Satellite Orbits

Keplerian motion
Perturbed motion
GPS orbits

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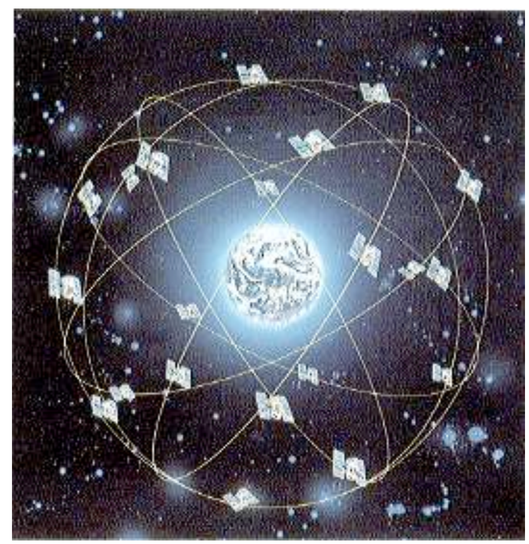

Orbits?

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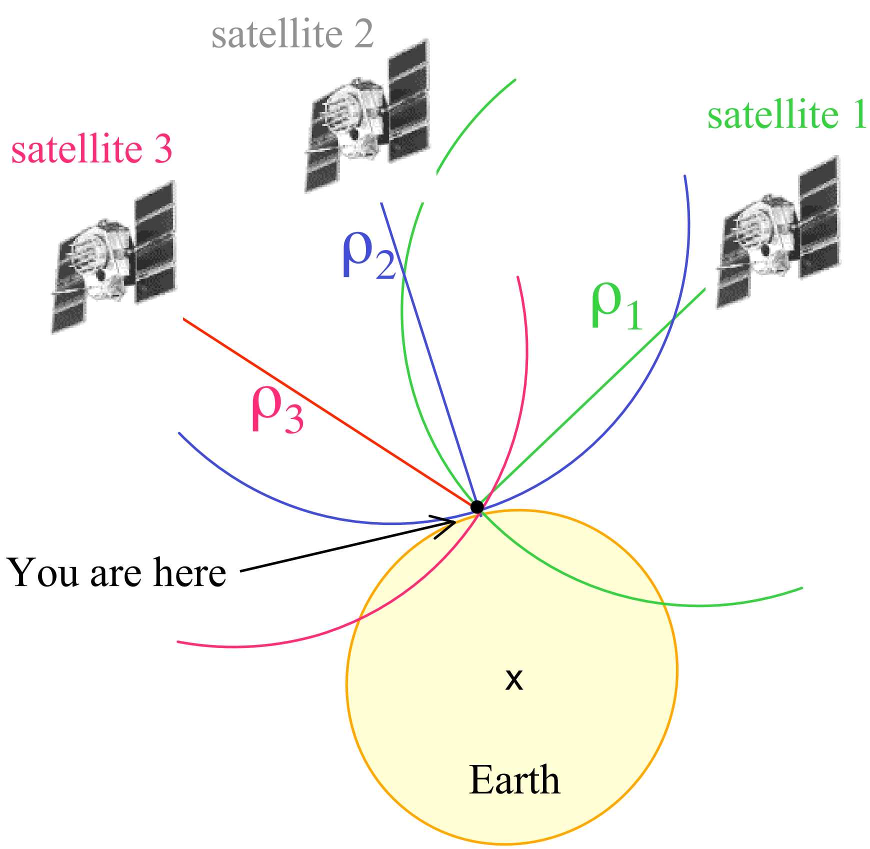
Satellite orbits/What for?



- Principle of GPS positioning:
  - Satellite 1 sends a signal at time t_{e1}
  - Ground receiver receives it signal at time t_r
  - The range measurement ρ_1 to satellite 1 is:
    - $\rho_1 = (t_r - t_{e1}) \times$ speed of light
    - We are therefore located on a sphere centered on satellite 1, with radius $\rho 1$
  - 3 satellites => intersection of 3 spheres

- In simple mathematical terms:

$$\rho_r^s = \sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2}$$

  - GPS receivers measure ρ_r^s
  - If the **position of the satellites in an Earth-fixed frame** (X_s, Y_s, Z_s) is known,
  - Then one can solve for (X_r, Y_r, X_r) (if at least 3 simultaneous range measurements)

Dynamics of satellite orbits

- Basic dynamics in an inertial frame described by: $\sum \vec{F} = m\vec{a}$
- Forces are:
  - Gravitational forces
  - Solar radiation pressure (drag is negligible for GPS)
  - Thruster firings (not directly modeled).
- Neglecting radiation pressure, one can write:

$$\vec{F} = \frac{G m_s m_E}{r^2}\vec{r} \qquad and \qquad \vec{F} = m_s \vec{a}$$

$$\Rightarrow \vec{a} + \frac{G m_E}{r^2}\vec{r} = \vec{0} \Leftrightarrow \boxed{\frac{d^2\vec{r}}{dt^2} = -\frac{G m_E}{r^2}\vec{r}}$$

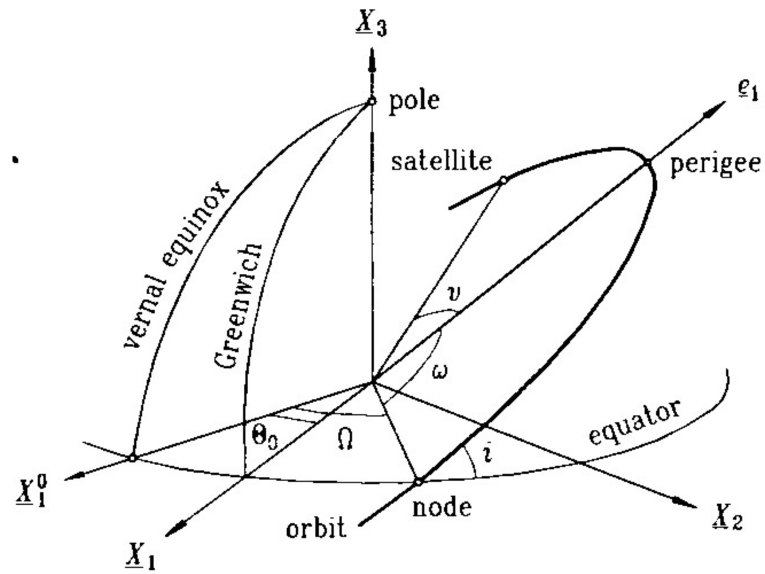
- r = geocentric position vector
- $a = d^2r/dt^2$, relative acceleration vector
- G = universal gravitational constant
- m_E = Earth's mass

