$ and conditional system $\pi_t(s^t|s^{t-1})$
  5. State Space: $\mathcal{S} = \{0, 1\}$
- For the stochastic process, we will focus on Markovian processess of order one

Planner's problem

- The planner's problem is given by:

$$\max_{c_{it}} \sum_{i \in I} \lambda_i \sum_{t=0}^{\infty} \sum_{s^t \in \mathcal{S}^t} \beta^t \pi_t(s^t) u(c_{it}(s^t))$$

$$\text{s.t.} \quad \sum_{i \in I} c_{it}(s^t) \leq \sum_{i \in I} e_{it}(s^t) \; \forall t, s^t \quad \text{(Resource constraint)},$$

- The associated lagrangian is given by:

$$\mathcal{L}(c_{it}(s^t); \theta(s^t)) = \sum_{t=0}^{\infty} \sum_{s^t} \left[\sum_{i \in I} \lambda_i \beta^t \pi_t(s^t) u(c_{it}(s^t)) + \theta(s^t) \sum_{i \in I} (e_{it}(s^t) - c_{it}(s^t)) \right]$$

- When combining FOC for $c_{1t}(s^t)$ and $c_{it}(s^t)$, and the resource constraint, we get the full insurance result:

$$\sum_{i \in I} (u')^{-1} \left(\frac{\lambda_1}{\lambda_i} u'(c_{1t}(s^t)) \right) = \sum_{i \in I} e_{it}(s^t)$$

Arrow-Debreu markets

- Agents trade dated history-contingent claims at **time zero** at price $Q_t(s^t)$
- The time zero budget constraint is given by

$$\sum_{t=0}^{\infty} Q_t(s^t)c_{it}(s^t) \leq \sum_{t=0}^{\infty} Q_t(s^t)e_{it}(s^t)$$

- Agent i has the following maximization problem:

$$\max_{c_{it}} \sum_{t=0}^{\infty} \sum_{s^t \in \mathcal{S}^t} \beta^t \pi_t(s^t) u(c_{it}(s^t))$$

$$\text{s.t. } \sum_{t=0}^{\infty} Q_t(s^t)c_{it}(s^t) \leq \sum_{t=0}^{\infty} Q_t(s^t)e_{it}(s^t)$$

- Does the FWT hold?

Sequential markets

- One-period securities are sufficient to implement Arrow-Debreu
- Let $a_{it}(s^t)$ be a claim to consumption at time t in history s_t and $q_t(s^t, s_{t+1})$ be the price of one unit of consumption in period $t+1$ contingent on state s_{t+1}
- The optimization problem of the household is given by:

$$\max_{c_{it}} \sum_{t=0}^{\infty} \sum_{s^t \in \mathcal{S}^t} \beta^t \pi_t(s^t) u(c_{it}(s^t))$$

$$\text{s.t. } c_{it}(s^t) + \sum_{s_{t+1}} q_t(s^t, s_{t+1}) a_{it}(s^t, s_{t+1}) \leq e_{it}(s^t) + a_{it}(s^t) \ \forall t, s^t$$

$$- a_{i,t+1}(s^{t+1}) \leq A_{i,t+1}(s^{t+1}) \ \forall t, s^t$$

- What is $A_{i,t+1}(s^{t+1})$?, under which conditions the borrowing constraint will not bind?
- Be careful setting up the lagrangian associated with the household optimization problem

Equivalence

- Arrow-Debreu and Sequential Markets are equivalent $\Rightarrow$ the time-zero trade price is equivalent to the product of the sequential prices

$$Q_t = \prod_{j=0}^{t} q_j$$

- Rishabh made a proof about the equivalence result by comparing allocations (i.e., state-contingent plans under both structures)
- Usually he asks to solve the AD problem and then the sequential problem
  - **Sanity check:** Allocations should be the same under both market structures

