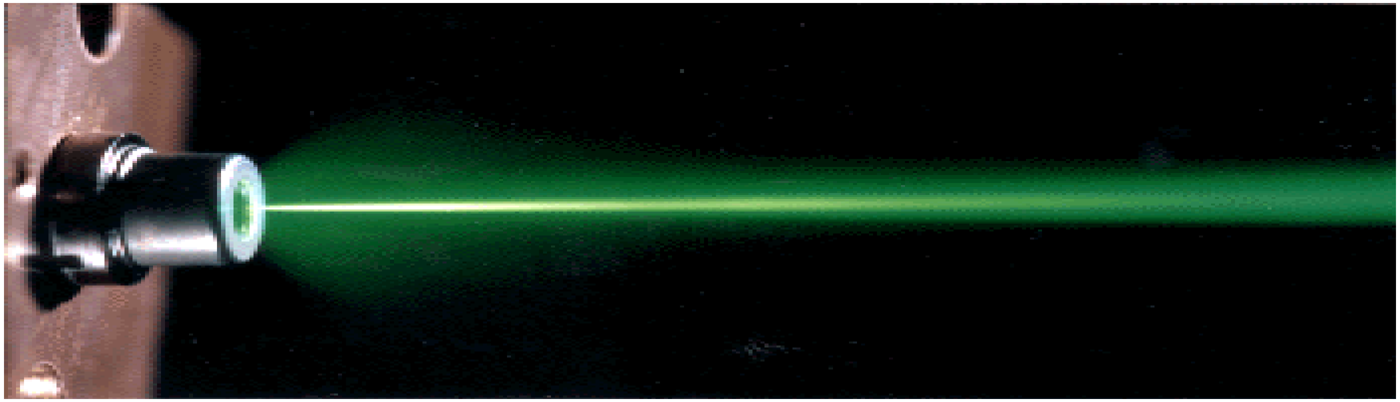
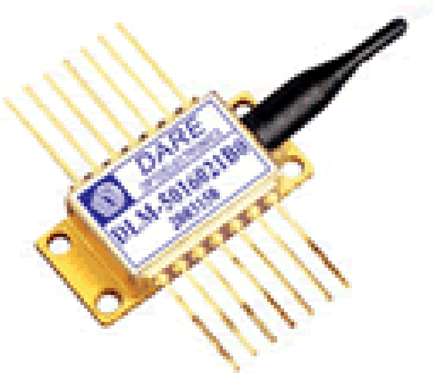
بسم الله الرحمن الرحیم

الکترونیک نوری
۱۴۰۳-۱



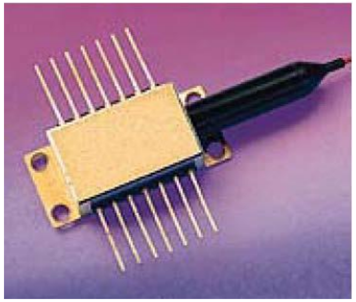
Optoelectronic



Dr. Kambiz Abedi

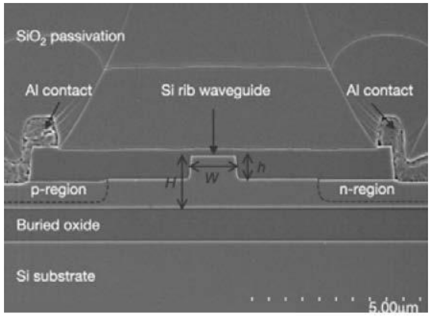
Optoelectronic Devices

Telecommunication laser



Blue laser

Raman waveguide laser



Optical fiber

LED traffic lights

Photodiodes

Solar cells

Name 10 Optoelectronic Components You Used Everyday











Course Syllabus

1) **مروری بر تئوریهای کلاسیک ومدرن**

- ✓ نظریه الکترومغناطیس
- ✓ قوانین انکسار و انعکاس نور (معادلات فرنل واسنل)
- ✓ اثر فوتوالکتریک
- ✓ تئوری پاشندگی
- ✓ ساختمان کریستالی نیمه هادیها
- ✓ تئوری تابش جسم سیاه
- ✓ نظریه کوانتوم مکانیک وتئوری باند، مفهوم جرم موثر و چگالی حالتها

2) **سیستم های فیبر نوری**

- ✓ نور هندسی و طبقه بندی فیبرهای نوری
- ✓ آنالیز یک فیبر نوری از دیدگاه موج
- ✓ موجبرهای مسطح دی الکتریک

3) **منابع نوری تکرنگ (لیزر)**

- ✓ انواع برهمکنش نور وماده
- ✓ معادلات انیشتین ، پدیده وارونگی

Course Syllabus

- ✓ انواع پمپینگ و معادلات نرخ
- ✓ محاسبات ضریب جذب
- ✓ طرز کار فبری پرو
- ✓ انواع لیزر

4) منابع نوری تکرنگ (دیود نوری)

- ✓ طرز ساخت دیودهای نوری LED
- ✓ کوپلینگ نوری از دیود به فیبر
- ✓ ویژگیهای دیودهای نوری از نظر پاسخ فرکانسی ومدولاسیون
- ✓ توان خروجی و عمر مفید

5) تقویت کننده های نوری

- ✓ نوع فبری پرو
- ✓ نوع موج رونده
- ✓ معادلات تقویت کنندگی سیگنال کوچک

Course Syllabus

6) صفحات نمایش نوری

- ✓ اثر لومینانس و فسفرسانس
- ✓ صفحات اشعه کاتدی CRT
- ✓ صفحات پلاسما
- ✓ صفحات نمایش کریستال مایع LCD

7) آشکار سازهای نوری

- ✓ فتو دیودهای p-n
- ✓ فتو دیودهای p-i-n
- ✓ فتو دیودهای APD
- ✓ فتوترانزیستورها

8) ملاحظات نویز

- ✓ نویز حرارتی ، تاریکی ، کوانتوم
- ✓ نویز در علائم آنالوگ و دیجیتال
- ✓ نویز در آشکارسازهای نوری

Course Syllabus

9) مدولاسیون نوری

- ✓ قطبش نور
- ✓ مفهوم دو شکستی
- ✓ اثرات الکترواپتیک Kerr و Pockels
- ✓ چرخش فارادی و مدولاسیون مگنتواپتیک
- ✓ شرط پراش براگ
- ✓ اثر اکوستواپتیک
- ✓ چند مثال کاربردی در مورد مدولاسیون نوری

Contents

- Wave Nature of Light – Conceptual Overview
  - Wave Equation, Refractive index, group and phase velocity, Poynting vector, Snell's law, Fresnel's equations, Optical Resonators, Optical Tunneling, Coherence, Diffraction
- Optoelectronic materials and heterostructure semiconductor devices
  - States of materials: solids, liquid, gas, LC
  - Crystals, interfaces, polycrystals
- Dielectric and Optical Waveguides

  Symmetric planer dielectric waveguide, modal and waveguide dispersion
- Optical processes and light propagation in crystals (polarization, refraction, reflection, transmission, Maxwell's equations and wave equations)

Contents

- Optical and electronic properties of semiconductors
- Light emitting diodes (material systems, physics of operation, structures, characteristics and reliability)
- Laser diodes (spontaneous and stimulated emission, gain and loss, structures, time response, characteristics)
- Optical detectors (optical absorption, physics of operation, structures, characteristics)
- Optical communication systems

References

1) J. Wilson, J.F.B. Hawkes, **Optoelectronics, An Introduction,** 1998

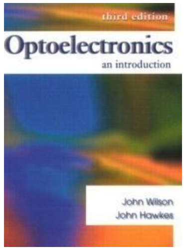
2) J. Singh, **Optoelectronics, An Introduction to Materials and Devices, McGraw-Hill,** 1996.

3) H. C. Cassy JR., M. B. Panish, **Heterostructure Lasers, Part A, Academic Press,** 1978.

4) J. M. Liu, **Photonic Devices, Cambridge University Press,** 2005.

5) G. P. Agrawal, **Fiber Optic Communication Systems, John Wiley & Sons,** 2002.

6) P. Bhattacharya, **Semiconductor Optoelectronic Devices, Prentice Hall International,** 2002.



References

7) A. Yariv, **Quantum Electronics,** 1989

8) A. Yariv, **Introduction to Optical Electronics,** 1976

9) J. M. Senior , **Optical Fiber Communications,** 2006

Optional Reading

1. Saleh and Teich, **Fundamentals of Photonics, 3rd ed.** Wiley Interscience 2019.
2. S. L. Chuang, **Physics of Photonic Devices, 2nd ed.** Wiley, 2009.
3. A. Yariv and P. Yeh, **Photonics: Optical electronics in Modern Communications, 6th ed.** Oxford University Press, 2007.
4. D. Birtalan and W. Nunley, **Optoelectrioncs: Infrared-Visible-Ultraviolet Devices and Applications, 2nd ed.,** CRC Press, 2009.

Journals

- IEEE Photonics Technology Letters
- IEEE Journal of Selected Topics in Quantum Electronics
- IEEE Quantum Electronics
- IEEE/OSA Journal of Lightwave Technology
- IEEE Photonics Journal
- Optical and Quantum Electronics
- Applied Optics
- Optics Express
- Optics Communications
- Fiber and Integrated Optics
- Optik
- Optoelectronics Letters
- Japanese J. of Applied Physics
- Solid State Electronics
- Modern Optics
- http://www.scimagojr.com/journalrank.php
- http://gen.lib.rus.ec/

Optical and Quantum Electronics (Netherlands)

Optical and Quantum Electronics publishes papers on the following topics:

a) semiconductors,

b) solid state and gas lasers,

c) optical communication systems,

d) fibres and planar waveguides,

e) non-linear optics,

f) optoelectronic devices,

g) ultra-fast phenomena,

h) optical storage,

i) optical materials,

j) photonic switching,

k) optics in computers and coherent optics

Seminars and Projects

1) Optical filters (ring resonator)
2) Optical splitters and couplers
3) Optical switches (based on SOA)
4) Terahertz Silicon Lasers
5) Photonic crystal laser
6) Optical negative-index metamaterials
7) RF/Microwave Photonics
8) Optical Networking
9) Terahertz-laser waveguides using Metamaterial
10) Quantum-Dot Ring Lasers
11) Plasmonic Waveguides and Gratings
12) Ultrafast Photonic Signal Processing
13) Multi-wavelength Semiconductor Fiber Lasers

Seminars and Projects

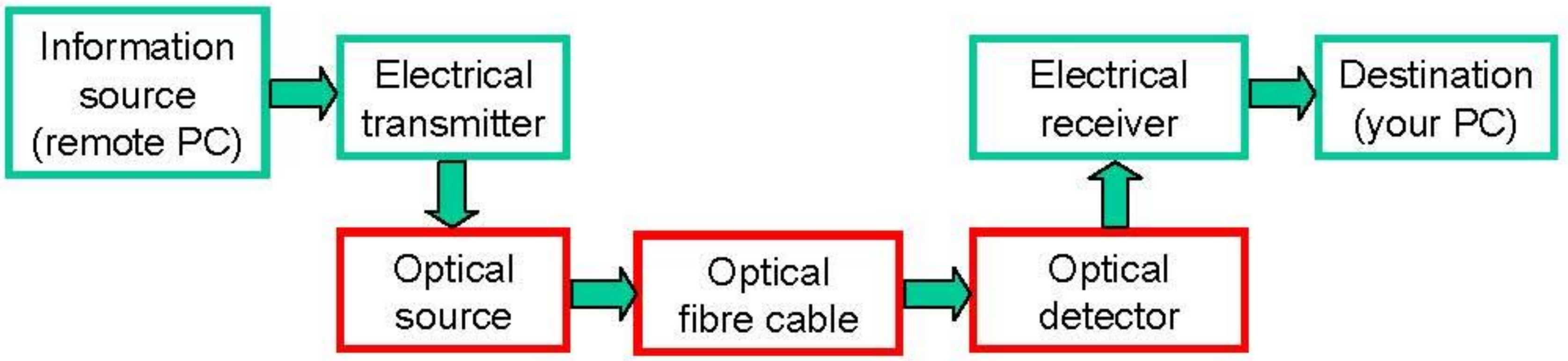
14) Slow-Light Photonic Crystal Devices
15) Slow light in silicon microring resonators
16) Transistor laser
17) All-optical logic gates
18) Active optical ring resonators
19) Silicon microring add-drop filter
20) Optical add-drop multiplexer
21) Ring-cavity surface-emitting lasers
22) Carbon Nanotubes
23) Nonlinear Optical Effects in Silicon Waveguides
24) Silicon Photonics
25) White LED and related technologies
26) Fiber-optic sensor and networks
27) Optical sensors and applications

Seminars and Projects

27) Nonlinear Optics and Nanophotonics
28) Interaction of light with sub-wavelength structures and optical metamaterials
29) Nanophotonics and Plasmonics
30) Metamaterials for optical and photonic applications
31) Terahertz metamaterial absorbers
32) Nanoplasmonics and Metamaterials
33) Metamaterial Surface Antenna Technology
34) Photonic Metamaterials

GENERAL COMMUNICATION SYSTEM

OPTICAL FIBRE COMMUNICATION SYSTEM



THIS COURSE

- Basics of optics and optical fibers
- Semiconductor optical sources
- Semiconductor optical detectors

Optoelectronics?

"Optoelectronics is the study and application of electronic devices that interact with light."

Major Optoelectronic Devices- Direct conversion between electrons and photons

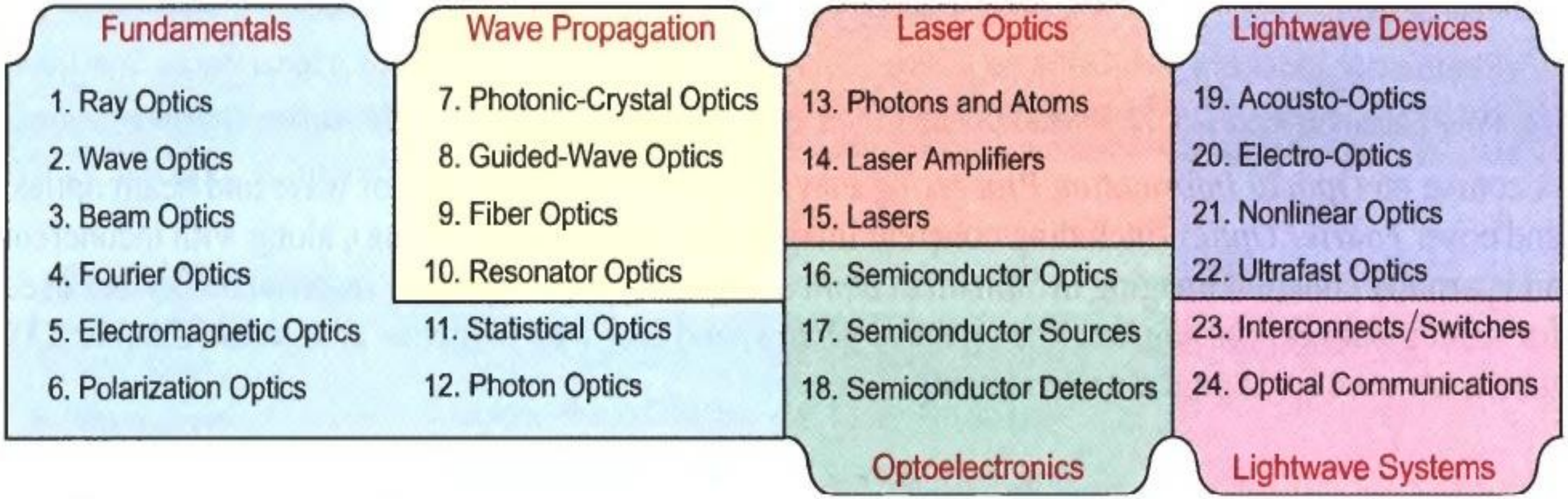
1) Light-emitting diodes (LEDs) (display, lighting,..)

2) Laser diodes (LDs) (data storage, telecommunication,…)

3) Photdiodes (PDs) (telecommunication,..)

