Satellite Launch and Alignment into Orbit

R. Chatterjee, V. Devaraj, F. Pineda, J. Shoemaker
Carnegie Mellon University
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Abstract

This paper focuses on the two major aspects of space exploration: satellite launch and satellite placement into orbit. The delicate nature of these calculations necessitates numerical methods and computer calculations. The margin of error is very small, so slight errors in cancelation, among other potential pitfalls, could derail the entire launch process. The same factors apply to placing a satellite into proper orbit. Ultimately, accuracy in both launch and alignment are crucial to satellite implementation.

1 Introduction

There are multiple factors to consider in the launching of satellites. One of these factors is a product of the Tsiolkovsky Rocket Equation, which is essentially derived from Newton's Law. Another crucial factor is the use of multi stage rockets to create an optimal energy to velocity balance.

Placing these launched satellites into orbit requires another series of delicate steps. A fundamental process necessary for orbit placement is the Hohmann transfer. There are a variety of different orbits to consider, including geocentric orbits and heliocentric orbits, which allows satellites to be utilized in different ways.

For all of the aforementioned equations and calculations, a high degree of numerical accuracy is extremely crucial. Millions of dollars are poured into these operations, and a slight miscalculation can result in the loss of valuable time and resources. This paper shows the thought process behind satellite launch/implementation and the derivation of numerical approximations to make these processes possible.

2 Rockets and Propulsion

This section follows the concepts and derivations found in [1].

2.1 Basics

The basic principle behind rocket propulsion is Newton's Third Law, which states that to every action there is a reaction of equal magnitude in the opposite direction. We have all experienced this with a balloon; if a balloon is inflated and then the air is let out, the balloon will fly through the air. In this case, the action is the air leaving the balloon and the reaction is the balloon flying through the air. Rockets are based on this principle; some mass is ejected from the rocket and the rocket itself is propelled forward. The man who formalized the analysis of the dynamics of rockets was Tsiolkovsky. The next section will derive the equations of motion of a rocket.

2.2 Tsiolkovsky's Rocket Equation

The equation that we start off with is the force of the thrust

$$F = mv_e \tag{1}$$

where F is the thrust the rocket is subject to, m the mass flow rate and v_e is the effective exhaust velocity. We have a mass flow rate since the fuel is being used up and the total mass is changing, this rate of change of mass is given by

$$m = \frac{dM}{dt} \tag{2}$$

Newton's law can also be written as

$$\frac{dv}{dt} = \frac{F}{M} \tag{3}$$

We can now substitute F from (1) and we get

$$\frac{dv}{dt} = v_e \frac{dM}{dt}\frac{1}{M} \tag{4}$$

Simplifying this we get to

$$dv = v_e \frac{dM}{M} \tag{5}$$

We can integrate this expression to find the velocity of the rocket V

$$\int_0^V dv = v_e \int_{M_0}^M \frac{dM}{M} \tag{6}$$

The solution of this is called the Tsiolkovsky Rocket Equation, or Rocket Equation informally

$$V = v_e \ln(\frac{M_0}{M}) \tag{7}$$

here M_0 is the initial mass of the rocket and propellant combined. Figure 1 shows the velocity of the rocket as a function of the mass ratio for different exhaust velocities. It is somewhat surprising to find

