Q: A laser emits a single-phase beam with a wavelength of λ, which is irradiated vertically from a flat mirror moving away at a speed v. Find the beet frequency between incident and re-emitted light.

Sol:

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \approx 1-\frac{v}{c} \xrightarrow{v<0} 1+\frac{v}{c} \xrightarrow{c=\frac{\lambda}{T}=\lambda\times f\rightarrow f=\frac{c}{\lambda}} \frac{f}{f'} = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \approx 1+\frac{v}{c}$$

$$f' = f \times \frac{1}{1+\frac{v}{c}}$$

The frequency that the mirror sees.

Now the frequency that the laser encounters:

$$f'' = f' \times \frac{1}{1+\frac{v}{c}} = f \times \left(\frac{1}{1+\frac{v}{c}}\right)^2$$

$$\rightarrow \Delta f = f''-f = f \times \left(\left(\frac{1}{1+\frac{v}{c}}\right)^2 - 1\right) = f \times \left(\frac{1}{1+\left(\frac{v}{c}\right)^2+2\frac{v}{c}} - 1\right)$$

$$= f \times \left(\frac{1-1-\left(\frac{v}{c}\right)^2-2\frac{v}{c}}{1+\left(\frac{v}{c}\right)^2+2\frac{v}{c}}\right) \xrightarrow{\left(\frac{v}{c}\right)^2 \ll 1} \left| f \times \left(\frac{-2\frac{v}{c}}{1+2\frac{v}{c}}\right)\right| \xrightarrow{\frac{v}{c}\ll 1} 2\frac{v}{c}f$$