4) Solar Cells (energy generation)

Fundamentals of Photonics



Optics

1) **Ray Optics (Background)**

2) **Wave Optics (Background)**

3) **Beam Optics**

4) **Fourier Optics**

5) **Electromagnetic Optics**

6) **Polarization and Crystal Optics**

7) **Guided -Wave Optics**

8) **Statistical Optics**

Lasers and Quantum Electronics

1) **Ray Optics (Background)**

2) **Wave Optics (Background)**

3) **Beam Optics**

4) **Resonator Optics**

5) **Photon Optics**

Optoelectronics

1) **Polarization and Crystal Optics (Background)**

2) **Photon Optics (Background)**

3) **Electro-Optics**

Optical Electronics and Communications

1) **Ray Optics (Background)**

2) **Wave Optics (Background)**

3) **Resonator Optics**

4) **Photon Optics**

Lightwave Devices

1) **Electromagnetic Optics (Background)**

2) **Resonator Optics (Background)**

3) **Photon Optics (Background)**

4) **Polarization and Crystal Optics**

5) **Guided -Wave Optics**

6) **Fiber Optics**

7) **Electro-Optics**

8) **Nonlinear Optics**

Fiber-Optic Communication or Lightwave Systems

1) **Electromagnetic Optics (Background)**

2) **Resonator Optics (Background)**

3) **Photon Optics (Background)**

4) **Polarization and Crystal Optics (Background)**

5) **Fiber Optics**

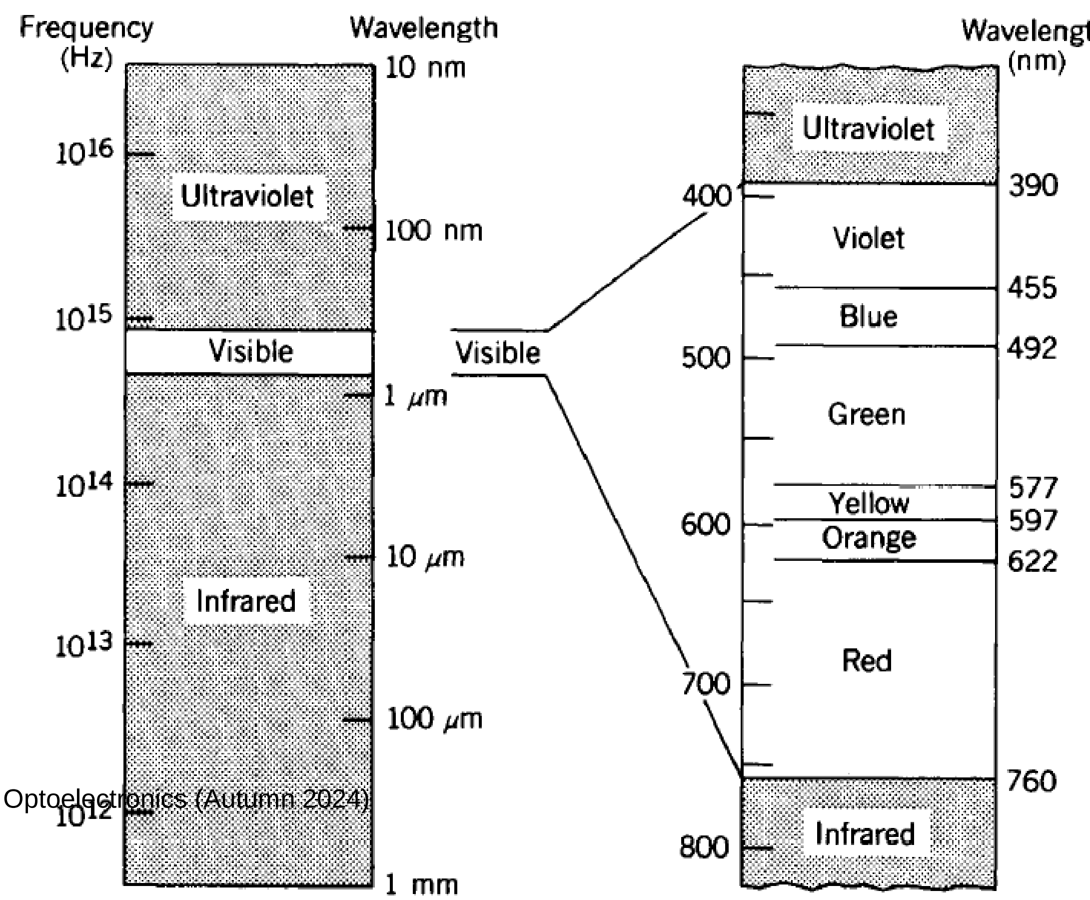
6) **Guided -Wave Optics**

Ray Optics

- ***Ray optics*** is the simplest theory of **light**. **Light** is described by rays that travel in different optical media in accordance with a set of **geometrical rules**. ***Ray optics*** is therefore also called **geometrical optics.**
- ***Ray optics*** is an **approximate theory**. Although it **adequately** describes **most of our daily experiences** with **light**, there are many phenomena that ray optics does not adequately describe.
- **Ray optics** is concerned with the location and direction of **light rays.** It is therefore useful in studying **image formation-**the collection of rays from each point of an object and their redirection by an optical component onto a corresponding point of an image.
- ***Ray optics*** permits us to determine conditions under which light is guided within a given medium, such as a ***glass fiber***.

WAVE OPTICS

❑***Light*** propagates in the form of waves. In free space, light waves travel with a constant speed $c_0 = 3.0 \times 10^8$ m/s (30 cm/ns or 0.3 mm/ps). The range of optical wavelengths contains three bands- ultraviolet (10 to 390 nm), visible (390 to 760 nm), and infrared (760 nm to 1 mm). The corresponding range of optical frequencies stretches from 3×10^{11} Hz to 3×10^{16} Hz.



WAVE OPTICS

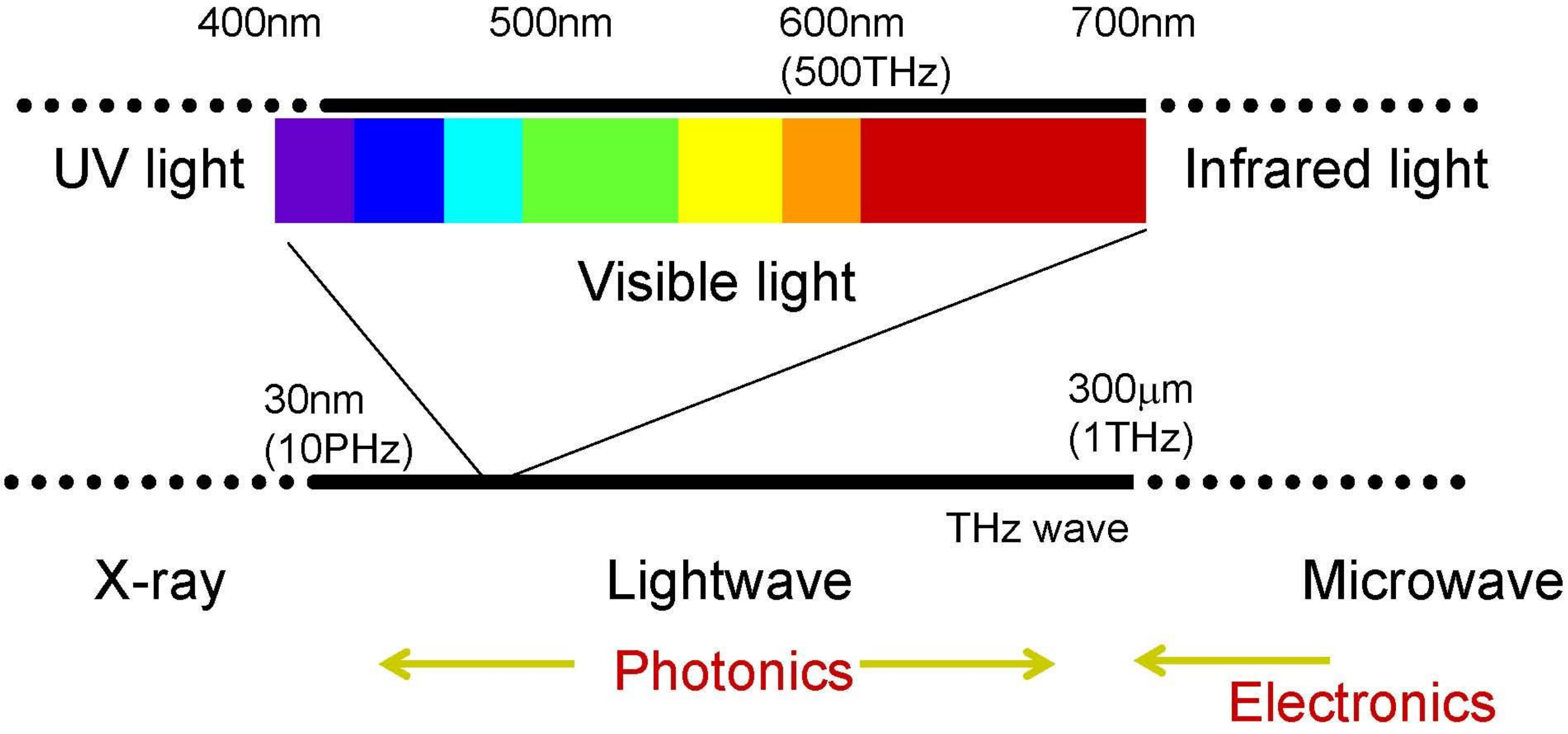
The **wave theory** of light encompasses the ray theory. Strictly speaking, **ray optics** is the limit of **wave optics** when the **wavelength** is infinitesimally **short**. However, the wavelength need not actually be equal to zero for the ray-optics theory to be useful. As long as the light waves propagate through and around objects whose dimensions are much greater than the wavelength, the ray theory suffices for describing most phenomena. Because the wavelength of visible light is much shorter than the dimensions of the visible objects encountered in our daily lives, manifestations of the wave nature of light are not apparent without careful observation.

ELECTROMAGNETIC OPTICS

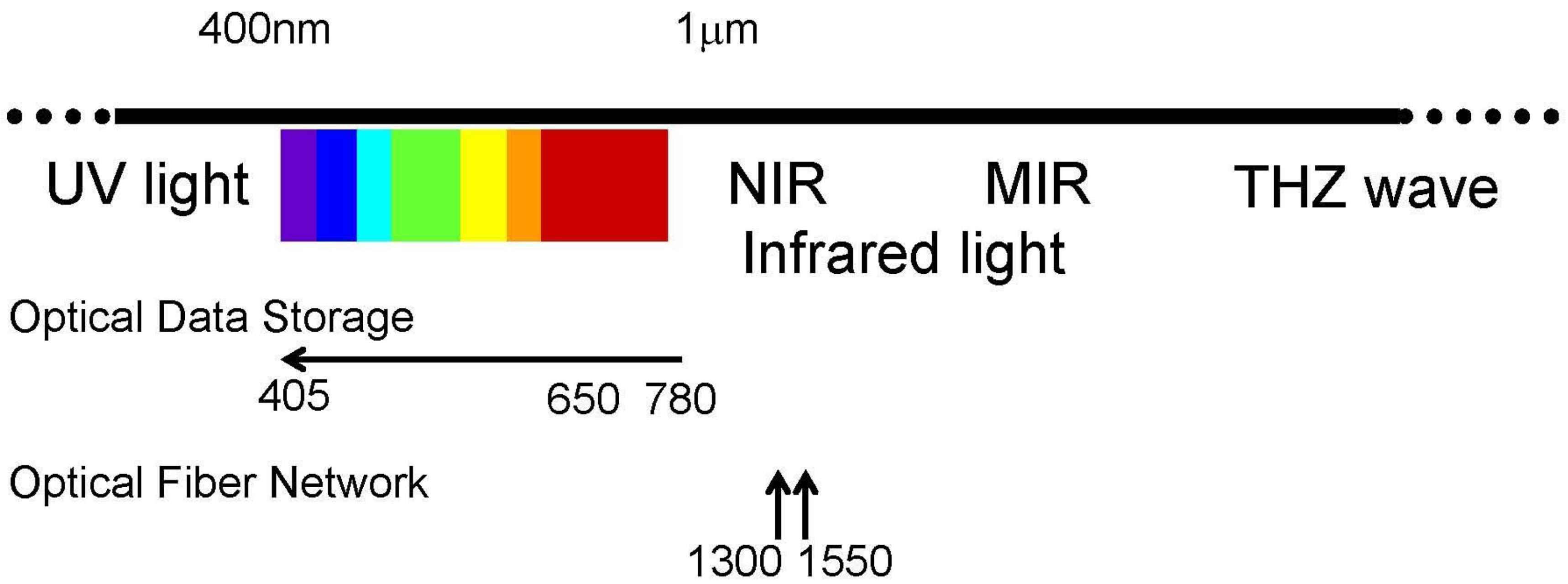
Light is an electromagnetic wave **phenomenon** described by the same theoretical principles that govern all forms of electromagnetic radiation. Optical frequencies occupy a band of the electromagnetic spectrum that extends from the **infrared** through the **visible** to the **ultraviolet**. Because the **wavelength of light** is relatively **short** (between **10 nm** and **1 mm**), the techniques used for *generating*, *transmitting*, and *detecting* optical waves have traditionally differed from those used for electromagnetic waves of longer wavelength. However, the recent miniaturization of **optical components** (e.g., **optical waveguides** and **integrated-optical devices**) has caused these differences to become less significant.

Lightwave

- Wavelength in vacuum (Frequency)



