Lab Session 02
Mathematical Modelling

Lab Exercise – Cruise Speed

Run the given code and explain the curve shown. Also change the parameters defined in the function. Observe the variation and comment.

Post lab Task 01, Code 01 - cruise_speed_test.m

```
v0 = 0;                                                    % initial speed
[t, v] = ode45('cruise_speed', [0, 200], v0);
plot(t, v);
grid on;
title('Cruise Speed Time Response to a Constant Friction Force Fa(t)');
```

Post lab Task 01, Code 02 - cruise_speed.m

```
function dvdt = cruise_speed(t, v)
M = 750;
B = 30;
Fa = 300;
dvdt = Fa/M - B/M * v;
```

Figure Generated by Codes 01 and 02

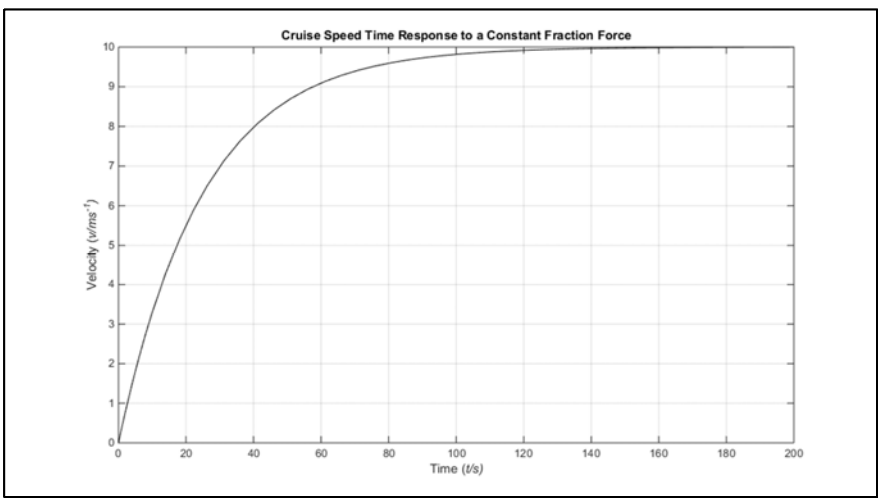


Figure 1: Cruise Speed Response Time generated by codes 01 and 02

Figure 1 shows the variation of a mass that is simultaneously under the action of an applied force, $F_a(t)$, and two opposing forces: its own inertia and the force of friction. The curve is a graphical representation of the numerical solution to the first order ODE describing this system

$$M\frac{dv(t)}{dt} + Bv(t) = F_a(t)$$
$$\Rightarrow \frac{dv(t)}{dt} = \frac{F_a(t)}{M} - \frac{B}{M}v(t) \ldots\ldots\ldots\ldots (1)$$

The parameters used in this system are as follows:

$$F_a(t) = 300\ (Applied\ Force)$$
$$B = 30\ (Frictional\ Coefficient)$$
$$M = 750\ (Mass)$$

Initially, the mass is at rest so its velocity $v(t)$ is zero, and the applied force $F_a(t)$ is much larger than the opposing frictional force $Bv(t)$ $(\because v(t) = 0)$. Thus, the initial rate of change of velocity is $\approx \frac{F_a(t)}{M}$, which is constant for the given system. As the velocity of the mass begins to increase, the frictional force $Bv(t)$ increases, and thus the magnitude of the term $\frac{Bv(t)}{M}$ in the expression for $\frac{dv(t)}{dt}$ begins to increase. Thus increasing velocity actually corresponds to an overall decrease in the rate of change of velocity.

From a purely mechanical perspective, this corresponds to the frictional force acting on the object increasing moving with velocity, eventually resulting the in the sum of opposing forces acting on the system to be equal to the applied force $F_a(t)$. At this point, all forces acting on the mass are balanced, and according to Newton's First Law of Motion, the acceleration and thus the rate of change of velocity are zero. Thus, the object continues to move at a constant or terminal velocity of 10 ms^{-1}.

Effects of Changing Parameters

To further investigate the system's response, each of the three variables – mass, frictional coefficient, and applied force – specified in the differential equation were varied as shown in the table below.

