CPSC 5310-01 (Machine Learning),
Homework 1
Priyadarshini Shanmugasundaram Murugan

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1) **A box contains three cards. One card is red on both sides, one card is green on both sides, and one card is red on one side and green on the other. We randomly select one card from this box, and we can know the color of the selected card's upper side. If this side is green, what is the probability that the other side of the card is also green?**

Given: 3 cards.
1 = red on both sides, 1 = green on both sides, one
card with red on one side and green on the other.

Solution:
Let C1 be the card having green on both sides, and C2 be the card having red on both sides.
and C3 be the card having red and green on each side.
Probability of randomly selecting a card is

$$p(C1) = 1/3$$
$$p(C2) = 1/3$$
$$p(C3) = 1/3$$

Let G be an event where the card selected shows a green color on the upper side, therefore.

$$P(G/A1) = 2/2 = 1$$
$$P(G/A2) = 0/2 = 0$$
$$P(G/A3) = 1/2$$

Using the concept of conditional probability, we get

$$P(C1/G) = \frac{P(C1) . P(G/C1)}{P(C1). P(G/C1) + P(C2). P(G/C2) + P(C3) . P(G/C3)}$$

$$\frac{1/3 . 1}{1/3 . 1 + 1/3 . 0 + 1/3. \frac{1}{2}}$$

$$= 1/3 / 1/2 \; (1/3 + 1/6 = 9/18 = 1/2)$$
$$= 1/3. 2$$

$$\mathbf{= 2/3}$$

the probability that the other side of the card is also green, given that the upper side is green, is 2/3

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2) **Suppose that the pdf of a random variable X is:**

$$f(x) = \begin{cases} cx^2, & 1 \le x \le 2 \\ 0, & \text{Otherwise} \end{cases}$$

a) What is the value of constant c?

To find the value of the constant c, we can use the property that the integral of the pdf over the entire sample space (in this case, from 1 to 2) should equal 1, as it represents the total probability.

So, we integrate the pdf $f(x)$ from 1 to 2 and set it equal to 1:

$\int_{-\infty}^{+\infty} f(x)\, dx = \int_1^2 cx^2\, dx = \frac{c}{3}x^3 \,|_1^2$

$$c(\frac{2^3}{3} - \frac{1^3}{3}) = 1$$

$$c(\frac{8}{3} - \frac{1}{3}) = 1$$

$$c(\frac{7}{3}) = 1$$

$$\boxed{c = \frac{3}{7}}$$

b) What is the probability of X being larger than 1.5?

P (X > 1.5) = $\int_1^2 \frac{3}{7}x^2\, dx = \frac{x^3}{7} \,|_{1.5}^2$

$$\boxed{\frac{8-3.375}{7} = \mathbf{0.660}}$$

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3) **Suppose that X is a random variable. If E(X) = μ, Var(X) = σ^2, then what is the value of E[X(X – 1)]?**

$$E[X(X-1)] = E[X^2 - X] = E[X^2] - E[X]$$

As we know $E[X^2] = E[X]^2 + Var(X)$

Therefore;

$$E[X(X-1)] = E[X^2 - X] = E[X^2] - E[X]$$

$$= E[X]^2 + Var(X) - E[X]$$

The value of E [X(X – 1) = $\mu^2 + \sigma^2 - \mu$ or $\mu(\mu - 1) + \sigma^2$

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4) **A probability distribution for** $x \geq 1$ and $a > 0$ **has the following pdf function:**

$$f(x) = \frac{a}{x^{a+1}}$$

Let us assume that X is a random variable. What is the expected value of this random variable?