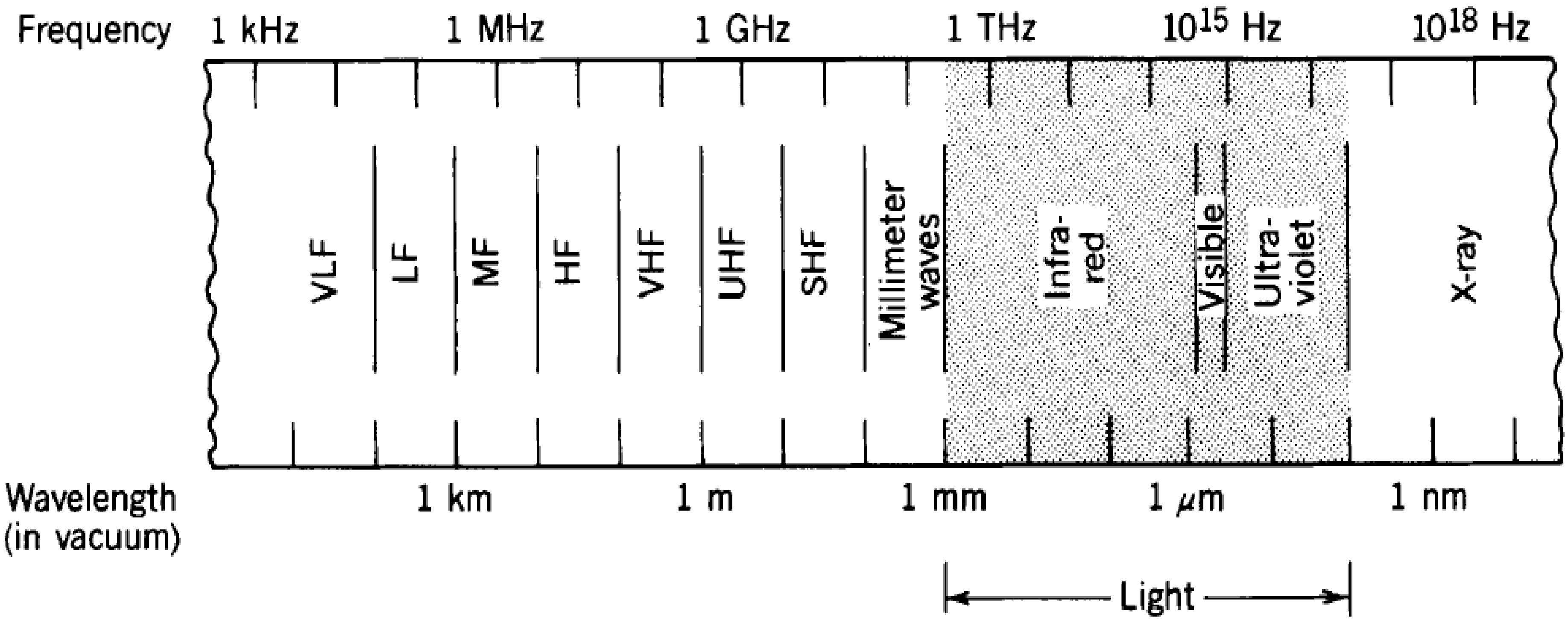
Wavelength Selection



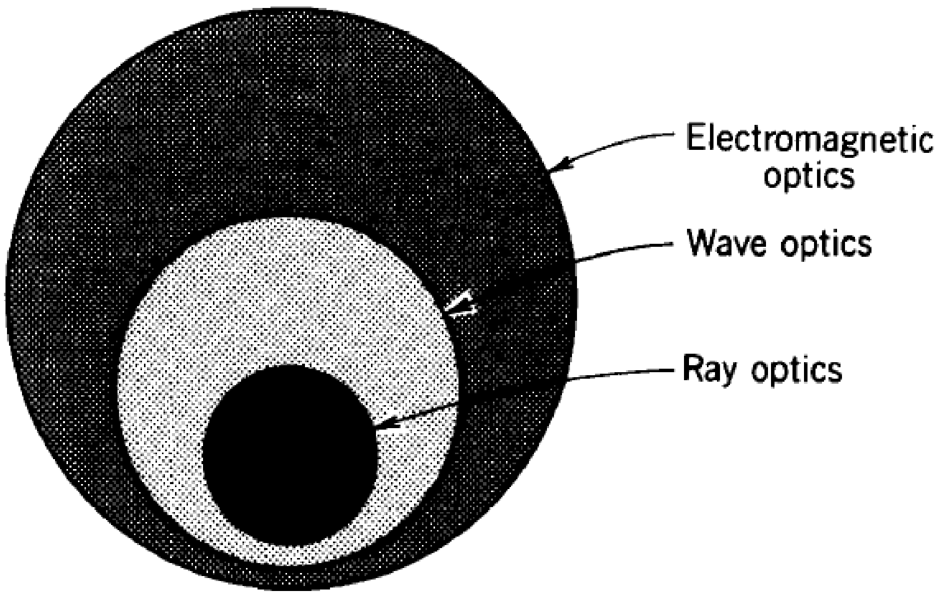
ELECTROMAGNETIC OPTICS

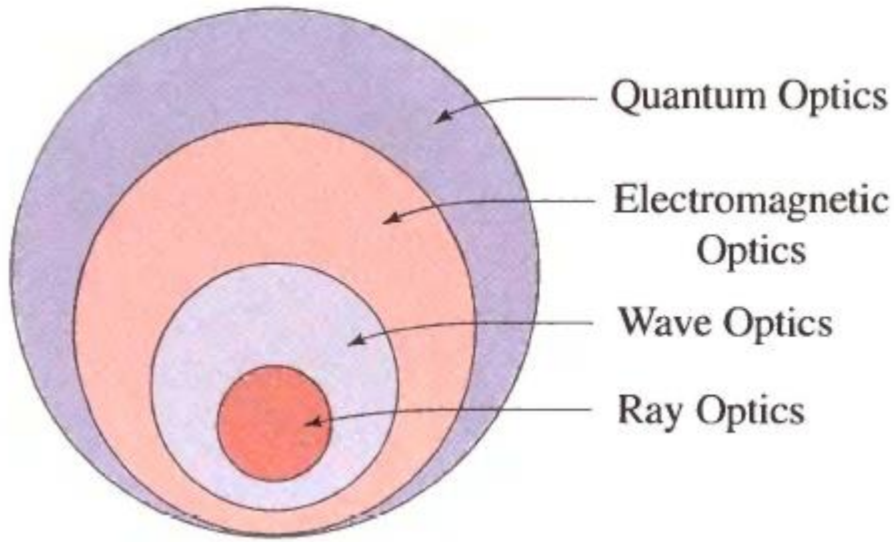
Electromagnetic radiation propagates in the form of two mutually coupled vector waves, an **electric-field** wave and a **magnetic-field** wave. The **wave optics theory** is an **approximation** of the electromagnetic theory, in which light is described by a single **scalar** function of **position** and **time** (the wavefunction). This **approximation** is adequate for paraxial waves under certain conditions. The **ray optics** approximation provides a further simplification valid in the **limit of short wavelengths**. Thus electromagnetic optics encompasses **wave optics**, which, in turn, encompasses **ray optics**.

The Electromagnetic Spectrum



ELECTROMAGNETIC OPTICS





Wave optics is the scalar approximation of electromagnetic optics.
Ray optics is the limit of wave optics when the **wavelength** is **very short**.

REVIEW OF THE ELECTROMAGNETIC THEORY OF LIGHT

1) Maxwell's equations: wave equation

Light is, according to classical theory, the flow of electromagnetic (EM) radiation through free space or through a medium in the form of **electric** and **magnetic** **fields**. Although electromagnetic radiation covers an extremely wide range, from **gamma rays** to **long radio waves**, the term "**light**" is **restricted** to the part of the electromagnetic spectrum that goes from the vacuum **ultraviolet** to the **far infrared**. This part of the spectrum is also called ***optical range***.

EM radiation propagates in the form of two mutually **perpendicular** and coupled **vectorial waves**: the **electric field** **E**(**r**, t) and the **magnetic field** **H**(**r**, t).

These **two** vectorial magnitudes depend on the **position** (**r**) and **time** (t).

Maxwell's equations in a material medium

Maxwell's equations form a set of **four coupled equations** involving the electric field vector and the magnetic field vector of the **light**, and are **based on** experimental evidence.
Two of them are scalar equations, and the other two are vectorial.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla . \mathbf{D} = \rho$$

$$\nabla . \mathbf{B} = 0$$

D(**r**,t) : ***electric displacement vector***
B(**r**,t) : ***magnetic flux density vector***

ρ(**r**,t) : ***charge density***
J(**r**,t) : ***current density vector***

If in the **medium** there are **no free electric charges**, **which is the most common situation** in **optics**, Maxwell's equations simplify in the form: $\nabla . \mathbf{D} = 0$

Maxwell's equations in a material medium

These relations are called *constitutive relations*, and **depend on** the **electric** and **magnetic properties** of the considered **medium**.

For a linear, **homogeneous** and **isotropic** medium, the *constitutive relations* are given by:

$$\mathbf{D} = \varepsilon \mathbf{E} \quad , \quad \mathbf{B} = \mu \mathbf{H} \quad , \quad \mathbf{J} = \sigma \mathbf{E}$$

ε is the **dielectric permittivity**, ***μ*** is the **magnetic permeability** and ***σ*** is the **conductivity** of the medium.

- A **homogeneous medium** implies that the **optical constants** of the medium ***ε***, ***μ*** and ***σ*** are not dependent of the position vector **r**.

- In an **isotropic medium** these optical constants are scalar magnitudes and **independent** of the **direction** of the vectors **E** and **H**, implying that the vectors **D** and **J** are **parallel** to the electric field **E**, and the vector **B** is **parallel** to the magnetic field **H**.

Maxwell's equations in a material medium

➢ In an **anisotropic** medium the **optical constants** must be **treated** as **tensorial magnitudes**, and the before mentioned **parallelism** is no longer **valid** in general.

➢ By using the constitutive relations for a linear, homogeneous and isotropic medium, Maxwell's equations can be written in terms of the electric field **E** and magnetic field **H** only

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla . \mathbf{E} = 0$$

$$\nabla . \mathbf{H} = 0$$

Maxwell's equations in a material medium

- By combining adequately these **four differential equations**, it is possible to obtain **two** differential equations in partial derivatives, one for the **electric field** and another for the **magnetic field**.

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{H} = \mu\sigma \frac{\partial \mathbf{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

- These two differential equations are known as ***wave equations*** for a **material medium**.

- The **solution of both equations** are not independent, because the **electric** and **magnetic fields** are related through Maxwell's equations.

Wave equation in dielectric media

- A **perfect** dielectric medium is defined as a material in which the conductivity is $\sigma = 0$.

- In this category fall most of the **substrate materials** used for **integrated optical devices**, such as **glasses**, **ferro-electric crystals** or **polymers**, while **metals** do not belong to this category because of their high conductivity.

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{H} = \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

- Each of these two **vectorial wave equations** can be separated on **three scalar wave equations**, expressed as:

$$\nabla^2 \xi = \mu\varepsilon \frac{\partial^2 \xi}{\partial t^2}$$

Wave equation in dielectric media

- The scalar variable $\xi(\mathbf{r}, t)$ may represent each of the six Cartesian components of either the electric and magnetic fields.

- The solution of this equation represents a wave that propagates with a speed *v* (*phase velocity*) given by:

$$\upsilon = \frac{1}{\sqrt{\mu\varepsilon}}$$

- For propagation in free space, and using the values for ε_0 and μ_0 we obtain:

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \approx 3.00\times10^{8}\,\mathrm{ms^{-1}}$$

- which corresponds to the **speed of light** in **free space** measured experimentally.

- The **speed of light** has been obtained **only** using values of **electric** and **magnetic** constants.

Wave equation in dielectric media

- The **propagation speed** of the electromagnetic waves in a **medium** v as function of the speed of light in **free space** c,

$$v \equiv \frac{c}{n}$$

- **n** represents the ***refractive index*** of the dielectric medium.

- The *refractive index* is related with the **optical constant** of the material medium and the dielectric permittivity and the magnetic permeability of the **free space** by:

$$n \equiv \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

- In most of the materials (non-magnetic materials), and in particular in **dielectric media**, the magnetic **permeability** is very close to that of free space: $\mu \approx \mu_0$.

$$n \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_r}$$

Wave equation in dielectric media

- ε_r : relative dielectric permittivity (dielectric constant), defined as the relation between the dielectric permittivity of the material medium and that of the free space.

Material	Refractive index	Wavelength (nm)
Glass (BK7)	1.51	633
Glass (ZBLAN)	1.50	633
Polymer (PMMA)	1.54	633
Silica (amorphous SiO_2)	1.45	633
Quartz (SiO_2)	1.55	633
Silicon nitride (Si_3N_4)	2.10	633
Calcium fluoride (CaF_2)	1.43	633
Lithium niobate ($LiNbO_3$)	2.28 (n_o) 2.20 (n_e)	633
Silicon (Si)	3.75	1300
Gallium arsenide (GaAs)	3.4	1000
Indium phosphide (InP)	3.17	1510

Refractive indices corresponding to materials commonly used in the fabrication of integrated photonic components

Wave equation in dielectric media

- The electromagnetic waves **transport** energy, and the **flux** of energy carried by the EM wave **is given** by the *Poynting vector* S, defined as:

$$S = \mathbf{E} \times \mathbf{H}$$

- On the other hand, the **intensity** (or irradiance) I of the EM wave, defined as **the amount of energy passing** through the **unit area** in the **unit of time**, is given by the **time average** of the **Poynting vector modulus**:

$$I = \langle |S| \rangle$$

- The electric and magnetic fields associated with the EM wave oscillate at very high frequency, and the apparatus used to detect that intensity (light detectors) cannot follow the instant values of the Poynting vector modulus.

Monochromatic waves

- The **time dependence** of the **electric** and **magnetic fields** within the wave equations admits solutions of the form of **harmonic functions**. **Electromagnetic waves** with such sinusoidal **dependence on** the time variable are called *monochromatic waves*, and are characterised by their **angular frequency** ω. In a general form, the electric and magnetic fields associated with a monochromatic wave can be expressed as:

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0(\mathbf{r})\cos\left[\omega t+\varphi(\mathbf{r})\right]$$

$$\mathbf{H}(\mathbf{r},t)=\mathbf{H}_0(\mathbf{r})\cos\left[\omega t+\varphi(\mathbf{r})\right]$$

- where the fields amplitudes $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{H}_0(\mathbf{r})$ and the initial phase φ(r) depend on the position r, but the **time dependence** is carried out only in the **cosine** argument through ωt.

Complex Notation of Monochromatic Waves

$$\mathbf{E}(\mathbf{r},t)=\mathrm{Re}\left[\mathbf{E}(\mathbf{r})e^{+i\omega t}\right]$$

$$\mathbf{H}(\mathbf{r},t)=\mathrm{Re}\left[\mathbf{H}(\mathbf{r})e^{+i\omega t}\right]$$

✓ **E(r)** and **H(r)** denote the *complex amplitudes* of the electric and magnetic fields, respectively.

✓The electromagnetic spectrum covered by **light** (optical spectrum) ranges from frequencies of 3 × 10^5 Hz corresponding to the far IR, to 6 × 10^{15} Hz corresponding to vacuum UV, being the frequency of visible light around 5 × 10^{14} Hz.

✓The **average** of the Poynting vector as a function of the *complex fields amplitudes* for **monochromatic waves**

$$\langle S\rangle=\left\langle \mathrm{Re}\left[\mathbf{E}e^{+i\omega t}\right]\times\mathrm{Re}\left[\mathbf{H}e^{+i\omega t}\right]\right\rangle=\mathrm{Re}\{\mathbf{S}\}$$

Complex Notation of Monochromatic Waves

- **S** has been defined as: $$\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}^*$$
- **S** is called the *complex Poynting vector*.
- The **intensity** carried by a **monochromatic EM** wave should be expressed as: $$I = \left|\mathrm{Re}\{\mathbf{S}\}\right|$$
- In the case of monochromatic waves, Maxwell's equations using the complex fields amplitudes **E** and **H** are simplified notably (a dielectric and non-magnetic medium, $\boldsymbol{\sigma = 0}$ and $\boldsymbol{\mu = \mu_0}$)

$$\nabla\times\mathbf{E} = -i\mu_0\omega\mathbf{H}$$

$$\nabla\times\mathbf{H} = i\varepsilon\omega\mathbf{E}$$

$$\nabla.\mathbf{E} = 0$$

$$\nabla.\mathbf{H} = 0$$

Complex Notation of Monochromatic Waves

✓ Now, if we **substitute** the **solutions** on the form of **monochromatic waves** in the **wave equation,** we obtain a new wave equation, **valid only** for **monochromatic waves**, known as the ***Helmholtz equation***:

$$\nabla^2\mathbf{E}(\mathbf{r})+k^2\mathbf{E}(\mathbf{r})=0 \qquad k\equiv\omega\left(\varepsilon\mu_0\right)^{1/2}=nk_0$$

$$\nabla^2\mathbf{H}(\mathbf{r})+k^2\mathbf{H}(\mathbf{r})=0 \qquad k_0\equiv\omega/c$$

✓ If the material medium is **inhomogeneous** the dielectric permittivity is **no longer** constant, but position dependent $\varepsilon=\varepsilon(\mathbf{r})$. The Helmholtz equations are **not longer** valid.

✓For a locally homogeneous medium, in which $\varepsilon(\mathbf{r})$ varies slowly for distances of $\sim 1/k$, those wave equations are **approximately** valid by now defining $k=n(\mathbf{r})k_0$, and $n(\mathbf{r})=[\varepsilon(\mathbf{r})/\varepsilon_0]^{1/2}$.