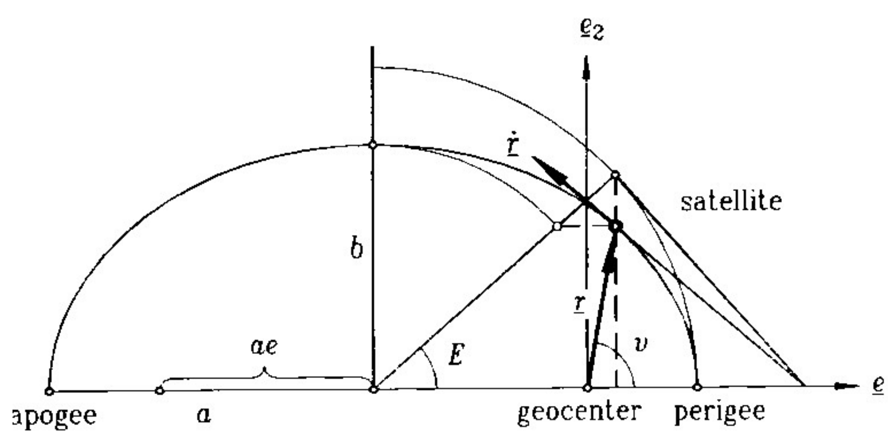
Keplerian motion

Analytical solution to this force model = Keplerian orbit (Leick, p.41-47):

- Six integration constants ⇒ 6 orbital parameters or Keplerian elements
- In an **inertial** reference frame, orbits can be described by an ellipse
- The orbit plane stays fixed in space
- One of the foci of the ellipse is the center of mass of the body
- These orbits are described by Keplerian elements:

a	Semi-major axis	Size and shape of orbit
e	Eccentricity	
Ω	Right ascension of ascending node	Orientation of the orbital plane in the inertial system
ω	Argument of perigee	
i	Inclination	
T_o	Epoch of perigee	Position of the satellite in the orbital plane





Keplerian motion

- Third Kepler's law (period^2/a^3=const.) relate mean angular velocity *n* and revolution period *P*:

$$n = \frac{2\pi}{P} = \sqrt{\frac{GM_E}{a^3}}$$

  – Me = Earth's mass, $GM_E = 3\ 986\ 005 \times 10^8\ m^3 s^{-2}$

- For GPS satellites:
  – Nominal semi-major axis *a* = 26 560 000 m
  – Orbital period = 12 sidereal hours (= 11h28mn UT), v=3.87 km/s
  – Therefore, positions of GPS satellites w.r.t. Earth's surface repeat every sidereal day

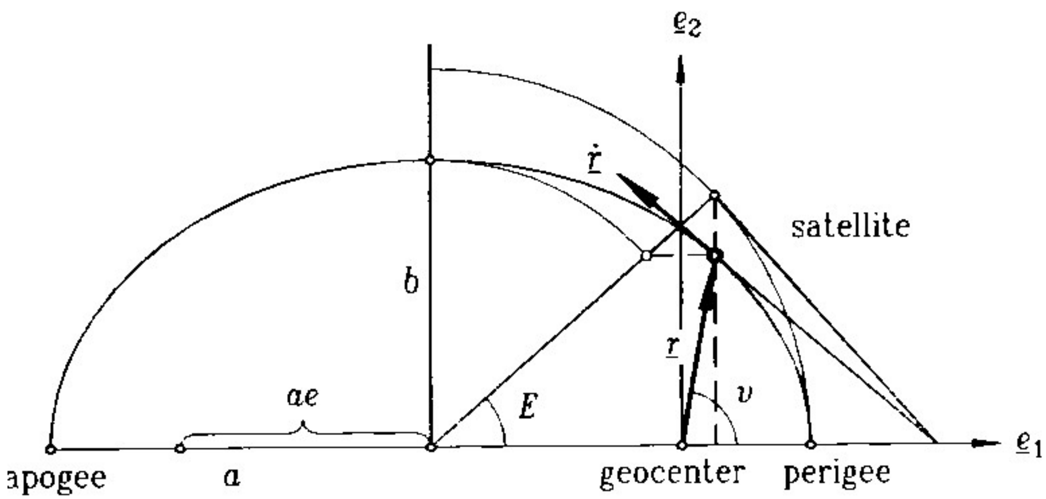
Keplerian motion

- Instantaneous position of a satellite on its orbit is defined by angular quantities known as *"anomalies"*:
  - Mean anomaly: $M(t) = n(t-To)$
  - Eccentric anomaly: $E(t) = M(t) + e \sin E(t)$
  - True anomaly: $v(t) = 2 \,\mathrm{atan}\, [((1+e)/(1-e))^{1/2} \tan(E(t)/2)]$
  - (To = time of perigee)

- In the coordinate system defined by the **orbital plane**:
  - $e1 = r \cos v = a \cos E - ae = a (\cos E - e)$
  - $e2 = r \sin v = (b/a)\, a \sin E = b \sin E = a (1-e^2)^{1/2} \sin E$
  - $e3 = 0$

$$\vec{r} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} a(\cos E - e) \\ a\sqrt{1-e^2}\sin E \\ 0 \end{bmatrix}$$

