Lab Session 06
Modelling Practical Mechanical Systems in Simulink

Exercise 01: Modelling Mass-Spring System in Simulink

1. **Draw the network diagram of a mechanical system.**
2. **Write differential equations describing its behaviour.**
3. **Use these equations to separate one or more suitable output variables.**
4. **Construct a Simulink model and properly label all signals with their corresponding gain blocks.**
5. **Obtain plots/graphs to understand the behaviour of the system.**
6. **Explain the results with proper justification.**

The mechanical system to be investigated consists of a mass M that is being acted on by a translational force F while a spring with damping coefficient K and a damper with damping coefficient B resist any changes in its state of motion. The system is shown in Figure 1.

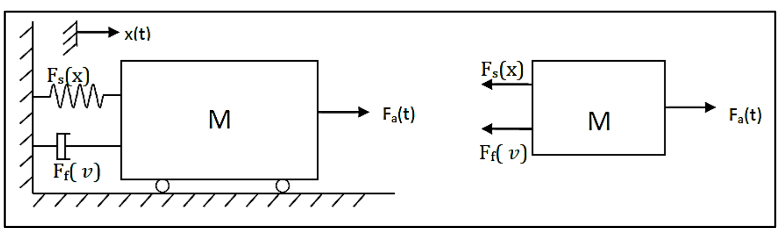

Figure 1: Simple Mass Spring system investigated in this lab session

Task 1 - Network Diagram

Using the force-voltage analogy, the mechanical network diagram of this system is shown in Figure 2.

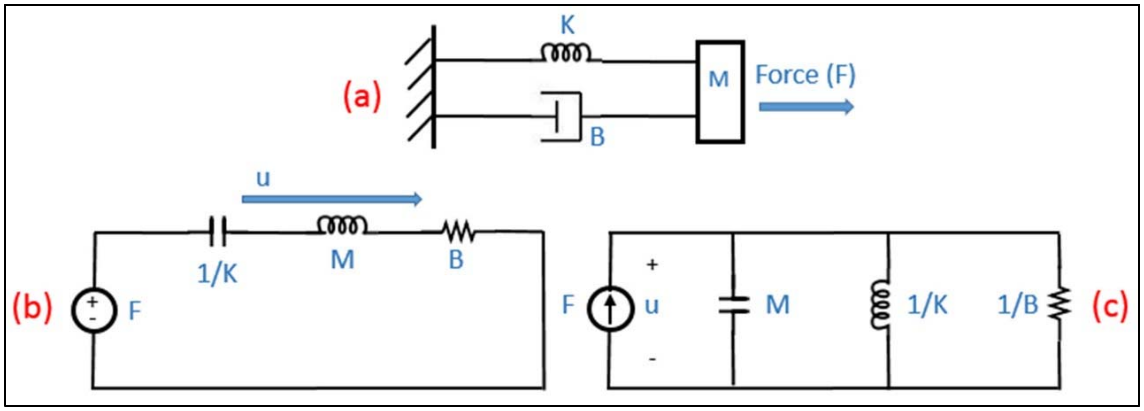

Figure 2: Mechanical network diagrams for the mass spring system. (b) and (c) are based on the force-voltage and force-current analogies respectively.

Task 2 and 3 - System Equations and Output Variables

The system has two energy storage elements - the Mass M and the spring with elastic coefficient K. As such, there are going to be two different state variables in the system.

The differential equations for this system are

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) = F_a(t)$$

$$\Rightarrow M\frac{dv}{dt} + Bv + Kx(t) = F_a(t)$$

$$\Rightarrow \boldsymbol{\frac{dv}{dt} = \frac{F_a(t)}{M} - \frac{B}{M}v - \frac{K}{M}x}$$

This is the **first state equation**.

Similarly,

$$\boldsymbol{\frac{dx}{dt} = v}$$

This is the **second state equation**

Thus the two state variables are the displacement x and the velocity v of the mass, and can adequately serve as output variables of the system as well.

Task 4 - Simulink Model

Figure 03 shows a Simulink model that was designed to investigate the response of the system in terms of its velocity and displacement. The input to the system is a unit step scaled by the value of the applied force F. The subsequent arrangement of adders, gain blocks, and integrator transfer function blocks calculates the displacement and velocity of the mass.

