تمرین 5 : $m_I \neq m_{II}$

اعمال شرط با $E>V_0$:

الف)

$$\psi_I(x)\Big|_{x=0} = \psi_{II}(x)\Big|_{x=0}$$

$$\frac{1}{m_I}\frac{d\psi_I(x)}{dx}\Big|_{x=0} = \frac{1}{m_{II}}\frac{d\psi_{II}(x)}{dx}\Big|_{x=0}$$

$$\begin{cases} A+B=C \\ (ik_I A - ik_I B)m_{II} = (ik_{II} C)\, m_I \quad (*) \end{cases}$$

ضریب عبور $t = \frac{C}{A}$ ضریب انعکاس $r = \frac{B}{A}$

احتمال عبور، تراگسیل $T = t^* t = \frac{CC^*}{AA^*}$ احتمال انعکاس $R = r^* r = \frac{BB^*}{AA^*}$

$$(*)\begin{cases} ik_I A + \cancel{ik_I B} = ik_I C \\ ik_I A - \cancel{ik_I B} = ik_{II} C \times \frac{m_I}{m_{II}} \end{cases} \Rightarrow 2\cancel{i}k_I A = \cancel{i}C\left(k_I + k_{II}\frac{m_I}{m_{II}}\right)$$

$$\Rightarrow \boxed{t = \frac{C}{A} = \frac{2k_I}{k_I + k_{II}\frac{m_I}{m_{II}}}} = \frac{2k_I m_{II}}{k_I m_{II} + k_{II} m_I}$$

$$\boxed{T = t^* t = \frac{4\,k_I^2}{\left(k_I + k_{II}\frac{m_I}{m_{II}}\right)^2}} = \frac{4\left(\frac{2m_I E}{\hbar^2}\right)}{\left[\sqrt{\frac{2m_I E}{\hbar^2}} + \sqrt{\frac{2m_{II}(E-V_0)}{\hbar^2}} \times \frac{m_I}{m_{II}}\right]^2}$$

$$(*)\begin{cases} -i k_{II} A - i k_{II} B = -i \cancel{k_{II} C} \\ \left(i k_I A - i k_I B\right)\frac{m_{II}}{m_I} = +i \cancel{k_{II} C} \end{cases} \Longrightarrow iA\left(k_I \frac{m_{II}}{m_I} - k_{II}\right) - iB\left(k_{II} + k_I \frac{m_{II}}{m_I}\right) = 0$$

$$\boxed{r = \frac{B}{A} = \frac{k_I \frac{m_{II}}{m_I} - k_{II}}{k_{II} + k_I \frac{m_{II}}{m_I}}}$$

$$\boxed{R = \frac{B^* B}{A^* A} = \frac{\left(k_I \frac{m_{II}}{m_I} - k_{II}\right)^2}{\left(k_{II} + k_I \frac{m_{II}}{m_I}\right)^2}}$$

سهم گرفتن
توابع
$$\Rightarrow \begin{cases} T = \frac{4\,k_I^2}{(k_I + k_{II})^2} = \frac{4}{\left(1 + \frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2} \\ R = \frac{\left(1 - \frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2}{\left(1 + \frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2} \end{cases}$$

