Q: Flat waveguide with refractive index of core $n_1 = 1.6$, shell $n_2 = 1.5$ is assumed:

a) What is the thickness of this waveguide so that there are two modes TM_0 and TM_1 at the wavelength of 1310^{nm}?

b) How many modes will the same waveguide have at a wavelength of 820^{nm}?

Sol:

a)

$$\frac{2\pi}{\lambda} d\sqrt{n_1^2 - n_2^2} > \frac{N\pi}{2}$$

$$\lambda = 1310^{nm}, N + 1 = 2 \rightarrow N = 1, n_1 = 1.6, n_2 = 1.5$$

$$\begin{cases} \min(N) = 1: \frac{2}{1310 * 10^{-9}} d\sqrt{1.6^2 - 1.5^2} > \frac{1}{2} \rightarrow d > 588^{nm} \\ \max(N) = 2: \frac{2}{1310 * 10^{-9}} d\sqrt{1.6^2 - 1.5^2} > \frac{2}{2} \rightarrow d > 1176^{nm} \end{cases}$$

$$588^{nm} < d < 1176^{nm}$$

Because the question in mind was to have only two modes, so we have obtained a range of thickness for it by using the formula.

b)

Now, based on the interval of the previous part, we get the minimum and maximum thickness of its modes.

$$\frac{2\pi}{\lambda} d\sqrt{n_1^2 - n_2^2} > \frac{N\pi}{2}$$

$$d = (588^{nm}, 1176^{nm}), \lambda = 820^{nm},, n_1 = 1.6, n_2 = 1.5$$

$$\begin{cases} \frac{2}{820 * 10^{-9}} 588 * 10^{-9}\sqrt{1.6^2 - 1.5^2} > \frac{N}{2} \rightarrow N + 1 < 2.59 \rightarrow N + 1 = 2 \\ \frac{2}{820 * 10^{-9}} 1176 * 10^{-9}\sqrt{1.6^2 - 1.5^2} > \frac{N}{2} \rightarrow N + 1 < 4.19 \rightarrow N + 1 = 4 \end{cases}$$

As it is known, in this period we have from 2 modes to 4 modes.