Chapter 2

Mission Analysis

As noted in Chapter 1, orbital and attitude dynamics must be considered as coupled. That is to say, the orbital motion of a spacecraft affects the attitude motion, and the attitude motion affects the orbital motion. The attitude $\rightarrow$ orbital coupling is not as significant as the orbital $\rightarrow$ attitude coupling. This fact is traditionally used as motivation to treat the orbit as a given motion and then investigate the attitude motion for a given orbit.

As long as the orbit is circular, the effects on attitude are reasonably straightforward to determine. If the orbit is elliptical, then the effects on the attitude dynamics are more complicated. In this chapter, I develop the basic geometric relationships necessary to investigate the effects of orbital motion on the spacecraft attitude or pointing requirements. The required topics in orbital dynamics are summarized in Appendix A, and some basic spherical geometry terms and relations are given in Appendix B.

I begin by describing the geometric quantities necessary to define pointing and mapping requirements. This space mission geometry is useful for understanding how to visualize attitude motion, and for determining important quantities such as the direction from the spacecraft to the sun. After developing these concepts, I show how a variety of uncertainties lead to errors and how estimates of these errors influence spacecraft design.

2.1 Mission Geometry

In the first approximation, an Earth-orbiting spacecraft follows an elliptical path about the Earth's mass center, and the Earth rotates about its polar axis. This approximation leads to a predictable motion of the satellite over the Earth, commonly illustrated by showing the satellite's *ground track*. Figure 2.1 shows the ground tracks for two satellites: (a) Zarya, the first component of the International Space Station, and satellite in a circular orbit with altitude 500 km, and inclination of 30°, and (b) a satellite in an elliptical orbit with periapsis altitude of 500

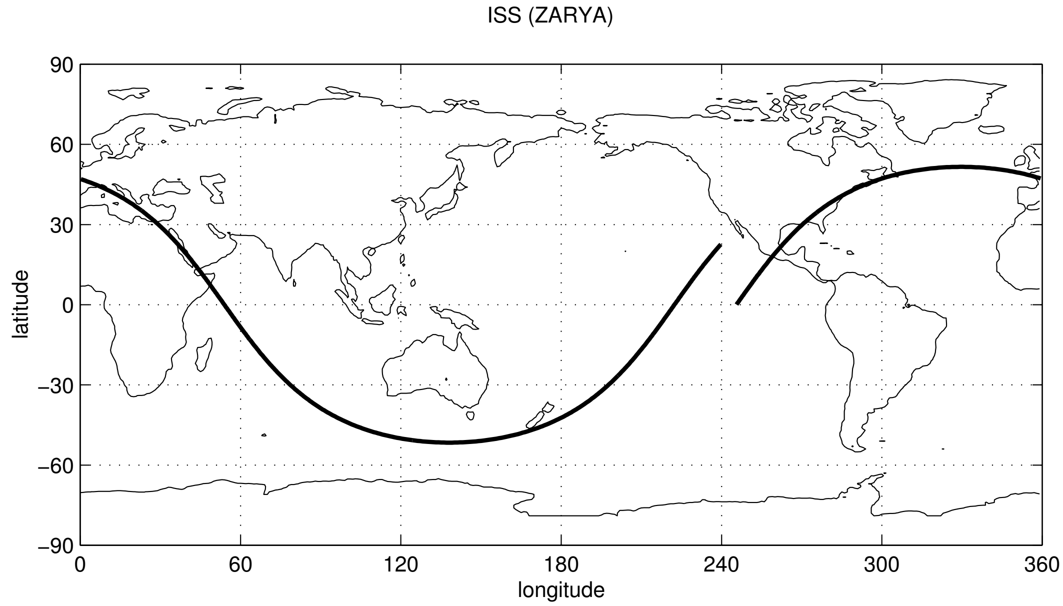

Figure 2.1: Ground Track for the International Space Station

km, apoapsis altitude of about 24,000 km, an eccentricity of 0.63, and inclination of about 26°. Similar plots can be created using Satellite Tool Kit or WinOrbit, which are available on the World Wide Web at http://www.stk.com and http://www.sat-net.com/winorbit/index.html, respectively.

2.1.1 Earth viewed from space

At any instant in time, the point on a ground track is defined as the point of intersection between the surface of the Earth and the line connecting the Earth center and the satellite. This point is called the *sub-satellite point* (SSP). The spacecraft can "see" the sub-satellite point and the area around the SSP. This area is called the *instantaneous access area* (IAA), and is always less than half of the surface area of the Earth. Figures 2.3 and 2.4 illustrate the IAA and related parameters. Figure 2.3 shows the Earth and satellite orbit. The satellite's altitude is denoted by H, so that the distance from the center of the Earth to the satellite is $R = R_\oplus + H$, where $R_\oplus$ is the radius of the Earth.

