Q: Two waves with the same amplitude, speed and frequency move together in a region of space. Brind's wave can be written as a sum of individual waves:

$$\psi(y,t) = Asin(ky + \omega t) + Asin(ky - \omega t + \pi)$$

Show using mixed exponentials

$$\psi(y,t) = 2Acos(ky)\sin(\omega t)$$

Sol:

$$\begin{aligned}\psi(y,t) = Asin(ky + \omega t) + Asin(ky - \omega t + \pi) &= A\left(Im\{e^{iky+} + e^{iky-\omega t+\pi}\}\right)\\ &= A\, Im\left\{e^{iky}\left(e^{i\omega t} - e^{-i\omega t}\right)\right\} = A\, Im\left\{e^{iky} * 2i\, sin(\omega t)\right\}\\ &= A\, Im\{(\cos(ky) + i\sin(ky)) * 2i\, sin(\omega t)\}\\ &= A\, Im\{\cos(ky) * 2i\, sin(\omega t) - \sin(ky) * 2sin(\omega t)\}\\ &= 2A\ \cos(ky) *\ sin(\omega t)\end{aligned}$$