Varied Quantity	Values				Constant 1		Constant 2	
	1	2	3	4	Parameter	Value	Parameter	Value
Mass (kg)	100	300	500	700	Force	300	Friction	30
Force (N)	100	300	500	700	Mass	750	Friction	30
Friction	10	30	50	70	Mass	750	Force	300

Effects of Varying Mass

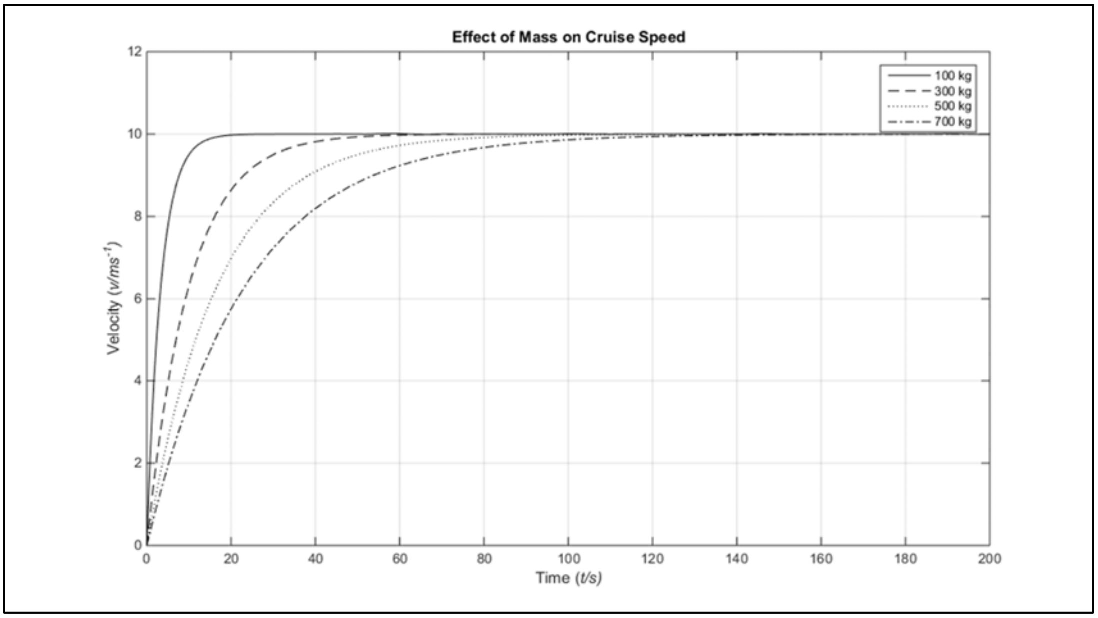


Figure 2: Variation of Cruise Speed with Mass

Effects of Varying Frictional Coefficient

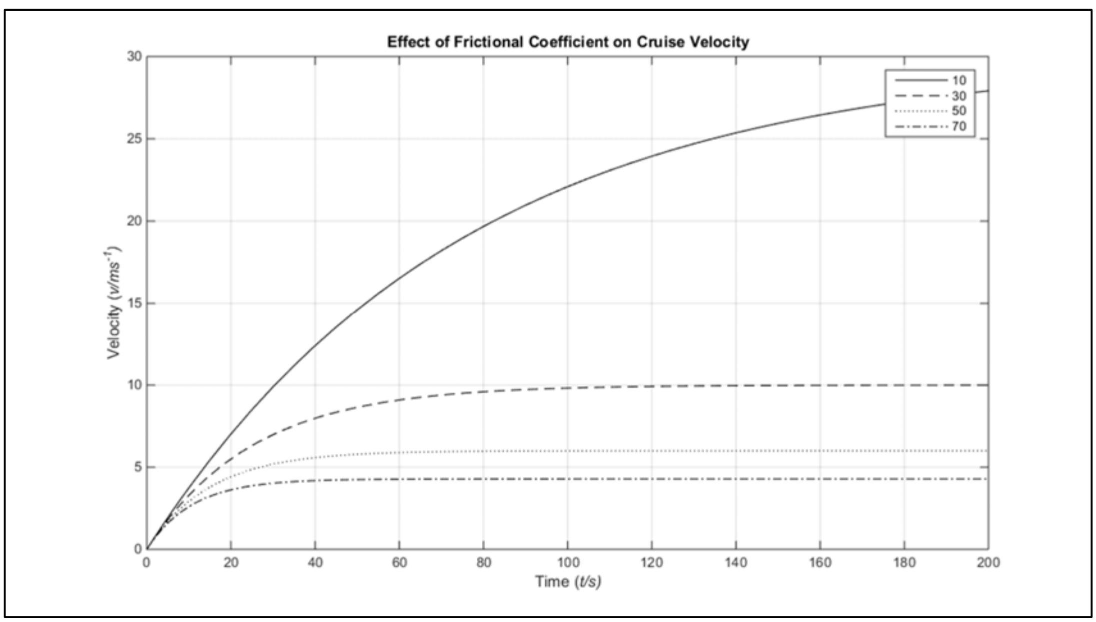


Figure 3: Variation of Cruise Speed with Frictional Coefficient

Effects of Varying Applied Force

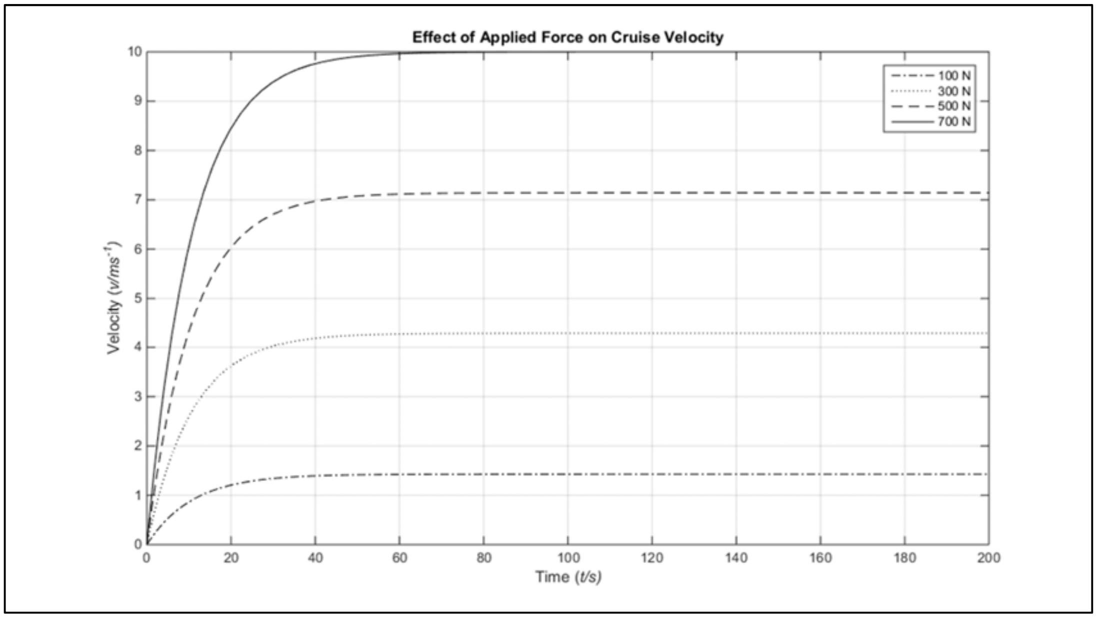


Figure 4: Variation of Cruise Velocity with Applied Force

Analysis of Results

Mass

Figure 2 shows that increasing the mass has no effect on the terminal velocity – the mass still reaches a final velocity of 10 ms^{-1}. However, the greater the mass, the more time it takes for the object to reach its terminal velocity. This is evident from the figure – when object attains terminal velocity by ≈ 20s, 60s, 80s, and 120s when its mass is 100 kg, 300 kg, 500 kg, and 700 kg respectively. This is because lower mass means lower inertia and a lower tendency for the object to change its state of rest or motion. Concretely, this means the rate of change of velocity $\frac{dv(t)}{dt}$ of the object is higher for a lower mass, which is evident from equation (1) where $\frac{dv(t)}{dt}\ \alpha \frac{1}{M}$ for a given applied force and frictional coefficient.

