Q:

a) Show that if the maximum positive displacement of a sine wave at t=0 is at a distance of x_0 cm from the origin, its initial phase angle is given by the following equation

$$\varphi_o = \frac{\pi}{2} - \left(\frac{2\pi}{\lambda}\right) x_0$$

where the wavelength λ is in centimeters.

b) Obtain the initial phase and draw the wave for $\lambda = 10\ cm$ and the initial distances x_0 equal to 0, $\frac{5}{6}$, $\frac{5}{2}$, 5 and $-\frac{1}{2}$ cm.

c) If we use the cosine function instead of the sine function, determine the initial phase angles for part (b).

Sol:

a)

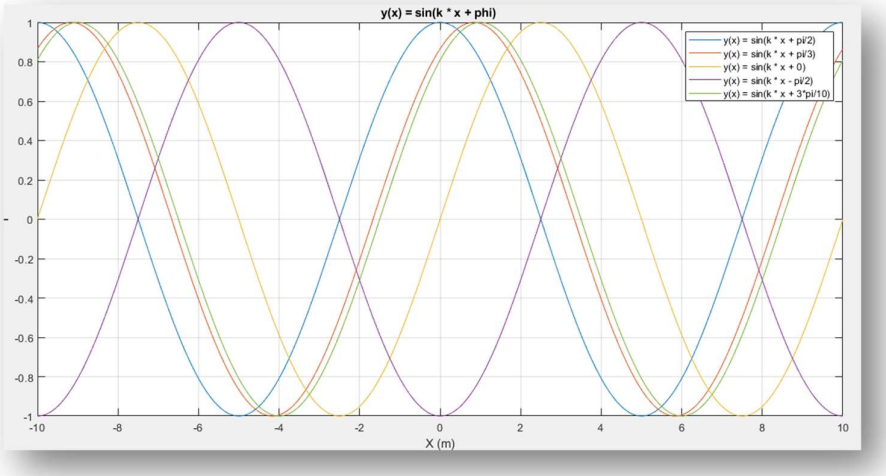
$$y = Asin(k(x + vt) + \varphi)$$

$$y = 1, t = 0 \rightarrow Asin(kx + \varphi) = 1 \rightarrow kx + \varphi = \frac{\pi}{2} \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{\lambda}x$$

b)

$$\lambda = 10$$

$$\begin{cases} x = 0 \rightarrow \varphi = \frac{\pi}{2} \\ x = \frac{5}{6} \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{10} * \frac{5}{6} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \\ x = \frac{5}{2} \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{10} * \frac{5}{2} = 0 \\ x = 5 \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{10} * 5 = -\frac{\pi}{2} \\ x = -\frac{1}{2} \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{10} * \frac{1}{2} = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10} \end{cases}$$



c) As it is known, the difference of sine or cosine is only $\frac{pi}{2}$, so we have to add $\frac{pi}{2}$ to the answers.

$$\lambda = 10$$

$$\begin{cases} x = 0 \rightarrow \varphi = \frac{\pi}{2} + \frac{\pi}{2} = \pi \\ x = \frac{5}{6} \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{10} * \frac{5}{6} + \frac{\pi}{2} = \frac{\pi}{2} - \frac{\pi}{6} + \frac{\pi}{2} = \frac{\pi}{3} + \frac{\pi}{2} = \frac{5\pi}{6} \\ x = \frac{5}{2} \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{10} * \frac{5}{2} + \frac{\pi}{2} = 0 + \frac{\pi}{2} = \frac{\pi}{2} \\ x = 5 \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{10} * 5 + \frac{\pi}{2} = -\frac{\pi}{2} + \frac{\pi}{2} = 0 \\ x = -\frac{1}{2} \rightarrow \varphi = \frac{\pi}{2} - \frac{2\pi}{10} * \frac{1}{2} + \frac{\pi}{2} = \frac{\pi}{2} - \frac{\pi}{5} + \frac{\pi}{2} = \frac{3\pi}{10} + \frac{\pi}{2} = \frac{8\pi}{10} \end{cases}$$