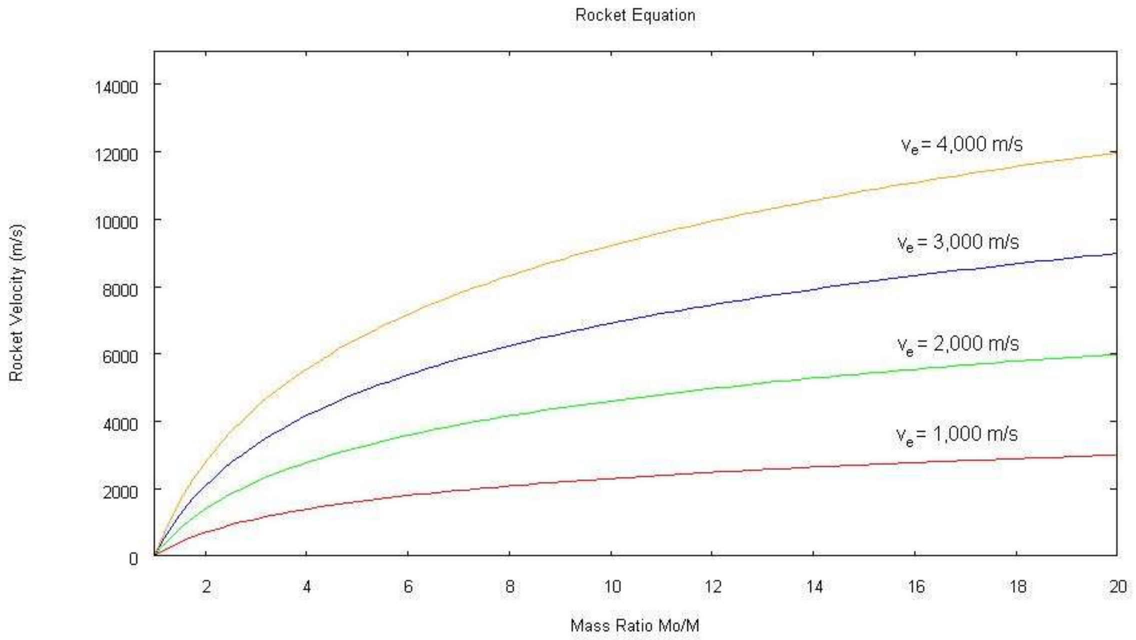

Figure 1: Tsiolkovsky's Rocket Equation

that the velocity depends only on two numbers, the exhaust velocity and the mass ratio. The exhaust velocity can be changed by the design of the rocket but mainly by the chemical composition of the fuel. In order to achieve a high velocity the mass ratio has to be large. This is why the size of the payload on a rocket is much smaller compared to the size of the rocket itself. In order to have a mass ratio of 5, for example, 80% of the initial mass would have to be fuel. Thus the design of a rocket has to be conducive to high mass ratios. This is why staging is such an important concept.

2.3 Multi Stage Rockets and Optimization

The basic idea behind staging is to discard empty fuel tanks after all the fuel in them has been used up. This way the mass ratio can be further increased. Lets take the example of a rocket divided in two stages and compare it to a single rocket. The mass ratio for a single rocket R_0 when all the fuel has been exhausted is

$$R_0 = \frac{M_S + M_F + M_P}{M_S + M_P} \tag{8}$$

The subscript P means propellant, F is for the fuel and S the structural mass. If we divide the rocket into two stages (each one with half of the total fuel) we will have two different mass ratios. The first one will be at the point when all the fuel in the first stage has been used, giving

$$R_1 = \frac{M_S + M_F + M_P}{M_S + 1/2M_F + M_P} \tag{9}$$

And the mass ratio when the fuel in the second stage has been used up is defined by

$$R_2 = \frac{1/2M_S + 1/2M_F + M_P}{1/2M_S + M_P} \tag{10}$$

To compare the performance of the different rockets we will look at their final velocity, substituting into equation (7) we get the following

$$V_0 = v_e \ln R_0 \tag{11}$$

$$V = v_e \ln R_1 + v_e \ln R_2 \tag{12}$$

To get a quantitative idea we can plug in some numbers just to exemplify. We can take the total fuel mass to be 100 tons, the mass of the payload of 1 ton and the structural mass will be 10% of the fuel mass. We take the exhaust velocity to be constant at 2700 ms^{-1}. Plugging these numbers into equations (11) and (12) we get that $V_0 = 5{,}959$ ms^{-1}, $V = 7{,}342$ ms^{-1}. It is easy to see the advantages of using multi-stage rockets over single rockets. It can be shown that the use of three stages in this case would give a final velocity of 8092 ms^{-1}.

This analysis begs the question, what number of stages is the optimal. Initially we would think that the more stages the better. However detailed calculations show that there is really no advantage to using more than 3 stages. An alternative technique to adding extra stages is to use strap on boosters to the sides of the main rocket. If the masses of the payloads are very different, the main rocket need not be altered. Instead, the strap-on boosters can be changed to deliver the extra propulsion needed.