$$* \quad P_j = \hbar k_j = m_j v_j \rightarrow v_j = \frac{\hbar k_j}{m_j}$$

مشاهده کن
توابع

$$\Rightarrow \begin{cases} T = \dfrac{4}{\left(1+\dfrac{k_{II}}{k_I}\dfrac{m_I}{m_{II}}\right)^2} = \dfrac{4}{\left(1+\dfrac{v_{II}\cancel{m_{II}}}{v_I\cancel{m_I}}\,\dfrac{\cancel{m_I}}{\cancel{m_{II}}}\right)^2} = \dfrac{4}{\left(1+\dfrac{v_{II}}{v_I}\right)^2} \\[3ex] R = \dfrac{\left(1-\dfrac{k_{II}\,m_I}{k_I\,m_{II}}\right)^2}{\left(1+\dfrac{k_{II}\,m_I}{k_I\,m_{II}}\right)^2} = \dfrac{\left(1-\dfrac{v_{II}\cancel{m_{II}}}{v_I\cancel{m_I}}\times\dfrac{\cancel{m_I}}{\cancel{m_{II}}}\right)^2}{\left(1+\dfrac{v_{II}\cancel{m_{II}}}{v_I\cancel{m_I}}\,\dfrac{\cancel{m_I}}{\cancel{m_{II}}}\right)^2} = \dfrac{\left(1-\dfrac{v_{II}}{v_I}\right)^2}{\left(1+\dfrac{v_{II}}{v_I}\right)^2} \end{cases}$$

تمرین ۵: $m_{I} \neq m_{II}$

الف) $E > U_0$ $\quad T , R$

ب) $E < U_0$ $\quad T , R$

T و R را به صورت E رسم نمایید.

حل: الف) در تمرین قبل این مقادیر را حساب کردیم.

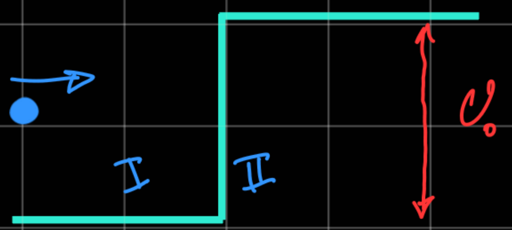
$$\boxed{T = t^* t = \frac{4 k_I^2}{\left(k_I + k_{II} \frac{m_I}{m_{II}}\right)^2}}$$

$$\boxed{R = \frac{\left(1 - \frac{k_{II}}{k_I} \frac{m_I}{m_{II}}\right)^2}{\left(1 + \frac{k_{II}}{k_I} \frac{m_I}{m_{II}}\right)^2}}$$

$$\boxed{Trans = \frac{\frac{4 k_I k_{II}}{m_I m_{II}}}{\left(\frac{k_I}{m_I} + \frac{k_{II}}{m_{II}}\right)^2}}$$

$$\boxed{Ref = \frac{\left(1 - \frac{m_I}{m_{II}} \frac{k_{II}}{k_I}\right)^2}{\left(1 + \frac{m_I}{m_{II}} \frac{k_{II}}{k_I}\right)^2}}$$

ب)



I) $\psi_I(x) = Ae^{+ik_I x} + Be^{-ik_I x}$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k_I = \sqrt{\frac{2m_I E}{\hbar^2}}$$

$$k_{II} = \sqrt{\frac{2m_{II}(E - U_0)}{\hbar^2}}$$

II) $\psi_{II}(x) = Ce^{+ik_{II} x} + De^{-ik_{II} x} \xrightarrow[k_{II}\text{ موهومی}]{} \psi_{II}(x) = Ce^{-k_{II} x}$

اعمال شرایط مرزی : تابع موج و مشتق آن در مرزها پیوسته باشند $m_I \neq m_{II}$

$$\begin{cases} A + B = C \\ (ik_I A - ik_I B)m_{II} = (-k_{II} C)m_I \end{cases}$$

ضریب عبور $t = \dfrac{C}{A}$ ضریب انعکاس $r = \dfrac{B}{A}$

احتمال عبور، گذار $T = t^* t = \dfrac{C C^*}{A A^*}$ احتمال انعکاس $R = r^* r = \dfrac{B B^*}{A A^*}$

$$\begin{cases} A \times ik_I + \cancel{B\, ik_I} = C \times i k_I \\[2ex] i k_I A - \cancel{i k_I}\, B = -k_{II}\, C \dfrac{m_I}{m_{II}} \end{cases}$$

