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Computing geodetic coordinates from geocentric coordinates

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Abstract. A closed-form algebraic method to transform geocentric coordinates to geodetic coordinates has previously been proposed. The validity domain of latitude and height formulae in the vicinity of the Earth's core is specified. A new expression of longitude is proposed, excluding indetermination and sensitivity to round-off error around the ±180 degrees longitude discontinuity.

Key words: Coordinate transformation – Geocentric coordinates – Geodetic coordinates

1 Notation

a, e	semi-major axis, eccentricity of reference ellipsoid
X, Y, Z	Cartesian geocentric coordinates
h, φ, λ	geodetic height, geodetic latitude, geodetic longitude

2 Validity domain of geodetic height and geodetic latitude

The geodetic coordinates h and φ can be computed from the geocentric coordinates by the following sequence of algebraic formulae (Vermeille 2002):

$$p = \frac{X^2 + Y^2}{a^2} \tag{1}$$

$$q = \frac{1 - e^2}{a^2} Z^2 \tag{2}$$

$$r = \frac{p + q - e^4}{6} \tag{3}$$

$$s = e^4 \frac{pq}{4r^3} \tag{4}$$

$$t = \sqrt[3]{1 + s + \sqrt{s(2 + s)}}$$

$$u = r\left(1 + t + \frac{1}{t}\right)$$

$$v = \sqrt{u^2 + e^4 q}$$

$$w = e^2 \frac{u + v - q}{2v}$$

$$k = \sqrt{u + v + w^2} - w$$

$$D = \frac{k\sqrt{X^2 + Y^2}}{k + e^2}$$

$$h = \frac{k + e^2 - 1}{k} \sqrt{D^2 + Z^2}$$

$$\varphi = 2 \arctan \frac{Z}{D + \sqrt{D^2 + Z^2}}$$

In a previous paper (Vermeille 2002), we have supposed s positive to solve an equation of degree 3 in u/r. This defines the validity domain of height and latitude: from Eq. (4), as p and q are positive, we conclude that r must be positive. Then Eq. (3) gives, from Eq. (1) and Eq. (2)

$$X^2 + Y^2 + (1 - e^2)Z^2 > a^2 e^4$$

Since

$$X^2 + Y^2 + (1 - e^2)Z^2 > (1 - e^2)(X^2 + Y^2 + Z^2)$$

a sufficient condition for $r > 0$ is

$$\sqrt{X^2 + Y^2 + Z^2} > \frac{ae^2}{\sqrt{1 - e^2}}$$

The height and latitude formulae will be valid if the distance between the point X, Y, Z and the origin, the centre of the Earth, is greater than 43 km. In practice, this is always true.

3 Geodetic longitude around ± 180 degrees

The value of λ

$$\lambda = 2 \arctan \frac{Y}{X + \sqrt{X^2 + Y^2}} \tag{5}$$

is indeterminate when $X \leq 0$ and $Y = 0$ and this formula is unusable around $\lambda = \pm 180$ degrees. We therefore propose a more suitable expression for λ.
When $Y \geq 0$, we write Eq. (5) as

$$\lambda = \frac{\pi}{2} - 2 \arctan 1 + 2 \arctan \frac{Y}{X + \sqrt{X^2 + Y^2}}$$

i.e.

$$\lambda = \frac{\pi}{2} - 2 \arctan \frac{X}{\sqrt{X^2 + Y^2} + Y} \tag{6}$$

where λ is continuous from $-\pi/2$ to $3\pi/2$.

In the same manner, when $Y < 0$ we write Eq. (5) as

$$\lambda = -\frac{\pi}{2} + 2 \arctan 1 + 2 \arctan \frac{Y}{X + \sqrt{X^2 + Y^2}}$$

i.e.

$$\lambda = -\frac{\pi}{2} + 2 \arctan \frac{X}{\sqrt{X^2 + Y^2} - Y} \tag{7}$$

where λ is continuous from $-3\pi/2$ to $\pi/2$.

In Eqs. (6) and (7), when $X < 0$ and $Y \simeq 0$ a round-off error may produce a value of arctan slightly less than $-\pi/4$. Then Eqs. (6) and (7) return a value of λ slightly exceeding $\pm\pi$ without discontinuity between $-\pi$ and $+\pi$.

4 Conclusion

Starting from Cartesian coordinates X, Y, Z, satisfying the following condition:

$$\sqrt{X^2 + Y^2 + Z^2} > \frac{ae^2}{\sqrt{1 - e^2}}$$

first we compute the value of h and φ by the following sequence of formulae:

$$p = \frac{X^2 + Y^2}{a^2}$$

$$q = \frac{1 - e^2}{a^2} Z^2$$

$$r = \frac{p + q - e^4}{6}$$

$$s = e^4 \frac{pq}{4r^3}$$

$$t = \sqrt[3]{1 + s + \sqrt{s(2 + s)}}$$

$$u = r(1 + t + \frac{1}{t})$$

$$v = \sqrt{u^2 + e^4 q}$$

$$w = e^2 \frac{u + v - q}{2v}$$

$$k = \sqrt{u + v + w^2} - w$$

$$D = \frac{k\sqrt{X^2 + Y^2}}{k + e^2}$$

$$h = \frac{k + e^2 - 1}{k} \sqrt{D^2 + Z^2}$$

$$\varphi = 2 \arctan \frac{Z}{D + \sqrt{D^2 + Z^2}}$$

Next, if we have not both X and $Y = 0$

when $Y \geq 0$

$$\lambda = \frac{\pi}{2} - 2 \arctan \frac{X}{\sqrt{X^2 + Y^2} + Y}$$

and when $Y < 0$

$$\lambda = -\frac{\pi}{2} + 2 \arctan \frac{X}{\sqrt{X^2 + Y^2} - Y}$$

Reference

Vermeille H (2002) Direct transformation from geocentric coordinates to geodetic coordinates. J Geod 76: 451–454