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Class: MSc CS Part I Roll no.: 45

Subject: Social Network Analysis

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**<u>Aim:</u>** Write a program to compute the following for a given a network:

- ♦ number of edges
- ♦ number of nodes
- ♦ degree of node
- ♦ node with lowest degree
- ♦ the adjacency list
- matrix of the graph.

## **Software(s) used:**

- ♦ R ver. 4.1.3
- ♦ RStudio ver. 2022.02.0+433

# **External packages required:**

♦ igraph

# **Description:**

• The igraph package: igraph is a library and R package for network analysis.

The main goals of the igraph library is to provide a set of data types and functions for:

- pain-free implementation of graph algorithms,
- ♦ fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.

- library(): library() loads and attach add-on packages.
- graph.formula():
  - ♦ Creating (small) graphs via a simple interface
  - ◆ This function is useful if you want to create a small (named) graph quickly, it works for both directed and undirected graphs.
- plot(): Use to plot any graph.
- ecount(): Returns the count of number of edges in graph
- vcount(): Returns the count of number of vertices in graph
- E():
  - ♦ Edges of a graph
  - ♦ An edge sequence is a vector containing numeric edge ids, with a special class attribute that allows custom operations: selecting subsets of edges based on attributes, or graph structure, creating the intersection, union of edges, etc.

#### • V():

- ♦ Vertices of a graph
- ◆ Create a vertex sequence (vs) containing all vertices of a graph.

### • degree():

- ♦ Degree and degree distribution of the vertices
- ♦ The degree of a vertex is its most basic structural property, the number of its adjacent edges.
- ♦ Mode-Character string, "out" for out-degree, "in" for indegree or "total" for the sum of the two. For undirected graphs this argument is ignored. "all" is a synonym of "total".