$$2 A i k_I = C\left(k_I i - k_{II}\frac{m_I}{m_{II}}\right)$$

$$t = \frac{C}{A} = \frac{2 i k_I}{i k_I - k_{II}\frac{m_I}{m_{II}}} = \frac{2}{1 + i\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}}$$

$$T = t t^* = \frac{4}{\left(1 + i\underbrace{\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}}_{x}\right)\left(1 - i\underbrace{\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}}_{x}\right)}$$

$$= \frac{4}{1 - ix + ix + x^2} = \frac{4}{1 + \left(\frac{k_{II}}{k_I}\frac{m_I}{m_{II}}\right)^2}$$

$$\begin{cases} A \times k_{II} + B \times k_{II} = C \times \cancel{k_{II}} \\ \left(i k_I A - i k_I B \right) \times \dfrac{m_{II}}{m_I} = -\cancel{k_{II}} C \end{cases}$$

$$A \left(i k_I \frac{m_{II}}{m_I} + k_{II} \right) + B \left(-i k_I \frac{m_{II}}{m_I} + k_{II} \right) = 0$$

$$r = \frac{B}{A} = -\frac{i k_I \frac{m_{II}}{m_I} + k_{II}}{-i k_I \frac{m_{II}}{m_I} + k_{II}}$$

$$R = r r^* = \frac{k_{II} + i k_I \frac{m_{II}}{m_I}}{k_{II} - i k_I \frac{m_{II}}{m_I}} \times \frac{k_{II} - i k_I \frac{m_{II}}{m_I}}{k_{II} + i k_I \frac{m_{II}}{m_I}}$$

$$= \frac{k_{II}^2 + k_I^2 \left(\frac{m_{II}}{m_I} \right)^2}{k_{II}^2 + k_I^2 \left(\frac{m_{II}}{m_I} \right)^2} = 1$$

□ تعادی جریان:

$$E < U_0$$

$$\psi_I(x) = A e^{+ik_I x} + B e^{-ik_I x}$$

$$\psi_{II}(x) = C e^{-k_{II} x}$$

$$j(x) = -i\frac{\hbar e}{2m}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right)$$

□ $$j_{input} = j_I = -\frac{ie\hbar}{2m_I}\Big[A^* e^{-ik_I x} \times \left(A\, ik_I e^{ik_I x}\right)$$

$$- A e^{+ik_I x} \times \left(-A^* ik_I e^{-ik_I x}\right)\Big]$$

$$= \frac{-ie\hbar}{2m_I}\left(ik_I |A|^2 + ik_I |A|^2\right) = \boxed{\frac{e\hbar k_I}{m_I}|A|^2}$$

□ $$j_R = -\frac{ie\hbar}{2m_I}\Big[B^* e^{+ik_I x} \times \left(-B\, ik_I e^{-ik_I x}\right)$$

$$- B e^{-ik_I x} \times \left(+B^* ik_I e^{+ik_I x}\right)\Big]$$

$$= \frac{-ie\hbar}{2m_I}\left[-ik_I BB^* - ik_I BB^*\right] = \boxed{\frac{-e\hbar}{m} k_I |B|^2}$$

III $\dot{J}_T = -\frac{ie\hbar}{2m_{II}} \Big[c e^{-k_{II}x} \times (-k_{II} c e^{-k_{II}x}) - c e^{-k_{II}x} \times (-k_{II} c e^{-k_{II}x}) \Big]$

$= 0$

$\text{Trans} = \dot{J}_T / \dot{J}_I \qquad \text{Ref} = \dot{J}_R / \dot{J}_I$

$T + R = 1$ current conservation

$\text{Trans} = 0$

$\text{Ref} = 1$

$T + R = 1 \quad \boxed{\checkmark}$

این نشان دهنده مفهوم تونل زنی و چون محیط دوم که یک

یک ترخ وجود ندارد.

In this assignment, we have to draw the wave functions and the probability of presence, of course we print the energy values in each level. ($m_1 \neq m_2$)

It is noteworthy that the results and calculation items are stated in the previous part.