Limited commitment

- Model of infinite horizon where we have two type of agents that differ on their endowments
- With commitment and under utilitarian social planner, we have that the planner's solution is perfect consumption smoothing for both type of agents
- There are clear incentives for the initial high type to not commit to that sequence of consumption (due to impatience)
- The main problem with limited commitment is that it hampers the ability to smooth consumption
- The key to solve this lack of commitment problem: **Voluntary Participation Constraint**

$$\underbrace{V_t^i \equiv \sum_{t=0}^{\infty} \beta^t u(c_t^i)}_{\text{Utility with commitment}} \geq \underbrace{\sum_{t=0}^{\infty} \beta^t u(e_t^i) \equiv V_t^{d,i}}_{\text{Utility with no commitment}},$$

Limited commitment (Cont.)

- Notice that the commitment problem comes from the high type agents (Why?) $\Rightarrow$ only VPC associated to the high type
- We can decentralized the planners solution by adding non-contingent debt with borrowing constraints
- The sequential borrowing constraints are given by

$$\begin{aligned} c_t^i + b_{t+1}^i = e_t^i + R_t b_t^i, \\ b_{t+1}^i \geq -\phi. \end{aligned}$$

- There is a problem of multiplicity of equilibria when $\phi = 0$

Arrow-Debreu markets

- The household problem with AD markets and VPC is given by

$$\max \sum_{t=0}^{\infty} \sum_{s^t \in \mathcal{S}^t} \beta^t \pi_t(s^t) u(c_t^i(s^t)),$$

$$s.t. \quad \sum_{t=0}^{\infty} \sum_{s^t \in \mathcal{S}^t} Q_t(s^t) c_t^i(s^t) = \sum_{t=0}^{\infty} \sum_{s^t \in \mathcal{S}^t} Q_t(s^t) e_t^i(s^t),$$

$$\sum_{j=t}^{\infty} \sum_{s^j \in \mathcal{S}^j} \beta^{j-t} \pi_j(s^t) u(c_j^i(s^j)) \geq \sum_{j=t}^{\infty} \sum_{s^j \in \mathcal{S}^j} \beta^{j-t} \pi_j(s^t) u(e_j^i(s^j)) \quad \forall t, s^t$$

- How does the planners problem looks like when we have time zero trading and VPC?

- Is the CE efficient? If so, why?

- Rishabh also talked about decentralizing the planners solution using sequential markets and introducing "not-too-tight" constraints (see Alvarez and Jermann (2000))

Spot markets (Intratemporal trade)

- Instead of imposing a harsh punishment such as permanent exclusion from capital markets (i.e., intertemporal trade) we will allow agents to engage in intratemporal trade (i.e., spot markets)
- For this particular setting, we have an agent with preferences for two goods (i.e., c_{1t} and c_{2t})
- Learn the definition of an allocation $\{(c_{1t}^A, c_{2t}^A), (c_{1t}^B, c_{2t}^B)\}_{t\geq 0}$ and price system $\{(q_{1t}, q_{2t})\}_{t\geq 0}$ that is spot market supporting
- Go through the material to understand the example and proof that Rishabh made regarding efficiency of CE (i.e., Kehoe-Levine proposition)

Optimal taxation

- What is the best thing that we can do with available tax instruments?
- To answer this question, we analyzed the Ramsey problem of optimal taxation for both fiscal and monetary policy
- The “beauty” of Ramsey taxation is that it allow us to simplify the problem of the planner by getting rid off prices and taxes and only **choosing allocations**

Fiscal policy

- These are the four steps that you need to do to solve for optimal fiscal policy

  1. Define a competitive equilibrium with taxes (also known as Tax Distorted Competitive Equilibrium (TDCE))

  2. Derive the Implementability Condition (IC)
     a. Obtain FOC from representative household problem
     b. Multiply by lagrange multiplier both sides of the sequential budget constraint and sum over $t \geq 0$ and states of nature $s^t \in \mathcal{S}^t$
     c. Combine the previous two points and simplify to get the IC