Monochromatic plane waves in dielectric media

- Consider the spatial dependence of the **electromagnetic fields**, For **monochromatic waves**, the solution for the spatial dependence, carried by the complex amplitudes **E(r)** and **H(r)**, can be obtained by solving the **Helmholtz equation** $\Longrightarrow$

$$\nabla^2\mathbf{E}(\mathbf{r})+k^2\mathbf{E}(\mathbf{r})=0$$

$$\nabla^2\mathbf{H}(\mathbf{r})+k^2\mathbf{H}(\mathbf{r})=0$$

- ***plane wave:*** One of the **easiest** and most **intuitive solutions** for the **Helmholtz equation** also the most frequently used in **optics.**

- The plane wave is characterised by its *wave vector* **k**, and the mathematical expressions for the **complex amplitudes** are:

$$\mathbf{E}(\mathbf{r})=\mathbf{E}_0 e^{-i\mathbf{kr}} \quad , \quad \mathbf{H}(\mathbf{r})=\mathbf{H}_0 e^{-i\mathbf{kr}}$$

- The magnitudes $\mathbf{E_0}$ and $\mathbf{H_0}$ are now constant vectors

Monochromatic plane waves in dielectric media

- ✓ Each of the Cartesian components of the complex amplitudes **E(r)** and **H(r)** will satisfy the **Helmholtz equation.**

- ✓ The modulus of the wave vector **k** is: $k = nk_0 = \left(\omega / c \right) n$

- ✓ ω is the angular frequency of the EM plane wave and n is the **refractive index** of the medium where the **wave propagates**.

$$\begin{cases} \mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{-i\mathbf{kr}} \\ \mathbf{H}(\mathbf{r}) = \mathbf{H}_0 e^{-i\mathbf{kr}} \end{cases} \Rightarrow \begin{cases} \mathbf{k} \times \mathbf{E}_0 = \omega \mu_0 \mathbf{H}_0 \\ \mathbf{k} \times \mathbf{H}_0 = -\omega \varepsilon \mathbf{E}_0 \end{cases}$$

$$\begin{cases} \nabla \times \mathbf{E} = -i \mu_0 \omega \mathbf{H} \\ \nabla \times \mathbf{H} = i \varepsilon \omega \mathbf{E} \end{cases}$$

- ✓ These **two formulae**, **valid only** for **plane monochromatic waves**

Monochromatic plane waves in dielectric media

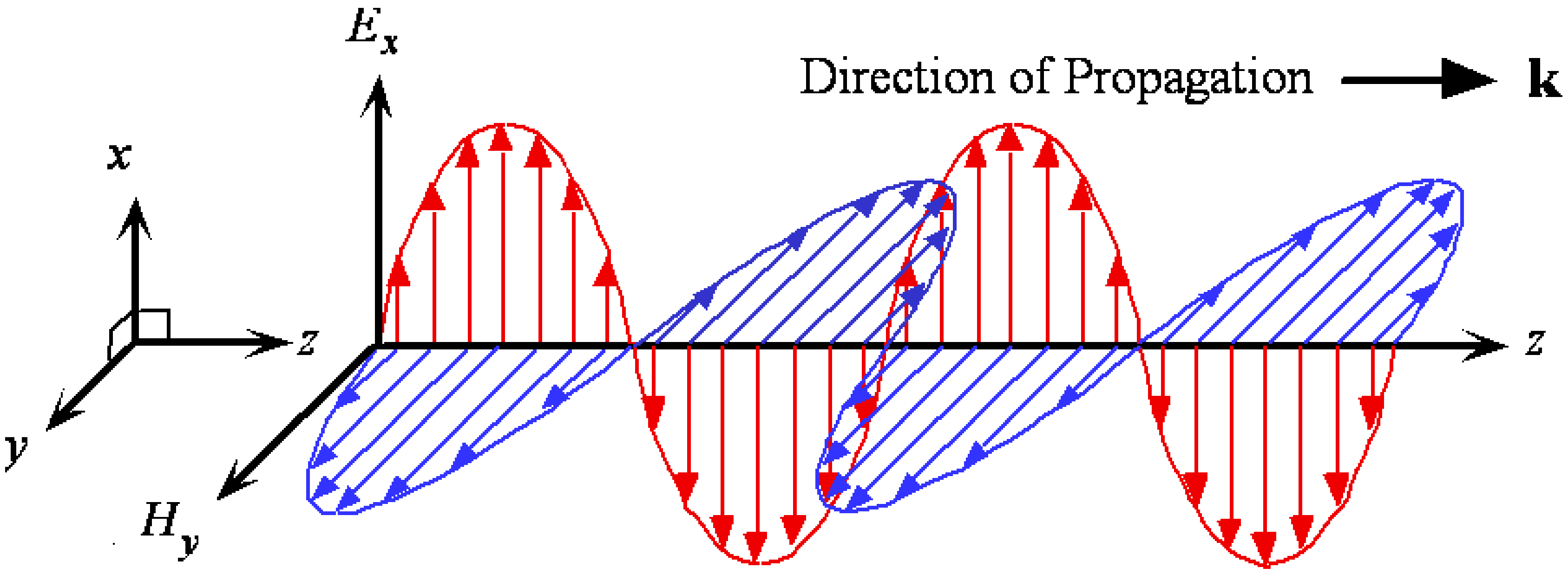
$$\mathbf{k}\times\mathbf{H}_0 = -\omega\varepsilon\mathbf{E}_0 \quad , \quad \mathbf{k}\times\mathbf{E}_0 = \omega\mu_0\mathbf{H}_0$$

- ✓ The **electric field** is **perpendicular** to the **magnetic field** and the wave vector **k**.
- ✓ The **magnetic field** is **perpendicular** to the **electric field** and the wave vector **k**.

- ✓ Therefore, one can conclude that **k**, **E** and **H** are **mutually orthogonal**, and because **E** and **H** *lie* on a **plane normal** to the propagation direction defined by **k**, such **wave** in called a ***transverse EM wave*** (**TEM**).
- ✓ The fact that these three vectors are perpendicular implies

$$\mathbf{H}_0 = \left(\omega\varepsilon/k\right)\mathbf{E}_0 \quad , \quad \left(k/\omega\mu_0\right)\mathbf{E}_0 = \mathbf{H}_0 \Rightarrow \quad k^2 = \omega^2\varepsilon\mu_0$$

The wave nature of light



An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and the direction of propagation, z.

Monochromatic plane waves in dielectric media

- When dealing with a monochromatic plane EM wave it is useful to characterise it by its radiation *wavelength* λ, defined as the distance between the two nearest points with equal phase of vibration, measured along the propagation direction. The wavelength is therefore expressed by:

$$\lambda = vT = v/\upsilon = \frac{2\pi}{k} = \frac{2\pi}{nk_0} = \frac{\lambda_0}{n}$$

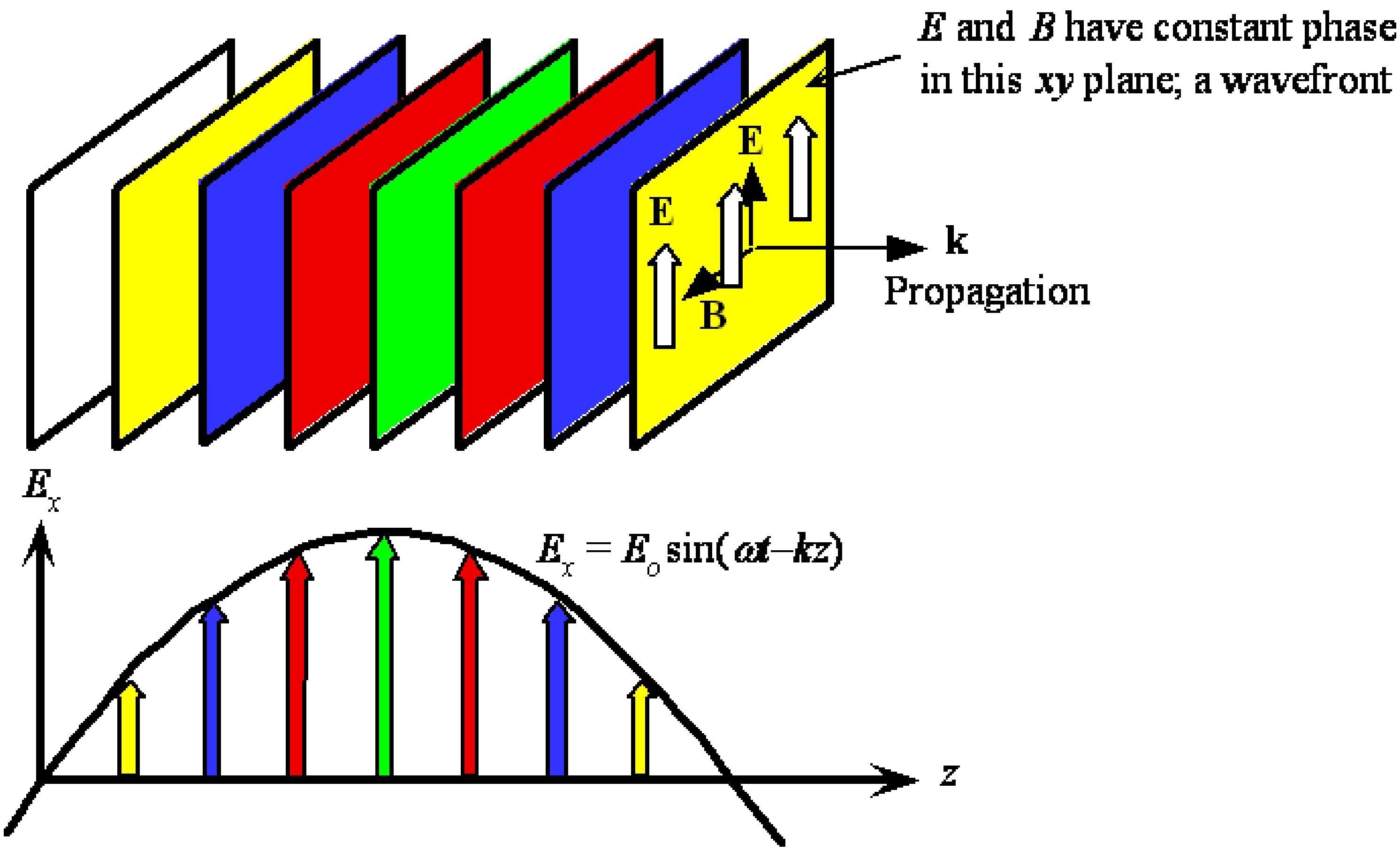
- λ_0 represents the wavelength of the EM wave in free space, given by:

$$\lambda_0 = cT = c/\upsilon = \frac{2\pi}{k_0}$$

Monochromatic plane waves in dielectric media

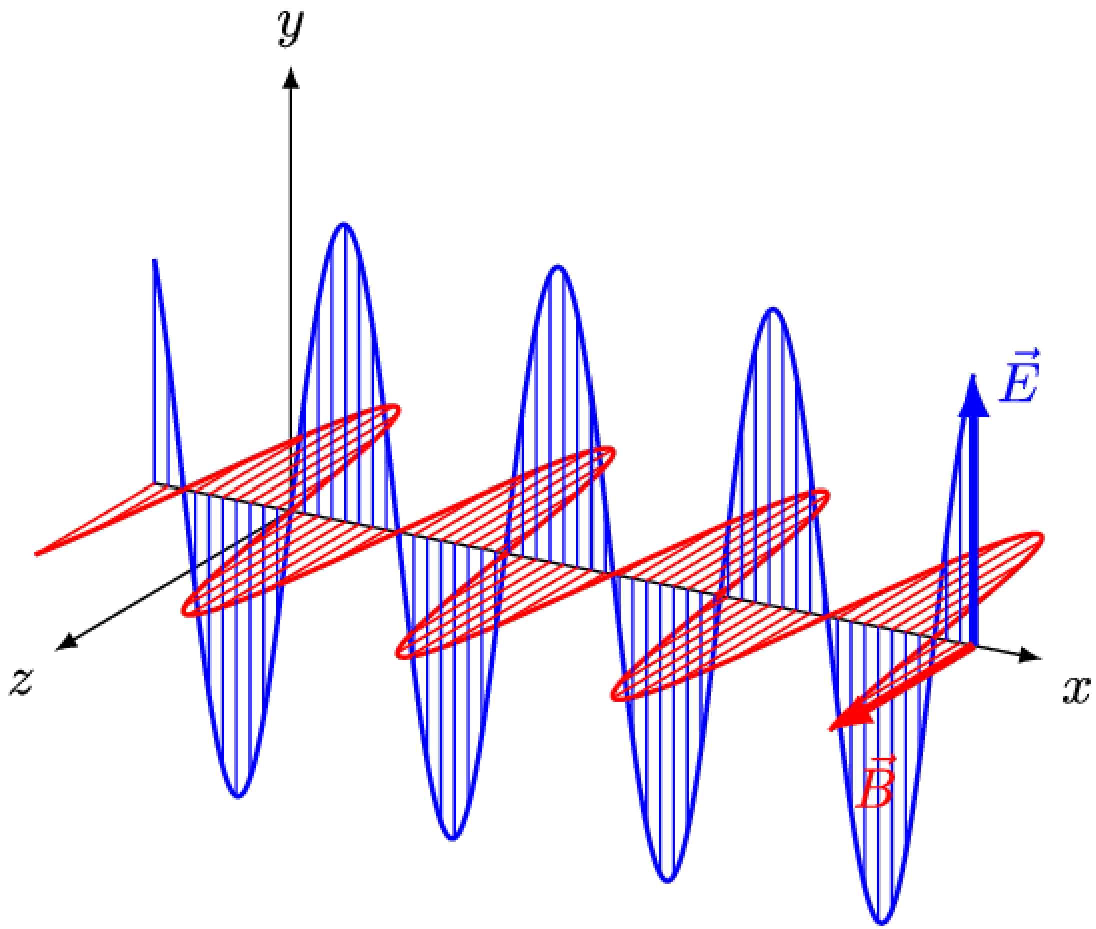
- ✓ It is worth remarking that when an **EM wave** passes from one medium to **another** its frequency remains **unchanged,** but as its phase velocity is **modified** due to its **dependence** on the **refractive index**, the **wavelength** associated with the EM wave should also **change**. Therefore, when the wavelength of an EM wave is given, it is usually referred to the wavelength of that radiation propagating through free space.

The wave nature of light



A plane EM wave travelling along z, has the same E_x (or B_y) at any point in a given xy plane. All electric field vectors in a given xy plane are therefore in phase. The xy planes are of infinite extent in the x and y directions.

The wave nature of light



Polarisation of electromagnetic waves

- A property associated with a transversal wave is its polarisation character, related to the closed curve described by the tip of the electric (or magnetic) field vector at a fixed point $\mathbf{r} = \mathbf{r_0}$ in the space.

- In order to analyse the polarisation character of an EM plane wave, let us assume, without loss of generality, that the EM wave propagates along the ***z*-axis**.

$$\mathbf{k} = k\mathbf{u}_z$$

- where we will use $\mathbf{u_x}$, $\mathbf{u_y}$ and $\mathbf{u_z}$ as the unitary vectors along the *x*, *y* and *z*-axis respectively.

- The simplest situation of an EM wave in which its associated **electric field** is along the *x*-axis

$$\mathbf{E} = E_0 \cos\left(\omega t - kz\right)\mathbf{u}_x$$

Polarisation of electromagnetic waves

- The magnetic field of this EM wave $\mathbf{H} = H_0 \cos(\omega t - kz)\mathbf{u}_y$

- The amplitude H_0 is related to the amplitude E_0 by:

$$H_0 = \left(k / \omega\mu_0 \right) E_0 = \left(\varepsilon / \mu_0 \right)^{1/2} E_0$$

- Note that the electric and magnetic fields are in phase, that is, if at a fixed time and at a particular plane $z = z_0$ (z being arbitrary) the electric field **E** reaches its maximum value, the magnetic field **H** will also be at its maximum value.

The *wave described* by above equations is said to be **linearly** polarised (or more specifically, linearly x-polarised) because the electric field vector **E** (or **H**) is always along a particular direction (x direction in this case)

Polarisation of electromagnetic waves

- Consider now a linearly y-polarised wave, with an addition phase of +π/2 described by:

$$\mathbf{E} = E_0 \cos\left(\omega t - kz + \frac{\pi}{2}\right)\mathbf{u}_y = -E_0 \sin(\omega t - kz)\mathbf{u}_y$$

$$\mathbf{H} = -H_0 \cos\left(\omega t - kz + \frac{\pi}{2}\right)\mathbf{u}_x = H_0 \sin(\omega t - kz)\mathbf{u}_x$$

$$H_0 = \left(k/\omega\mu_0\right)E_0 = \left(\varepsilon/\mu_0\right)^{1/2} E_0$$

- Because Maxwell's equations **are linear**, a **linear combination** of several solutions will also be a solution.

$$\mathbf{E} = E_0\left[\cos(\omega t - kz)\mathbf{u}_x - \sin(\omega t - kz)\mathbf{u}_y\right]$$

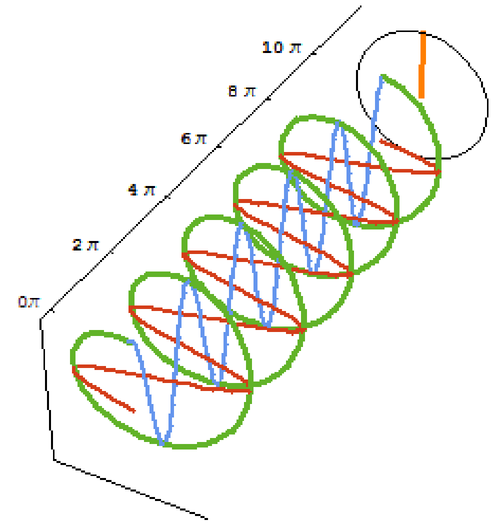
$$\mathbf{H} = H_0\left[\cos(\omega t - kz)\mathbf{u}_y + \sin(\omega t - kz)\mathbf{u}_x\right]$$

Polarisation of electromagnetic waves

- study the curve described by the tip of the electric field vector at a fixed plane, for instance, the plane defined by z = 0. At this position, the time dependence of the fields is:

$$\mathbf{E}_x = E_0 \cos(\omega t) \quad \text{and} \quad \mathbf{E}_y = -E_0 \sin(\omega t)$$

$$\mathbf{H}_x = H_0 \sin(\omega t) \quad \text{and} \quad \mathbf{H}_y = H_0 \cos(\omega t)$$



- The modulus of the electric and magnetic field vector is therefore:

$$\mathbf{E}^2 = \mathbf{E}_x^2 + \mathbf{E}_y^2 = E_0^2$$

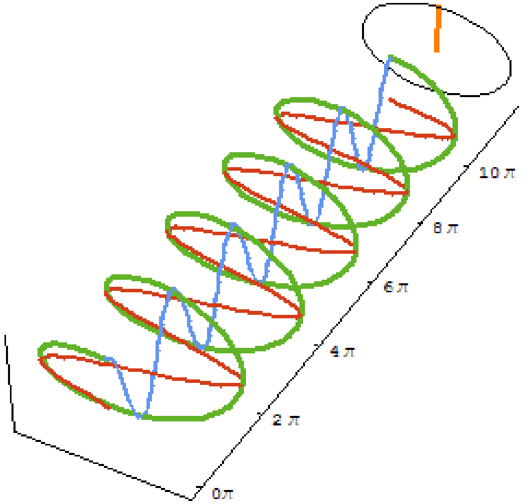
$$\mathbf{H}^2 = \mathbf{H}_x^2 + \mathbf{H}_y^2 = H_0^2$$

- which indicates that, at a fixed plane, the tip of the electric field vector (and the magnetic field vector) describe a *circle* (*circularly polarised*).