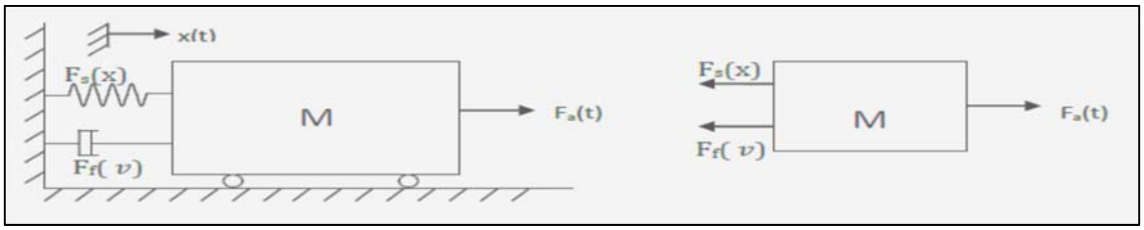
Frictional Coefficient

Figure 3 shows that the relationship between the frictional coefficient B and the cruise speed is a little more complicated. Lower frictional coefficients result in an overall higher cruise velocity, but it takes longer for said cruise velocity to be attained. For instance, with a frictional coefficient of 10 $N.s/m$ leads to the highest cruise velocity of ~28 m/s which is barely attained by 200s, whereas the highest frictional coefficient of 70 N.s/m leads to the object attaining a much lower cruise velocity of ~4 m/s albeit in only 20 seconds. This is because equation (1) shows the higher the value of the coefficient B, the more quickly the rate of change of velocity $\frac{dv(t)}{dt}$ reaches its minimum value of zero. Intuitively, this is because a higher coefficient of viscous friction means the frictional forces acting on the object at any given time are larger, which means it takes less time for the cumulative effects of these frictional forces to be equal in magnitude but opposite in direction to the applied force, leading to zero net force acting on the mass.

