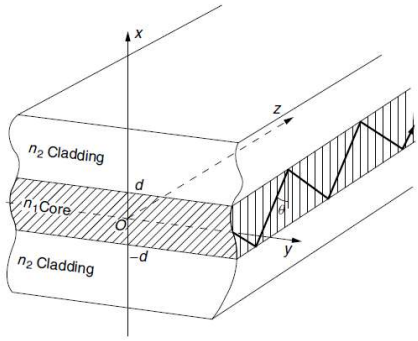
Q: Consider a flat waveguide in which the following magnetic and electric fields propagate in the z direction:

$$\mathbf{E} = \mathbf{E}_0(x, y)e^{j(\beta z - \omega t)}$$
$$\mathbf{H} = \mathbf{H}_0(x, y)e^{j(\beta z - \omega t)}$$

Show that for the correct operation of the waveguide (the light is limited inside the waveguide) the following relationship must hold:

$$\boldsymbol{n_1 k > \beta > n_2 k}$$



Sol: First, we pay attention to the **Helmholtz** equation:

$$\nabla^2 H + \left(n_{1,2}k\right)^2 H = 0 \rightarrow \frac{\partial^2 H}{\partial x^2} + \left(n_{1,2}^2 k^2 - \beta^2\right)H = 0$$

Now we have to check it in two environments, one in the core environment and the other in the cladding environment:

$$\begin{cases} Core: n_1^2 k^2 - \beta^2 = K^2 > 0 \;\; |x| < d \rightarrow \quad H = A * cosKx + B * sinKx \\ Cladding: n_2^2 k^2 - \beta^2 = -\gamma^2 < 0 \;\; |x| > d \rightarrow \quad H = C * e^{-\gamma x} + D * e^{\gamma x} \end{cases}$$

$$\begin{cases} Core: n_1^2 k^2 - \beta^2 > 0 \rightarrow n_1 k > \beta \\ Cladding: n_2^2 k^2 - \beta^2 < 0 \rightarrow n_2 k < \beta \end{cases} \rightarrow \boldsymbol{n_1 k > \beta > n_2 k}$$