Polarisation of electromagnetic waves

- On a general form, if two linearly polarised waves, mutually perpendicular, are superposed, having the same propagation direction and frequency, but with different amplitudes and relative phases, at a generic plane (for instance, at z = 0), we will have

$$\mathbf{E}_x = E_{01}\cos(\omega t - \theta_1) \qquad \text{and} \qquad \mathbf{E}_y = -E_{02}\cos(\omega t - \theta_2)$$

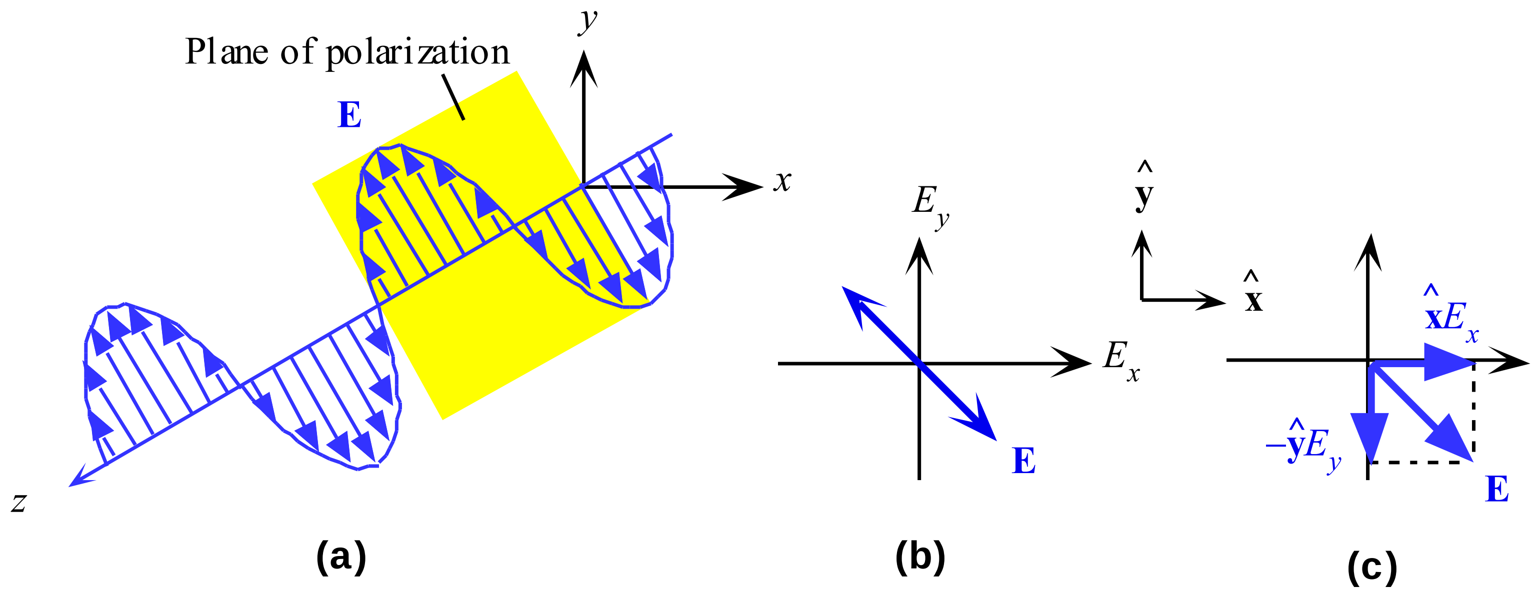


- For such a wave, the relation between the Cartesian components of the electric field is

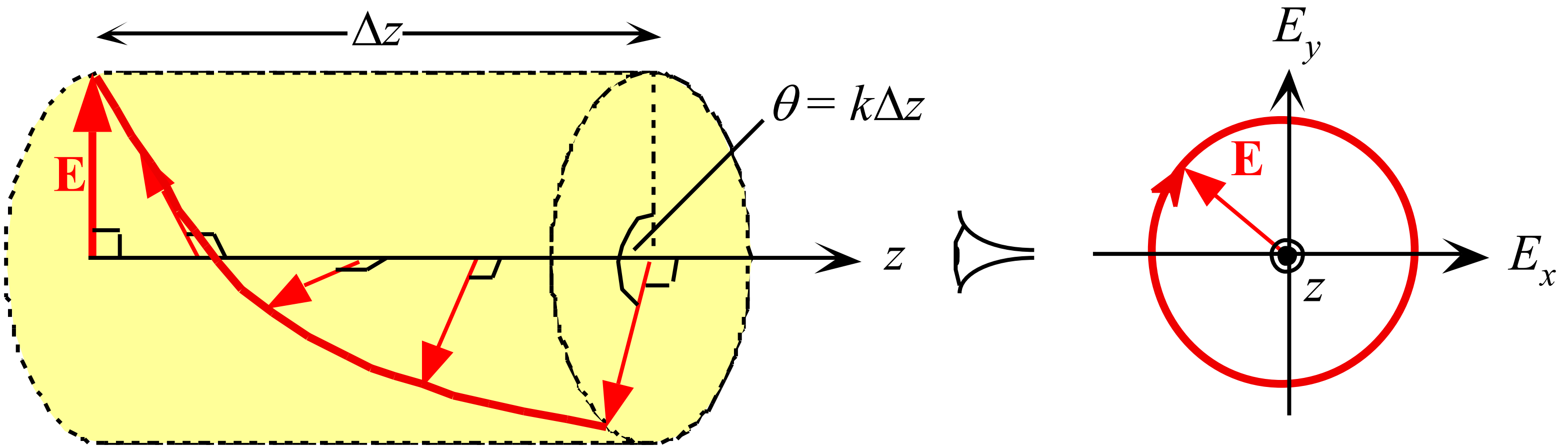
$$\left(\mathbf{E}_x / E_{01}\right)^2 + \left(\mathbf{E}_y / E_{02}\right)^2 - 2\left(\mathbf{E}_x / E_{01}\right)\left(\mathbf{E}_y / E_{02}\right)\cos(\theta) = \sin^2\theta$$

$$\theta = \theta_2 - \theta_1$$

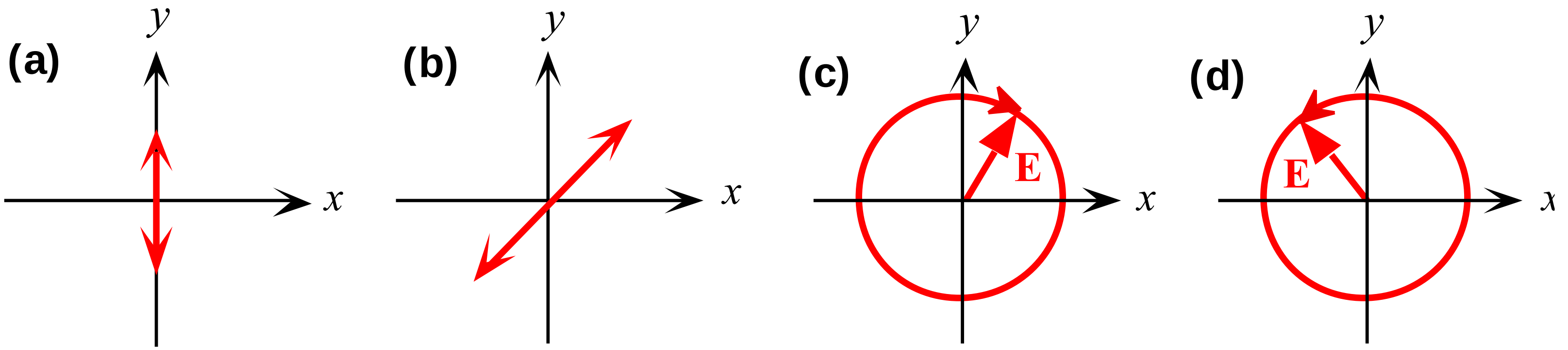
- $\theta = (\theta_2 - \theta_1)$: the relative phase between $\mathbf{E}_\mathbf{x}$ and $\mathbf{E}_\mathbf{y}$
- This equation represents an *ellipse*, being the curve drawn by the electric field (*elliptically polarised wave*)



(a) A linearly polarized wave has its electric field oscillations defined along a line perpendicular to the direction of propagation, z. The field vector **E** and z define a *plane of polarization*. (b) The E-field oscillations are contained in the plane of polarization. (c) A linearly polarized light at any instant can be represented by the superposition of two fields E_x and E_y with the right magnitude and phase.



A *right circularly polarized light*. The field vector **E** is always at right angles to z , rotates clockwise around z with time, and traces out a full circle over one wavelength of distance propagated.



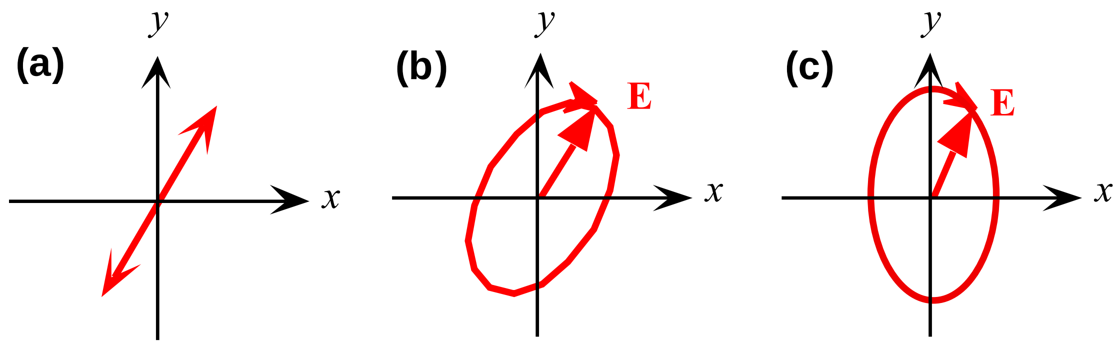
(a) $E_{xo} = 0$, $E_{yo} = 1$, $\phi = 0$

(b) $E_{xo} = 1$, $E_{yo} = 1$, $\phi = 0$

(c) $E_{xo} = 1$, $E_{yo} = 1$, $\phi = \pi/2$

(d) $E_{xo} = 1$, $E_{yo} = 1$, $\phi = -\pi/2$

Examples of linearly, (a) and (b), and circularly polarized light (c) and (d); (c) is right circularly and (d) is left circularly polarized light (as seen when the wave directly approaches a viewer)

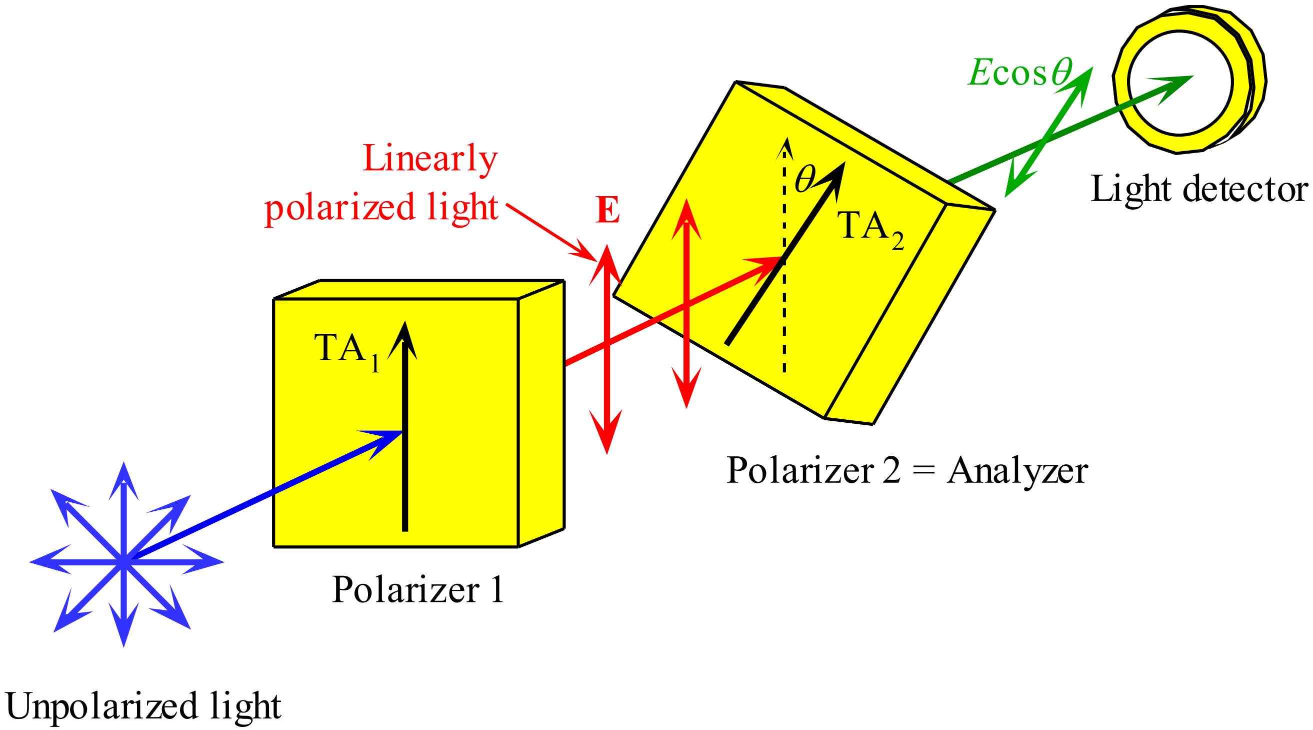


$E_{xo} = 1$
$E_{yo} = 2$
$\phi = 0$

$E_{xo} = 1$
$E_{yo} = 2$
$\phi = \pi/4$

$E_{xo} = 1$
$E_{yo} = 2$
$\phi = \pi/2$

(a) Linearly polarized light with $E_{yo} = 2E_{xo}$ and $\phi = 0$. (b) When $\phi = \pi/4$ (45°), the light is right elliptically polarized with a tilted major axis. (c) When $\phi = \pi/2$ (90°), the light is right elliptically polarized. If E_{xo} and E_{yo} were equal, this would be right circularly polarized light.



Randomly polarized light is incident on a Polarizer 1 with a transmission axis TA_1. Light emerging from Polarizer 1 is linearly polarized with **E** along TA_1, and becomes incident on Polarizer 2 (called "analyzer") with a transmission axis TA_2 at an angle θ to TA_1. A detector measures the intensity of the incident light. TA_1 and TA_2 are normal to the light direction.

WAVE EQUATIONS

✓Let us assume that an electromagnetic field oscillates at a single angular frequency ω (• (in radians per meter). Vector **A**, which designates an **electromagnetic field**, is exp

$$\mathbf{A}(\mathbf{r}, t) = \mathrm{Re}\{\bar{\mathbf{A}}(\mathbf{r}) \exp(j\omega t)\}.$$

✓we can write the following phaser expressions

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\{\bar{\mathbf{E}}(\mathbf{r}) \exp(j\omega t)\},$$

$$\mathbf{H}(\mathbf{r}, t) = \mathrm{Re}\{\bar{\mathbf{H}}(\mathbf{r}) \exp(j\omega t)\},$$

$$\mathbf{D}(\mathbf{r}, t) = \mathrm{Re}\{\bar{\mathbf{D}}(\mathbf{r}) \exp(j\omega t)\},$$

$$\mathbf{B}(\mathbf{r}, t) = \mathrm{Re}\{\bar{\mathbf{B}}(\mathbf{r}) \exp(j\omega t)\}.$$

✓For simplicity we denote $\bar{\mathbf{E}}$, … in the phaser representation as **E**, **H**, **D**, and **B**.

