Q: A moving harmonic wave in the $+x$ direction at $t = 0$ has a displacement of 13 units at $x = 0$ and a displacement of -7.5 units at $x = \frac{3\lambda}{4}$. Write the wave function at $t = 0$.

Sol:

$$y = A\sin(k(x + vt) + \varphi)$$

$$\begin{cases} t = 0, x = 0 \rightarrow 13 = A\sin(\varphi) \\ t = 0, x = \dfrac{3\lambda}{4} \rightarrow -7.5 = A\sin\left(2\pi * \dfrac{3}{4} + \varphi\right) = A\sin\left(\pi * \dfrac{3}{2} + \varphi\right) \end{cases}$$

$$\begin{cases} 13 = A\sin(\varphi) \\ -7.5 = -A\cos(\varphi) \end{cases} \rightarrow \begin{cases} 13 = A\sin(\varphi) \\ 7.5 = A\cos(\varphi) \end{cases}$$

$$\rightarrow \frac{\sin(\varphi)}{\cos(\varphi)} = \frac{13}{7.5} \rightarrow \varphi = \tan^{-1}\left(\frac{13}{7.5}\right) = 60^o = \frac{2\pi}{3}$$

$$13 = A\sin\left(\frac{2\pi}{3}\right) \rightarrow A = \frac{13}{\frac{\sqrt{3}}{2}} = 15$$

$$t = 0 \rightarrow y = A\sin(kx + \varphi) = 15 * \sin\left(\frac{2\pi}{\lambda}x + \frac{2\pi}{3}\right)$$