Lab Session 03 – Modelling Translational Mechanical Systems

Exercise 01

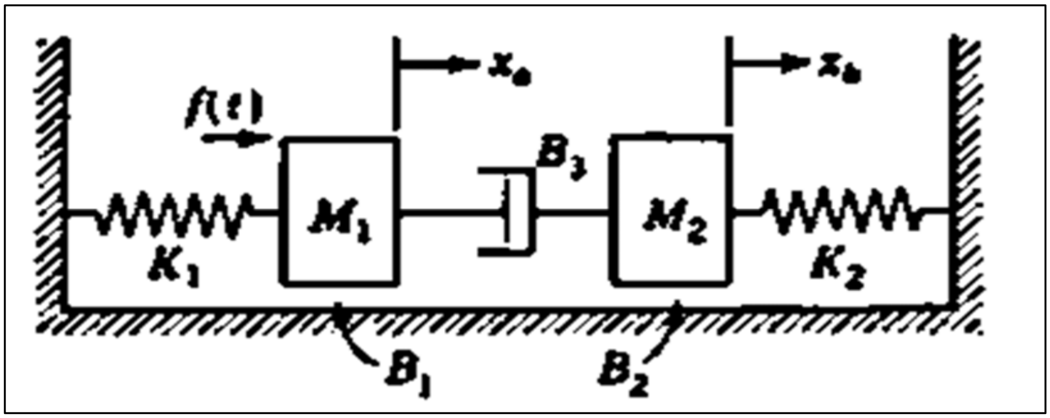

Figure 1: Multiple Element Translational Mechanical System for Exercise 01

A) Write the differential equations for the given Multiple-Element Mechanical Translational System in terms of the two displacements x_1 and x_2. Obtain the state equations for the system where x_2 is the system's output.

Differential Equation for Mass M_1

$$M_1\frac{d^2x_1}{dt^2}+(B1+B3)\frac{dx_1}{dt}-B3\frac{dx_2}{dt}+kx_1(t)=f(t)$$
$$M_1\frac{dv_1}{dt}+(B1+B3)v_1(t)-B3v_2(t)+kx_1(t)=f(t)\ldots(1)$$

Differential Equation for Mass M_2

$$M_2\frac{d^2x_2}{dt^2}+(B2+B3)\frac{dx_2}{dt}-B3\frac{dx_1}{dt}+kx_2(t)=0$$
$$M_2\frac{dv_2}{dt}+(B2+B3)v_2(t)-B3v_1(t)+kx_2(t)=0\ldots(2)$$

Differentiated variables in the differential equations for this system include$x_1(t), v_1(t)$, $x_2(t), and\ v_2(t)$. Assuming these variables as state variables and forming state equations.

State Equation for $x_2(t)$

$$\frac{dx_2}{dt} = v_2(t)$$

State Equation for $v_2(t)$ obtained by making $\frac{dv_2}{dt}$ subject of equation (2)

$$\frac{dv_2}{dt} = -\frac{K_2}{M_2}x_2(t) - \frac{(B_2 + B_3)}{M_2}v_2(t) + \frac{B_3}{M_2}v_1(t)$$

State Equation for $x_1(t)$

$$\frac{dx_1}{dt} = v_1(t)$$

State Equation for $v_1(t)$

$$\frac{dv_1}{dt} = \frac{B_3}{M_1}v_2(t) - \frac{K_1}{M_1}x_1(t) - \frac{(B_1 + B_3)}{M_1}v_2(t) + \frac{1}{M_1}f(t)$$

<u>Combining these equations into a state space representation</u>

$$\dot{\mathrm{x}} = A\mathrm{x} + B\mathrm{u}$$

$$\begin{bmatrix} \frac{dx_2}{dt} \\ \frac{dv_2}{dt} \\ \frac{dx_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_2}{M_2} & -\frac{(B_2 + B_3)}{M_2} & 0 & \frac{B_3}{M_2} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{B_3}{M_1} & -\frac{K_1}{M_1} & -\frac{(B_1 + B_3)}{M_1} \end{bmatrix} \begin{bmatrix} x_2 \\ v_2 \\ x_1 \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_1} \end{bmatrix} f(t)$$

Continued on next page.

B) Plot the positions and the speeds in separate graphs.

