

**Q:** Write the following plane waves in mixed representation. Using this representation, show that the summation of these waves is a standing wave given by the relation  $E_R = (2E_0 \sin(kx)) \cos(\omega t)$ .

$$E_1 = E_0 \sin(kx - \omega t)$$

$$E_2 = E_0 \sin(kx + \omega t)$$

**Sol:**

$$E_1 = E_0 \sin(kx - \omega t) = \text{Im}\{E_0 e^{i(kx - \omega t)}\}$$

$$E_2 = E_0 \sin(kx + \omega t) = \text{Im}\{E_0 e^{i(kx + \omega t)}\}$$

$$\begin{aligned} E_T = E_1 + E_2 &= \text{Im}\{E_0 e^{i(kx - \omega t)} + E_0 e^{i(kx + \omega t)}\} = \text{Im}\{E_0 e^{ikx} (e^{-i\omega t} + e^{i\omega t})\} \\ &= \text{Im}\{E_0 e^{ikx} \times 2\cos(\omega t)\} = 2E_0 \text{Im}\{(\cos(kx) + i\sin(kx)) \times \cos(\omega t)\} \\ &= 2E_0 \text{Im}\{\cos(kx) \cos(\omega t) + i\sin(kx) \cos(\omega t)\} = 2E_0 \sin(kx) \cos(\omega t) \end{aligned}$$