$$\|\vec{r}\| = a(1 - e\cos E)$$



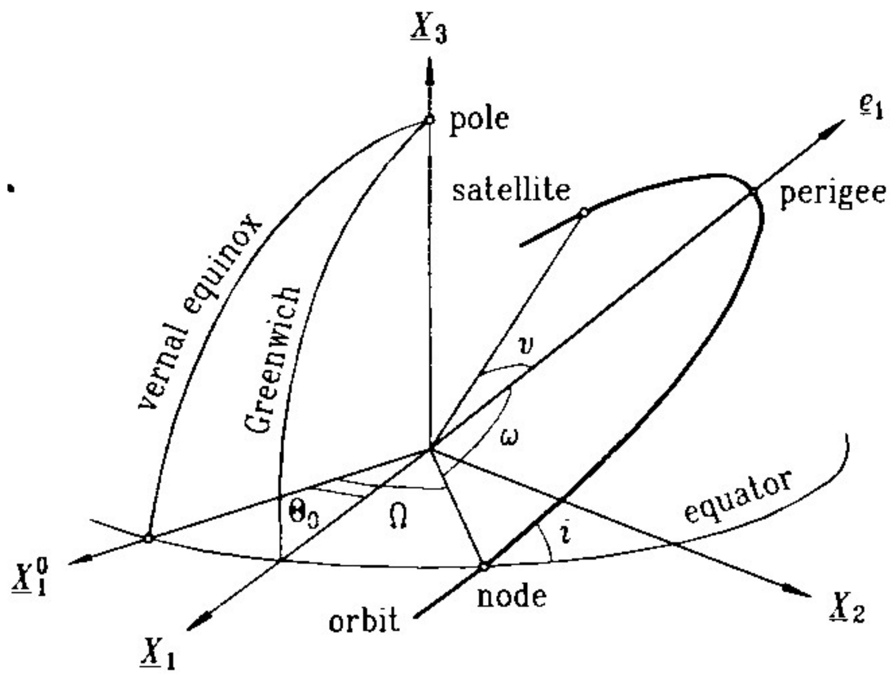
Keplerian motion

In the **Earth centered inertial system** (X_1^0, X_2^0=X_2, X^O_3=X_3), r relates to ρ through the combination of 3 rotations:

$$\vec{\rho} = R\vec{r}$$

$$R = R_3\{-\Omega\}R_1\{-i\}R_3\{-\omega\}$$

$$R = \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i & \sin\Omega\sin i \\ \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i & -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i & -\cos\Omega\sin i \\ \sin\omega\sin i & \cos\omega\cos i & \cos i \end{bmatrix}$$



Keplerian motion

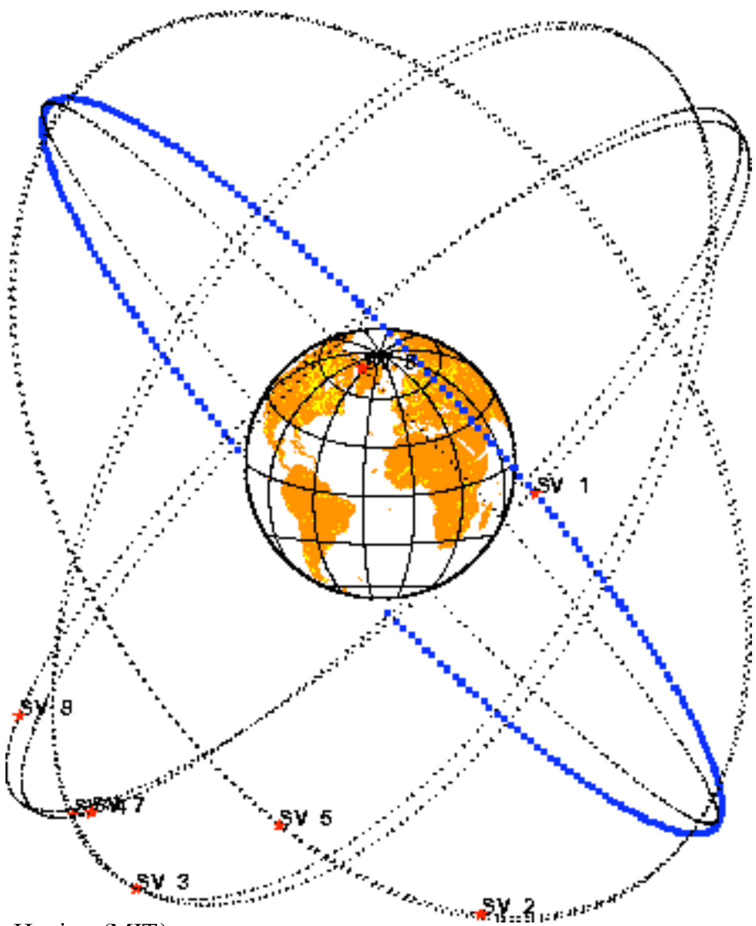


Fig. T. Herring (MIT)

GPS orbit, inertial frame

- GPS orbit in an inertial frame (+Earth rotation)
- To compute site position on the Earth, we need to know the satellite orbit to an Earth-fixed frame!

Keplerian motion

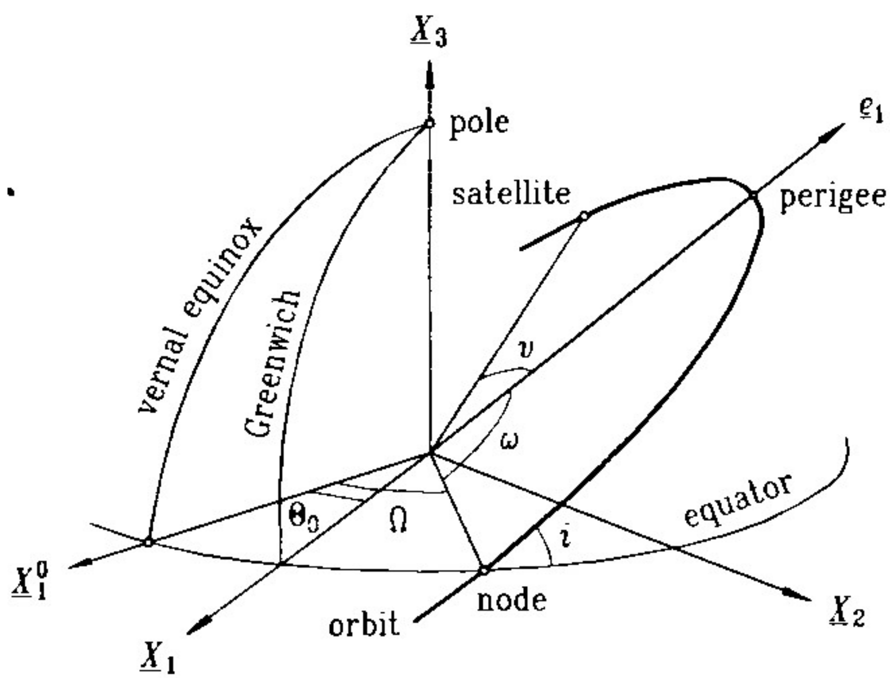
To convert ρ to an **Earth fixed system** (X_1, X_2, X_3), one needs an additional rotation of Θ_o (related to the Greenwich Sidereal Time):

$$\textit{defining}: \quad l = \Omega - \Theta_o$$

$$\vec{\rho} = R'\vec{r}$$

$$R' = R_3\{\Theta_o\}R = R_3\{\Theta_o\}R_3\{-\Omega\}R_1\{-i\}R_3\{-\omega\}$$

$$R' = \begin{bmatrix} \cos l \cos\omega - \sin l \sin\omega \cos i & -\cos l \sin\omega - \sin l \cos\omega \cos i & \sin l \sin i \\ \sin l \cos\omega + \cos l \sin\omega \cos i & -\sin l \sin\omega + \cos l \cos\omega \cos i & -\cos l \sin i \\ \sin\omega \sin i & \cos\omega \cos i & \cos i \end{bmatrix}$$



