Lab Session 10
Block Diagram Reduction

Exercise 1

For the following multi-loop feedback system, get the closed loop transfer function and the corresponding pole-zero map of the system. Also find the system's performance characteristics using `stepinfo()`.

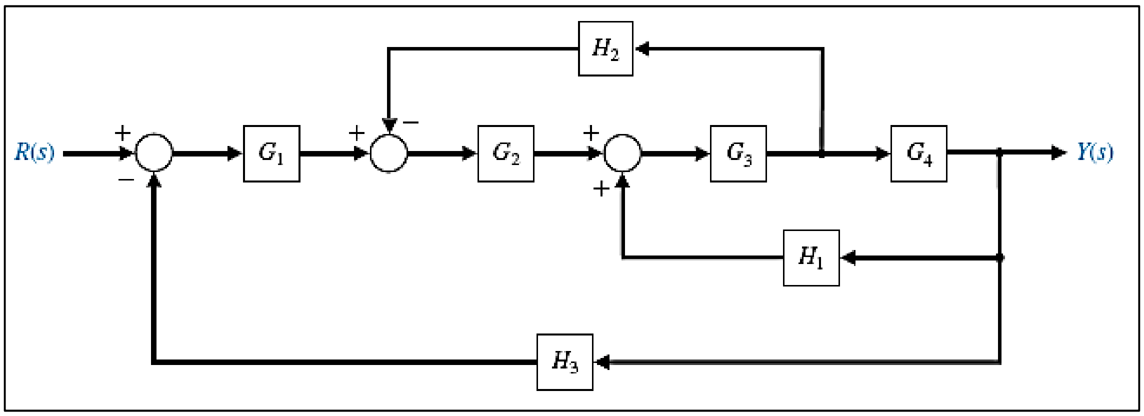

Figure 1: Multi-loop Feedback System for Exercise 1

List of Transfer Functions

$$G_1(s) = \frac{1}{s+10}$$

$$G_2(s) = \frac{1}{s+1}$$

$$G_3(s) = \frac{s^2+1}{s^2+4s+4}$$

$$G_4(s) = \frac{s+1}{s+6}$$

$$H_1(s) = \frac{s+1}{s+2}$$

$$H_2(s) = 2$$

$$H_3(s) = 1$$

MATLAB Program

Code 01 shows a MATLAB program that uses built-in MATLAB block reduction functionality to derive a closed loop transfer function for the multi-loop system shown in Figure 01. The program first clears the MATLAB workspace to ensure variables left over from previous executions of the program do not affect the current execution's results. It then uses the built-in MATLAB function `tf` to create objects representing each of the individual loop gains/transfer functions listed above.

Code 01: Block Reduction for System in Exercise 01

```
%%  FCS Lab 10 Exercise 01 - Reduction of a multiploop feedback system
%   Saad Mashkoor Siddiqui, EE-16163, Section D, TE-EE 16-17

%%  Prepare workspace and IDE for program
clear all; close all; clc;

%%  Define individual transfer functions
G1 = tf([1], [1 10]);
G2 = tf([1], [1, 1]);
G3 = tf([1, 0, 1], [1, 4, 4]);
G4 = tf([1, 1], [1, 6]);
H1 = tf([1, 1], [1, 2]);
H2 = [2];
H3 = [1];

%%  Block reduction
G5 = feedback(series(G3, G4), H1, +1);
G6 = feedback(series(G2, G5), H2/G4, -1);
G_sys= feedback(series(G1, G6), H3, -1);

%%  Output final transfer function
G_sys

%%  Pole-Zero Map
pzmap(G_sys, 'k')

%%  Visualizing the system's step response
stepinfo(G_sys)
```

The system's overall transfer function, `G_sys`, is calculated as follows:

1. Move the take-off point connected to H_2 after G_2, thereby scaling H_2 to $\frac{H_2}{G_4}$.
2. Find the transfer function for the series combination of G_3 and G_4 and H_1. This is G_5.
3. Find the transfer function for the feedback looped formed by the series combination of G_2 and G_5 with $\frac{H_2}{G_4}$ as a feedback gain. This is G_6.

4. The overall system's closed-loop gain is therefore formed by the series combination of G_1 and G_6 with the feedback gain H_3. This is G_{sys}.