$$E[X] = \int_1^{\infty} x \cdot \frac{a}{x^{a+1}} \mathrm{d}x = 1$$

Using Linearity we will get

$$E[X] = a \int_1^{\infty} \frac{1}{x^a}$$

Using $\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1}$ *we get*

$$\left[a \cdot \frac{x^{1-a}}{1-a} \right]_1^{\infty}$$

substituting the value of x *with* ∞ *and* 1

$$\frac{a}{a-1} \; for \; (a > 1)$$

Considering 3 *Test cases*

$$a = 0 \,, \quad \frac{a}{a-1} = \frac{0}{-1} = 0$$

$$a < 1, \; a = -1 = -\frac{1}{-1-1} = -\frac{1}{-2} = \frac{1}{2}, \quad a = -2, \; = -\frac{2}{-2-1} = \frac{2}{3}$$

$$a > 0, \; a = 1 = \frac{1}{1-1} = -\frac{1}{0} = \infty \,, \quad a = 2, \; = -\frac{2}{2-1} = 2 \,, \quad a = 3, \; = -\frac{3}{3-1} = \frac{3}{2}$$

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5) **Using the probability axioms to prove that**

a) $\boldsymbol{A \subseteq B \Rightarrow P(A) \leq P(B)}$

Using conditional probability.

$A \subseteq B \rightarrow \quad B = A \cup (B - A)$

$\rightarrow P(B) = P(A \cup (B - A) = P(A) + P(B - A) \; P(B - A) \geq 0$

$\rightarrow P(A) \leq P(B)$

b) $\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

$A \cup B = A \cup (B \cap \bar{A})$..(1)

$P(A \cup B) = P(A) + P(B \cap \bar{A})$......................... (2)

$B = (B \cap \bar{A}) \cup (B \cap A)$ ....(3)

$P(B) = P(B \cap \bar{A}) + P(A \cap B)$ ....(4)

$P(B \cap \bar{A}) = P(B) - P(A \cap B)$

Substitute $P(B \cap \bar{A})$ into (2) to get (6). ....(5)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$..............(6)

6) **Write a Python program for the following purpose.**

If we are tossing a coin, what is the number of tosses before observing three heads? Perform this experiment one thousand times and plot a graph for it. The horizontal axis is the experiment number, and the vertical axis is the average number of tosses to get three heads. For example after ten experiments we should find the average of ten experiments.

```
import random
import matplotlib.pyplot as plt

# Simulate a Coin-toss experiment until 3 heads are obtained
1 usage
def simulate_experiment():
    heads = 0
    tosses = 0
    while heads < 3:
        toss = random.randint( a: 0,  b: 1)  # 0 for tails, 1 for heads
        tosses += 1
        heads += toss

    return tosses
1 usage
def main():
    trials = 1000
    tosses_list = []
    average_list = []
    # Perform the 1000 trials
    for i in range(1, trials):
        tosses = simulate_experiment()
        average_tosses = sum(tosses_list) / i
        average_list.append(average_tosses)
        tosses_list.append(tosses)
# finding the Average number of Tosses and Trials
    print("Average number of trials:", trials)
    print("Average:")
    for i, average in enumerate(average_list, start=1):
        print(f"{average: .2f}", end=" ")
        if i % 30 == 0:
            print()
# Graph Plotting
    plt.plot(average_list)
    plt.title('Average number of tosses: ' + str(sum(tosses_list) / trials))
    plt.xlabel("Trial Number")
    plt.ylabel("Number of Tosses to get Heads")
    plt.show()
if __name__ == "__main__":
    main()
```

```
C:\Users\priya\PycharmProjects\pythonProject\venv\Scripts\python.exe C:\Users\priya\PycharmProjects\pythonProject\main.py
Average number of trials: 1000
Average:
0.00 4.00 4.33 5.00 5.20 6.17 6.14 6.25 6.00 5.80 5.91 5.75 5.77 5.79 5.60 5.75 5.65 5.83 5.84 5.85 5.81 5.68 5.57 5.67 5.60 5.54 5.48 5.39 5.45 5.50
5.55 5.47 5.48 5.44 5.40 5.50 5.43 5.42 5.36 5.33 5.37 5.33 5.37 5.52 5.49 5.48 5.53 5.60 5.63 5.68 5.67 5.75 5.70 5.69 5.69 5.80 5.81 5.78 5.73 5.70
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