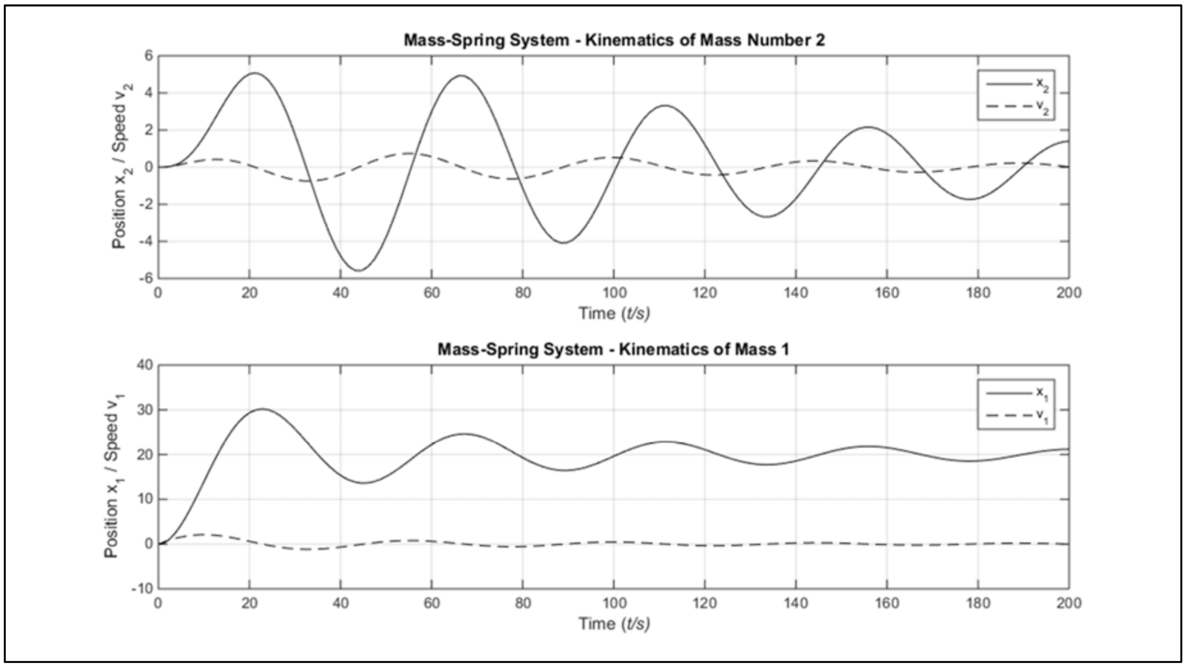

Figure 2: Multiple Element Mechanical System Response

C) Change the values of each parameters, such as

- **Mass M_1 and M_2,**
- **Friction coefficients B_1, B_2 & B_3 and**
- **Elastic characteristic K_1 & K_2.**
- **Compare the results and describe the effects.**

Variation in Mass

Test	Mass 1 (kg)	Mass 2 (kg)	Observations
1	750	750	This result was used as the standard. Transient response for both masses' displacement and velocity has large amplitude and almost dies out by 200s.
2	7500	7500	Almost no oscillations in velocity. Large oscillations in displacement. Transient displacement duration is large.
3	75	75	Faster, rapid oscillations in velocity and displacement of both masses. Transient response dies out fairly quickly. Amplitude of oscillations in displacement > that of velocity.

4	750	75	Large amplitude of oscillations in displacement of both masses. Transient response duration 110s. Velocity oscillations no longer as insignificant as test 3.
5	75	750	Small, short lived oscillations for mass 1. Transient response for mass 2 larger in both amplitude and durations.

Variation in Frictional Coefficients

Test	B1 (N.s/m)	B2 (N.s/m)	B3 (N.s/m)	Observations
1	20	20	30	Base case. Large oscillations in displacement 2 compared to displacement 1. Velocity oscillations small in both cases. Displacement 2 transient response has shorter duration than that of 1.
2	200	200	300	Displacement 2 increases to peak value and then steadily decreases to mean value. Displacement 1 likewise rises to peak value steadily with no oscillations. Clearly overdamped. Very little variation in velocities of both masses.
3	2	2	3	Clearly underdamped. Transient response does not die out in 200s. Rather, for mass 2, amplitude of displacement and velocity oscillations seems to increase with time. Amplitude of oscillations in displacement 1 decreases very slowly. Steady rise in oscillation amplitude for velocity 1.

Variation in Elastic Characteristics

Test	K1 (N.s/m)	K2 (N.s/m)	Observations
1	15	15	Base case. Displacement 1 amplitude oscillations greater than those of displacement 2. Transient response 2 eliminated by 200s.
2	150	150	Frequency of oscillations has increased substantially. Transient response for both masses still decays almost completely by 200s. Mass 2's oscillations have a larger amplitude.
3	1.5	1.5	For both masses, amplitude of velocity oscillations is much smaller than that of displacement. Mass 2 has larger displacement oscillations than mass 2. Some evidence of

			overdamping as displacement oscillations have yet to dissipate by 200 s.
4	150	1..5	Displacement and velocity of mass 2 oscillate rapidly, but with considerably smaller amplitude, with transient response almost eliminated by 100 s. Displacement of mass 2 rises amidst lots of fluctuations to peak value and then decays to steady state value. Fluctuations also seen in velocity of mass 1.
5	1.5	150	Mass 1's response now appears critically damped, with little variation in the velocity. The displacement rises steadily to its mean value by 100 s. Fluctuations still present in the response for mass 2, which is similar to that seen in test 4.