- max() and min():
  - ♦ Maxima and Minima
  - ◆ Returns the (regular or parallel) maxima and minima of the input values.
- get.adjacency(): Convert a graph to an adjacency matrix
- get.adjlist():
  - ♦ Adjacency lists
  - ♦ Create adjacency lists from a graph, either for adjacent edges or for neighboring vertices

```
library(igraph)

u_graph - graph.formula(A - B, A - C, A - D, B - C, B - F, C -
D, C - E, C - F, D - E, E - F, F - G, G - H)

d_graph - graph.formula(A + B, A + D, A -+ C, B -+ C, B -+
E, B -+ F, C -+ D, C -+ F, D -+ E)

e_count(u_graph)

e_count(d_graph)

v_count(d_graph)

E(u_graph)

E(u_graph)

V(u_graph)

V(u_graph)
```

```
degree(u_graph)
degree(u_graph, mode = "in")
degree(u_graph, mode = "out")

degree(d_graph)
degree(d_graph, mode = "in")
degree(d_graph, mode = "out")

V(u_graph)$name[degree(u_graph) = min(degree(u_graph))]
V(d_graph)$name[degree(d_graph, mode = "in") =
min(degree(d_graph, mode = "in"))]
V(d_graph)$name[degree(d_graph, mode = "out") =
min(degree(d_graph, mode = "out"))]

get.adjacency(u_graph)
get.adjlist(u_graph)
get.adjlist(u_graph)
get.adjlist(d_graph)
```

```
> library(igraph)
> u_graph ← graph.formula(A - B, A - C, A - D, B - C, B - F, C - D, C - E, C - F, D - E, E - F, F - G, G - H)
> d_graph ← graph.formula(A ++ B, A ++ D, A -+ C, B -+ C, B -+ E, B -+ F, C -+ D, C -+ F, D -+ E)
> ecount(d_graph)
[1] 11
> ecount(d_graph)
[1] 11
> vcount(u_graph)
[1] 8
> vcount(d_graph)
[1] 6
> E(u_graph)
+ 12/12 edges from 535b776 (vertex names):
[1] A -- B A -- C A -- D B -- C B -- F C -- D C -- F C -- E D -- E F -- E F -- G G -- H
> E(d_graph)
+ 11/11 edges from 5365b0b (vertex names):
[1] A -> B A -> D A -> C B -> A B -> C B -> E B -> F D -> A D -> E C -> D C -> F
```

```
> V(u_graph)
+ 8/8 vertices, named, from 535b776:
[1] A B C D F E G H
> V(d_graph)
+ 6/6 vertices, named, from 5365b0b:
[1] A B D C E F
 degree(u_graph)
A B C D F E G H
3 3 5 3 4 3 2 1
 degree(u_graph, mode = "in")
 BCDFEGH
 3 5 3 4 3 2 1
 degree(u_graph, mode = "out")
 BCDFEGH
 3 5 3 4 3 2 1
 degree(d_graph)
ABDCEF
5 5 4 4 2 2
```

```
> degree(d_graph, mode = "in")
A B D C E F
2 1 2 2 2 2
> degree(d_graph, mode = "out")
A B D C E F
3 4 2 2 0 0
>
> V(u_graph)$name[degree(u_graph) == min(degree(u_graph))]
[1] "H"
> V(d_graph)$name[degree(d_graph, mode = "in") == min(degree(d_graph, mode = "in"))]
[1] "B"
> V(d_graph)$name[degree(d_graph, mode = "out") == min(degree(d_graph, mode = "out"))]
[1] "E" "F"
>
```

```
> get.adjacency(u_graph)
8 x 8 sparse Matrix of class "dgCMatrix"
 ABCDFEGH
A . 1 1 1 . . .
B 1 . 1 . 1 . .
C 1 1 . 1 1 1 .
D 1 . 1 . . 1
F.11..11.
 . . 111. . .
G
> get.adjacency(d_graph)
6 x 6 sparse Matrix of class "dgCMatrix"
  ABDCEF
A . 1 1 1 . .
B 1 . . 1 1 1
D1...1.
Ε
```

```
> get.adjlist(u_graph)
$A
+ 3/8 vertices, named, from 535b776:
[1] B C D

$B
+ 3/8 vertices, named, from 535b776:
[1] A C F

$C
+ 5/8 vertices, named, from 535b776:
[1] A B D F E

$D
+ 3/8 vertices, named, from 535b776:
[1] A C E

$F
+ 4/8 vertices, named, from 535b776:
[1] B C E G
```

```
$E
+ 3/8 vertices, named, from 535b776:
[1] C D F
$G
+ 2/8 vertices, named, from 535b776:
[1] F H
$H
+ 1/8 vertex, named, from 535b776:
[1] G
> get.adjlist(d_graph)
$A
+ 5/6 vertices, named, from 5365b0b:
[1] B B D D C
$B
+ 5/6 vertices, named, from 5365b0b:
[1] A A C E F
```

```
$D
+ 4/6 vertices, named, from 5365b0b:
[1] A A C E

$C
+ 4/6 vertices, named, from 5365b0b:
[1] A B D F

$E
+ 2/6 vertices, named, from 5365b0b:
[1] B D

$F
+ 2/6 vertices, named, from 5365b0b:
[1] B C
```

## **<u>Aim:</u>** Perform following tasks:

- ♦ View data collection forms and/or import onemode/two-mode datasets.
- ♦ Basic Networks matrices transformations.

## **Software(s) used:**

- ♦ R ver. 4.1.3
- ♦ RStudio ver. 2022.02.0+433

## **External packages required:**

♦ igraph

## **Description:**

- The igraph package: igraph is a library and R package for network analysis.
  - The main goals of the igraph library is to provide a set of data types and functions for:
  - ♦ pain-free implementation of graph algorithms,
  - ♦ fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
- getwd(): Used to get the absolute filepath of the current R session.
- require(): library() and require() load and attach addon packages.

