Q: Using the necessary terms for $k.r$, write down the functions describing a plane sine wave in three dimensions in which the wavelength and velocity are explicitly entered for the following propagation directions.

a) Along the $z+$ axis.
b) Along the line $z = 0, y = x$.
c) Perpendicular to the planes $x + y + z = const$

Sol:

$$\vec{k} = k_x\hat{\imath} + k_y\hat{\jmath} + k_z\hat{z}\,, \vec{r} = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{z}$$

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \frac{2\pi}{\lambda}$$

a)

$$k_z = k = \frac{2\pi}{\lambda}$$

$$k_x = k_y = 0$$

$$\psi = Asin(k_z z - \omega t) = Asin(\frac{2\pi}{\lambda} z - \omega t)$$

b)

$$k_x = k_y = \frac{\sqrt{2}\pi}{\lambda}$$

$$k_z = 0$$

$$\psi = Asin\left(\frac{2\sqrt{2}\pi}{\lambda} x \pm \omega t\right)$$

c)

$$k_z = k_x = k_y = \frac{2\pi}{\sqrt{3}\lambda}$$

$$\psi = Asin\left(\frac{2\pi}{\sqrt{3}\lambda}(x + y + z) \pm \omega t\right)$$