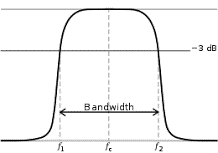
***Q:*** According to the Q-factor formula, which is as follows:

Prove that this formula can also be defined as:

***Sol:***

I will say an introduction that will be an interesting summary of this issue, we used the energy relationship to define the Q-factor in a way that we stated that the input energy becomes the Q-factor; Now, exactly the same problem can be expressed for frequency or wavelength, pay attention as follows:



Imagine that we sent a light with a frequency and we expected to have a sharp vertical line at the output, but it didn't, but it was shown with a bandwidth at the output, which is the beginning of our next definition.

Now let's analyze ourselves step by step:

**Q-factor in Terms of Energy**

The Q-factor is a measure of how efficiently a resonator stores energy compared to how much energy it loses. It can be expressed as:

- is the angular frequency of the resonant mode.

- is the energy stored in the resonator.

- is the rate at which energy is lost from the resonator.

**Q-factor and Frequency Bandwidth**

Another common way to express the Q-factor is in terms of the resonant frequency and the bandwidth of the resonance:

- is the resonant frequency.

- is the full width at half maximum (FWHM) of the resonance, representing the bandwidth over which the energy drops to half its peak value.

**Deriving the Relationship**

To relate these two expressions, we need to understand how energy decay translates to bandwidth. In a resonator, the stored energy decays exponentially over time as:

where is the resonant angular frequency.

The energy decay rate can be expressed as:

Thus, substituting this into the Q-factor definition, we get:

This confirms the consistency of the definition. Now, let's connect this to the frequency domain.

**Frequency Domain Perspective**

In the frequency domain, the Q-factor is defined as the ratio of the resonant frequency to the bandwidth

This comes from the relationship between energy storage and loss in the frequency domain. The bandwidth represents the range of frequencies over which the resonator effectively stores energy.

**Wavelength Perspective**

For light or electromagnetic waves, we often use wavelength instead of frequency. The relationship between frequency and wavelength is:

where is the speed of light. The bandwidth in terms of wavelength can be related to the bandwidth in terms of frequency

Therefore, the Q-factor in terms of wavelength is:

By understanding that:

- The Q-factor is a measure of energy stored versus energy lost.

- In the frequency domain, it is the ratio of the resonant frequency to the bandwidth.

- The relationship between frequency and wavelength allows us to express the Q-factor in terms of wavelength and its bandwidth.

Therefore, the Q-factor can equivalently be expressed as:

This shows the direct relationship between the energy decay perspective and the frequency (or wavelength) bandwidth perspective of the Q-factor.

***The concept was very interesting to me*** 😊