Decisions about the attitude control system must be made based on the requirements of the spacecraft mission. As described in Chapter 1, ACS requirements generally arise from the need to point the payload or some other subsystem in a particular direction. For example, a communications satellite's antenna must point at its terrestrial counterpart. Similarly, a remote sensing satellite's instrument must point at its subject. More generally, solar panels must point at the sun, and thermal radiators must point away from the sun. Some sensitive optical instruments must avoid pointing near the sun, moon, or earth, as the light from these objects could damage the instruments. Additional requirements include the range of possible pointing

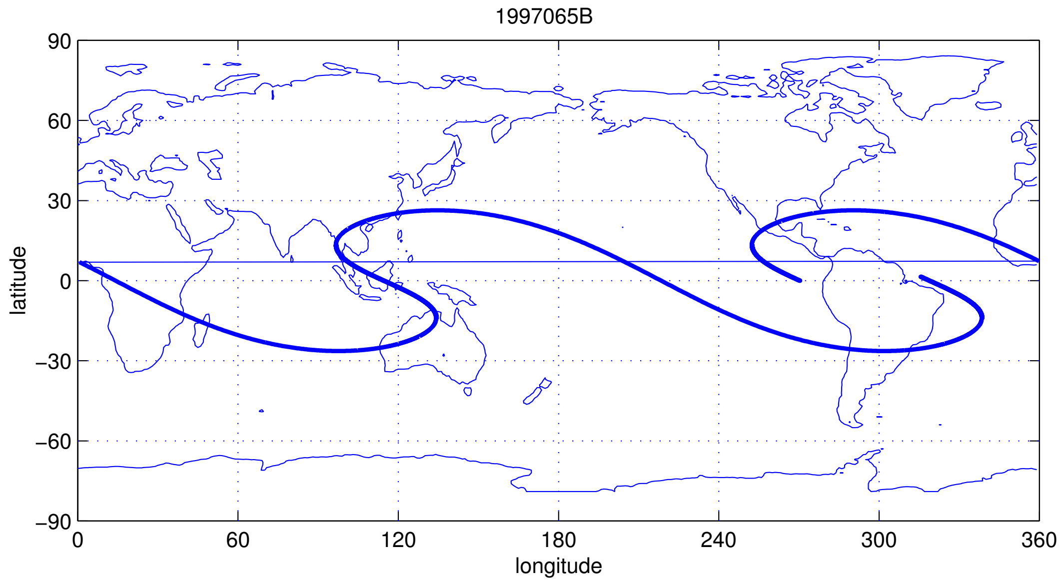

Figure 2.2: Ground Track for the Falcon Gold Satellite

directions, pointing accuracy and stability, pointing knowledge accuracy, and slew rate.

From orbital dynamics (Appendix A), we know how to follow a satellite in its orbit. That is, we can readily compute $\vec{\mathbf{r}}(t)$ and $\vec{\mathbf{v}}(t)$ for a given orbit. We can also compute the orbit ground track in a relatively straightforward manner. The following algorithm computes latitude and longitude of the sub-satellite point (SSP) as functions of time.

Algorithm 2.1
Initialize

orbital elements: a, e, i, ω, Ω, ν_0
Greenwich sidereal time at epoch: θ_{g0}
period: $P = 2\pi\sqrt{a^3/\mu}$
number of steps: N
time step: $\Delta t = P/(N-1)$

for $j = 0$ *to* $N-1$

Compute

Greenwich sidereal time: $\theta_g = \theta_{g0} + j\omega_\oplus \Delta t$
position vector: $\mathbf{r}$
latitude: $\delta_s = \sin^{-1}(r_3/r)$
longitude: $L_s = \tan^{-1}(r_2/r_1) - \theta_g$

At any point in its orbit, the spacecraft can see a circular region around the sub-satellite point. This region is known as the instantaneous access area (IAA). The IAA sweeps out a swath as the spacecraft moves in its orbit. Knowing how to determine