- read.csv(): Reads a file in table format and creates a data frame from it, with cases corresponding to lines and variables to fields in the file.
- head(): Returns the first part of a vector, matrix, table, data frame or function. Since head() and tail() are generic functions, they may also have been extended to other classes.
- graph.data.frame(): This function creates an igraph graph from one or two data frames containing the (symbolic) edge list and edge/vertex attributes.
- get.adjacency(): Sometimes it is useful to work with a standard representation of a graph, like an adjacency matrix.
- plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.

```
require("igraph")

edges - read.csv("C:/Temp/input_files/edges.csv")

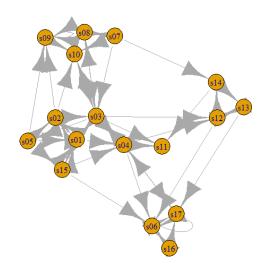
nodes - read.csv("C:/Temp/input_files/nodes.csv")

head(edges)
head(nodes)

dir_graph - graph.data.frame(d = edges, vertices = nodes, directed = T)
get.adjacency(dir_graph)

plot(dir_graph)
```

```
require("igraph")
  edges ← read.csv("C:/Temp/input_files/edges.csv")
 head(edges)
  from to weight
                       type
1 s01 s02
               10 hyperlink
2 s01 s02
               12 hyperlink
  s01 s03
               22 hyperlink
  s01 s04
               21 hyperlink
5
  s04 s11
               22
                    mention
  s05 s15
               21
                    mention
 head(nodes)
                    media media.type type.label audience.size
1 s01
                 NY Times
                                      Newspaper
                                                            20
2 s02
          Washington Post
                                   1
                                      Newspaper
                                                            25
3 s03 Wall Street Journal
                                                            30
                                   1
                                      Newspaper
4 s04
                USA Today
                                   1
                                      Newspaper
                                                            32
5 s05
                 LA Times
                                   1
                                      Newspaper
                                                            20
6 s06
            New York Post
                                                            50
                                      Newspaper
```



# **<u>Aim:</u>** Compute the following node level measures:

- **♦** Density
- **♦** Degree
- ♦ Reciprocity
- **♦** Transitivity
- **♦** Centralization
- ♦ Clustering.

# **Software(s) used:**

- ♦ R ver. 4.1.3
- ♦ RStudio ver. 2022.02.0+433

## **External packages required:**

♦ igraph

## **Description:**

• The igraph package: igraph is a library and R package for network analysis.

The main goals of the igraph library is to provide a set of data types and functions for:

- ◆ pain-free implementation of graph algorithms,
- ♦ fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.

- library(): library() and require() load and attach addon packages.
- graph.famous(): Create an igraph graph from a list of edges, or a notable graph.
- ecount(): Returns the count of number of edges in graph
- vcount(): Returns the count of number of vertices in graph
- graph.formula(): This function is useful if you want to create a small (named) graph quickly, it works for both directed and undirected graphs.
- plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.
- reciprocity(): Calculates the reciprocity of a directed graph.
- dyad.census(): Classify dyads in a directed graphs. The relationship between each pair of vertices is measured. It can be in three states: mutual, asymmetric or non-existent.
- adjacent.triangles(): Count how many triangles a vertex is part of, in a graph, or just list the triangles of a graph.
- transitivity(): Transitivity measures the probability that the adjacent vertices of a vertex are connected. This is sometimes also called the clustering coefficient.
- degree(): The degree of a vertex is its most basic structural property, the number of its adjacent edges.