3 Orbits

The next concern when putting artificial satellites in space is putting them in the right place. This means that an orbit must be selected. The type of orbit desired will depend on the purpose of the satellite. Very precise calculations are necessary to put a satellite in the correct orbit. Not only must the orbit be right, but also other satellites already in orbit must be accounted for since any collision could be disastrous.

3.1 Different Types of Orbits

It is commonly believed that satellites in space deal with a lack of gravity, when in fact this is not the case. Earth's gravity keeps these satellites in orbit. It turns out that we are not dealing with a lack of gravity, but with changes in the gravitational field. The gravitational field of any object is radial; thus it is possible for smaller objects to orbit around larger masses in circular (or elliptical) trajectories. Spacecrafts and satellites always move in orbits when they are in space. There are many different types of orbits when we talk about satellites. These orbits include, low earth orbit, geostationary orbit, sun synchronous orbit, among others. Depending on the purpose of the satellite the orbit will be different. For example for a telecommunications satellite a geostationary orbit may be the best. Since the satellite will always be in the same point in the sky and it could give continuous coverage to a particular area. This type of orbit, however, may not be very useful for a spy satellite, where we would want the satellite to 'sweep' through large areas. The defining factor of each type of orbit will be its radius and, as we will see, the angular momentum of the object in orbit.

We will start our analysis with a circular orbit. We know from basic physics that the force keeping objects in circular motion is the centripetal force which is given by

$$F = \frac{mV^2}{r} \tag{13}$$

where m is the mass of the object in orbit, V is its speed and r is the radius of the object. The centripetal force is the generic name given to any force that is causing the circular motion. When dealing with the Earth-satellite system, the gravitational force is what is causing the circular motion, this force is given by

$$F = \frac{GM_E m}{r^2} \tag{14}$$

here M_E is the mass of the Earth and G is the gravitational constant. If we set (13) equal to (14) we can solve for the speed of the object and we get

$$V = \sqrt{\frac{GM_E}{r}} \tag{15}$$

This result may be somewhat surprising seeing as how the mass of the object in orbit does not affect its speed. Another important quantity when dealing with orbits is an object's angular momentum, seeing as how this quantity is conserved throughout the orbit (this we know from conservation of momentum, a law in physics which must always be obeyed). The angular momentum is given by

$$h = mrV \tag{16}$$

We can define the orbit of an object with two parameters. the first is the angular momentum of the object and the second will be the minimum distance of the orbit to the focus, since orbits are not all circular, some are elliptical. For a satellite to be placed in the correct orbit it is necessary to first get the object to the correct distance and, as given by the equations above, ensure that the satellite has the right parameters for that particular orbit. In fact there are several stages to put a satellite into its correct orbit. The first