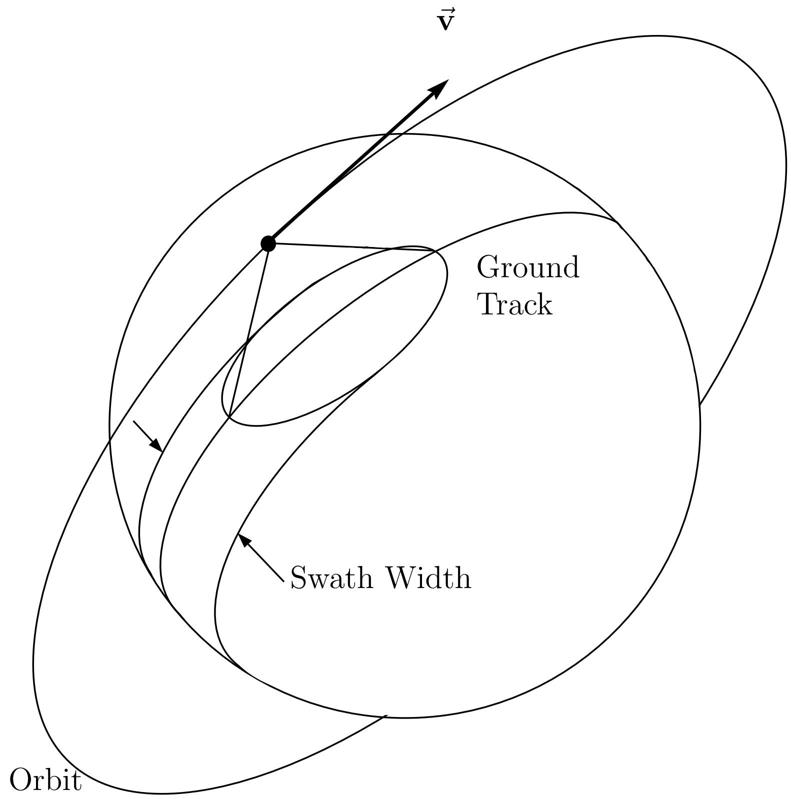

Figure 2.3: Earth Viewed by a Satellite

the SSP, let us develop some useful concepts and algorithms for spacecraft looking at the Earth.

The instantaneous access area is the area enclosed by the small circle on the sphere of the Earth, centered at the SSP and extending to the horizon as seen by the spacecraft. Two angles are evident in this geometry: the Earth central angle, λ_0, and the Earth angular radius, ρ. These are the two non-right angles of a right triangle whose vertices are the center of the Earth, the spacecraft, and the Earth horizon as seen by the spacecraft. Thus the two angles are related by*

$$\rho + \lambda_0 = 90^\circ \tag{2.1}$$

From the geometry of the figure, these angles may be computed from the relation

$$\sin\rho = \cos\lambda_0 = \frac{R_\oplus}{R_\oplus + H} \tag{2.2}$$

*We usually give common angles in degrees; however, most calculations involving angles require radians.

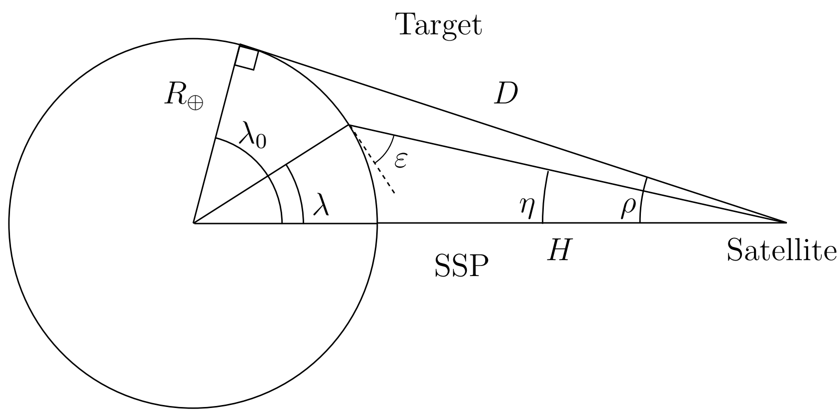

Figure 2.4: Geometry of Earth-viewing

where $R_\oplus$ is the radius of the Earth, and H is the altitude of the satellite. The IAA may be calculated as

$$\text{IAA} = K_A(1 - \cos\lambda_0) \tag{2.3}$$

where

$$\begin{array}{lcll} K_A & = & 2\pi & \text{area in steradians} \\ K_A & = & 20,626.4806 & \text{area in deg}^2 \\ K_A & = & 2.55604187 \times 10^8 & \text{area in km}^2 \\ K_A & = & 7.45222569 \times 10^7 & \text{area in nmi}^2 \end{array}$$

