تمرین ۵ : $m_I \neq m_{II}$

الف) $E > U_0$ $\qquad$ $T \,,\, R$

ب) $E < U_0$ $\qquad$ $T \,,\, R$

T و R را بر حسب E رسم نمائید.

حل: الف) در تمرین قبل این مقادیر را حساب کرده‌ایم.

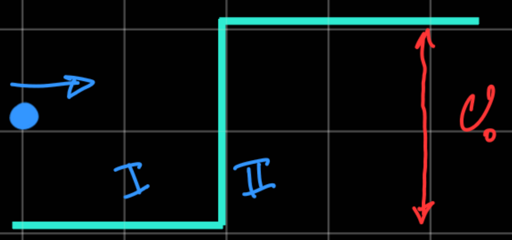
$$T = t^* t = \frac{4 k_I^2}{\left(k_I + k_{II}\frac{m_I}{m_{II}}\right)^2}$$

$$R = \frac{\left(1 - \frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2}{\left(1 + \frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2}$$

$$\text{Trans} = \frac{\frac{4 k_I k_{II}}{m_I m_{II}}}{\left(\frac{k_I}{m_I} + \frac{k_{II}}{m_{II}}\right)^2}$$

$$\text{Ref} = \frac{\left(1 - \frac{m_I}{m_{II}}\frac{k_{II}}{k_I}\right)^2}{\left(1 + \frac{m_I}{m_{II}}\frac{k_{II}}{k_I}\right)^2}$$

ب)



$$\mathrm{I})\quad \psi_I(x) = Ae^{+ik_I x} + Be^{-ik_I x}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow K_I = \sqrt{\frac{2m_I E}{\hbar^2}}$$

$$K_{II} = \sqrt{\frac{2m_{II}(E - V_0)}{\hbar^2}}$$

$$\mathrm{II})\quad \psi_{II}(x) = Ce^{+ik_{II}x} + De^{-ik_{II}x} \xRightarrow[K_{II}\ \text{موهومی}]{} \psi_{II}(x) = Ce^{-k_{II}x}$$

اعمال شرایط مرزی: تابع موج و مشتق آن در مرزها پیوسته باشه $m_I \neq m_{II}$

$$\begin{cases} A + B = C \\ \left(ik_I A - ik_I B\right)m_{II} = \left(-k_{II} C\right)m_I \end{cases}$$

ضریب عبور $t = \dfrac{C}{A}$ ضریب انعکاس $r = \dfrac{B}{A}$

احتمال عبور، گذار $T = t^* t = \dfrac{CC^*}{AA^*}$ احتمال انعکاس $R = r^* r = \dfrac{BB^*}{AA^*}$

$$\begin{cases} A \times ik_I + \cancel{B\, ik_I} = C \times ik_I \\ ik_I A - \cancel{ik_I B} = -k_{II} C \dfrac{m_I}{m_{II}} \end{cases}$$

$$2A\, ik_I = C\left(k_I i - k_{II}\frac{m_I}{m_{II}}\right)$$

$$t = \frac{C}{A} = \frac{2ik_I}{ik_I - k_{II}\frac{m_I}{m_{II}}} = \frac{2}{1 + i\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}}$$

$$T = tt^* = \frac{4}{\left(1 + i\underbrace{\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}}_{x}\right)\left(1 - i\underbrace{\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}}_{x}\right)}$$

$$= \frac{4}{1 - ix + ix + x^2} = \boxed{\frac{4}{1 + \left(\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2}}$$

$$\begin{cases} A \times k_{II} + B \times k_{II} = C \times \cancel{k_{II}} \\ \left(i k_I A - i k_I B \right) \times \dfrac{m_{II}}{m_I} = -\cancel{k_{II}}\, C \end{cases}$$

$$A\left(i k_I \frac{m_{II}}{m_I} + k_{II} \right) + B\left(-i k_I \frac{m_{II}}{m_I} + k_{II} \right) = 0$$

$$r = \frac{B}{A} = -\frac{i k_I \frac{m_{II}}{m_I} + k_{II}}{-i k_I \frac{m_{II}}{m_I} + k_{II}}$$

$$R = r r^* = \frac{k_{II} + i k_I \frac{m_{II}}{m_I}}{k_{II} - i k_I \frac{m_{II}}{m_I}} \times \frac{k_{II} - i k_I \frac{m_{II}}{m_I}}{k_{II} + i k_I \frac{m_{II}}{m_I}}$$

$$= \frac{k_{II}^2 + k_I^2 \left(\frac{m_{II}}{m_I} \right)^2}{k_{II}^2 + k_I^2 \left(\frac{m_{II}}{m_I} \right)^2} = 1 \quad \boxed{\checkmark}$$

□ چگالی جریان:

$$E < U_0$$

$$\psi_I(x) = A e^{+ik_I x} + B e^{-ik_I x}$$

$$\psi_{II}(x) = C e^{-k_{II} x}$$

$$j(x) = -i \frac{\hbar e}{2m}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right)$$

□ $$j_{input} = j_I = -\frac{ie\hbar}{2m_I}\Big[A^* e^{-ik_I x} \times \left(A\, ik_I e^{ik_I x}\right)$$

$$- A e^{+ik_I x} \times \left(-A^* ik_I e^{-ik_I x}\right)\Big]$$

$$= \frac{-ie\hbar}{2m_I}\left(ik_I |A|^2 + ik_I |A|^2\right) = \boxed{\frac{e\hbar k_I}{m_I}|A|^2}$$

□ $$j_R = -\frac{ie\hbar}{2m_I}\Big[B^* e^{+ik_I x} \times \left(-B\, ik_I e^{-ik_I x}\right)$$

$$- B e^{-ik_I x} \times \left(+B^* ik_I e^{+ik_I x}\right)\Big]$$

$$= \frac{-ie\hbar}{2m_I}\left[-ik_I BB^* - ik_I BB^*\right] = \boxed{\frac{-e\hbar}{m} k_I |B|^2}$$

III $$\dot{j}_T = -\frac{ie\hbar}{2m_{II}} \left[c e^{-k_{II}x} \times (-k_{II} c e^{-k_{II}x}) - c e^{-k_{II}x} \times (-k_{II} c e^{-k_{II}x}) \right]$$

$$= 0$$

$$Trans = \dot{j}_T / \dot{j}_I \qquad Ref = \dot{j}_R / \dot{j}_I$$

$$T + R = 1$$ current conservation

$$Trans = 0$$

$$Ref = 1$$

$$T + R = 1 \quad \boxed{\checkmark}$$

این نشان دهنده مفهوم تونل زنی و چون محیط دوم فقط لـ

یک تخن وجود ندارد.