Transfer Function

The system's overall transfer function as calculated by MATLAB is as follows:

```
                s^5 + 4 s^4 + 6 s^3 + 6 s^2 + 5 s + 2
  ---------------------------------------------------------------
  12 s^6 + 205 s^5 + 1066 s^4 + 2517 s^3 + 3128 s^2 + 2196 s + 712
```

Pole-Zero Map

The system's pole-zero map is as follows:

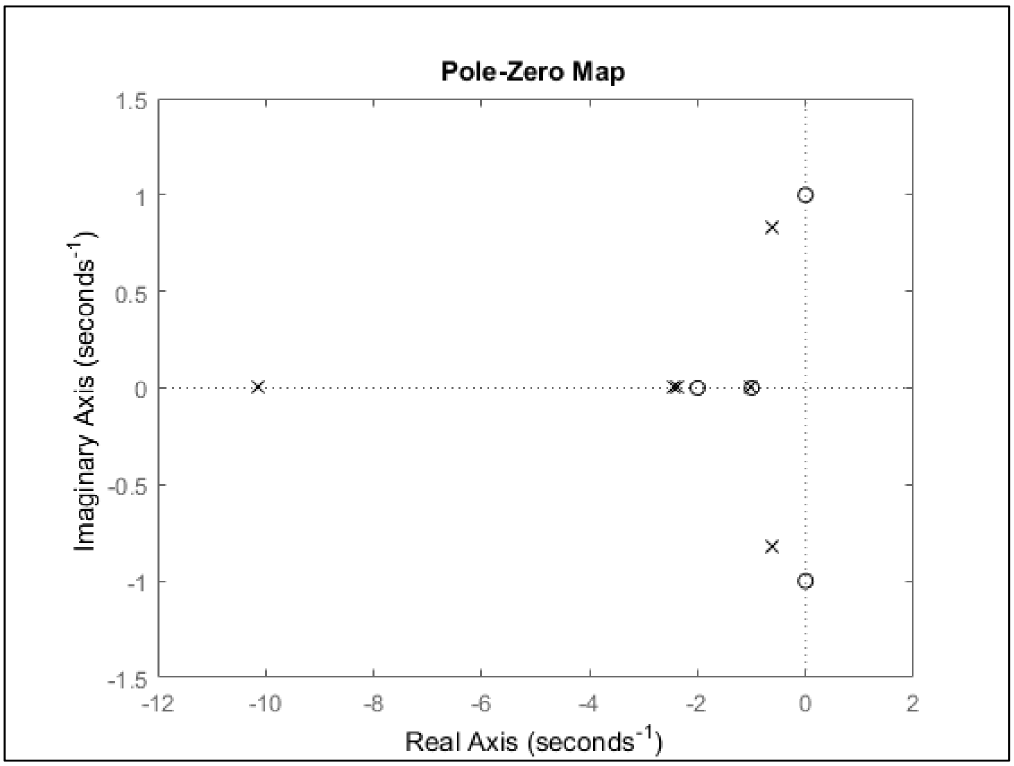

Figure 2: Pole-Zero map for Exercise 01

Step Response

Using MATLAB's `stepinfo()` function, the system's performance characteristics were determined to be as follows:

Table 1: Step Response Performance Characteristics - Exercise 01

Property	Value	Property	Value
Rise Time	0.0356s	Overshoot	75.7308
Settling Time	6.2253s	Undershoot	0
Settling Minimum	8.5167e-04	Peak	0.0049
Settling Maximum	0.0049	Peak Time	0.1730s

Exercise 2

For the following multi-loop feedback system, get the closed loop transfer function and the corresponding pole-zero map of the system. Also find the system's performance characteristics using `stepinfo()`.