Keplerian motion

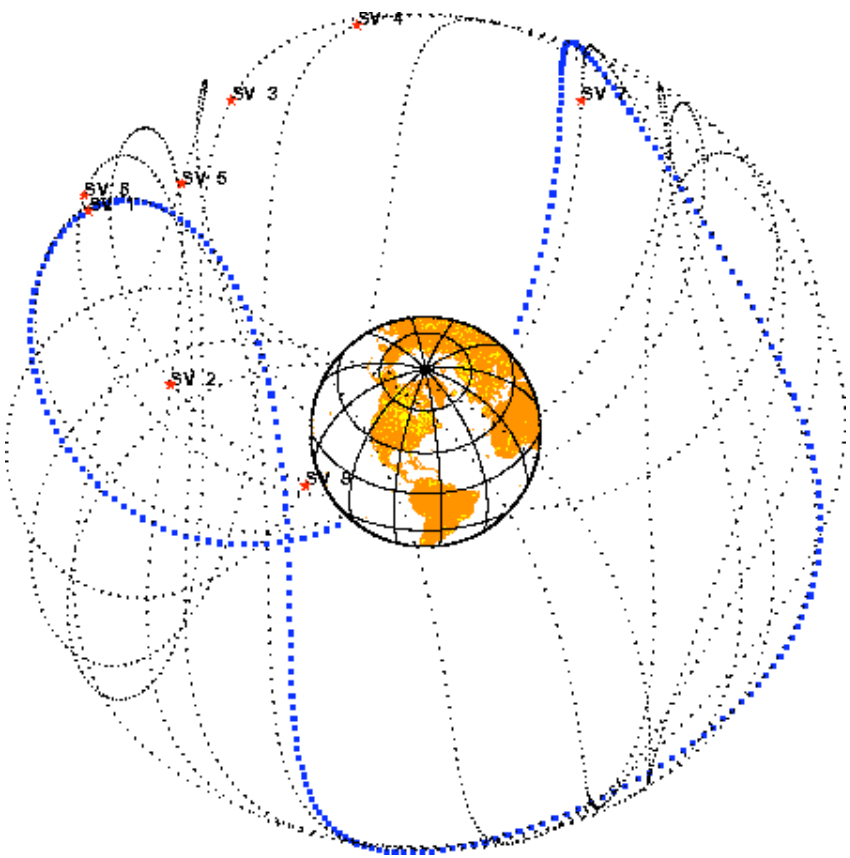


Fig. T. Herring (MIT)

GPS orbit, Earth-fixed frame

1. Broadcast ephemerides (in GPS signal) contain **orbital parameters** *M, e, a, Ω, i, ω* , *n* (=angular velocity)

2. Orbital parameters used to compute orbit in **inertial frame**

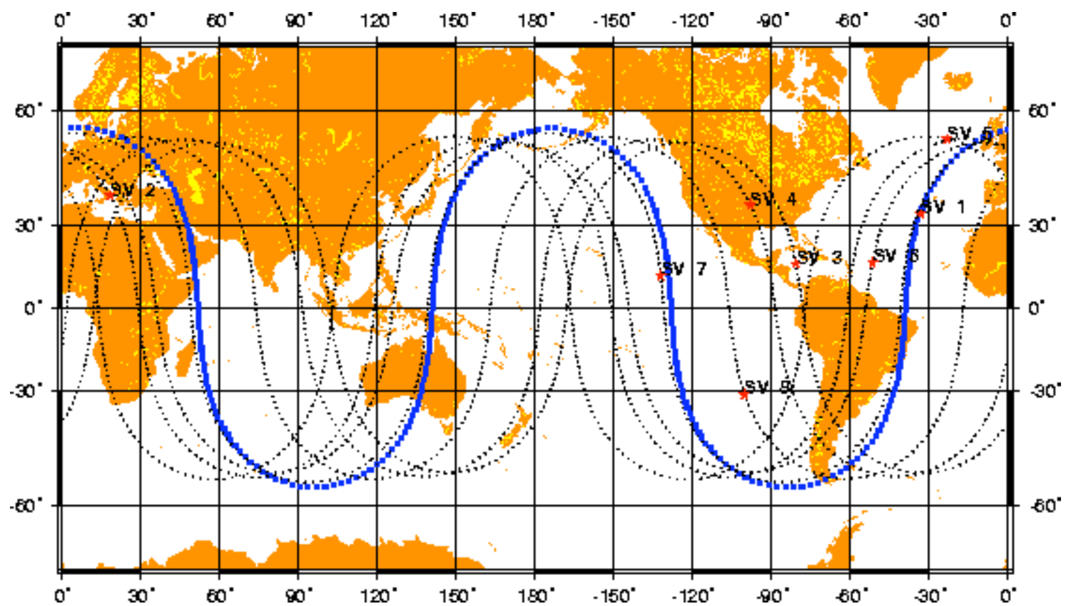
3. Orbit in inertial frame (Earth rotating) is **rotated to terrestrial frame** (Earth fixed)