Applied Force

The larger the applied force, the higher the cruise speed and the longer it takes it attain said cruise speed. Figure 4 clearly shows that an applied force of 700 N leads to a much larger cruise velocity (10 m/s) compared to that of a 100 N force (only 1.5 m/s). This is because the larger the applied force, the longer it takes for the opposing viscous friction to become equal to it in magnitude. Thus the object accelerates for a longer duration before the opposing force becomes equal to the applied force, resulting in higher cruise velocity.

Post-lab Exercise (Exercise 02) – Mass Spring System



$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx^r(t) = F_a(t)$$

A) Use codes 01 and 02 to plot the position and speed in separate graphs. Change the value of `r` to 2 and 3. Superpose the results and compare with the linear case r = 1, and plot all three cases in the same plot with different figures for velocity and displacement.

Post lab Task 02, Code 01 - mass_spring.m

```
function dXdt = mass_spring(t, X)
M = 750;                                    % (kg)
B = 15;                                     % (N.s/m)
Fa = 300;                                   % (N)
K = 15;                                     % (N/m)
r = 1;                                      % dX/dt
dXdt(1, 1) = X(2);
dXdt(2, 1) = -B/M*X(2) - K/M * X(1)^r + Fa/M;
```

Post lab Task 02, Code 02 - mass_spring_test.m

```
clear all; close all; clc;
X0=[0;0];
[t,X]=ode45('mass_spring',[0 200],X0);
figure;
plot(t,X(:,1)); Plot the first variable "position"
xlabel('Time(t)');
ylabel('Position');
title('Mass spring system');
legend('Position '); grid;
figure;
plot(t,X(:,2),'r'); Plot the second variable "speed"
xlabel('Time(t)'); ylabel('Speed');
title('Mass spring system');
legend('Speed ');
grid;
```