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} = -j\omega \mu_0 \mathbf{H},$$

$$\nabla \times \mathbf{H} = j\omega \mathbf{D} = j\omega \varepsilon \mathbf{E},$$

$$\nabla \cdot \mathbf{H} = 0,$$

$$\nabla \cdot (\varepsilon_r \mathbf{E}) = 0,$$

✓where it is assumed that $\mu_r = 1$ and $\rho = 0$.

Wave Equation for Electric Field E

✓ Applying a vectorial rotation operator $\nabla \times$

$$\nabla \times (\nabla \times \mathbf{E}) = -j\omega\mu_0 \nabla \times \mathbf{H}.$$

✓ Using the vectorial formula $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A},$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}.$$

✓ The symbol ∇^2 is a Laplacian defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

✓ Since $\nabla \cdot (\varepsilon_r \mathbf{E}) = 0,$

$$\nabla \cdot (\varepsilon_r \mathbf{E}) = \nabla \varepsilon_r \cdot \mathbf{E} + \varepsilon_r \nabla \cdot \mathbf{E} = 0,$$

✓ We obtain

$$\nabla \cdot \mathbf{E} = -\frac{\nabla \varepsilon_r}{\varepsilon_r} \cdot \mathbf{E}.$$

Wave Equation for Electric Field E

$$\nabla\times(\nabla\times\mathbf{E}) = \nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E} = -\nabla\left(\frac{\nabla\varepsilon_r}{\varepsilon_r}\cdot\mathbf{E}\right) - \nabla^2\mathbf{E}.$$

✓ Thus, for a medium with the relative permittivity ε_r. The vectorial wave equation for the electric field **E** is

$$\nabla^2\mathbf{E} + \nabla\left(\frac{\nabla\varepsilon_r}{\varepsilon_r}\cdot\mathbf{E}\right) + k_0^2\varepsilon_r\mathbf{E} = \mathbf{0}.$$

✓where k_0 is the wave number in a vacuum and is expressed as

$$k_0 = \omega\sqrt{\varepsilon_0\mu_0} = \frac{\omega}{c_0}.$$

Wave Equation for Electric Field E

✓ Only **two components** of the **electric field** are required. If the **transverse components** e_x and e_y are known, the **longitudinal** component may be calculated by: $\nabla \cdot (\varepsilon_r \mathbf{E}) = 0,$

✓For the case that refractive index profile of an optical waveguide does not vary along the propagation direction. Therefore, it makes sense to separate the **electric field** into **transverse** and **longitudinal components**, and assume z-dependence of $e^{-j\beta z}$.

$$\mathbf{E}(x, y, z) = \left(\mathbf{E_t} + \hat{\mathbf{z}} E_z\right) e^{-j\beta z} \quad , \quad \mathbf{E_t}(x,y) = \hat{x} E_x + \hat{y} E_y$$

✓where $\beta = k_0 n_{eff}$ is the **propagation constant** and n_{eff} is the **effective index**. The full-vector wave equation can be written in terms of the transverse components,

$$\nabla^2 \mathbf{E_t} + \nabla \left(\frac{\nabla(\varepsilon_r)}{\varepsilon_r} . \mathbf{E_t} \right) + k_0^2 \varepsilon_r \mathbf{E_t} = \beta^2 \mathbf{E_t}$$

Wave Equation for Electric Field E

✓ The longitudinal component E_z can be computed from $\mathbf{E_t}$ using the divergence relation:

$$j\beta E_Z = \nabla . \mathbf{E_t} + \frac{1}{\varepsilon_r} \nabla \left(\varepsilon_r \right) . \mathbf{E_t}$$

✓When the relative permittivity $\boldsymbol{\varepsilon_r}$ is constant in the medium, this **vectorial wave equation** can be reduced to the **Helmholtz equation**

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = \mathbf{0}.$$

```
 L = 2*Lb+Lw;     % [A]
  Nx =npts;
  xvec = linspace(-L/2,L/2,Nx);
  dx=xvec(2)-xvec(1);
V = zeros(size(xvec));
 index_b1=find(xvec<-Lw/2);
index_b2=find(xvec>Lw/2);
index_w=find(xvec>=-Lw/2 & xvec<=Lw/2);
 %-------------------------------------------------------
t0=[(h^2)/(2*m*((dx*1e-10)^2))/e]*ones(size(xvec));    % h equal h_bar
%------------------------------------------------------------------------------
% define a potential energy profile.....
V(index_b1) = V0;
  V(index_w) =  0;
  V(index_b2) = V0;
   cdiag = 2*t0.*+V;
  udiag = -t0(1:end-1);
  ldiag = -t0(2:end);
H = diag(cdiag) + diag(udiag,1) + diag(ldiag ,-1);
[PSI,D]=eig(H);
% These are the energies
% E = diag(E);
[E, Ind]=sort(diag(D));
```

Light propagation in absorbing media

❖ An **absorbing medium** is characterised by the fact that **the energy** of the **EM radiation** is **dissipated** in it.

❖ This would **imply** that the **amplitude** of a **plane EM** wave **decreases** *exponentially* as the **wave propagates** along the absorbing medium.

❖ The **dielectric permittivity** is **no longer** a **real number**, but a **complex quantity** ε_c.

❖ The electric field by $\mathbf{D} = \varepsilon_c \mathbf{E}$, **will not** be in phase with the electric field in general.

❖ ***complex refractive index :***

$$n_c = \sqrt{\frac{\varepsilon_c}{\varepsilon_0}} = n - i\kappa$$

❖ ***n*** is the *real refractive index*, and κ is called the *absorption index*.

Light propagation in absorbing media

❖ The *complex wavevector :*

$$\left.\begin{aligned}\mathbf{k}_c^2 &\equiv \omega^2\varepsilon_c\mu = n_c^2 k_0^2\\ \mathbf{k}_c &\equiv \mathbf{k} - i\mathbf{a}\end{aligned}\right\} \Rightarrow \begin{cases}\mathbf{k}^2 - \mathbf{a}^2 = k_0^2\left(n^2 - \kappa^2\right)\\ \mathbf{ka} = k_0^2 n\kappa\end{cases}$$

❖**k** represents the **real wavevector**, and **a** is called the **attenuation** vector.

❖ The **electric field** for a **plane monochromatic wave** in **absorbing** medium

$$\mathbf{E}\left(\mathbf{r},t\right) = \mathrm{Re}\left[\mathbf{E}_0 e^{i\left(\omega t - \mathbf{k}_c\mathbf{r}\right)}\right] = \mathrm{Re}\left[\mathbf{E}_0 e^{-\mathbf{ar}} e^{i\left(\omega t - \mathbf{kr}\right)}\right]$$

❖The planes of constant amplitude will be determined by the condition **ar** = constant. and therefore they will be planes **perpendicular** to the attenuation vector **a**.

❖ The planes of equal phase will be defined by the condition of **kr** = constant, and thus the phase front will be **planes perpendicular** to the real wavevector **k.**

Light propagation in absorbing media

❖ In general, these two planes will not be coincident, and in this case the EM wave is said to be an *inhomogeneous wave*.

❖ In absorbing media, the vectors **k** and **a** are parallel, and such a wave is called *a homogeneous wave*.

❖ The vectors $\mathbf{k_c}$, **k** and **a** are related to the optical constant of the medium

$$\mathbf{k} = n\mathbf{k_0} \quad , \quad \mathrm{a} = \kappa\mathbf{k}_0 \quad , \quad \mathbf{k}_c \equiv (n - i\kappa)\mathbf{k_0}$$

❖The electric field :

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{E}_0 e^{i(\omega t - n_c \mathbf{k}_0 \mathbf{r})}\right] = \mathrm{Re}\left[\mathbf{E}_0 e^{-\kappa \mathbf{k}_0 \mathbf{r}} e^{i(\omega t - n\mathbf{k}_0 \mathbf{r})}\right]$$

❖**One important aspect** concerning **light propagation** in **absorbing** media is the **intensity variation** suffered by the wave as it **propagates**.

The intensity of light in absorbing media

- we assume that the propagation is along the z-axis; in this case, the intensity takes the form:

$$I(z) = \frac{1}{2c\mu_0} |\mathbf{E}_0|^2 e^{-2\kappa k_0 z}$$

- $I_0 = \frac{1}{2c\mu_0} |\mathbf{E}_0|^2$: the **intensity** associated with the wave at the plane $z = 0$,

$$I(z) = I_0 e^{-2\kappa k_0 z}$$

- The **intensity** of the wave **decreases** *exponentially* as a function of the **propagation distance**. $I(z) = I_0 e^{-\alpha z}$

- The *absorption coefficient* $\boldsymbol{\alpha}$, defined as: $\alpha \equiv 2\kappa k_0 = 2\kappa \frac{\omega}{c} \quad (\mathrm{m}^{-1})$

- The **light attenuation** in *decibels* (dB), $1\ \mathrm{dB} \equiv 10 \log \left(I_0 / I \right) = 4.3 \alpha d$

Metallic media

- A high electrical conductivity σ (compared with $\varepsilon\omega$)

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \Rightarrow \nabla^2 \mathbf{E}(\mathbf{r}) + \overbrace{\omega^2 \mu \underbrace{\left(\varepsilon - i\,\sigma/\omega\right)}_{\varepsilon_{\mathrm{G}}}}^{k_c^2} \mathbf{E}(\mathbf{r}) = 0$$

- $\boldsymbol{\varepsilon_G}$ (clearly, a **complex quantity**) is known as generalised **dielectric permittivity.**

- The **Helmholtz equation** is still **valid.**

- Transparent dielectric medium ($\kappa = 0$),

EM Waves at Planar Dielectric Interfaces

- **Boundary conditions at the interface**

  ✓Another *important aspect* in the study of **light propagation** is the behaviour of **EM waves passing** from **one medium** to **another**.

  ✓The **behaviour** of an **EM monochromatic plane wave** travelling through a *homogeneous* medium, incident on a second *homogeneous* medium, separated from the former by a planar interface.

  ✓The equations that determine the reflection and transmission coefficients can be studied separately in two groups:

1) The **electric field** of the incident EM wave has **only** a **parallel component** with respect to the **incident plane** (the magnetic field being perpendicular to that plane)
2) the electric vector has only the component perpendicular to the incident plane

EM Waves at Planar Dielectric Interfaces

- The **relations** between the **incident**, **reflected** and **transmitted** waves are obtained by setting the adequate **boundary conditions** for the **fields** at **the planar interface**, which are derived directly from **Maxwell's equations.**

- $$\left.\begin{aligned}\nabla.\mathbf{D}=0\\ \nabla.\mathbf{B}=0\end{aligned}\right\} \Rightarrow \begin{cases}\left(\mathbf{D}^{\text{Normal}}\right)_{\mathbf{Medium1}}=\left(\mathbf{D}^{\text{Normal}}\right)_{\mathbf{Medium}2}\\ \left(\mathbf{B}^{\text{Normal}}\right)_{\mathbf{Medium1}}=\left(\mathbf{B}^{\text{Normal}}\right)_{\mathbf{Medium}2}\end{cases}$$ **at interface**

- $$\left.\begin{aligned}\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\\ \nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\end{aligned}\right\} \Rightarrow \begin{cases}\left(\mathbf{E}^{\text{Tangential}}\right)_{\mathbf{Medium1}}=\left(\mathbf{E}^{\text{Tangential}}\right)_{\mathbf{Medium}2}\\ \left(\mathbf{H}^{\text{Tangential}}\right)_{\mathbf{Medium1}}=\left(\mathbf{H}^{\text{Tangential}}\right)_{\mathbf{Medium}2}\end{cases}$$ at interface

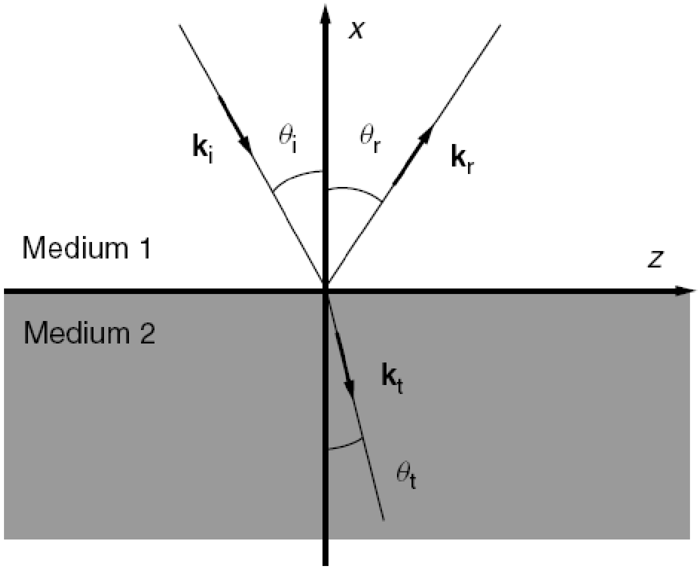
EM Waves at Planar Dielectric Interfaces

- The dielectric media are characterized by their optical constant (ε_1,μ_1) and (ε_2,μ_2),

$$\mathbf{E}_i(\mathbf{r},t)=\mathbf{E}_i e^{i(\omega_i t-\mathbf{k}_i\mathbf{r})}$$

$$\mathbf{E}_r(\mathbf{r},t)=\mathbf{E}_r e^{i(\omega_r t-\mathbf{k}_r\mathbf{r})}$$

$$\mathbf{E}_t(\mathbf{r},t)=\mathbf{E}_t e^{i(\omega_t t-\mathbf{k}_t\mathbf{r})}$$



- Apply the **condition** of the **continuity** of the **tangential component** of the electric field across the interface

$$\left[\mathbf{E}_i(\mathbf{r},t)+\mathbf{E}_r(\mathbf{r},t)\right]^{\text{Tangential}}=\left[\mathbf{E}_t(\mathbf{r},t)\right]^{\text{Tangential}}$$

$$\left[\mathbf{E}_i e^{i(\omega_i t-\mathbf{k}_i\mathbf{r})}+\mathbf{E}_r e^{i(\omega_r t-\mathbf{k}_r\mathbf{r})}\right]^{\text{Tangential}}=\left[\mathbf{E}_t e^{i(\omega_t t-\mathbf{k}_t\mathbf{r})}\right]^{\text{Tangential}}$$

- As this relation should be valid for any instant of time, it follows that:

$$\omega_i=\omega_r=\omega_t$$

EM Waves at Planar Dielectric Interfaces

- The **condition** of equal spatial dependence on the exponents **at the interface**

$$k_{iy}y + k_{iz}z = k_{ry}y + k_{rz}z = k_{ty}y + k_{tz}z \qquad \text{(at the interface } \boldsymbol{x} = \mathbf{0}\text{)}$$

- This result indicates that the **tangential** component of the wavevectors (for the **incident**, **reflected** and **transmitted** **waves**) **must be equal**:

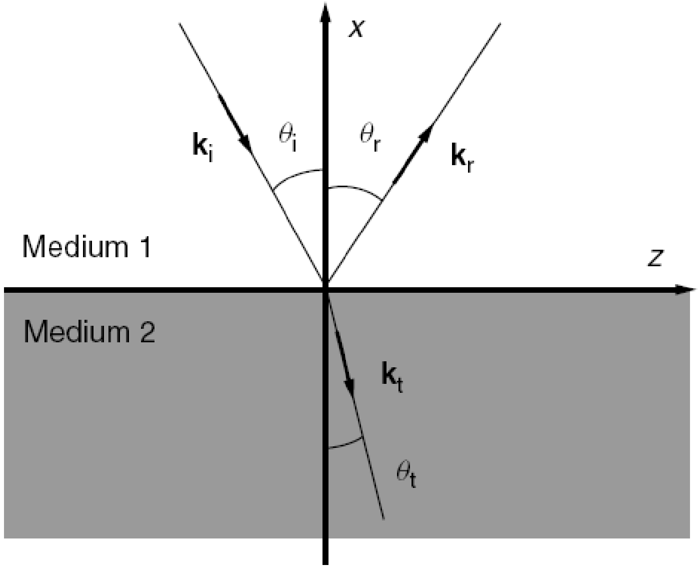
$$[\mathbf{k}_i]^T = [\mathbf{k}_r]^T = [\mathbf{k}_t]^T$$

- In other words, at the **boundary** only the **perpendicular** component of the wavevectors **can change**.
- Thus, the vectors $\mathbf{k_r}$ and $\mathbf{k_t}$ must **lie** in the plane defined by the $\mathbf{k_i}$ vector and the **normal** to the **plane** of the **interface**.

EM Waves at Planar Dielectric Interfaces

- This plane, **perpendicular** to the **plane** that separates both media, is called the ***incident plane***, and all the wavevectors lie on it.