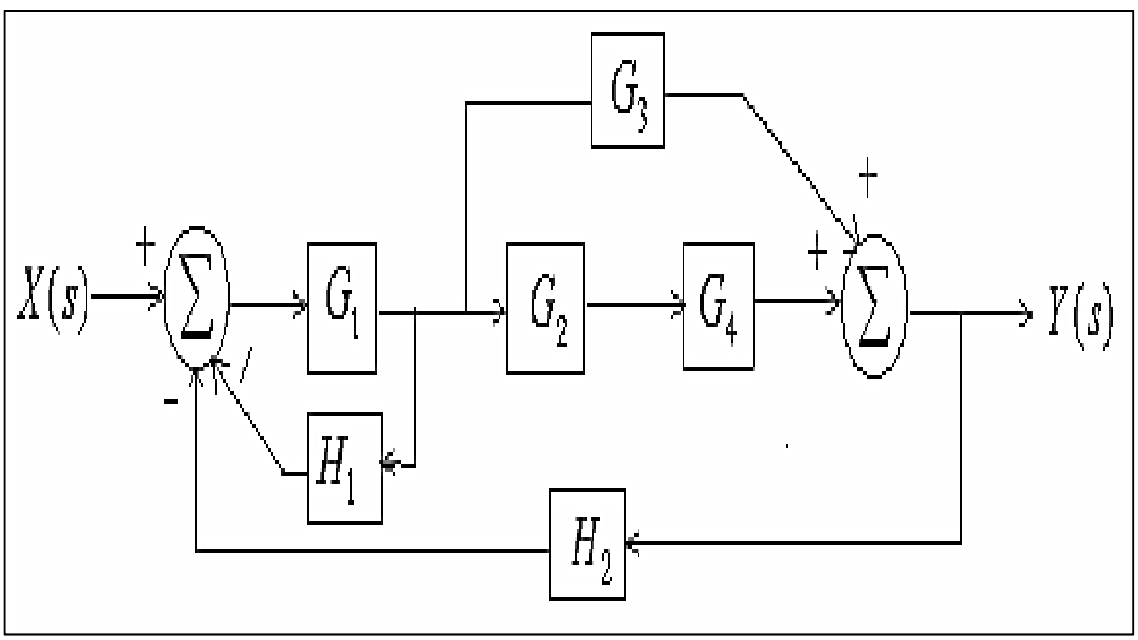

Figure 3: Multi-loop Feedback System for Exercise 02

List of Transfer Functions

$$G_1(s) = \frac{1}{s+10}$$

$$G_2(s) = \frac{1}{s+1}$$

$$G_3(s) = \frac{s^2+1}{(s+2)^2}$$

$$G_4(s) = \frac{s+1}{s+6}$$

$$H_1(s) = \frac{s+1}{s+2}$$

$$H_2(s) = 1$$

MATLAB Program

Code 02 shows a MATLAB program that uses roughly the same workflow as Code 01 to obtain an overall closed-loop transfer function for the system shown in Figure 02. The only difference is the way the individual loop and feedback gains are combined.

Code 02: Block Reduction for System in Exercise 02

```
%%  FCS Lab 10 Exercise 02 - Reduction of a multiploop feedback system
%   Saad Mashkoor Siddiqui, EE-16163, Section D, TE-EE 16-17

%%  Prepare workspace and IDE for program
clear all; close all; clc;

%%  Define all transfer functions
G1 = tf([0, 1], [1, 10]);
G2 = tf([0, 1], [1, 1]);
G3 = tf([1, 0, 1], [1, 4, 4]);
G4 = tf([1, 1], [1, 6]);
H1 = tf([1, 1], [1, 2]);
H2 = [1]

%%  Block diagram reduction
G5 = series(G2, G4);
G6 = parallel(G5, G3);
G7 = feedback(G1, H1, -1);
G8 = series(G7, G6);
G_sys = feedback(G8, H2, -1);

%%  Echo system's transfer function
G_sys

%%  Create pole-zero map and get step response parameters
```

```
pzmap(G_sys, 'k'); stepinfo(G_sys);
```

Specifically, the steps used by the program to find the overall transfer function are as follows:

1. Find the series combination of G_2 and G_4. This is G_5.
2. G_5 is added in parallel with G_3 to form G_6.
3. Simplify the feedback loop formed by G_1 and H_1 to yield G_7.
4. Find the overall system transfer function G_{sys} by reducing the feedback loop formed by the series combination of G_6 and G_7 along with H_2.

Transfer Function

The overall transfer function of the system as calculated by MATLAB is

```
            s^5 + 10 s^4 + 28 s^3 + 39 s^2 + 40 s + 20
  ----------------------------------------------------------
  s^6 + 25 s^5 + 212 s^4 + 805 s^3 + 1537 s^2 + 1444 s + 524
```

Pole-Zero Map

The system's pole-zero map is as follows:

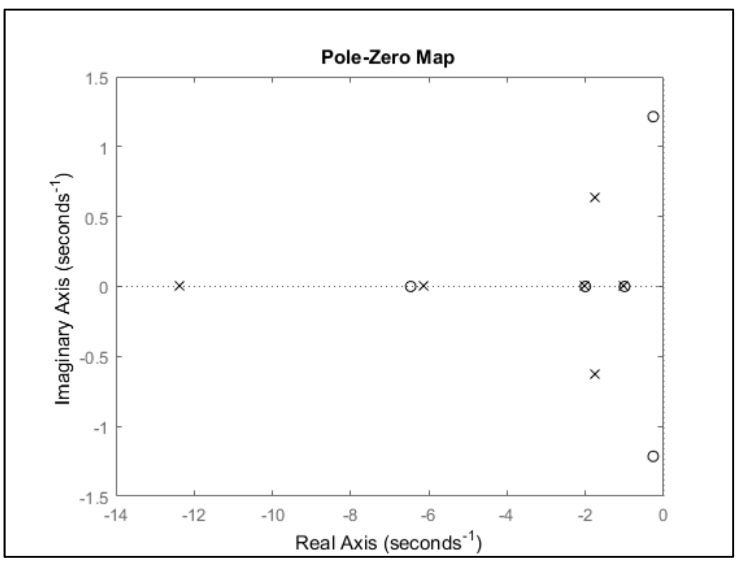

Figure 4: Pole-Zero Map for Multi-loop Feedback System 02

Step Response

Using MATLAB's `stepinfo()` function, the system's performance characteristics were determined to be as follows:

Table 2: Step Response Performance Characteristics - Exercise 01

Property	Value	Property	Value
Rise Time	0.0453s	Overshoot	38.5441
Settling Time	3.4245s	Undershoot	0
Settling Minimum	0.0165	Peak	0.0529
Settling Maximum	0.0529	Peak Time	0.1563s