تمرین 5 : $m_I \neq m_{II}$

حالت $E > V_0$ را حل کنید.

الف)

$$\psi_I(x)\Big|_{x=0} = \psi_{II}(x)\Big|_{x=0}$$

$$\frac{1}{m_I}\frac{d\psi_I(x)}{dx}\Big|_{x=0} = \frac{1}{m_{II}}\frac{d\psi_{II}(x)}{dx}\Big|_{x=0}$$

$$\begin{cases} A + B = C \\ (ik_I A - ik_I B)\,m_{II} = (ik_{II} C)\,m_I \end{cases} \quad (*)$$

ضریب عبور $t = \frac{C}{A}$ ضریب انعکاس $r = \frac{B}{A}$

احتمال عبور ذره $T = t^* t = \frac{CC^*}{AA^*}$ احتمال انعکاس $R = r^* r = \frac{BB^*}{AA^*}$

$$(*) \begin{cases} ik_I A + ik_I B = ik_I C \\ ik_I A - ik_I B = ik_{II} C \times \frac{m_I}{m_{II}} \end{cases}$$

$$\Rightarrow 2ik_I A = ik_I C\left(k_I + k_{II}\frac{m_I}{m_{II}}\right)$$

$$\Rightarrow \boxed{t = \frac{C}{A} = \frac{2k_I}{k_I + k_{II}\frac{m_I}{m_{II}}}} = \frac{2k_I m_{II}}{k_I m_{II} + k_{II} m_I}$$

$$T = t^* t = \frac{4k_I^2}{\left(k_I + k_{II}\frac{m_I}{m_{II}}\right)^2} = \frac{4\left(\frac{2m_I E}{\hbar^2}\right)}{\left[\sqrt{\frac{2m_I E}{\hbar^2}} + \sqrt{\frac{2m_{II}(E-V_0)}{\hbar^2}} \times \frac{m_I}{m_{II}}\right]^2}$$

$$(*) \begin{cases} -i k_{II} A - i k_{II} B = -i k_{II} C \\ \left(i k_I A - i k_I B\right)\frac{m_{II}}{m_I} = +i k_{II} C \end{cases} \Longrightarrow iA\left(k_I \frac{m_{II}}{m_I} - k_{II}\right) - iB\left(k_{II} + k_I \frac{m_{II}}{m_I}\right) = 0$$

$$r = \frac{B}{A} = \frac{k_I \frac{m_{II}}{m_I} - k_{II}}{k_{II} + k_I \frac{m_{II}}{m_I}}$$

$$R = \frac{B^* B}{A^* A} = \frac{\left(k_I \frac{m_{II}}{m_I} - k_{II}\right)^2}{\left(k_{II} + k_I \frac{m_{II}}{m_I}\right)^2}$$

همجرم کردن
توابع

$$\Rightarrow \begin{cases} T = \frac{4k_I^2}{(k_I + k_{II})^2} = \frac{4}{\left(1 + \frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2} \\ R = \frac{\left(1 - \frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2}{\left(1 + \frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2} \end{cases}$$

$$* \quad P_j = \hbar k_j = m_j v_j \rightarrow v_j = \frac{\hbar k_j}{m_j}$$

ساده‌تر کردن ⇒
توابع

$$T = \frac{4}{\left(1+\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2} = \frac{4}{\left(1+\frac{v_{II}\,m_{II}}{v_I\,m_I}\,\frac{m_I}{m_{II}}\right)^2} = \frac{4}{\left(1+\frac{v_{II}}{v_I}\right)^2}$$

$$R = \frac{\left(1-\frac{k_{II}\,m_I}{k_I\,m_{II}}\right)^2}{\left(1+\frac{k_{II}\,m_I}{k_I\,m_{II}}\right)^2} = \frac{\left(1-\frac{v_{II}\,m_{II}}{v_I\,m_I}\times\frac{m_I}{m_{II}}\right)^2}{\left(1+\frac{v_{II}\,m_{II}}{v_I\,m_I}\,\frac{m_I}{m_{II}}\right)^2} = \frac{\left(1-\frac{v_{II}}{v_I}\right)^2}{\left(1+\frac{v_{II}}{v_I}\right)^2}$$