- barabasi.game(): The BA-model is a very simple stochastic algorithm for building a graph.
- watts.strogatz.game(): Generate a graph according to the Watts-Strogatz network model.
- graph.union(): The union of two or more graphs are created. The graphs may have identical or overlapping vertex sets.
- simplify(): Simple graphs are graphs which do not contain loop and multiple edges.

```
library("igraph")

kite - graph.famous("Krackhardt_Kite")

vcount(kite)
ecount(kite)
ecount(kite) / (vcount(kite) * (vcount(kite) - 1) / 2)

dir_graph - graph.formula(A + B, A + D, A -+ C, B -+ C, B -+ E, B -+ F, C -+ D, C -+ F, D -+ E)
plot(dir_graph)
reciprocity(dir_graph)
dyad.census(dir_graph)
mutual - dyad.census(dir_graph)$mut
```

```
mutual / (ecount(dir_graph))

atri - adjacent.triangles(kite)
plot(kite, vertex.label = atri)

transitivity(kite, type = "local")
adjacent.triangles(kite) / (degree(kite) * (degree(kite) - 1) / 2)

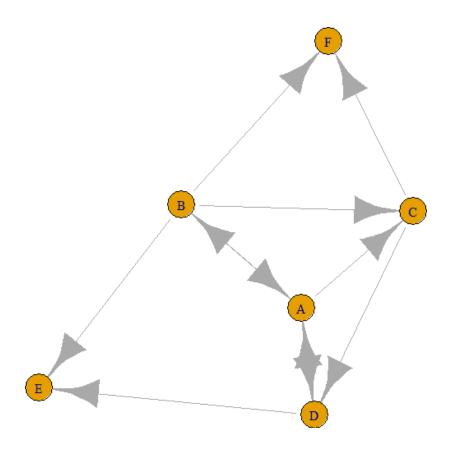
graph_2 - barabasi.game(50, p = 2, directed = F)
graph_1 - watts.strogatz.game(1, size = 100, nei = 5, p = 0.05)
graph - graph.union(graph_1, graph_2)
graph - simplify(graph)
plot(graph)
```

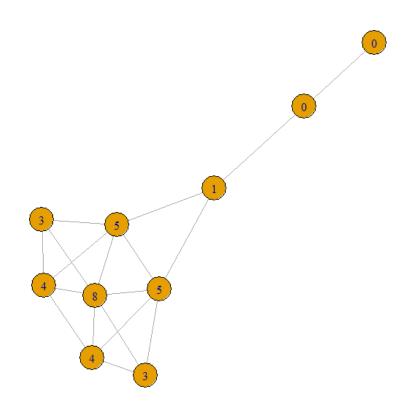
```
> library("igraph")
> 
   kite ← graph.famous("Krackhardt_Kite")
> vcount(kite)
[1] 10
> ecount(kite)
[1] 18
> ecount(kite) / (vcount(kite) * (vcount(kite) - 1) / 2)
[1] 0.4
> 
   dir_graph ← graph.formula(A ++ B, A ++ D, A -+ C, B -+ C, B -+ E, B -+ F, C -+ D, C -+ F, D -+ E)
> plot(dir_graph)
> reciprocity(dir_graph)
[1] 0.3636364
```

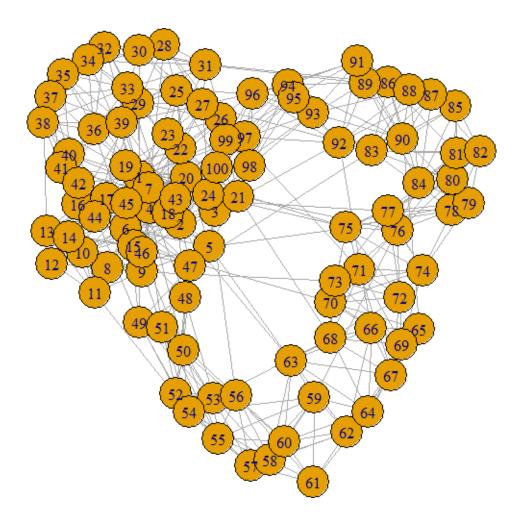
```
> reciprocity(dir_graph)
[1] 0.3636364
> dyad.census(dir_graph)
$mut
[1] 2

$asym
[1] 7

$null
[1] 6
> mutual ← dyad.census(dir_graph)$mut
> mutual / (ecount(dir_graph))
[1] 0.1818182
> atri ← adjacent.triangles(kite)
> plot(kite, vertex.label = atri)
> transitivity(kite, type = "local")
[1] 0.66666667 0.66666667 1.00000000 0.53333333 1.00000000 0.50000000 0.33333333 0.000000000
[10] NaN
```







## **<u>Aim:</u>** For a given network find the following:

- ♦ Length of the shortest path from a given node to another node
- ♦ The density of the graph
- ◆ Draw egocentric network of node G with chosen configuration parameters.