4. In order to do this accurately over long time periods, one need to know UT1, nutations, and polar motion!

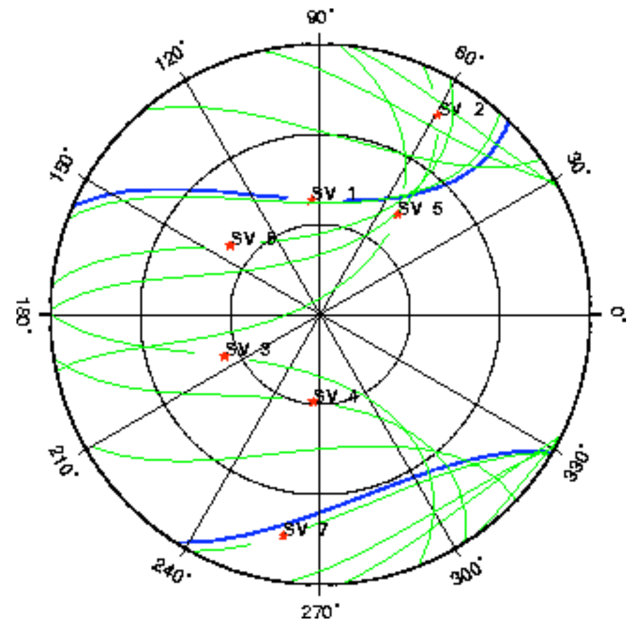
5. And take into account orbit perturbations (Earth's gravity field, solar pressure, attraction from Sun and Moon): also provided in broadcast ephemerides

Keplerian motion

Other representations of GPS orbits



GPS orbit in Earth-fixed frame =
"ground track"



GPS orbit in
topocentric frame =
"sky plot"

Perturbed motions

Central gravitational force = main force acting on GPS satellites, but there are other significant perturbations:

- Gravitational forces:
  - Non sphericity of the Earth gravitational potential:
    - Major contribution from the Earth's flattening (J2)
    - GPS orbits high (20 000 km) and attraction force attenuates rapidly with altitude => only a few terms of the Earth's gravitational potential are necessary for modeling GPS orbits
  - Third body effect: direct attraction of Moon and Sun => lunar and solar ephemerides necessary to model GPS orbits
  - [Tidal effects of Sun and Moon => deforms Earth => modifies Earth's gravitational potential: negligible for GPS satellites]
- Non-gravitational forces:
  - Solar radiation pressure:
    - Impact on the satellite surfaces of photons emitted by the Sun and reflected by the Earth surface: can be modeled, knowing the 3D geometry and the attitude of the satellite.
    - Effects on GPS satellite position: 5-10 m
    - Eclipse periods = satellite in the Earth's shadow (1-2/year, lasts about 1 hr): transition to Sun light difficult to model, usually this part of the orbit is simply edited out!
  - [Atmospheric drag = negligible for GPS satellites]
  - Satellite maneuvers

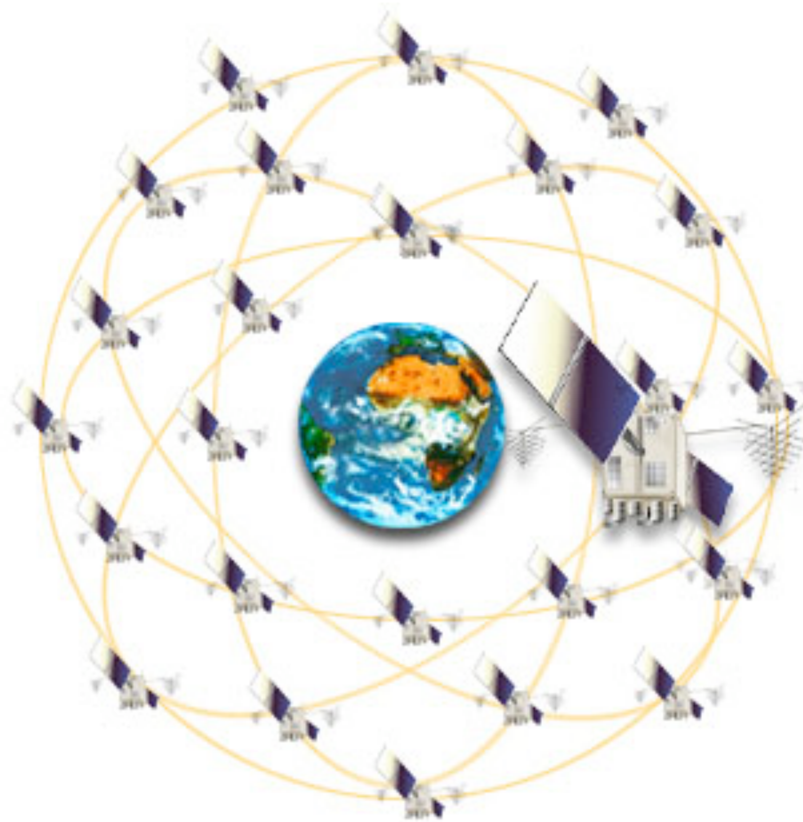
Perturbed motions

Term	**Acceleration (m/sec^2)**
Central	0.6
J2	5×10^{-5}
Other gravity	3×10^{-7}
Third body	5×10^{-6}
Earth tides	10^{-9}
Ocean tides	10^{-10}
Drag	~0
Solar radiation	10^{-7}
Albedo radiation	10^{-9}

GPS orbits

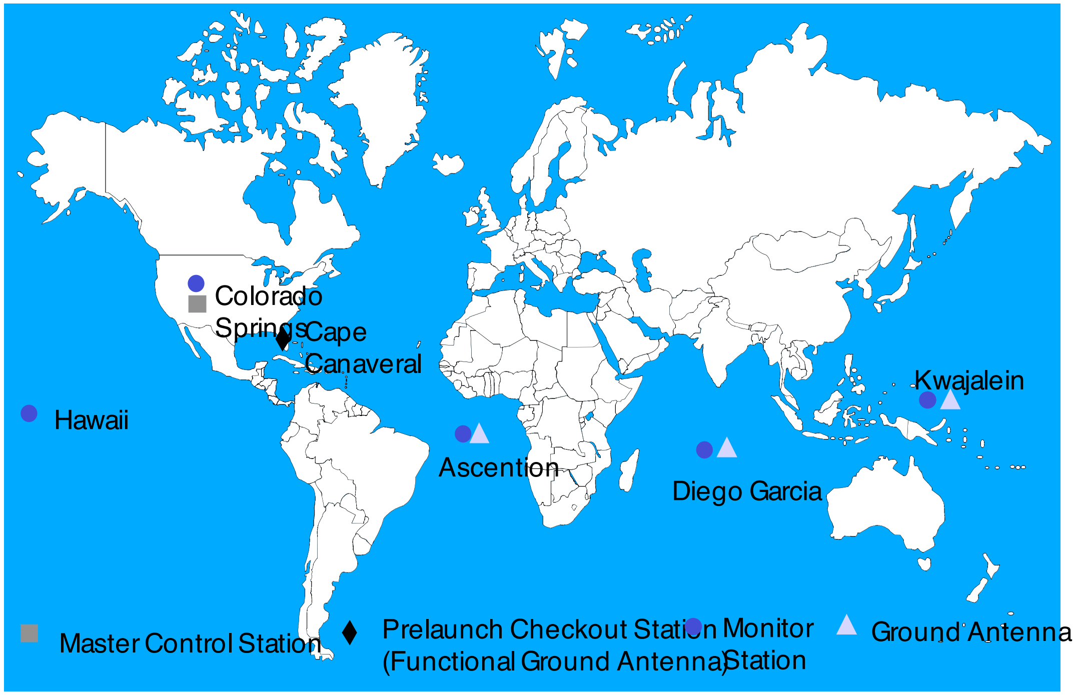
Orbit characteristics:

- Semi-major axis = 26400 km
- Period = 12 sidereal hour
- Inclination = 55.5 degrees
- Eccentricity near 0 (largest 0.02)
- 6 orbital planes with 4-5 satellites per plane



GPS orbits

- GPS control segment = 5 monitoring stations, including master control station
- Computes and uploads **broadcast ephemerides** into the satellites



GPS broadcast ephemeris

- Broadcast ephemeris: distributed to users via a “navigation message” included in the signal sent by the GPS satellites
- Information contained is:
  - Keplerian elements with periodic terms added to account for solar radiation and gravity perturbations
  - Periodic terms are added for argument of perigee, geocentric distance and inclination
- Broadcast ephemeris:
  - Decoded by all GPS receivers and for geodetic receivers
  - Distributed in ASCII format in *Receiver Independent Exchange* format (RINEX)
  - Form [4-char][Day of year][Session].[yy]n
  - e.g. brdc0120.02n

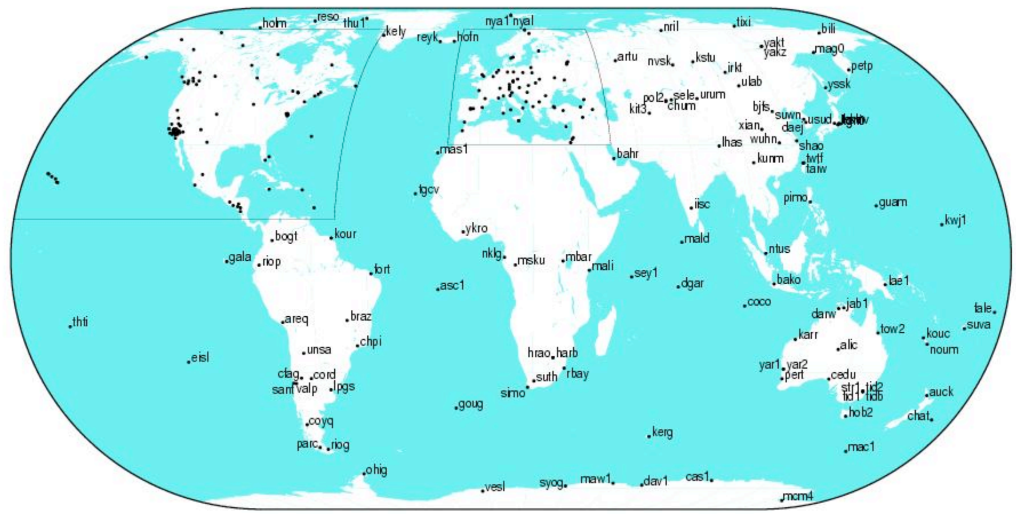
GPS broadcast ephemeris

- Accuracy of broadcast ephemeris:
  - ~ 10 m
  - Can be degraded by the DoD to 300 m
  - $db/b = dr/r \Rightarrow$ if $dr = 200$ m ($r = 20\,000$ km):
    - $d = 100$ km $\Rightarrow dr = 1$ m, not sufficient for geophysical applications
    - $d = 1$ km $\Rightarrow dr = 1$ cm, sufficient for surveying
- Broadcast ephemerides are not accurate enough for most geophysical applications.
- Requirements for geophysical applications (mm – cm level):
  - Accurate orbits: e.g. 10 cm orbit error $\Rightarrow$ 0.5 mm error on a 100 km baseline
  - Independent from the DoD

Precise GPS orbits and the IGS

IGS = International GPS Service for Geodynamics:

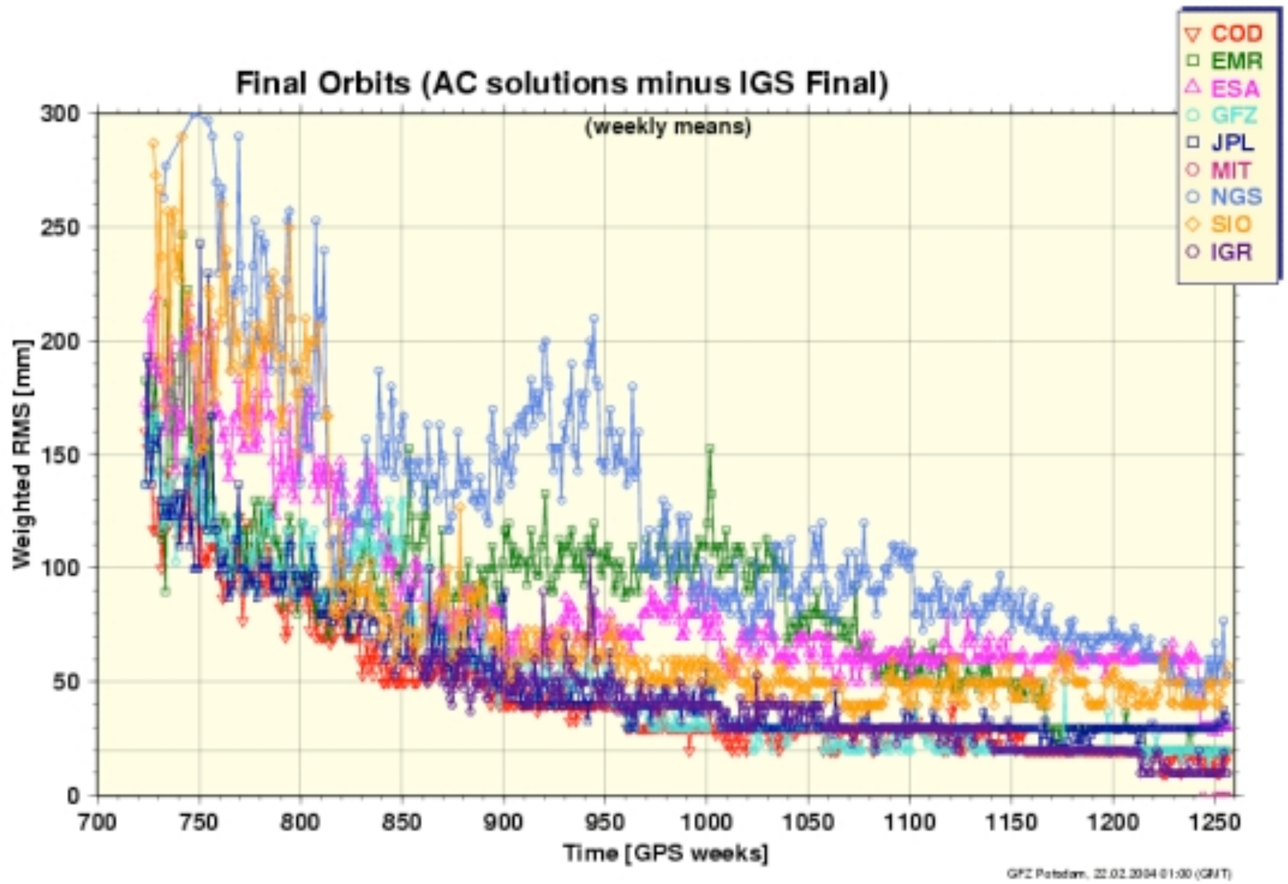
- International service of the IAG (International Association of Geodesy)
- Coordinates data archiving and processing of a global control network of dual-frequency permanent GPS stations
- Test campaign in 1992, routine operations since 1994
- Provide precise GPS satellite orbits to the scientific community
- IGS also provides:
  - Satellite clock corrections
  - Earth rotation parameters
  - Precise coordinates of control stations
  - Atmospheric parameters



Orbits Type	Accuracy/clock accuracy	Latency	Updates	Sample interval
Broadcast	~260 cm/~7 ns	Real-time		daily
Final	< 5 cm/0.1 ns	~13 days	Weekly	15 min
Rapid	5 cm/0.2 ns	17 hours	Daily	15 min
Predicted (ultra-rapid)	~25 cm/~7 ns	Real-time	Twice daily	15 min