- if we choose the **incident plane** as the *x-z* plane, in this case the *y* components of the **wavectors** are **null:**



$$k_{iz}z = k_{rz}z == k_{tz}z \qquad ==>>$$

$$k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t$$

$$\left.\begin{aligned} k_{\mathrm{i}} &= \omega\left(\varepsilon_1\mu_1\right)^{1/2} = k_r \\ k_{\mathrm{t}} &= \omega\left(\varepsilon_2\mu_2\right)^{1/2} \end{aligned}\right\} \Rightarrow \quad \theta_i = \theta_r$$

(*law of reflection*)

$$k_i \sin\theta_i = k_t \sin\theta_t \qquad \textbf{(\textit{Transmission law})}$$

Snell's law

- If the two **homogeneous** media are **non-magnetic** *($\mu_1 \approx \mu_2 \approx \mu_0$) and* ***non-absorbing materials*** *(real refractive* indices)

$$\left.\begin{aligned}\left(\varepsilon_1/\varepsilon_0\right)^{1/2} &= \left(\varepsilon_{r1}\right)^{1/2} = n_1 \\ \left(\varepsilon_2/\varepsilon_0\right)^{1/2} &= \left(\varepsilon_{r2}\right)^{1/2} = n_2\end{aligned}\right\} \quad k_i \sin\theta_i = k_t \sin\theta_t \quad \Rightarrow \left(n_1 \sin\theta_i = n_2 \sin\theta_t \right)$$

- ***Snell's law*** is valid for **dielectric materials**.

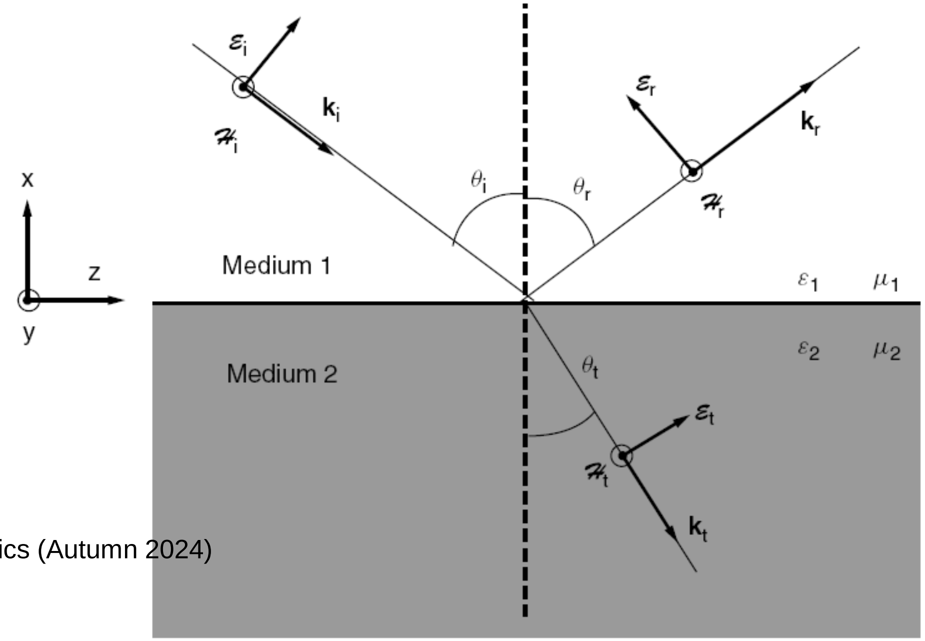
- In the case of **absorbing media**, the equation $k_{iz}z = k_{rz}z == k_{tz}z$ is still **valid**, and is the correct relation to obtain the transmitted wave.

Reflection and transmission coefficients: reflectance and transmittance

- The **relations** between the **electric field** amplitude for the **incident**, **reflected** and **transmitted waves,**

- **Transverse magnetic incidence (TM incidence or p waves)**
  1) The **electric field vector** associated with the **incident monochromatic plane** wave **lies** on the **incident plane.** **(Parallel Polarization)**

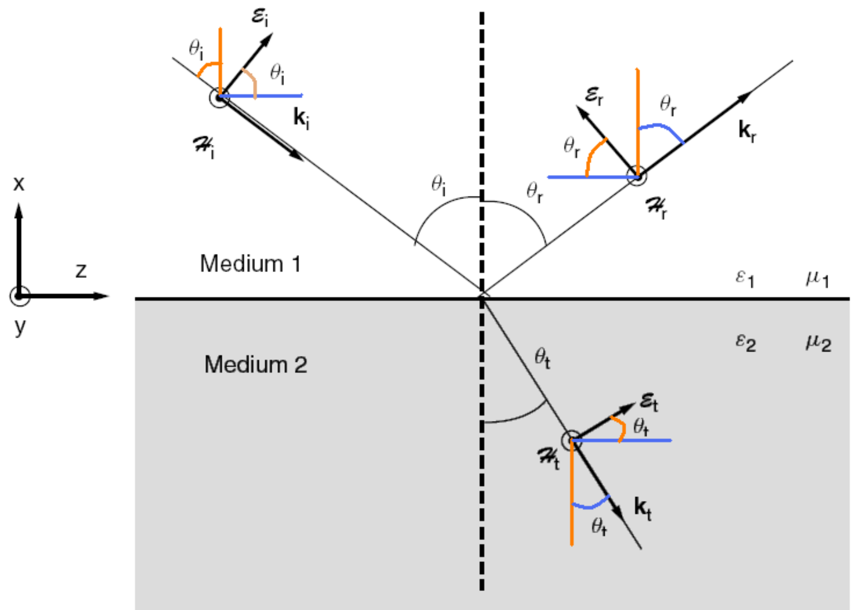
  2) and the **magnetic field** vector is **perpendicular** to **both vectors**



Transverse Magnetic incidence (TM incidence or p waves)

$$\mathbf{E}_i \equiv \mathbf{E}_i^{\parallel} \equiv \left[\mathbf{E}_{ix}, 0, \mathbf{E}_{iz}\right]$$

$$\mathbf{H}_i \equiv \mathbf{H}_i^{\perp} \equiv \left[0, \mathbf{H}_{iy}, 0\right]$$



- The symbols || and ⊥ denote vectors parallel and perpendicular to the incident plane, respectively.
- As the **electric field** vector is **paralle** to the incidence plane, the **TM incidence** is also called **parallel incidence**.
- The **condition** of the continuity of the **tangential component** of the **electric field** at the interface:

$$\left(\mathbf{E}^{\text{Tangential}}\right)_{\textbf{Medium1}} = \left(\mathbf{E}^{\text{Tangential}}\right)_{\textbf{Medium}\,2} \Rightarrow \mathbf{E}_{iz} + \mathbf{E}_{rz} = \mathbf{E}_{tz}$$

$$\left[\mathbf{E}_i e^{i(\omega_i t - \mathbf{k}_i \mathbf{r})} \cos\theta_i - \mathbf{E}_r e^{i(\omega_r t - \mathbf{k}_r \mathbf{r})} \cos\theta_r\right]_{x=0} = \left[\mathbf{E}_t e^{i(\omega_t t - \mathbf{k}_t \mathbf{r})} \cos\theta_t\right]_{x=0}$$

TM incidence (p waves)

- The **temporal** and **spatial dependences** of the **exponentials** are equal (at $x = 0$) $\quad \mathbf{E}_i \cos\theta_i - \mathbf{E}_r \cos\theta_i = \mathbf{E}_t \cos\theta_t$

- The condition of **continuity** of the normal component of the dielectric displacement vector

$$\left(\mathbf{D}^{\text{Normal}}\right)_{\textbf{Medium1}} = \left(\mathbf{D}^{\text{Normal}}\right)_{\textbf{Medium}\,2} \quad \Rightarrow \quad \mathbf{D}_{ix} + \mathbf{D}_{rx} = \mathbf{D}_{tx}$$

$$\varepsilon_1 \mathbf{E}_i \sin\theta_i + \varepsilon_1 \mathbf{E}_r \sin\theta_i = \varepsilon_2 \mathbf{E}_t \sin\theta_t$$

- The relation between the **electric field** amplitudes of the **reflected** and **incident** waves

$$r_{TM}\left(\Gamma_{\|}\right) \equiv \frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

- r_{TM} denotes the ***reflection coefficient*** for **parallel polarisation.**
- ***Fresnel equation*** **for the parallel polarization**

TM incidence (p waves)

- r_{TM} can also be written in several equivalent forms, one of which is

$$r_{TM}\left(\Gamma_{\parallel}\right) \equiv \frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{n_2^2\cos\theta_i - n_1\sqrt{n_2^2 - n_1^2\sin^2\theta_i}}{n_2^2\cos\theta_i + n_1\sqrt{n_2^2 - n_1^2\sin^2\theta_i}}$$

- r_{TM} is real when θ_i is smaller than the critical angle, $\sin\phi_c = \dfrac{n_2}{n_1}$

- r_{TM} can be positive or negative depending on the incidence angle.

- $\Gamma_{\parallel}$ vanishes if $n_1 > n_2$ and if the incidence angle is $\tan\theta_i = \dfrac{n_2}{n_1}$

- The incidence angle is commonly known as the **polarizing** or **Brewster angle**.

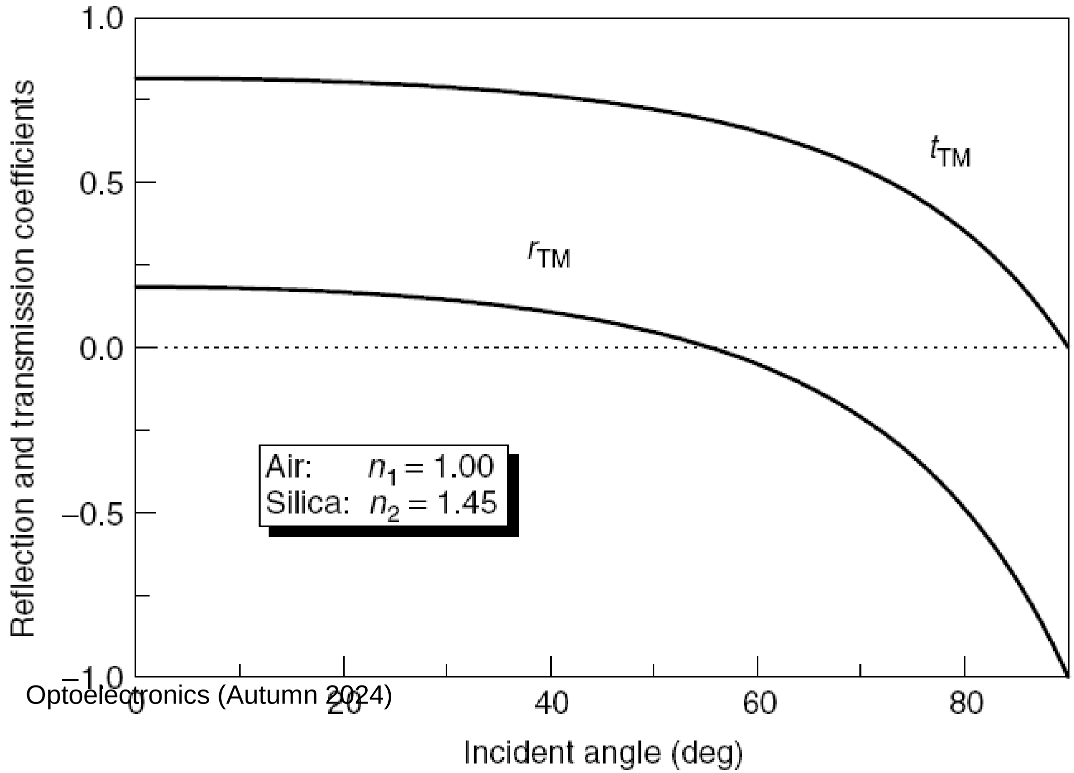
TM incidence (p waves)

- The **relation** between the **amplitude** between the transmitted and **incident** waves

$$t_{TM} \equiv \frac{\mathbf{E}_t}{\mathbf{E}_i} = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t}$$

- t_{TM} : the ***transmission coefficient*** for parallel polarisation.

- In general r_{TM} and t_{TM} can be **complex magnitudes.**



TM incidence (p waves)

- Although the **reflection** and **transmission** **coefficients** give us valuable information concerning the **relation** between the electric field amplitudes of the **incident**, **reflected** and **transmitted** waves, in many cases the relevant parameter is the fraction of the incidente energy that is reflected and transmitted at the interface, defined through **reflectance** and **transmittance**.

- The *reflectance R* is defined as the quotient between the reflected energy in an unit of time over a differential area, and the incident energy per unit of time over the same area at the interface.

- The *transmittance is* defined as the quotient between the transmitted energy per unit of time over a differential area and the incident energy in that unit of time over the same area.

Reflectance and Transmittance for TM incidence

$$\left.\begin{aligned} R_{TM} &= \left|\frac{\mathbf{E}_r}{\mathbf{E}_i}\right|^2 = \left|r_{TM}\right|^2 = \left(\frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t}\right)^2 \\ T_{TM} &= \left|\frac{\mathbf{E}_t}{\mathbf{E}_i}\right|^2 = \left|t_{TM}\right|^2 = \frac{4n_1n_2\cos\theta_i\cos\theta_t}{\left(n_2\cos\theta_i + n_1\cos\theta_t\right)^2} \end{aligned}\right\} \Rightarrow \quad R_{TM} + T_{TM} = 1$$

- From equation $R_{TM} = \left(\frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t}\right)^2$ it follows that the reflectance will vanish for the condition $n_2\cos\theta_i = n_1\cos\theta_t$

- By combining R_{TM} with the **Snell's law**, one obtains that the **reflectance** is **zero** for an **incident angle** that fulfils the equation:

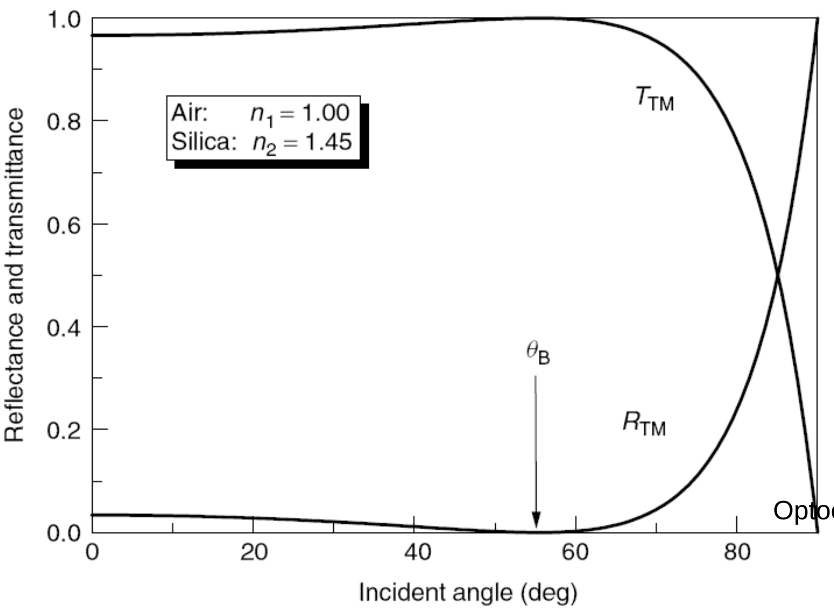
$$\tan\theta_i = n_2/n_1$$

Reflectance and Transmittance for TM incidence

➢This angle, for which $R_{TM} = 0$, is called **Brewster's angle** θ_B or the polarising angle, because the reflected wave will be linearly polarised for an incident wave with arbitrary polarisation state.