#### **Software(s) used:**

- ♦ R ver. 4.1.3
- ♦ RStudio ver. 2022.02.0+433

# **External packages required:**

♦ igraph

## **Description:**

- The igraph package: igraph is a library and R package for network analysis.
  - The main goals of the igraph library is to provide a set of data types and functions for:
  - pain-free implementation of graph algorithms,
  - ♦ fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
- library(): library() and require() load and attach addon packages.

- as.matrix(): matrix() creates a matrix from the given set of values.as.matrix() attempts to turn its argument into a matrix.
- read.table(): Reads a file in table format and creates a data frame from it, with cases corresponding to lines and variables to fields in the file.
- colnames() and rownames(): Retrieve or set the row or column names of a matrix-like object.
- is.na(): The generic function is.na() indicates which elements are missing.
- graph.adjacency(): graph\_from\_adjacency\_matrix() is a flexible function for creating igraph graphs from adjacency matrices.
- plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.
- shortest.paths(): shortest\_paths() calculates one shortest path (the path itself, and not just its length) from or to the given vertex.
- print(): print() prints its argument and returns it invisibly.
- graph.formula(): This function is useful if you want to create a small (named) graph quickly, it works for both directed and undirected graphs.
- graph.density(): The density of a graph is the ratio of the number of edges and the number of possible edges.

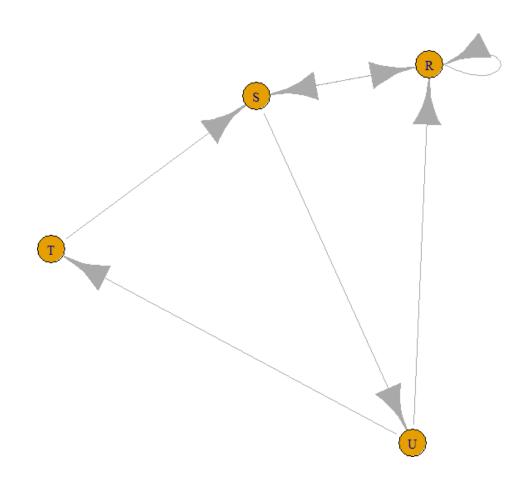
• simplify(): Simple graphs are graphs which do not contain loop and multiple edges.