Position Figure

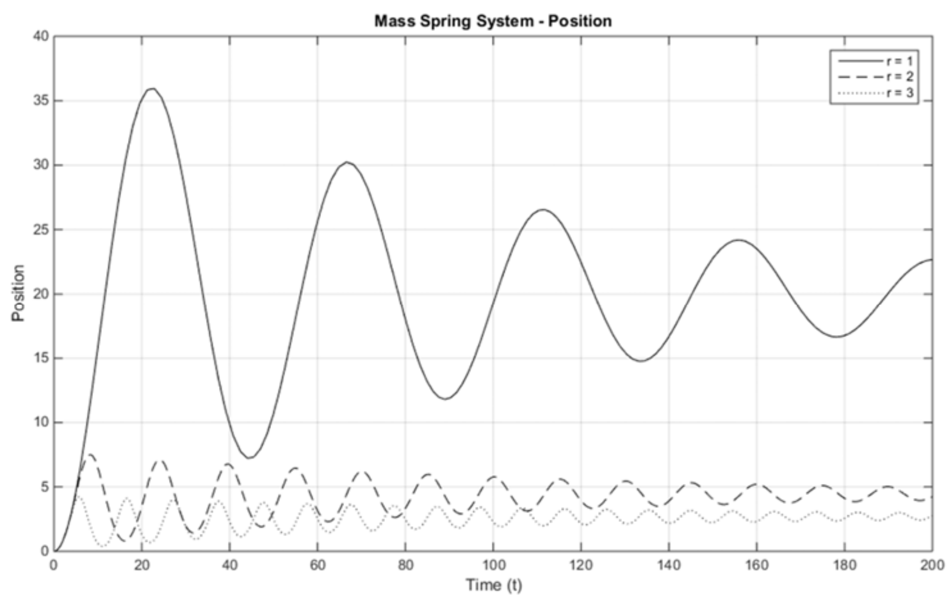


Figure 5: Variation of Position with Spring Order

Speed Figure

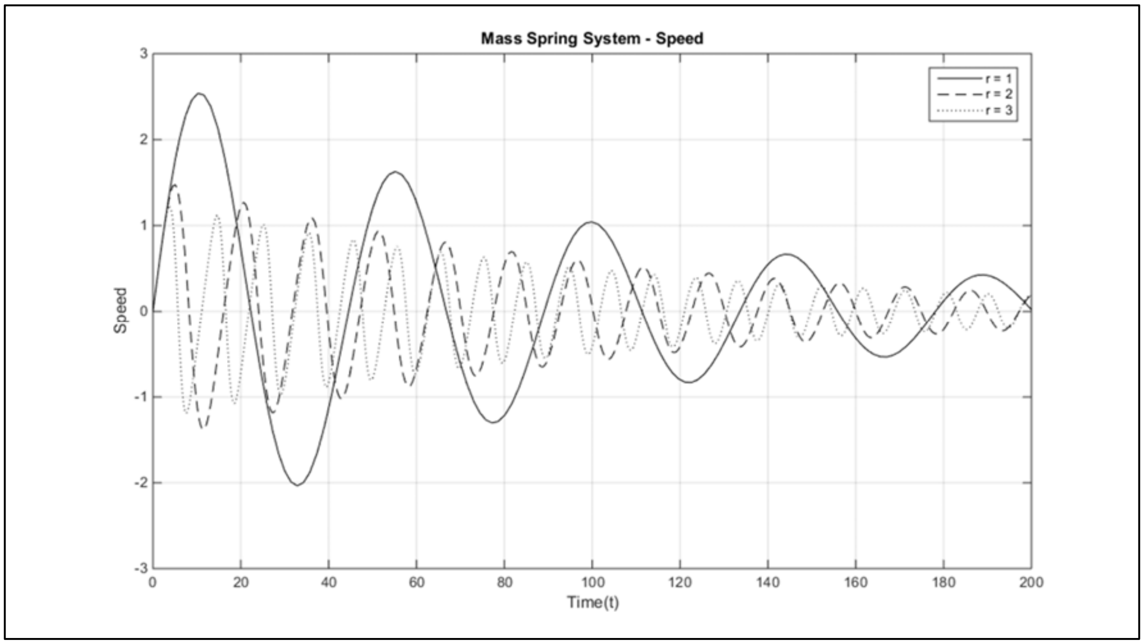


Figure 6: Speed Variation with Spring Order

Analysis of Results

- As the order of the spring increases, the amplitude, period, and mean value of the mass's displacement decrease. The linear spring has the largest amplitude of velocity and displacement.
- There is a significant difference in the mean position between linear springs and quadratic/cubic springs, but no such difference in mean value exists for velocity.
- Oscillations occur more rapidly in higher order springs, but also dissipate more quickly. This could be because of the higher order spring's restoring force being much larger for the same displacement.
- The variation in both position and velocity shows clear evidence of damping.

B) Change the values of each parameter such as Mass M, Friction Coefficient B, and Elastic Coefficient K. Compare the results and describe the effects.