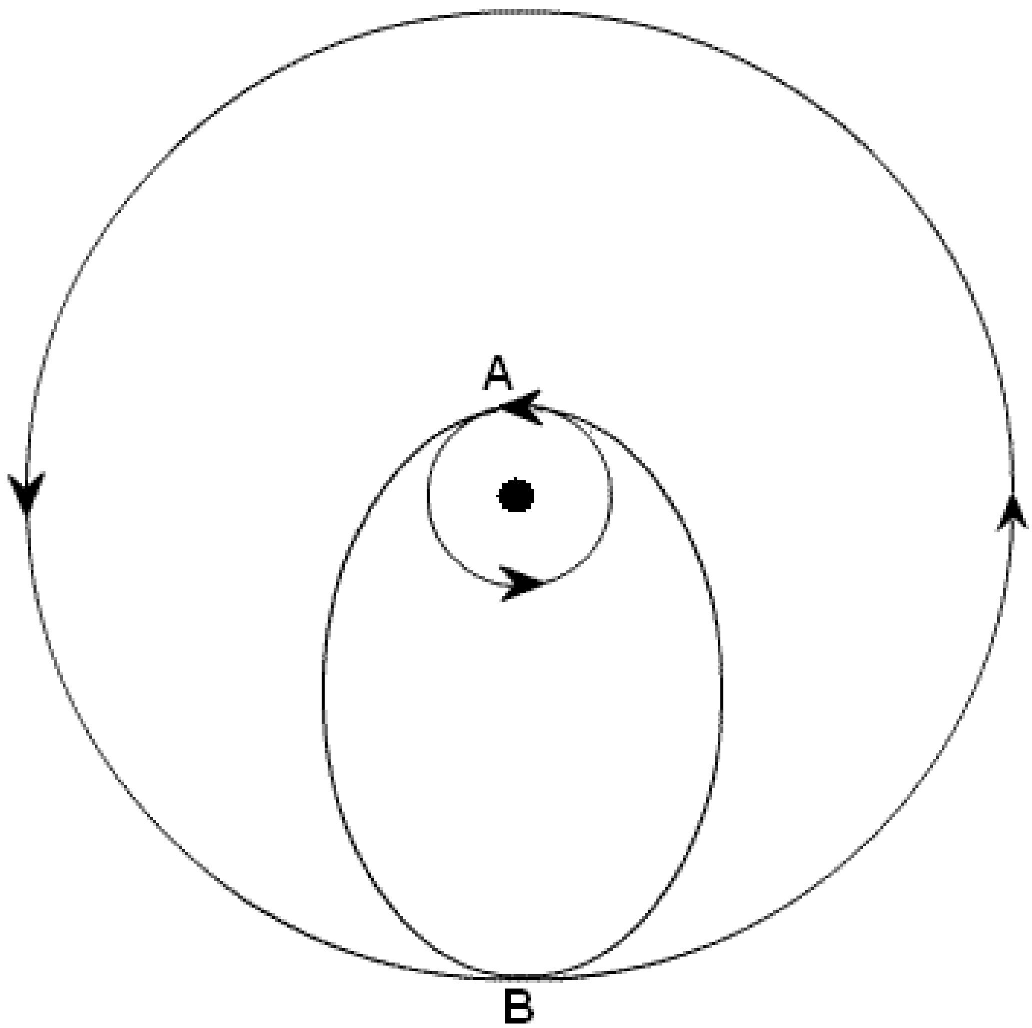

Figure 2: Diagram of a Hohmann transfer orbit

step is to take the satellite from the ground and put it into Low Earth Orbit, which is defined as an orbit 100 to 1200 miles above the Earth's surface. In the next step the satellite is taken into a transfer orbit (See fig. 2), and finally the satellite is put into its final orbit. This process is called a Hohmann transfer. The concept behind this is to put the vehicle, or satellite in this case, in an elliptical transfer orbit that fits in between the initial low orbit and the final desired orbit. This change of orbits is achieved by changing the speed of the satellite, as we have seen the speed of the satellite will change its angular momentum which is one of the parameters which specifies the orbit, so by changing the speed, it is possible to change the orbit. In order to find the speed changes necessary, the calculations use another conserved quantity in these orbits, the specific energy. The specific energy ε is defined as

$$\varepsilon = \frac{E}{m} = \frac{V^2}{2} - \frac{\mu}{r} \tag{17}$$

where $\mu = GM_{Earth}$. The first term corresponds to the kinetic energy of the object, and the second is its potential energy from being in a gravitational field. The value for the velocity at which this specific energy is zero is known as the escape velocity since the object will have enough energy to escape the gravitational field and not come back. Since we are talking about ellipses we must change this equation to

$$\varepsilon = -\frac{\mu}{2a} = -\frac{\mu}{r_a + r_p} \tag{18}$$

here 2a is the distance from the apogee to the perigee, and r_a, r_p are the distances from the focus to the apogee and the perigee of the ellipse. We will now work out the example for a transfer orbit between Low Earth Orbit (LEO) and Geosynchronous orbit.