Generally, a spacecraft can point an instrument at any point within its IAA; however, near the horizon, a foreshortening takes place that distorts the view. The operational effects of this distortion are handled by introducing a minimum elevation angle, ε_{min} that reduces the usable IAA. Figure 2.4 shows the relationship between elevation angle, ε, Earth central angle, λ, Earth angular radius, ρ, nadir angle, η, and range, D. These variables are useful for describing the geometry of pointing a spacecraft at a particular target. The target must be within the IAA, and the spacecraft elevation angle ε is measured "up" from the target location to the satellite. Thus if the target is on the horizon, then $\varepsilon = 0$, and if the target is at the SSP, then $\varepsilon = 90°$; therefore ε is always between 0 and 90°. The angle λ is the angle between the position vectors of the spacecraft and the target, so it can be computed using

$$\vec{\mathbf{r}}_s \cdot \vec{\mathbf{r}}_t = r_s r_t \cos\lambda \tag{2.4}$$

where $\vec{\mathbf{r}}_s$ is the position vector of the spacecraft and $\vec{\mathbf{r}}_t$ is the position vector of the target. Clearly $r_s = R_\oplus + H$, and $r_t = R_\oplus$, so

$$\cos\lambda = \frac{\vec{\mathbf{r}}_s \cdot \vec{\mathbf{r}}_t}{R_\oplus(R_\oplus + H)} \tag{2.5}$$

Since we already know the latitude, δ, and longitude, L of both the SSP and the target, we can simplify these expressions, using

$$\vec{\mathbf{r}}_s = (R_\oplus + H)\left(\cos\delta_s \cos L_s \hat{\mathbf{I}}' + \cos\delta_s \sin L_s \hat{\mathbf{J}}' + \sin\delta_s \hat{\mathbf{K}}'\right) \tag{2.6}$$

$$\vec{\mathbf{r}}_t = R_\oplus \left(\cos\delta_t \cos L_t \hat{\mathbf{I}}' + \cos\delta_t \sin L_t \hat{\mathbf{J}}' + \sin\delta_t \hat{\mathbf{K}}'\right) \tag{2.7}$$

Carrying out the dot product in Eq. (2.5), collecting terms, and simplifying, leads to

$$\cos\lambda = \cos\delta_s \cos\delta_t \cos\Delta L + \sin\delta_s \sin\delta_t \tag{2.8}$$

where $\Delta L = L_s - L_t$. Clearly λ must be between 0 and 90°.

Knowing λ, the nadir angle, η, can be found from

$$\tan\eta = \frac{\sin\rho\sin\lambda}{1 - \sin\rho\cos\lambda} \tag{2.9}$$

Knowing η and λ, the elevation angle can be determined from the relationship

$$\eta + \lambda + \varepsilon = 90^\circ \tag{2.10}$$

Also, the range, D, to the target may be found using

$$D = R_\oplus \frac{\sin\lambda}{\sin\eta} \tag{2.11}$$

2.2 Summary of Earth Geometry Viewed From Space

The following formulas are useful in dealing with Earth geometry as viewed by a spacecraft. In these formulas, the subsatellite point has longitude L_s and latitude δ_s, and the target point has longitude L_t and latitude δ_t. The notation δ_t' is used to denote the *colatitude* $90^\circ - \delta_t$. The angular radius of the Earth is ρ, the radius of the Earth is R_E, and the satellite altitude is H. The Earth central angle is λ, Az is the azimuth angle of the target relative to the subsatellite point, η is the nadir angle, and ε is the grazing angle or spacecraft elevation angle. This notation is used in Larson and Wertz's *Space Mission Analysis and Design*, 2nd edition, 1992, which is essentially the only reference for this material.

$$\sin\rho = R_E/(R_E + H)$$

Spacecraft viewing angles $(L_s, \delta_s, L_t, \delta_t) \mapsto (\lambda, \mathrm{Az}, \eta)$

$$\Delta L = |L_s - L_t|$$

$$\begin{aligned}
\cos\lambda &= \sin\delta_s \sin\delta_t + \cos\delta_s \cos\delta_t \cos\Delta L \quad (\lambda < 180^\circ) \\
\cos \mathrm{Az} &= \frac{\sin\delta_t - \cos\lambda \sin\delta_s}{\sin\lambda \cos\delta_s} \\
\tan\eta &= \frac{\sin\rho \sin\lambda}{1 - \sin\rho\cos\lambda}
\end{aligned}$$