<u>Effect of Varying Mass on Position and Velocity</u>

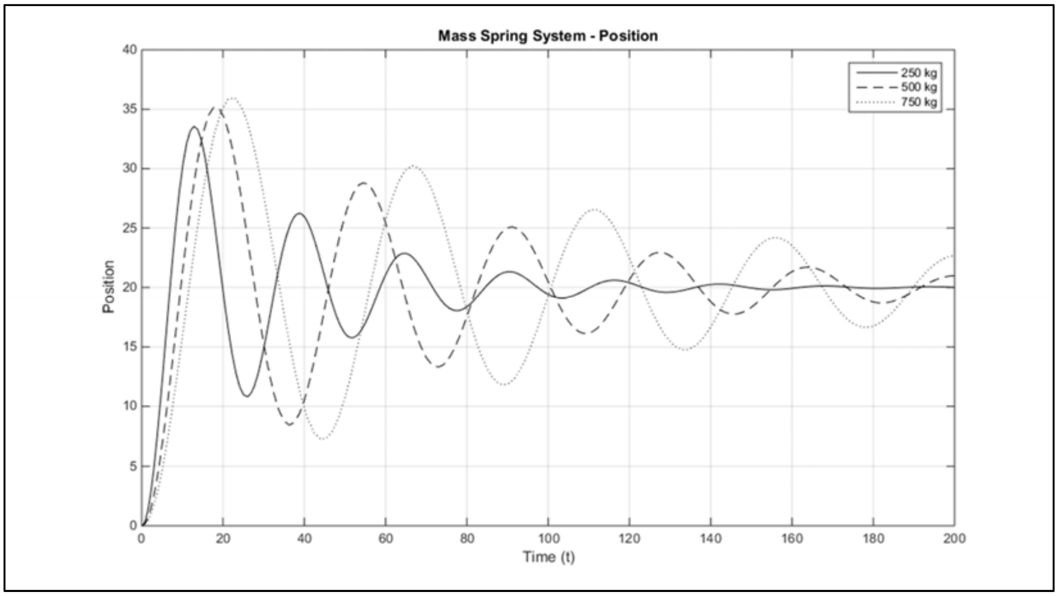


Figure 7a: Variation of Position with Mass

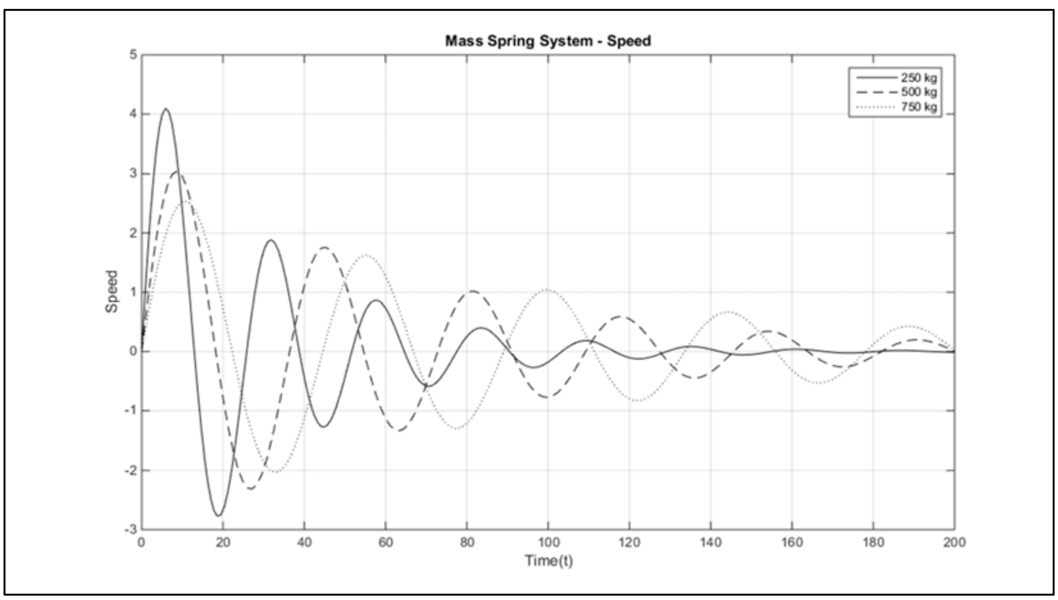


Figure 7b: Variation of Speed with Mass

Effect of Varying Friction Coefficient on Position and Velocity

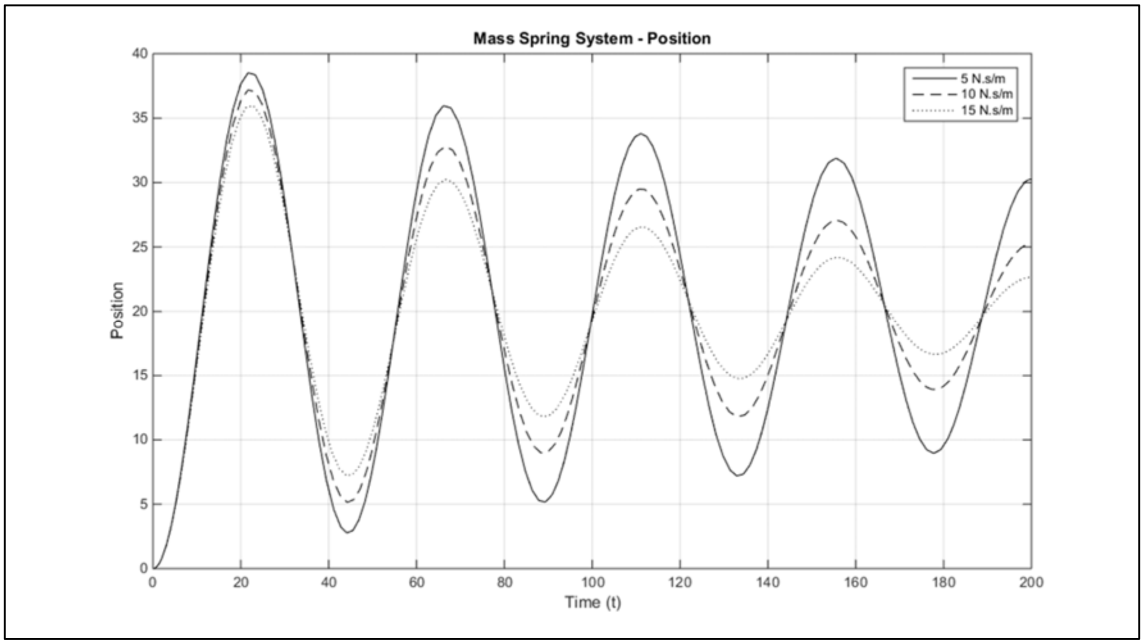


Figure 8a: Variation of Position with Frictional Coefficient

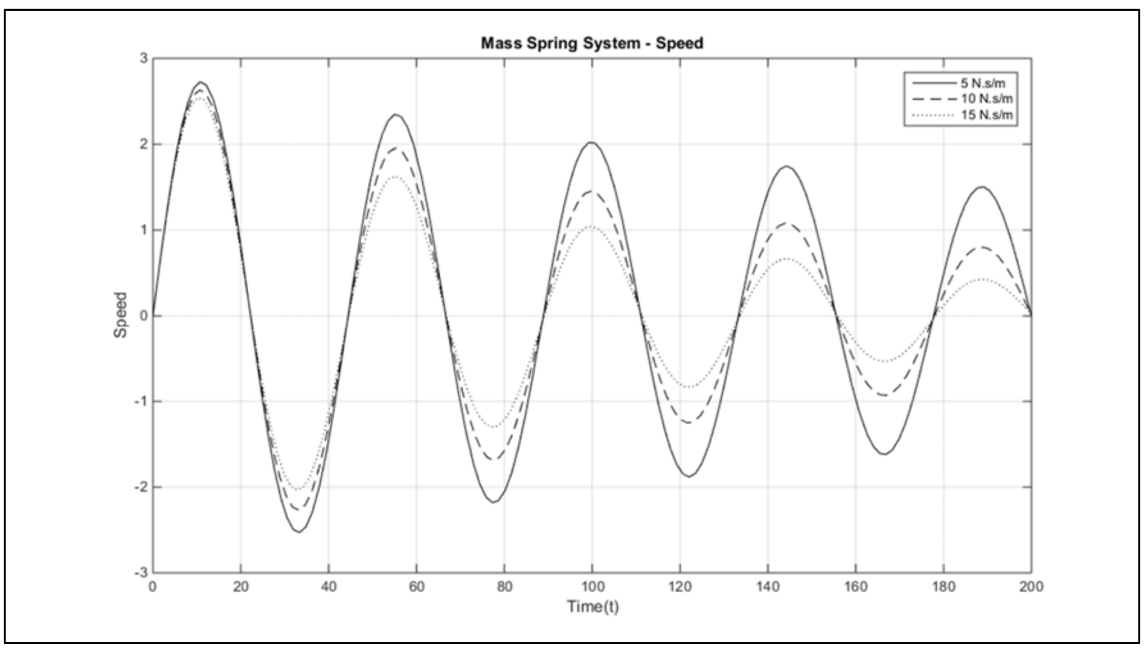


Figure 8b: Variation of Velocity with Frictional Coefficient

Effect of Elastic Coefficient on Position and Velocity

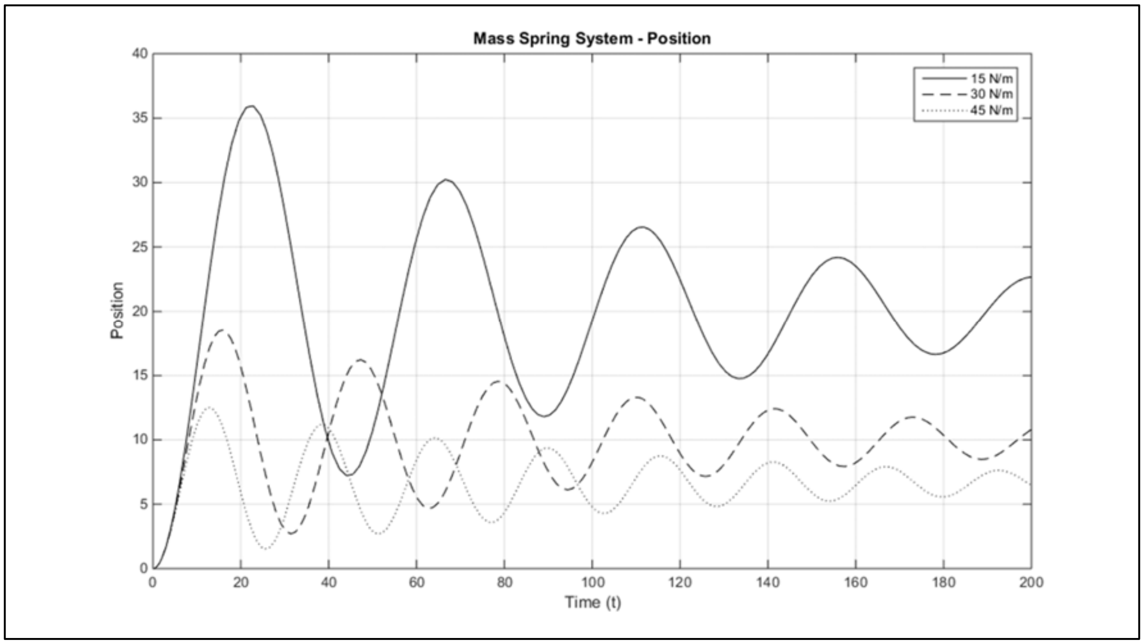


Figure 9a: Variation of Position with Elastic Coefficient

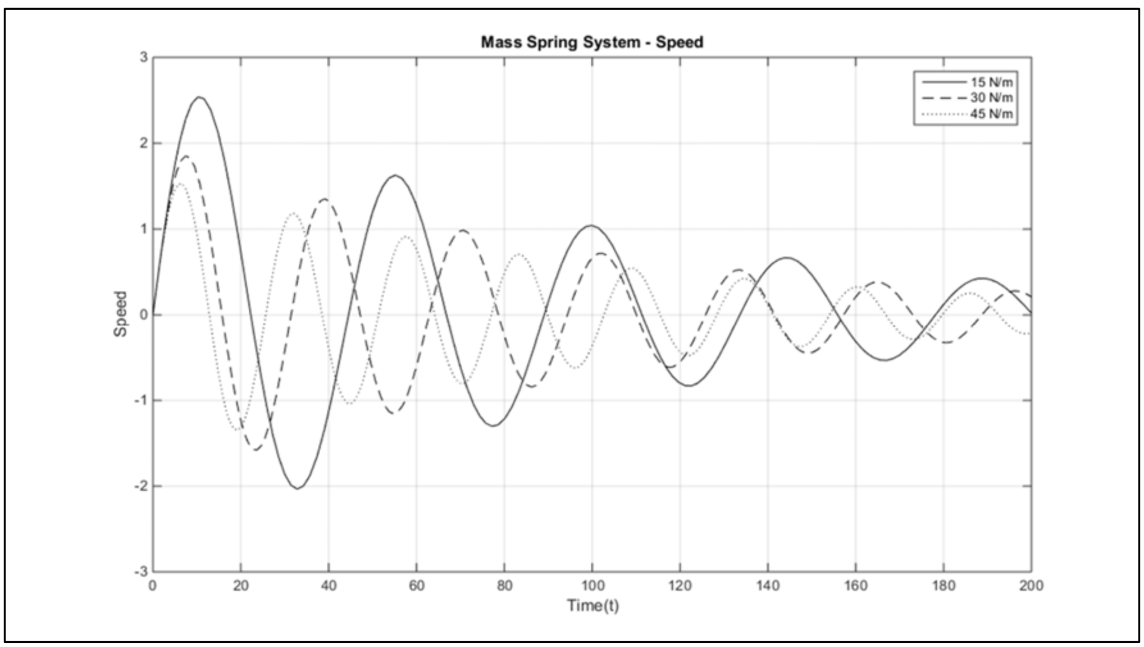


Figure 9b: Variation of Speed with Elastic Coefficient

Analysis of Results

Mass

- The smaller the mass, the quicker the oscillations in displacement and velocity die out.
- Large mass leads to smaller amplitude of displacement oscillations, but larger amplitude of velocity oscillations.
- There is a significant delay/phase shift in the relative displacements/velocities of the object when the mass increases.

Frictional Coefficient

- There is no relative phase shift/delay between the positions and velocities of the object for different frictional coefficients – these quantities reach their peaks and troughs at the same instants, regardless of frictional coefficient.
- The frequency of oscillations in displacement and velocity remain are independent of the frictional coefficient.
- The higher the frictional coefficient, the larger the effect of damping – the amplitudes of oscillations in displacement and velocity decrease more quickly when the frictional coefficient is higher.
- The object does not reach its mean position or zero velocity for any of the investigated frictional coefficients within 200 seconds

Elastic Coefficient

- The higher the elastic coefficient, the more pronounced the damping effect.
- This is why the higher elastic coefficient leads to a lower amplitude of oscillations in displacement and velocity, as well as a higher frequency of oscillations.
- The mean value of position is still around 20 m, and the mean velocity is still 0 m/s.