  3. Define a Ramsey Equilibrium as a policy rule $\{\pi(s^t)\}_{t\geq 0, s^t}$ and allocation $\{x(s^t)\}_{t\geq 0, s^t}$ such that:
     a. The allocation solves the planner's problem subject to resource constraint and Implementability Condition
     b. For every policy, the allocation and corresponding price system constitute a Competitive Equilibrium

  4. Solve the Ramsey problem by redefining the utility function as a combination of the instantaneous utility function and the IC

- **Note:** When asked to impose some constraint to the government tax structure (e.g., taxes cannot differ between agents) $\Rightarrow$ incorporate an extra constraint in Ramsey's problem to account for that specific tax system (e.g., Final of 2021)

Monetary policy

- Framework: Two-type of goods with cash-in-advance constraint
- The procedure to solve the Ramsey policy with Monetary Policy is basically the same:
    1. Derive the IC by combining the FOC of the household problem, the budget constraint, and the CIA constraint
    2. Write down the Ramsey problem in which the government/planner will maximize utility subject to resource constraint, the IC and an extra condition that captures the "zero-lower bound" constraint for the interest rate (i.e., $R(s^t) \geq 1$ for all t and s^t)
    3. Solve Ramsey problem by relaxing the problem (i.e., ignoring the "zero-lower bound" constraint and the verify that it is satisfied) and compare with the household efficiency conditions
- **Main result:** under homothetic preferences for cash and credit goods, the optimal monetary policy is to set $R = 1$ for all $t \geq 0 \Rightarrow \pi \approx -\rho$ (Friedman rule)

Private Information

- The static Mirrleesian model can be thought as a mechanism design problem in which the principal faces uncertainty about the true type of the agents
- We studied two examples:
    1. Two types of agents
    2. Continuum of types using Myerson result: GIC $\Leftrightarrow$ LIC
- We derive the important Diamond-Saez-Mirrless condition for optimal marginal taxation and look at an example where the types come from a Pareto distribution

Example 1: Two type of agents

- Two type of agents (i.e., high and low) that value consumption and they get disutility of working. High type agent is more productive than the low type agent
- General procedure:
  1. Solve the planner's problem when information is public
  2. Assume that information is private:
     a. Define the Incentive Compatibility Constraint (IC) for each type of agents:

     $$u(c(\theta)) - v\left(\frac{y(\theta)}{\theta}\right) \geq u(c(\hat{\theta})) - v\left(\frac{y(\hat{\theta})}{\theta}\right), \quad \forall \theta, \hat{\theta} \in \Theta = \{\theta_H, \theta_L\}.$$

     b. Solve the planner's problem subject to resource constraint and IC constraints
     c. We can drop the IC for the agent that it not binding and validate that the allocation indeed satisfies the IC constraint
  3. We can decentralize the planner's solution with private information by using taxes or non-linear transfers
  4. Compare efficiency conditions and determine marginal tax rate
- It is useful to understand the proof of slackness of IC for low-type

Example 1: Two type of agents (Cont.)

Proof.

From the planner's problem with private information that $c(\theta_H) > c(\theta_L)$ and $y(\theta_H) > y(\theta_L)$. Assume, towards a contradiction, that IC for low-type is not satisfied