$$\begin{aligned}
\text{Earth coordinates} & \quad (L_s, \delta_s, \mathrm{Az}, \eta) \mapsto (\lambda, \delta_t, \Delta L) \\
\cos\varepsilon &= \frac{\sin\eta}{\sin\rho} \\
\lambda &= 90^\circ - \eta - \varepsilon \\
\cos\delta_t' &= \cos\lambda \sin\delta_s + \sin\lambda \cos\delta_s \cos \mathrm{Az} \quad (\delta_t' < 180^\circ) \\
\cos\Delta L &= \frac{\cos\lambda - \sin\delta_s \sin\delta_t}{\cos\delta_s \cos\delta_t}
\end{aligned}$$

The Instantaneous Access Area (IAA) is

$$\mathrm{IAA} = K_A(1 - \cos\lambda_0)$$

where

$$\begin{aligned}
K_A &= 2\pi && \text{area in steradians} \\
K_A &= 20,626.4806 && \text{area in deg}^2 \\
K_A &= 2.55604187 \times 10^8 && \text{area in km}^2 \\
K_A &= 7.45222569 \times 10^7 && \text{area in nmi}^2
\end{aligned}$$

2.3 Error Budget

The attitude or orientation of a spacecraft usually arises as either a pointing problem requiring control, or a mapping problem requiring attitude determination. Example pointing problems would be to control the spacecraft so that a particular instrument (camera, antenna, *etc.*) points at a particular location on the surface of the Earth or at a particular astronomical object of interest. Mapping problems arise when the accurate location of a point being observed is required. In practice, most spacecraft operations involve both pointing and mapping. For example, we may command the spacecraft to point at Blacksburg and take a series of pictures. Afterwards we may need to determine the actual location of a point in one of the pictures.

In order to describe precisely the errors associated with pointing and mapping, we need to define some terms associated with the geometry of an Earth-pointing spacecraft. In this figure, the angle ϵ is the elevation, ϕ is the azimuth angle with respect to the orbital plane, η is the nadir angle, and λ is the Earth central angle. The distance from the center of the Earth to the satellite is R_s, and to the target point is R_T. We define the errors in position as ΔI, ΔC, and ΔR_s, where ΔI is the in-track error, ΔC is the cross-track error, and ΔR_s is the radial error. The instrument

Table 2.1: Sources of Pointing and Mapping Errors[2]

Spacecraft Position Errors		
ΔI	In- or along-track	Displacement along the spacecraft's velocity vector
ΔC	Cross-track	Displacement normal to the spacecraft's orbit plane
ΔR_S	Radial	Displacement toward the center of the Earth (nadir)
Sensing Axis Orientation Errors (in polar coordinates about nadir)		
$\Delta\eta$	Elevation	Error in angle from nadir to sensing axis
$\Delta\phi$	Azimuth	Error in rotation of the sensing axis about nadir
Other Errors		
ΔR_T	Target altitude	Uncertainty i the altitude of the observed object
ΔT	Clock error	Uncertainty in the real observation time

axis orientation error is described by two angles: $\Delta\eta$ and $\Delta\phi$. Two additional error sources are uncertainty of target altitude and clock error: ΔR_T and ΔT.

Based on these error sources, the approximate mapping and pointing errors are described in Tables 2.1 – 2.2. This development follows that in Larson and Wertz.[2]

The errors defined in Table 2.1 lead to mapping and pointing errors as described in Table 2.2.

2.4 References and further reading

Mission analysis is closely related to space systems design. Wertz's handbook[1] provides substantial coverage of all aspects of attitude determination and control systems. The more recent volume edited by Larson and Wertz[2] updates some of the material from Ref. 1, and gives an especially useful treatment of space mission geometry and its influence on design. Brown's text[3] focuses mostly on orbital analysis, but includes a chapter on "Observing the Central Body." The Jet Propulsion Laboratory's *Basics of Space Flight*[4] covers interplanetary mission analysis in detail, but does not provide much on attitude control systems.

Bibliography

[1] J. R. Wertz, editor. *Spacecraft Attitude Determination and Control*. D. Reidel, Dordrecht, Holland, 1978.

[2] Wiley J. Larson and James R. Wertz, editors. *Space Mission Analysis and Design*. Microcosm, Inc., Torrance, CA, second edition, 1995.