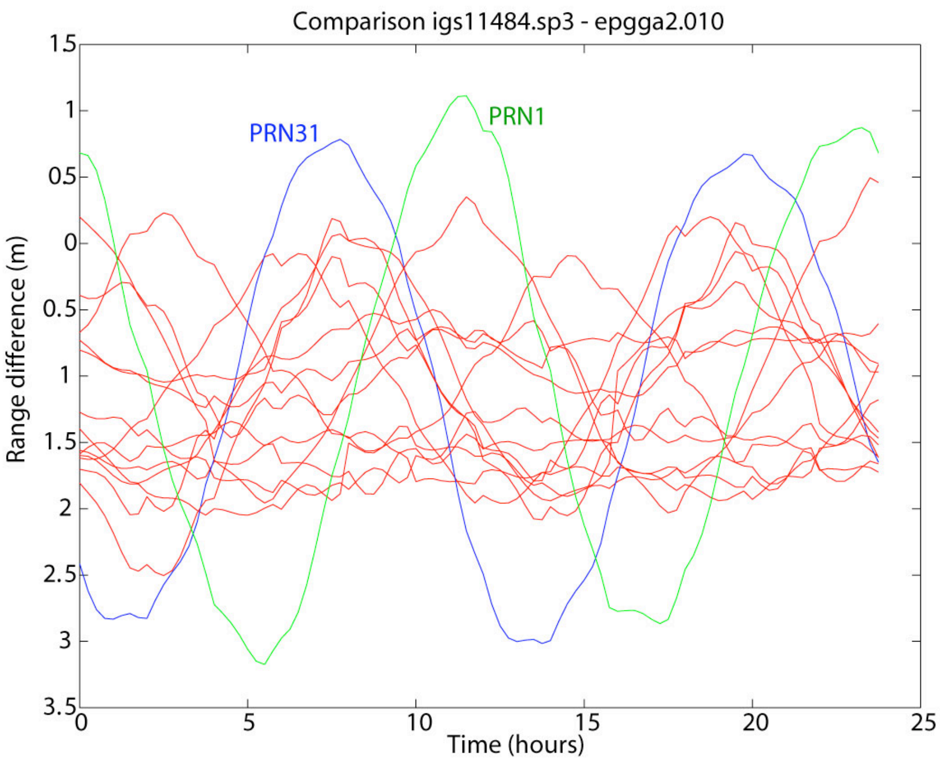
http://igscb.jpl.nasa.gov/

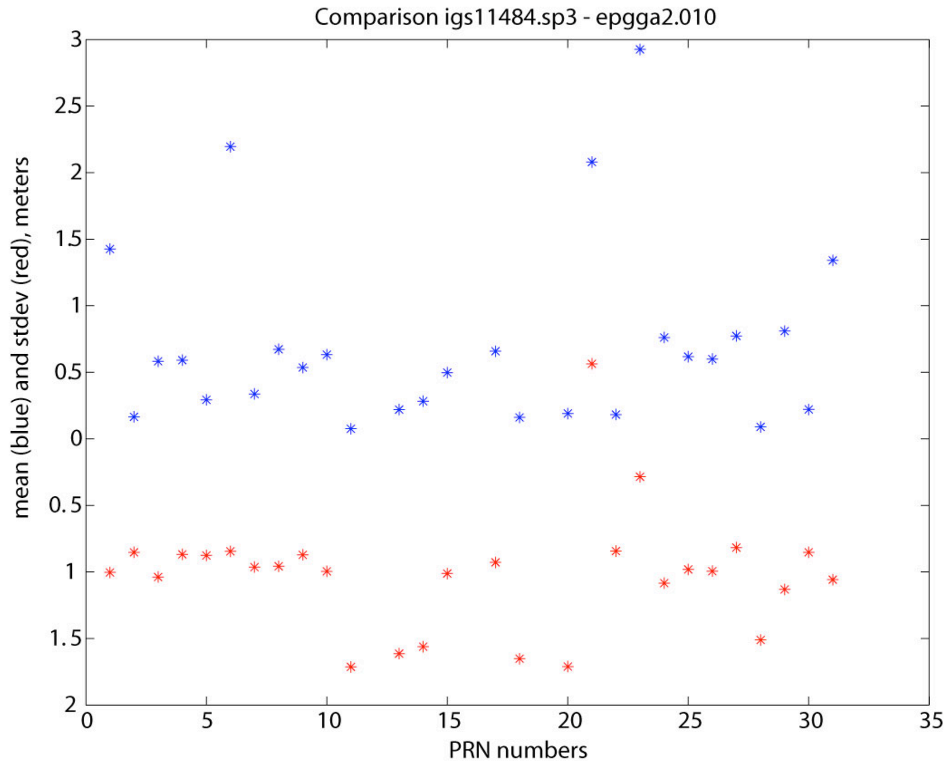
IGS orbits



Final Orbits (AC solutions minus IGS Final)
(weekly means)
COD
EMR
ESA
GFZ
JPL
MIT
NGS
SIO
IGR
Weighted RMS [mm]
300
250
200
150
100
50
0
700
750
800
850
900
950
1000
1050
1100
1150
1200
1250
Time [GPS weeks]

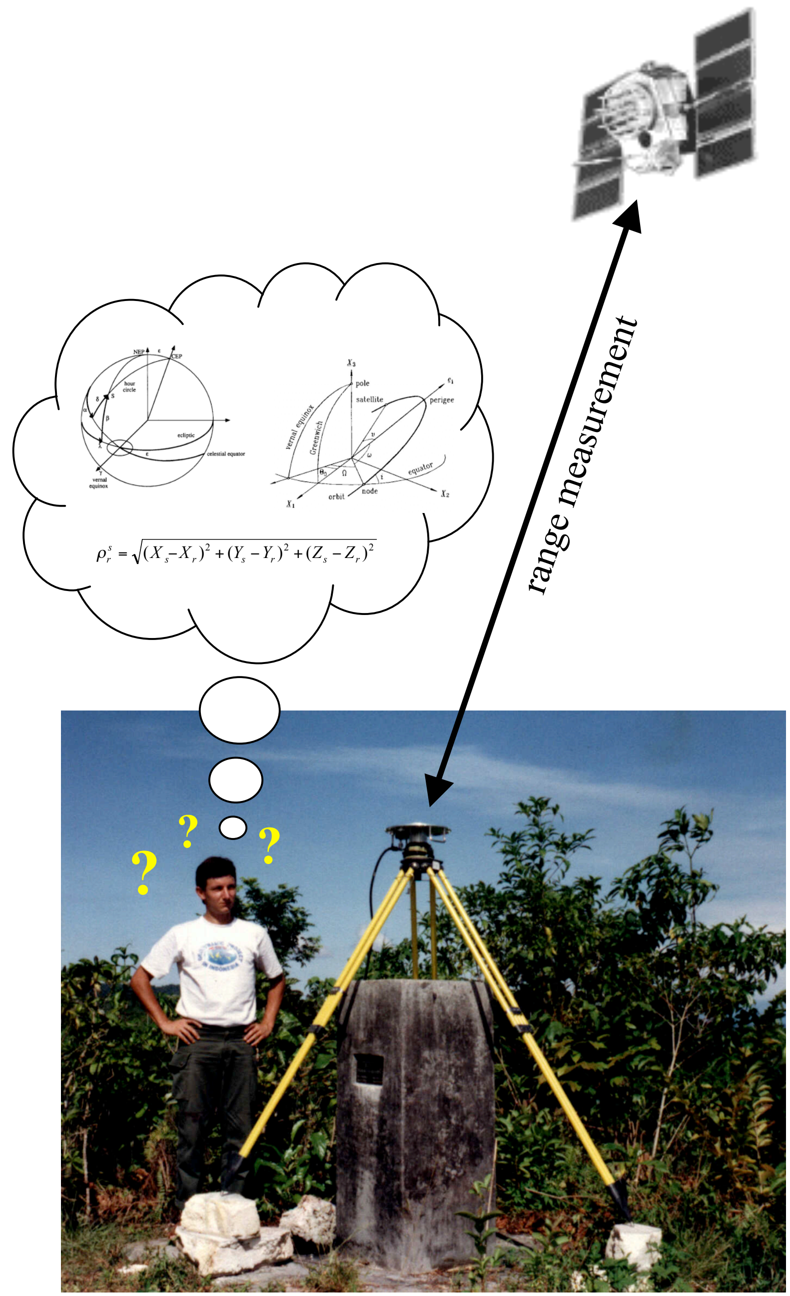
Comparison IGS - broadcast





Summary

- Objective: Estimate the position of a point on the Earth's surface.
- Requires:
  - Satellite-receiver range measurement = from GPS signal
  - Location of satellite in Earth-fixed frame, requires:
    - Orbital parameters = GPS signal
    - Conversion into Earth-fixed frame
  - Relation between inertial frame and terrestrial frame = Earth Orientation parameters (= precession, nutation, UT1, polar motion)
  - Knowledge of perturbation forces:
    - Moon and Sun = lunar and solar ephemerides
    - Earth gravity field and tides



Elements of bibliography

- Fundamentals of Geophysics, W. Lowrie, Cambridge University Press
- Physics of the Earth, F. Stacey, Wiley and Sons.
- GPS Satellite Surveying, A. Leick, John Wiley and Sons.
- GPS, Theory and Practive, B. Hofmann-Wellenhof et al., Springer-Verlag.
- Linear Algebra, Geodesy, and GPS, G. Strang and K. Borre, Wellesley-Cambridge Press.
- http://hpiers.obspm.fr/eop-pc/earthor/EOP.html
- http://einstein.gge.unb.ca/tutorial/tutorial.htm
- http://igscb.jpl.nasa.gov/
- http://www.oso.chalmers.se/~loading/
- http://www.colorado.edu/geography/gcraft/notes/coordsys/coordsys.html
- http://lareg.ensg.ign.fr/ITRF/