➢For the particular case of **normal incidence** ($\theta_i = 0$), the formula for the reflectance is simplified to:

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$



➢ Reflectance and transmittance for TM incidence corresponding to the interface air–silica (n_1 = 1.00, n_2 = 1.45). For an incident angle at $\theta_i = \theta_B$ the reflectance vanishes, corresponding to an angle of 55.4°

Brewster's angle or polarization angle (θ_p)

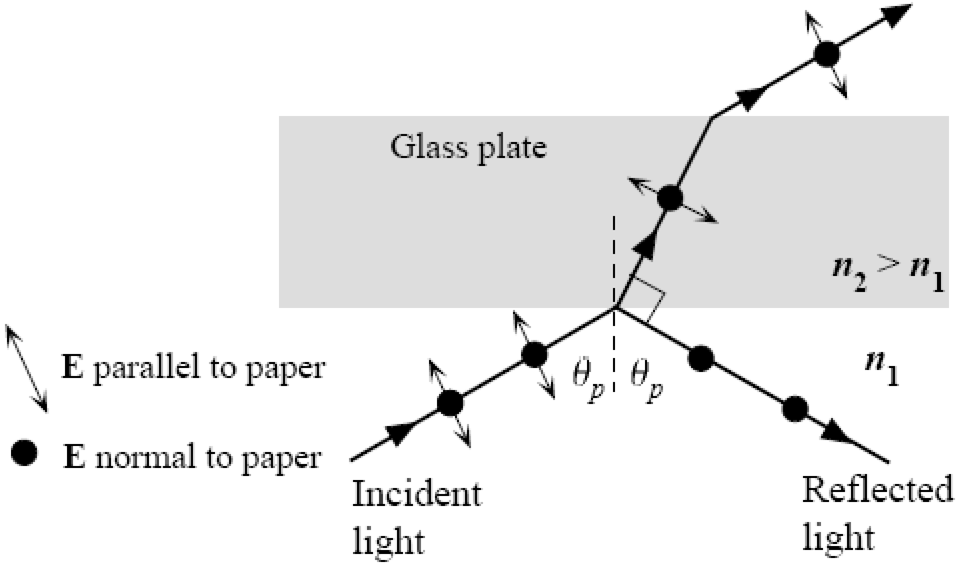
- is **the angle** of incidence which results in the **reflected wave** having **no electric field** in **the plane of incidence** (plane defined by the incident **ray** and the normal to the surface). The electric field oscillations are in the plane perpendicular to the plane of incidence. When the angle of incidence of a light wave is equal to the polarization angle θ_p, the field in the reflected wave is then always perpendicular to the plane of incidence and hence well-defined. The reflected wave is then plane polarized. This special angle is given by

$$\tan\theta_p = \frac{n_2}{n_1}$$

- In addition, the transmitted (the refracted) wave has a greater field amplitude in the plane of incidence. By using a pile of glass plates inclined at the Brewster angle, one can construct a polarizer that provides a reasonable polarized light with the field in the plane of incidence.

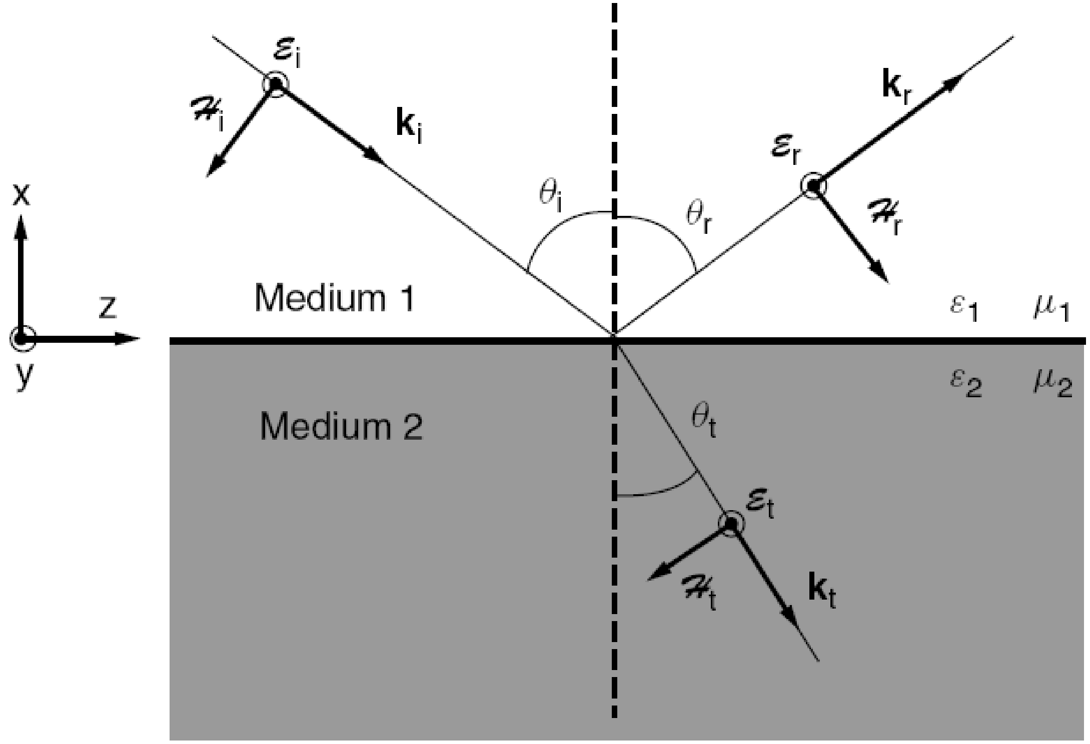
Brewster's angle or polarization angle (θ_p)

➢ When **an unpolarized light wave** is incident at the **Brewster angle**, the reflected wave is polarized with its optical field normal to the plane of incidence, that is parallel to the surface of the glass plate. The angle between the refracted (transmitted) beam and the reflected beam is 90 .



Transverse Electric Incidence (TE incidence or n waves)

➢ The **electric field** vector of the incident wave is **perpendicular** to the **incident plane.**



Reflection and **transmission** corresponding to **TE** incidence **perpendicular polarisation**). While the **electric field** vectors are perpendicular to the incident plane (*x* –*z* plane*)*, the wavevectors and the magnetic field vectors lie in that plane

TE incidence

- The electric and magnetic field vectors associated with the incident wave are:

$$\mathbf{E}_i \equiv \mathbf{E}_i^{\perp} \equiv \left[0, \mathbf{E}_{iy}, 0\right]$$

$$\mathbf{H}_i \equiv \mathbf{H}_i^{\parallel} \equiv \left[\mathbf{H}_{ix}, 0, \mathbf{H}_{iz}\right]$$

- The continuity of the tangential component of the electric field across the boundary $\mathbf{E}_{iy} + \mathbf{E}_{ry} = \mathbf{E}_{ty}$

- To obtain the reflection and transmission coefficients it is necessary to find a second relation between the electric field amplitudes.

- The condition of continuity of the tangential component of the magnetic field vector at the interface: $\mathbf{H}_{iz} + \mathbf{H}_{rz} = \mathbf{H}_{tz}$

TE incidence

- by relating the magnetic field vectors with the electric field vectors by using equation $\mathbf{k} \times \mathbf{E}_0 = \omega\mu_0 \mathbf{H}_0$

- After straightforward calculations, the boundary condition $\mathbf{H}_{iz} + \mathbf{H}_{rz} = \mathbf{H}_{tz}$ becomes: $k_{ix}\left(\mathbf{E}_{iy} - \mathbf{E}_{ry}\right) = k_{tx}\mathbf{E}_{ty}$

- The reflection and transmission coefficients for TE incidence are obtained as a function of the wavevectors:

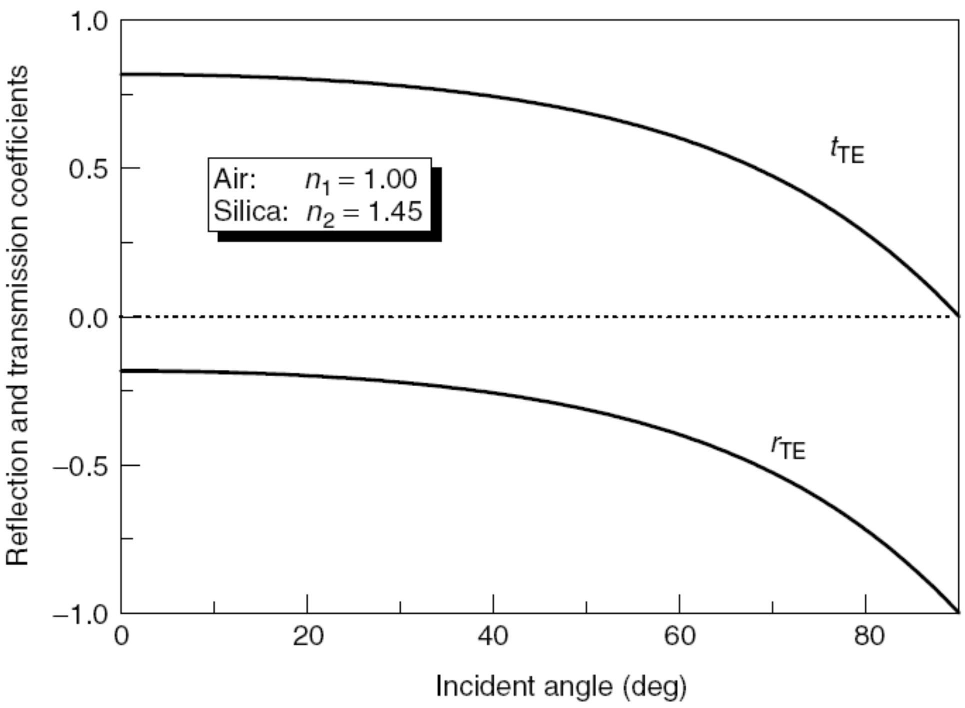
$$r_{TE} \equiv \frac{E_r}{E_i} = \frac{k_{ix} - k_{tx}}{k_{ix} + k_{tx}} \quad , \quad t_{TE} \equiv \frac{E_t}{E_i} = \frac{2k_{ix}}{k_{ix} + k_{tx}}$$

- These coefficients can be expressed in a more convenient form as a function of the incident and refracted angles and the refractive indices of the two media by using Snell's law:

$$r_{TE} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \quad , \quad t_{TE} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

TE incidence

➢ The reflection and transmission coefficients as a function of the incident angle in the case of air–silica interface for TE incidence, where both coefficients are real in the whole range of incident angles.



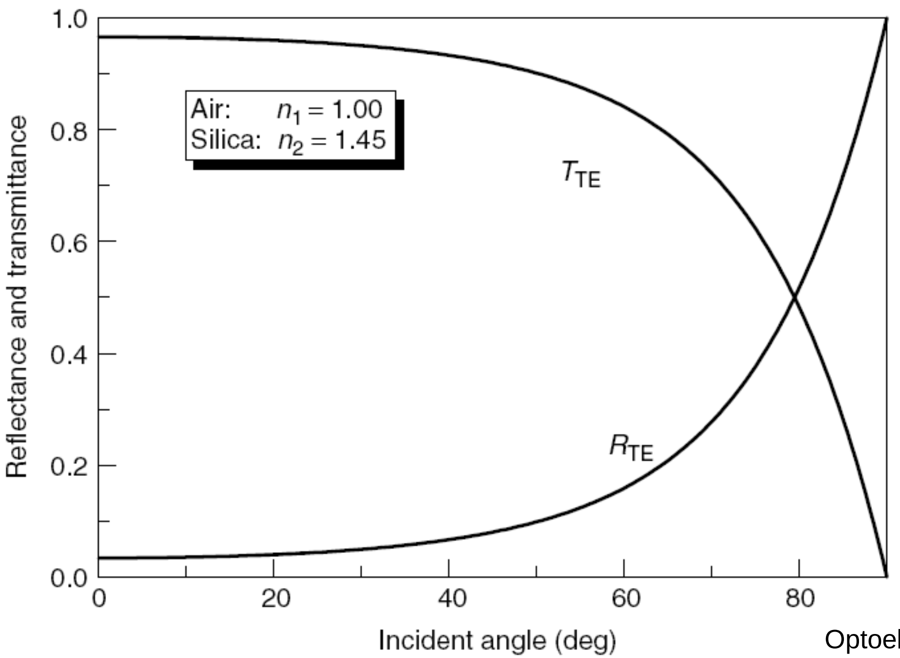
➢ As can be seen, the transmission coefficient is positive, indicating that the direction of the electric field vector of the transmitted wave is coincident to that of the incident wave.

➢ By contrast, the electric field vector associated with the reflected wave is reversed in respect to that of the incident wave, indicating a phase shift of π in the reflected wave.

Reflectance and Transmittance for TE incidence

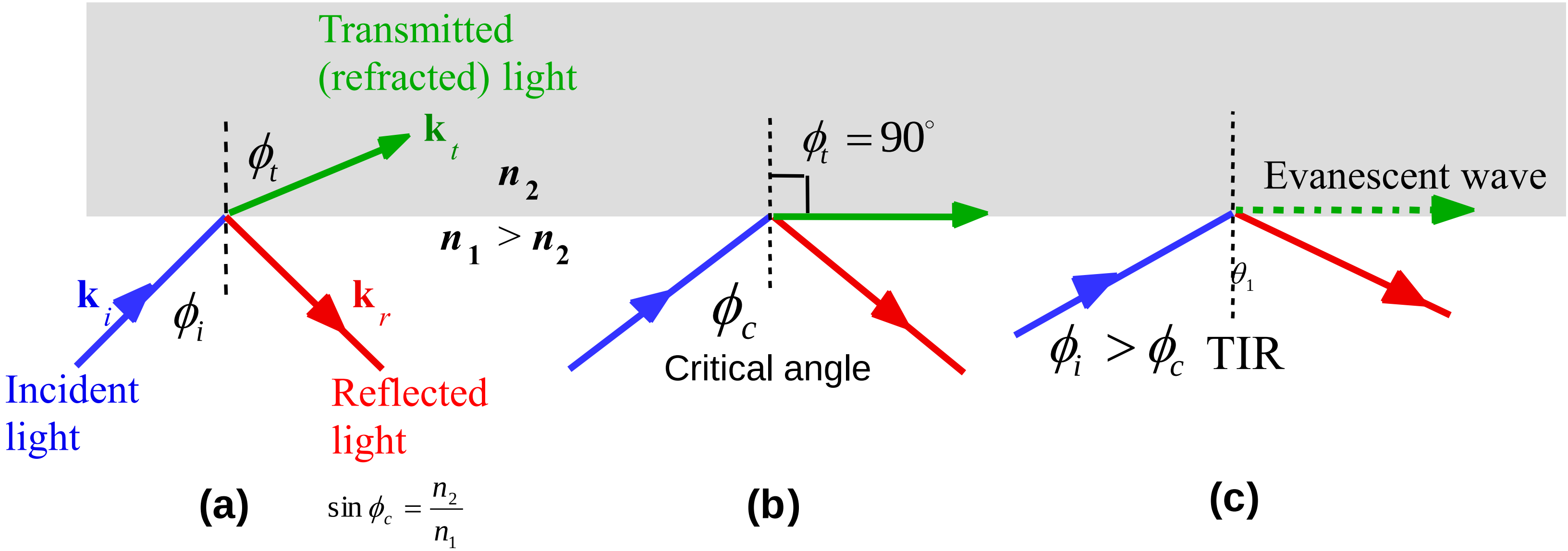
$$R_{TE} = \left|\frac{\mathbf{E}_r}{\mathbf{E}_i}\right|^2 = \left|r_{TE}\right|^2 = \left(\frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}\right)^2$$

$$T_{TE} = \left|\frac{\mathbf{E}_t}{\mathbf{E}_i}\right|^2 = \left|t_{TE}\right|^2 = \frac{4n_1 n_2\cos\theta_i\cos\theta_t}{\left(n_1\cos\theta_i + n_2\cos\theta_t\right)^2}$$



- In TE incidence the reflectance is a monotonous increasing function of the incident angle. Therefore, if a beam of non-polarised light is incident at an angle of θ_B, the interface only will reflect the TE component of such radiation, and thus the reflected wave will be linearly polarised with the electric field vector perpendicular to the incident plane. This is the reason why Brewster's angle is also called the polarising angle, and this phenomenon can be used to design polarisation devices.

Total internal reflection, Critical angle



Light wave travelling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to ϕ_c, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected. (a) $\phi_i < \phi_c$ (b) $\phi_i = \phi_c$ (c) $\phi_i > \phi_c$ and total internal reflection (TIR).

$$\sin\phi_c = \frac{n_2}{n_1}$$

EXAMPLE

Water has an index of refraction $n = 1.33$. The index of refraction of ordinary glass is approximately $n = 1.5$. For most semiconductors, such as *Si*, *GaAs*, and InP, the index of refraction is often in the range between 3 and 4, depending on the optical wavelength and the material. Here we take a nominal value of $n = 3.5$ for a semiconductor. Find the reflectivities at normal incidence, the Brewster angles, and the critical angles for these media at their interfaces with air.

- $R = 0.02$ for water, $R = 0.04$ for ordinary glass, and R typically falls in the range of 0.3 and 0.32 for a semiconductor.

- $\theta_B \approx 54^\circ$ for water, $\theta_B \approx 56^\circ$ for ordinary glass, and θ_B is typically around 74° for a semiconductor.

- $\theta_c \approx 49^\circ$ for water, $\theta_c \approx 42^\circ$ for ordinary glass, and θ_c is around 17° for a semiconductor.