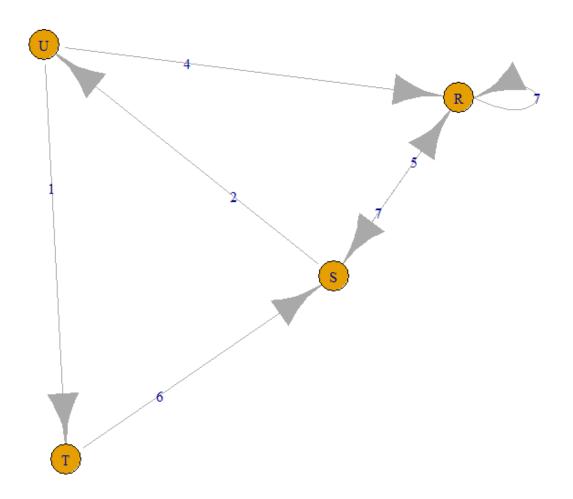
# **Source Code:**

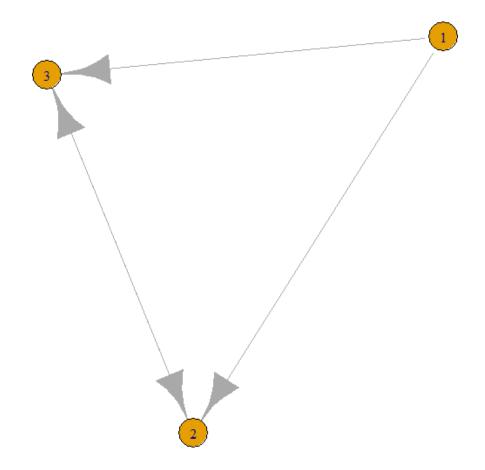
```
library(igraph)
matt - as.matrix(read.table(text=
                               "node R S T U
                                     R 7 5 0 0
                                     S 7 0 0 2
                                     T 0 6 0 0
                                     U 4 0 1 0", header=T))
nms - matt[, 1]
matt - matt[, -1]
colnames(matt) - rownames(matt) - nms
matt[is.na(matt)] - 0
g - graph.adjacency(matt,weighted=TRUE)
plot(g)
s.paths - shortest.paths(g, algorithm = "dijkstra")
print(s.paths)
shortest.paths(g, v="R", to="S")
plot(g, edge.label=E(g)$weight)
```

```
dg - graph.formula(1-+2, 1-+3, 2 +3)
plot(dg)
graph.density(dg, loops=TRUE)
graph.density(simplify(dg), loops=FALSE)
```

```
> shortest.paths(g, v="R", to="S")
S
R 5
> plot(g, edge.label=E(g)$weight)
> dg 		 graph.formula(1-+2, 1-+3, 2++3)
> plot(dg)
> graph.density(dg, loops=TRUE)
[1] 0.4444444
> graph.density(simplify(dg), loops=FALSE)
[1] 0.6666667
> |
```







<u>Aim:</u> Write a program to distinguish between a network as a matrix, a network as an edge list and a network as a sociogram (or "network graph") using 3 distinct networks representatives of each.

### **Software(s) used:**

- ♦ R ver. 4.1.3
- ♦ RStudio ver. 2022.02.0+433

## **External packages required:**

♦ igraph

## **Description:**

- The igraph package: igraph is a library and R package for network analysis.
  - The main goals of the igraph library is to provide a set of data types and functions for:
  - pain-free implementation of graph algorithms,
  - ♦ fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
- library(): library() and require() load and attach addon packages.

- graph.formula(): This function is useful if you want to create a small (named) graph quickly, it works for both directed and undirected graphs.
- plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.
- get.adjacency(): Sometimes it is useful to work with a standard representation of a graph, like an adjacency matrix.
- E(): An edge sequence is a vector containing numeric edge ids, with a special class attribute that allows custom operations: selecting subsets of edges based on attributes, or graph structure, creating the intersection, union of edges, etc.
- get.adjedgelist(): Create adjacency lists from a graph, either for adjacent edges or for neighboring vertices.

```
library(igraph)
```

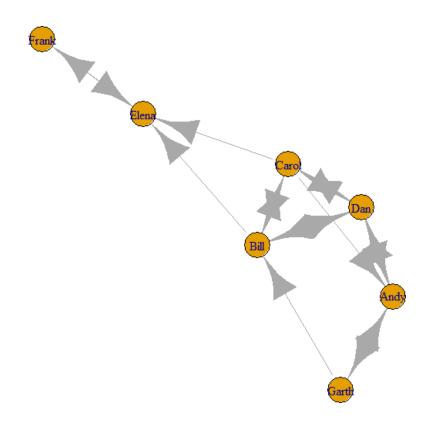
plot(sociogram)

```
get.adjacency(sociogram)
E(sociogram)
get.adjedgelist(sociogram)
```

```
7 x 7 sparse Matrix of class "dgCMatrix"
      Andy Garth Bill Elena Frank Carol Dan
Andv
Garth
                    1
Bill
                          1
                                       1
                                           1
Elena
                                 1
Frank
Carol
                          1
                                           1
Dan
```

```
E(sociogram)
+ 16/16 edges from f262a60 (vertex names):
[1] Andy →Garth Andy →Dan Garth→Andy
[5] Bill →Elena Bill →Carol Bill →Dan
                                                 Garth→Bill
                                                 Elena→Frank
 [9] Frank→Elena Carol→Andy Carol→Bill Carol→Elena
[13] Carol→Dan
                   Dan \rightarrowAndy Dan \rightarrowBill Dan \rightarrowCarol
> get.adjedgelist(sociogram)
$Andy
+ 5/16 edges from f262a60 (vertex names):
[1] Andy →Garth Andy →Dan Garth→Andy
[5] Dan →Andy
                                               Carol→Andy
$Garth
+ 3/16 edges from f262a60 (vertex names):
[1] Garth→Andy Garth→Bill Andy →Garth
$Bill
+ 6/16 edges from f262a60 (vertex names):
[1] Bill →Elena Bill →Carol Bill →Dan
                                                Garth→Bill
[5] Carol\rightarrowBill Dan \rightarrowBill
```

```
$Elena
+ 4/16 edges from f262a60 (vertex names):
[1] Elena→Frank Bill →Elena Frank→Elena Carol→Elena
$Frank
+ 2/16 edges from f262a60 (vertex names):
[1] Frank→Elena Elena→Frank
$Carol
+ 6/16 edges from f262a60 (vertex names):
[1] Carol→Andy Carol→Bill Carol→Elena Carol→Dan
[5] Bill →Carol Dan →Carol
$Dan
+ 6/16 edges from f262a60 (vertex names):
[1] Dan →Andy Dan →Bill Dan →Carol Andy →Dan
[5] Bill →Dan Carol→Dan
```