$$
\begin{aligned}
u(c(\theta_L)) - v\left(\frac{y(\theta_L)}{\theta_L}\right) &\le u(c(\theta_H)) - v\left(\frac{y(\theta_H)}{\theta_L}\right), \\
\iff v\left(\frac{y(\theta_H)}{\theta_L}\right) - v\left(\frac{y(\theta_L)}{\theta_L}\right) &\le u(c(\theta_H)) - u(c(\theta_L)), \\
\underbrace{\iff}_{FTC} \frac{1}{\theta_L}\int_{y(\theta_L)}^{y(\theta_H)} v'\left(\frac{z}{\theta_L}\right) dz &\le u(c(\theta_H)) - u(c(\theta_L)), \\
\underbrace{\iff}_{\substack{\text{convexity of } v(\cdot) \\ \text{and } \theta_H > \theta_L}} \frac{1}{\theta_H}\int_{y(\theta_L)}^{y(\theta_H)} v'\left(\frac{z}{\theta_H}\right) dz < \frac{1}{\theta_L}\int_{y(\theta_L)}^{y(\theta_H)} v'\left(\frac{z}{\theta_L}\right) dz &\le u(c(\theta_H)) - u(c(\theta_L)), \\
\iff v\left(\frac{y(\theta_H)}{\theta_H}\right) - v\left(\frac{y(\theta_L)}{\theta_H}\right) &< u(c(\theta_H)) - u(c(\theta_L)), \\
\iff u(c(\theta_H)) - v\left(\frac{y(\theta_H)}{\theta_H}\right) &> u(c(\theta_L)) - v\left(\frac{y(\theta_L)}{\theta_H}\right)!
\end{aligned}
$$

□

Example 2: Continuum of types

- We have a type space with a continuum of possible elements Θ (still we impose that Θ is a compact set such that $\Theta = [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} < \bar{\theta}$)

- The GIC is given by

$$\mathcal{U}(\theta) \equiv u(c(\theta)) - v\left(\frac{y(\theta)}{\theta}\right) \geq u(c(\hat{\theta})) + v\left(\frac{y(\hat{\theta})}{\theta}\right), \quad \forall \theta, \hat{\theta} \in \Theta.$$

- What is the main problem associated with the GIC?

- Myerson theorem provides equivalance between GIC and LIC, where LIC are given by:

$$\text{LIC 1}: \quad y(\theta) \text{ is increasing in } \theta,$$
$$\text{LIC 2}: \quad \mathcal{U}'(\theta) = \frac{y(\theta)}{\theta^2} v'\left(\frac{y(\theta)}{\theta}\right)$$

- Also, it is useful to understand the tricks made to prove that LIC $\Rightarrow$ GIC (similar tricks as in the previous slide)

Diamond-Saez-Mirrlees Condition

- The planner's problem with private information under a continuum of types is given by

$$
\begin{aligned}
\max_{c,y} \int_{\Theta} \mathcal{W}\left(u(c(\theta)) - v\left(\frac{y(\theta)}{\theta}\right)\right) dF(\theta),& \\
s.t. \quad \int_{\Theta} c(\theta) dF(\theta) \leq \int_{\Theta} y(\theta) dF(\theta),& \\
\mathcal{U}(\theta) = u(c(\theta)) - v\left(\frac{y(\theta)}{\theta}\right),& \\
\mathcal{U}'(\theta) = \frac{y(\theta)}{\theta^2} v'\left(\frac{y(\theta)}{\theta}\right),& \\
y(\theta) \text{ is increasing in } \theta.&
\end{aligned}
$$

- After working the FOC, we the famous Diamond-Saez-Mirrlees condition:

$$
\frac{T'(y)}{1-T'(y)} = u'(c(\theta)) \underbrace{\frac{1-F(\theta)}{\theta f(\theta)}}_{\substack{\text{Represents the Hazard Rate:} \\ \text{Fat tails imply } T'(y)>0}} \underbrace{\left[1 + \frac{y(\theta)}{\theta} \frac{v''\left(\frac{y(\theta)}{\theta}\right)}{v'\left(\frac{y(\theta)}{\theta}\right)}\right]}_{\substack{\text{Labor supply elasticity} \\ \text{More inelastic labor supply } T'(y)>0}} \underbrace{\int_{\theta}^{\bar{\theta}} \left[\frac{1}{u'(c(z))} - \frac{\mathcal{W}'(\mathcal{U}(z))}{\lambda}\right] \frac{dF(z)}{1-F(\theta)}}_{\substack{\text{Redistributional concerns} \\ \text{Planner that loves redistribution } T'(y)>0}}.
$$