Q: According to the equations of B, C and D in micro-ring amplifiers, calculate the ratios $\left|\frac{B}{A}\right|^2$ and $\left|\frac{D}{A}\right|^2$ and draw its output.

Sol:

$$\begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} \hat{t} & \hat{\kappa} \\ -\hat{\kappa}^* & \hat{t}^* \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix}$$

$$\begin{cases} D = ae^{i\theta}\mathrm{C} \\ a = e^{-\alpha(2\pi r)} \\ \theta = \beta.2\pi r = k.n_{eff}.2\pi r = \dfrac{2\pi.n_{eff}.2\pi r}{\lambda} = 4\pi^2 n_{eff}\dfrac{r}{\lambda} \end{cases}$$

- ***A*** ***is chosen to be equal to*** *1*

$$\begin{cases} B = tA + \kappa D \\ C = -\kappa^* A + t^* D = -\kappa^* A + t^* a e^{i\theta}\mathrm{C} \rightarrow C = -\dfrac{\kappa^*}{1 - t^* a e^{i\theta}} \end{cases}$$

$$\rightarrow D = -\frac{\kappa^* a e^{i\theta}}{1 - t^* a e^{i\theta}} = -\frac{a\kappa^*}{e^{-i} - t^* a}$$

$$\rightarrow B = tA + \kappa D = t - \frac{a|\kappa|^2}{e^{-i\theta} - t^* a} = \frac{te^{-i\theta} - |t|^2 a - a|\kappa|^2}{e^{-i\theta} - t^* a} = \frac{te^{-i\theta} - a(|t|^2 + |\kappa|^2)}{e^{-i\theta} - t^* a}$$
$$= \frac{te^{-i\theta} - a(1)}{e^{-i\theta} - t^* a} = \frac{te^{-i\theta} - a}{e^{-i\theta} - t^* a}$$

- According to the initial assumption : $\boldsymbol{t = |t|e^{-i\varphi}}$

$$I_t = \left|\frac{B}{A}\right|^2 = \left|\frac{\frac{te^{-i\theta} - a}{e^{-i\theta} - t^* a}}{1}\right|^2 = \left|\frac{t\big(\cos(\theta) - i\sin(\theta)\big) - a}{\cos(\theta) - i\sin(\theta) - t^* a}\right|^2$$

$$= \left|\frac{|t|\big(\cos(\theta+\varphi) - i\sin(\theta+\varphi)\big) - a}{\cos(\theta) - i\sin(\theta) - |t|e^{i\varphi}a}\right|^2 = \left|\frac{|t|\big(\cos(\theta+\varphi) - i\sin(\theta+\varphi)\big) - a}{\cos(\theta) - i\sin(\theta) - |t|\big(\cos(\varphi) + i\sin(\varphi)\big)a}\right|^2$$

$$= \frac{(t\cos(\theta+\varphi) - a)^2 + t^2\sin^2(\theta+\varphi)}{\big(-|t|a\cos(\varphi) + \cos(\theta)\big)^2 + (-\sin(\theta) - |t|\mathrm{asin}\,(\varphi))^2}$$

$$= \frac{t^2\cos^2(\theta+\varphi) + a^2 - 2at\cos(\theta+\varphi) + t^2\sin^2(\theta+\varphi)}{\cos^2(\theta) + t^2a^2\cos^2(\varphi) - 2a|t|\cos(\theta)\cos(\varphi) + \sin^2(\theta) + |t|^2\mathrm{a}^2\sin^2(\varphi) + 2a|t|\sin(\theta)\sin(\varphi)}$$

$$= \frac{t^2 + a^2 - 2atcos(\theta + \varphi)}{1 + a^2t^2 - 2a|t|(\cos(\theta)\cos(\varphi) - \sin(\theta)\sin(\varphi))} = \frac{a^2 + t^2 - 2at\cos(\theta + \varphi)}{1 + a^2t^2 - 2at\cos(\theta + \varphi)}$$

assumption : $\boldsymbol{t = |t|e^{-i\varphi}, \kappa = |\kappa|e^{-i\psi}}$

$$I_i = \left|\frac{D}{A}\right|^2 = \left|-\frac{\frac{a\kappa^*}{e^{-i\theta} - t^*a}}{1}\right|^2 = \left|-\frac{a\kappa^*}{e^{-i\theta} - t^*a}\right|^2 = \left|-\frac{a|\kappa|e^{-i\psi}}{e^{-i\theta} - |t|e^{-i\varphi}a}\right|^2$$

$$= \left|-\frac{a|\kappa|(\cos(\psi) - isin(\psi))}{\cos(\theta) - isin(\theta) - |t|(\cos(\varphi) + isin(\varphi))a}\right|^2$$

$$= \frac{a^2|\kappa|^2}{(\cos(\theta) - a|t|\cos(\varphi))^2 + (-a|t|sin(\varphi) - sin(\theta))^2}$$

$$= \frac{a^2|\kappa|^2}{\cos^2(\theta) + a^2|t|^2\cos^2(\varphi) - 2a|t|\cos(\theta)\cos(\varphi) + a^2|t|^2\sin^2(\varphi) + \sin^2(\theta) + 2a|t|sin(\theta)sin(\varphi)}$$

$$= \frac{a^2(1 - t^2)}{1 + a^2t^2 - 2at\cos(\theta + \varphi)}$$

Now we calculate two equations based on the equations 😊