Table 2.2: Pointing and Mapping Error Formulas[2]

Source	Magnitude	Magnitude of Mapping Error (km)	Magnitude of Pointing Error (rad)	Direction of Error
Attitude Errors:				
Azimuth	$\Delta\phi$ (rad)	$\Delta\phi D \sin\eta$	$\Delta\phi \sin\eta$	Azimuthal
Nadir Angle	$\Delta\eta$ (rad)	$\Delta\eta D / \sin\varepsilon$	$\Delta\eta$	Toward nadir
Position Errors:				
In-track	ΔI (km)	$\Delta I (R_T/R_S) \cos H$	$(\Delta I/D) \sin Y_I$	Parallel to ground track
Cross-track	ΔC (km)	$\Delta C (R_T/R_S) \cos G$	$(\Delta C/D) \sin Y_C$	Perpendicular to ground track
Radial	ΔR_S (km)	$\Delta R_S \sin\eta / \sin\varepsilon$	$(\Delta R_S/D) \sin\eta$	Toward nadir
Other Errors:				
Target altitude	ΔR_T (km)	$\Delta R_T / \tan\varepsilon$	—	Toward nadir
S/C Clock	ΔT (s)	$\Delta T V_e \cos\ \mathrm{lat}$	$\Delta T (V_e/D) \cos\ \mathrm{lat} \sin J$	Parallel to Earth's equator

$\sin H = \sin\lambda \sin\phi$
$\sin G = \sin\lambda \cos\phi$
$V_e = 464$ m/s (Earth rotation velocity at equator)
$\cos Y_I = \cos\phi \sin\eta$
$\cos Y_C = \sin\phi \sin\eta$
$\cos J = \cos\phi_E \cos\varepsilon$, where ϕ_E = azimuth relative to East

[3] Charles D. Brown. *Spacecraft Mission Design.* AIAA, Reston, Virginia, second edition, 1998.

[4] Dave Doody and George Stephan. *Basics of Space Flight Learners' Workbook.* Jet Propulsion Laboratory, Pasadena, 1997. http://www.jpl.nasa.gov/basics.

2.5 Exercises

1. A satellite is in a circular Earth orbit with altitude 500 km. Determine the instantaneous coverage area if the minimum elevation angle is $\epsilon_{\text{min}} = 10°$.
2. Derive Eqs. (2.9) and (2.11), and give a geometrical explanation for Eq. (2.10).

2.6 Problems

1. Make a ground track plot for a satellite with the following two-line element set:

```
COSMOS 2278
1 23087U 94023A   98011.59348139  .00000348  00000-0  21464-3 0  5260
2 23087  71.0176  58.4285 0007185 172.8790 187.2435 14.12274429191907
```

2. The following two-line element set (TLE) is for the International Space Station:

```
ISS (ZARYA)
1 25544U 98067A   99026.49859894 -.00001822  00000-0 -18018-4 0  2532
2 25544  51.5921 190.3677 0004089  55.0982 305.0443 15.56936406 10496
```

Detailed information about the TLE format can be found in the "handout" on the course webpage: Appendix A: Orbits. Another good source is on the web at http://celestrak.com, which is where I got the information in the appendix, and where I obtained this TLE.

For this assignment, you should ignore Earth oblateness effects, as well as the "perturbation" terms in the TLE ($\dot{n}$, $\ddot{n}$, and B^*).

(a) What are the date and the Eastern Standard Time of epoch?

(b) What are the orbital elements of Zarya? (a, e, i, ω, Ω, and ν_0, with a in km, and angles in degrees)

(c) What are the latitude and longitude of the sub-satellite point (SSP) at epoch?

(d) What is the area, in km^2, of the instantaneous access area (IAA) at epoch? What fraction of the theoretical maximum IAA is this area?

(e) If the minimum elevation angle is $\varepsilon_{min} = 10°$, then what is the "reduced" IAA at epoch?

(f) What are the latitude and longitude of the SSP the next time (after epoch) the station passes through apoapsis?

3. **Programming Project.** A useful MatLab function would compute the sub-satellite point latitude and LST for a given set of orbital elements, not necessarily circular. A calling format could be

```
[lat,lst] = oe2ssp(oe,dt)
```

The argument `dt` represents a Δt from epoch and could be optional, and it should work if `dt` is a "vector." If `dt` is omitted, then the function should return latitude and LST at epoch. If `dt` is a vector, then the returned values of `lat` and `lon` should be vectors of the same length. The function should also have an optional argument for specifying the gravitational parameter μ, so that the function can be used with different systems of units. How would you need to modify this function so that it provided latitude and longitude instead of latitude and LST? What if you wanted to use it for planning missions about other planets?