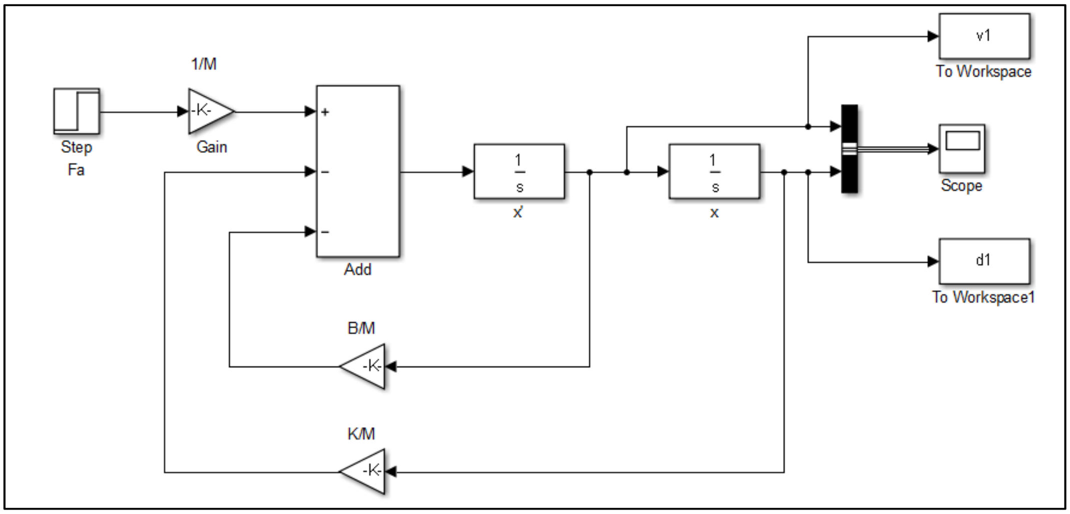

Figure 3: Simulink Model for Mass Spring System

Task 5 - System Plots

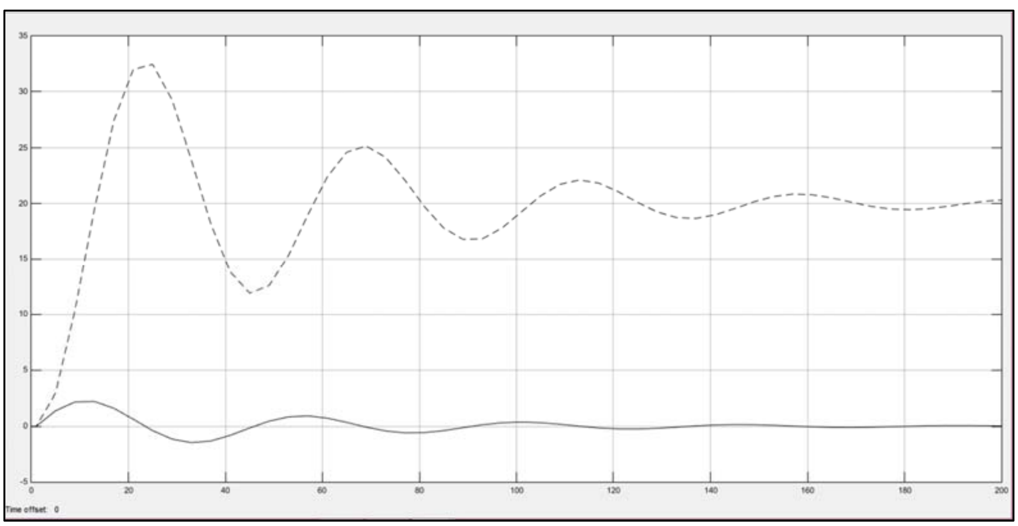

Figure 4: Mass Spring System Response - Position (dotted) an and Speed (Solid)

Figure 4 shows the output generated by the system for the mass-spring system by Simulink. The parameter values used to obtain this response are as follows:

- Mass = 750 kg
- Damping Coefficient $B = 30$ Ns/m
- Spring Coefficient $K = 15$ N/m
- Applied Force $F = 300$ N

Tasks 6 and 7 - System Behaviour and Justification

The plots clearly show that the applied force causes the mass to move in the direction of the applied force (a net positive displacement) and that in its steady state condition, the mass is 20 units away from its mean position. The mass's begins to move from its original position (denoted by 0) and oscillates rapidly about its steady state position, suggesting the system is underdamped.

The magnitude of both the oscillations as well as the steady state value of the mass's velocity is much lower than its displacement. It, too, has oscillations, which suggests that the mass moves both towards and away from the direction of the applied force with decreasing speed until it eventually comes to rest. This is made evident by the velocity having a steady state value of 0 m/s. Also, even though there is a slight delay between changes in velocity and the corresponding changes in displacement, the mass's position becomes roughly constant when its velocity becomes 0 m/s.

Exercise 2 - Investigating the Effect of System Parameters

Change the values of each parameters, such as Mass M, Friction coefficients B and Elastic characteristic *K* to understand their effects on the system's overall performance.

The table below shows the different values that were used for each of the parameters F, M, B, and K.