<u>Aim:</u> Write a program to exhibit structural equivalence, automatic equivalence, and regular equivalence from a network.

## **Software(s) used:**

- ♦ R ver. 4.1.3
- ♦ RStudio ver. 2022.02.0+433

## **External packages required:**

- ♦ igraph
- ♦ sna

## **Description:**

- The igraph package: igraph is a library and R package for network analysis.
  - The main goals of the igraph library is to provide a set of data types and functions for:
  - pain-free implementation of graph algorithms,
  - ♦ fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
- The sna package: sna is a package containing a range of tools for social network analysis. Supported functionality includes node and graph-level indices, structural distance and covariance methods, structural equivalence detection, p\* modeling, random graph generation, and 2D/3D network visualization (among other things).

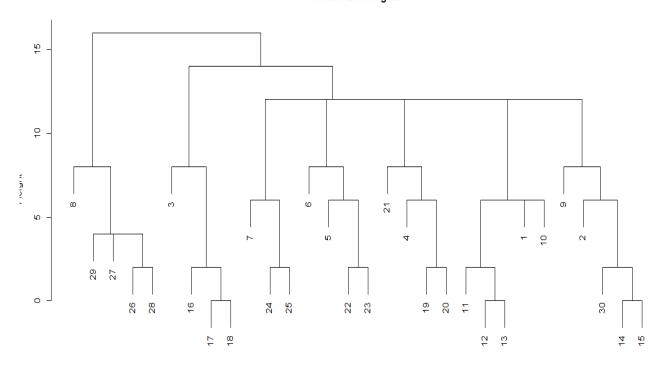
- library(): library() and require() load and attach addon packages.
- read.csv(): Reads a file in table format and creates a data frame from it, with cases corresponding to lines and variables to fields in the file.
- equiv.clust(): equiv.clust() uses a definition of approximate equivalence (equiv.fun()) to form a hierarchical clustering of network positions.
- plot(): Draw a scatter plot with decorations such as axes and titles in the active graphics window.
- sedist(): sedist() uses the graphs indicated by g in the arguments to assess the extent to which each vertex is structurally equivalent.
- cmdscale(): Classical multidimensional scaling (MDS) of a data matrix.
- as.dist(): This function computes and returns the distance matrix computed by using the specified distance measure to compute the distances between the rows of a data matrix.
- blockmodel(): Given a set of equivalence classes and one or more graphs, blockmodel will form a blockmodel of the input graph(s) based on the classes in question

```
library(sna)
library(igraph)
links2 - read.csv("C:/Temp/input_files/edges1.csv", header=T,
row.names=1)
eq -equiv.clust(links2)
plot(eq)
g.se -sedist(links2)

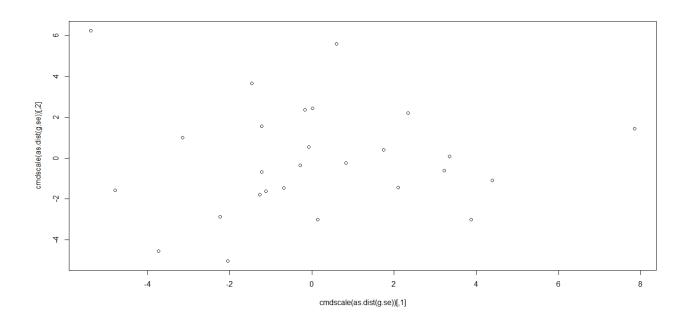
plot(cmdscale(as.dist(g.se)))
b -blockmodel(links2,eq,h=10)
plot(b)
```

```
> library(sna)
> library(igraph)
> links2 ← read.csv("C:/Temp/input_files/edges1.csv", header=T, row.names=1)
> eq←equiv.clust(links2)
> plot(eq)
>
> g.se←sedist(links2)
> plot(cmdscale(as.dist(g.se)))
> b←blockmodel(links2,eq,h=10)
> plot(b)
```