We will suppose that our satellite is going from a LEO with an altitude of 250km to a geosynchronous orbit at an altitude of 36,000km. Our conservation laws require that the specific energy of the satellite be the same at the point at which the LEO intersects the transfer oribit and at the point where the geosynchronous orbit intersects the transfer orbit. And it must be the same as the specific energy of the transfer orbit as given by (18) these conditions give us

$$\varepsilon_{transfer} = -\frac{\mu}{r_a + r_p} = \frac{(v^{leo}_{transf})^2}{2} - \frac{\mu}{r_{leo}} = \frac{(v^{geo}_{transf})^2}{2} - \frac{\mu}{r_{geo}} \tag{19}$$

Since we know angular momentum is conserved we can use (16) to get

$$r_{leo} v^{leo}_{transf} = r_{geo} v^{geo}_{transf} \tag{20}$$

Here the masses have cancelled since we are dealing with the same object. Next we define the difference in speeds at the transfer points.

$$\Delta v_{geo} = v_{geo} - v^{geo}_{transf} \tag{21}$$

$$\Delta v_{leo} = v_{leo} - v^{leo}_{transf} \tag{22}$$

Using the numbers we have and (15) we get the following numbers

$$\Delta v_{leo} = 2.449 \; km/sec$$

$$\Delta v_{geo} = 1.474 \; km/sec$$

Looking at the definitions for the difference of the speeds and finding that they are positive we can see that at the transfer points our satellite has to slow down to get into a higher orbit. For this, rockets on the satellite must slow it down by these exact velocities at the transfer points. We can see that it is crucial for these calculations to be very accurate seeing as how a small mistake may put the satellite in the wrong transfer orbit or may cause it to collide with another satellite in orbit.

3.2 Keeping the Satellite in its orbit

Throughout the life of a satellite it will be necessary for it to adjust its speed, altitude or direction where it is pointing. For this satellites are equipped with small rockets so that ground controllers can adjust the satellite if necessary. If the orbit of a satellite is lower than 100 miles from the Earth's surface, atmospheric drag will have a large effect; this is why satellite orbits are above this altitude [2].

3.3 Approximations in this Section

There are some factors that have been ignored in this section that make these calculations more complicated. For example we have only taken into account Earth's gravitational field, we would have to analyze whether the Moon has a non-negligible effect. In fact when the Apollo 11 lunar module was going to land on the Moon, Armstrong had to take control of the vehicle since in the initial calculations NASA engineers had not taken into account the gravitational effects of Mars. For this reason it is necessary to come up with methods to do these calculations taking into account several effects, we must also answer the question of when these effects are negligible. Should we take into account the gravitational effects of Venus, the Sun; where do we draw the line? There is a large problem when dealing with the numbers for different gravitational effects, this is because the distance between the satellite and Earth is orders of magnitude smaller than the one between the satellite and Mars, so we will encounter additions of numbers that are many orders of magnitude different, and we might encounter some cancelation problems. For this reason when putting a satellite into orbit several very accurate simulations are necessary. Any error could render the satellite useless and an investment of around a billion dollars would be lost.

4 Conclusions

To study rocket propulsion, we derived the Tsiolkovsky Rocket Equation, which calculates velocity based on the exhaust velocity and the mass ratio of the payload to the rocket. To increase the mass ratio, multi-stage rockets are used, which carry extra fuel tanks that are discarded after they exhaust their fuel. Detailed calculations show that having a 3-stage rocket creates the optimal mass ratio for rocket propulsion.

Once rockets have been launched, we need to determine how to put them in the appropriate orbit. Based on the satellites intended use, there are a variety of orbits, which vary due to their different radii and angular momentum. We derived the equations to determine the optimal velocity and angular momentum to place satellites into orbit. Precise calculations are needed for the multi-stage process to transfer satellites from low-earth orbit, after their escape from the earths atmosphere, to their final orbit, in a process known as a Hohman transfer. If the escape velocity is reached, the satellite would travel out of orbit, and become useless. If the satellite doesn't reach its desired orbit, it ceases to be useful for its intended function. Thus, determining accurate velocities is extremely important in launching satellites into orbit.

5 References

[1] Turner, M. 2000. Rocket and Spacecraft Propulsion. Praxis Publishing. United Kingdom

[2] Cook, W. 1996. Launching Satellites in Orbit. America Online. United States

[3] Morel, B. 2006. Lectures from 19-430 Civilian and Military Applications of Space. Carnegie Mellon University