Parameter	Value 1	Value 2	Value 3
Mass (M)/kg	100	500	750
Friction (B)/N.s.m^{-1}	3	30	300
Elastic Coefficient (K)/N.m^{-1}	1.5	15	150
Force (F)/N	3	300	3000

The results of the simulation were plotted with MATLAB are as follows

<u>Varying Mass</u>

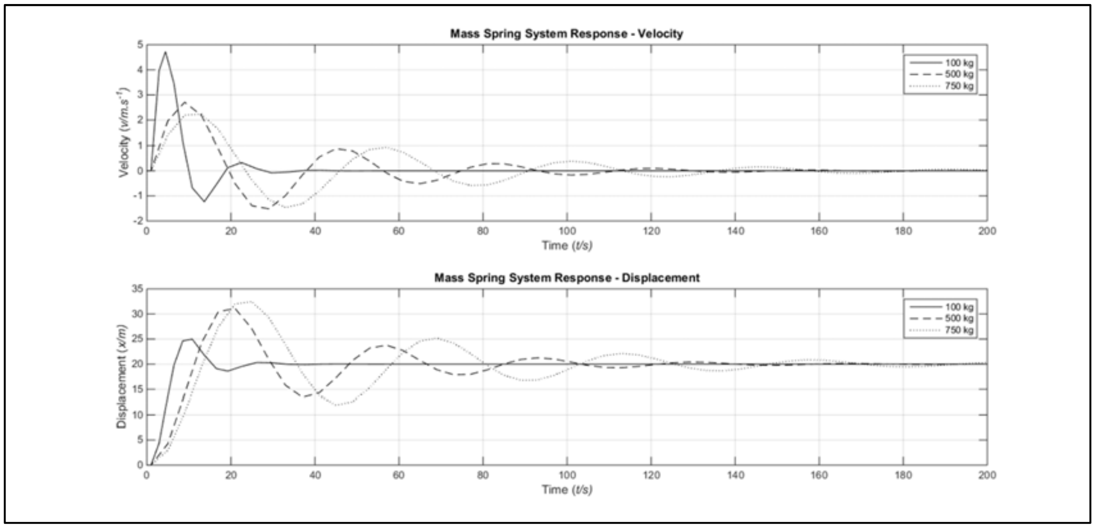

Figure 5: Variation of System Response with Mass

<u>Varying Coefficient of Viscous Friction</u>

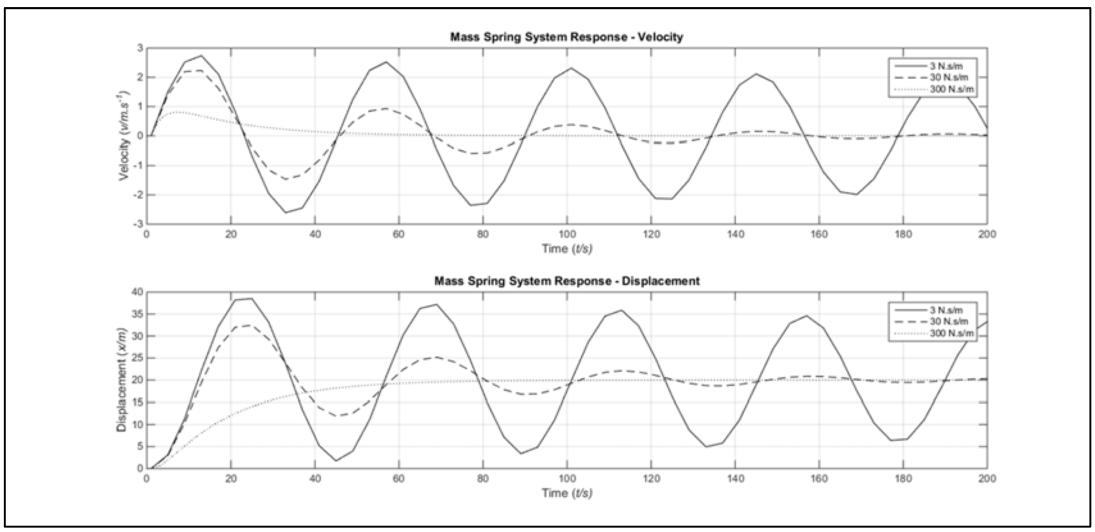

Figure 6: Variation of System Response with Coefficient of Viscous Friction

Varying Elastic Coefficient

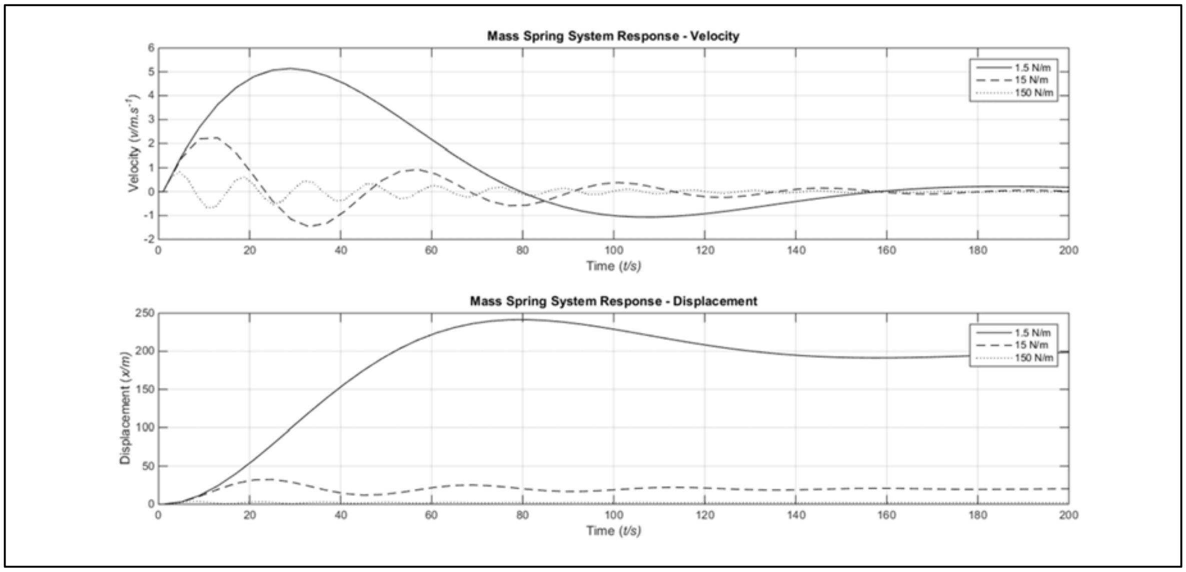

Figure 7: Variation of System Response with Elastic Coefficient

Varying Applied Force

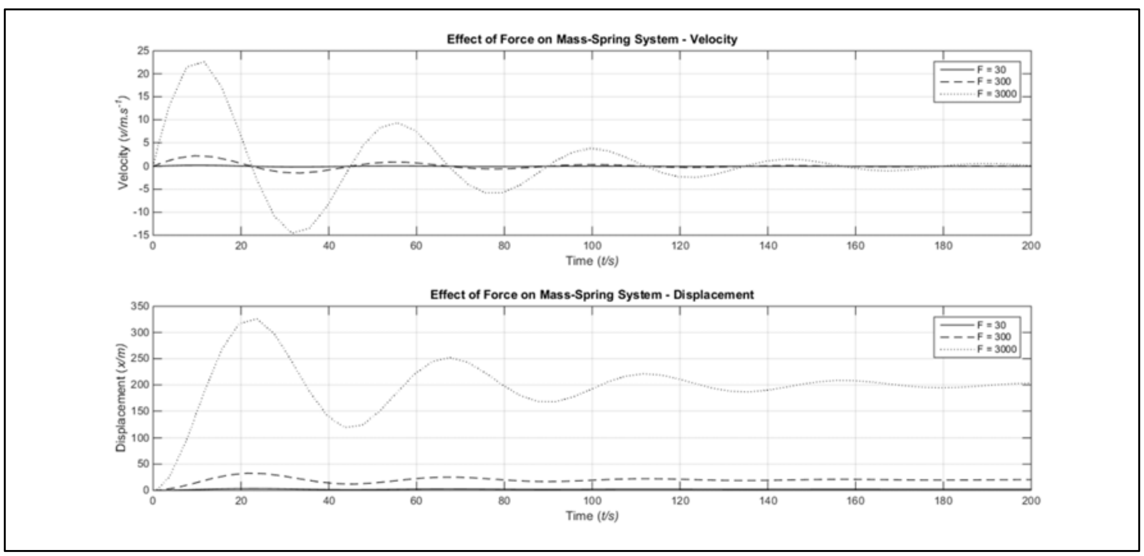

Figure 8: Variation of System Response with Applied Force

Analysis of Results

- As the mass increases, the amplitude of the oscillations in velocity decreases while that of displacement increase. In both cases, the duration of the transient response/oscillations increases with mass.
- The ideal coefficient of friction for M = 750 kg, F = 300N, and K = 15 N/m seems to be 300 N.s/m, as both the displacement and the velocity steadily increase and decrease respectively to their steady state values, with little to no oscillations.
- A friction coefficient of 3 N.s/m is clearly insufficient and shows evidence of underdamping, as oscillations in displacement and velocity have large, slowly decaying amplitudes. The rate of decay increases with increasing coefficient of friction.
- There is a significant difference in the steady state displacement when the spring constant is 1.5 N/m compared to those with spring constants of 15 and 150 N/m.
- The transient velocity response dies out almost instantly when the spring constant is 150 N/m, and takes only slightly longer in the case of the displacement.
- Increasing the value of spring constants decreases transient amplitude and duration.