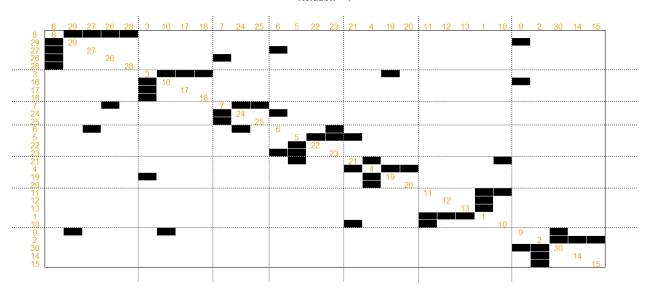
#### Cluster Dendrogram







#### Relation - 1



**<u>Aim:</u>** Perform SVD analysis of a network.

### **Software(s) used:**

- ♦ R ver. 4.1.3
- ♦ RStudio ver. 2022.02.0+433

## **External packages required:**

♦ igraph

# **Description:**

• The igraph package: igraph is a library and R package for network analysis.

The main goals of the igraph library is to provide a set of data types and functions for:

- ◆ pain-free implementation of graph algorithms,
- ♦ fast handling of large graphs, with millions of vertices and edges, allowing rapid prototyping via high level languages like R.
- matrix(): matrix() creates a matrix from the given set of values.
- c(): Combines values into a vector or list.
- print(): print() prints its argument and returns it invisibly.

• svd(): Compute the singular-value decomposition of a rectangular matrix.

## **Source Code:**

```
[,1] [,2] [,3] [,4]
1 1 0 0
              0
              0
                   Θ
        1
   1
       0
                   0
              1
  1
       0
                   0
   ī
                   0
        0
   1
        0
             0
                   1
   1
                   1
        0
              0
              0
        Θ
```

```
$d
[1] 3.464102e+00 1.732051e+00 1.732051e+00 1.922963e-16
$u
            [,1]
                                      [,3]
                       [,2]
                                                    [,4]
 [1,] -0.3333333 0.4714045 -1.741269e-16 7.760882e-01
 [2,] -0.3333333  0.4714045 -3.692621e-16 -1.683504e-01
 [3,] -0.3333333 0.4714045 -5.301858e-17 -6.077378e-01
 [4,] -0.3333333 -0.2357023 -4.082483e-01 6.774193e-17
[5,] -0.3333333 -0.2357023 -4.082483e-01 6.774193e-17
 [6,] -0.3333333 -0.2357023 -4.082483e-01 6.774193e-17
 [7,] -0.3333333 -0.2357023 4.082483e-01 5.194768e-17
[8,] -0.3333333 -0.2357023 4.082483e-01 5.194768e-17 [9,] -0.3333333 -0.2357023 4.082483e-01 5.194768e-17
$v
          [,1] [,2]
                                    [,3] [,4]
[1,] -0.8660254 0.0000000 -4.378026e-17 0.5
[2,] -0.2886751  0.8164966 -2.509507e-16 -0.5
[3,] -0.2886751 -0.4082483 -7.071068e-01 -0.5
[4,] -0.2886751 -0.4082483 7.071068e-01 -0.5
```