We have to consider two conditions:

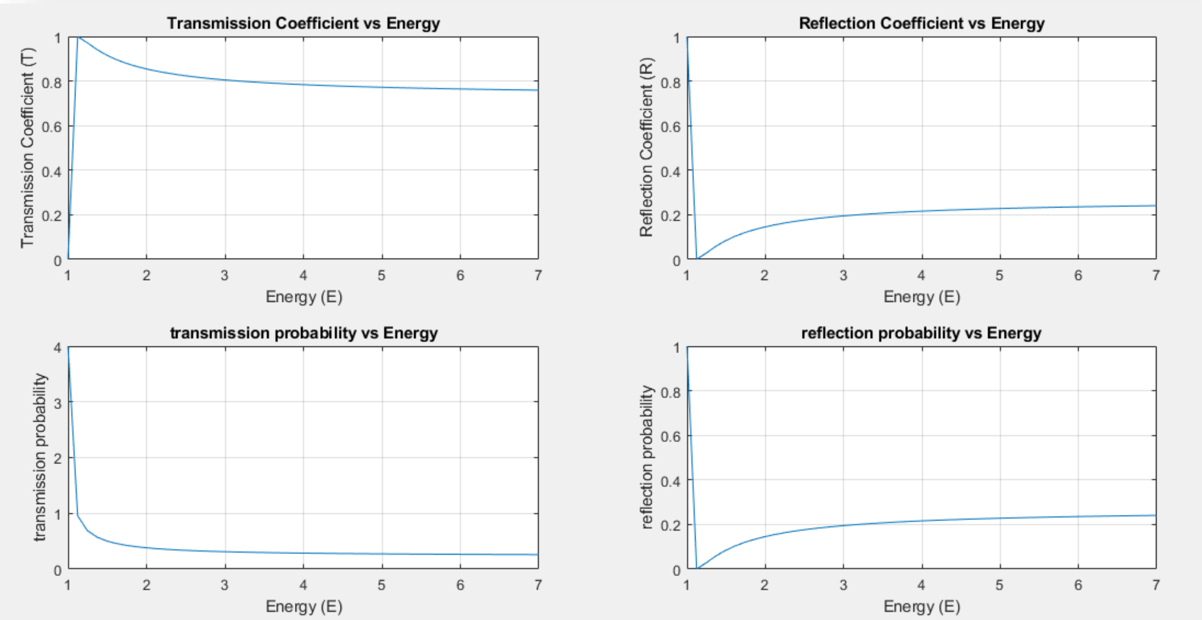
1) $E > U_0$

```
k1 = sqrt(2 * m1 * E / H^2);
k2 = sqrt(2 * m2 * (E - U0) / H^2);

% Formula transmission, reflection coefficient
Transmission = ((4 * k1 .* k2) / (m1 .* m2)) ./ ((k1 ./ m1) + (k2 ./ m2)).^2;
Reflection = 1 - Transmission;

T = 4 ./ (1 + k2 .* m1 ./ (k1 .* m2)).^2;
R = (1 - k2 * m1 ./ k1 * m2).^2 ./ (1 + k2 * m1 ./ k1 * m2).^2;
```

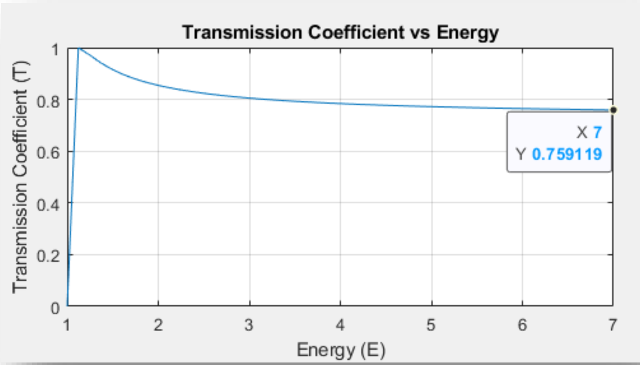
Enter the following data:

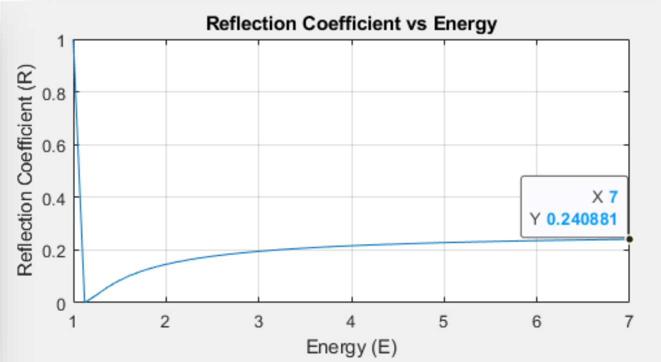
```
Enter the U0: 1
Enter the m1: 10
Enter the m2: 1
```

Now the results will be as follows:



As it is known, for Transmission, as much energy is transmitted (starting from U0) and it continues until the transmission becomes more, i.e. 1, now with the increase of energy, as discussed in the analytical part, it reaches a constant value and decreases.





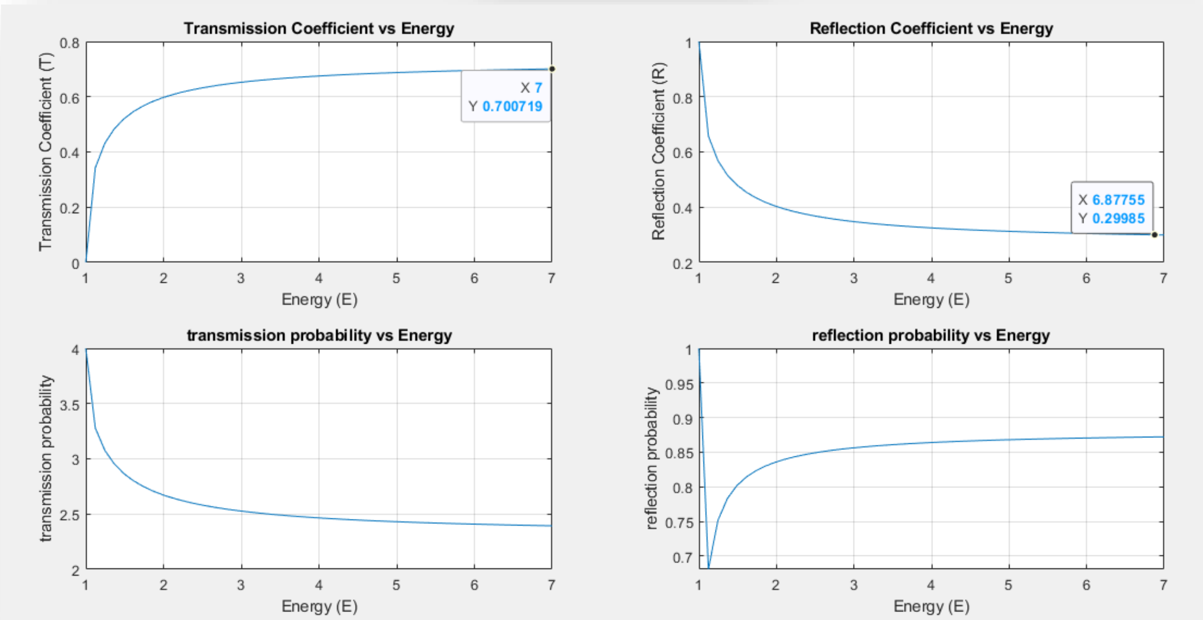
As it is known, for the Reflection function, it is its inverse because it follows the relationship

$$\boldsymbol{Transmission + Reflection = 1}$$

Two other graphs are displayed based on its formulas.

Now, if we put the mass values as below, the output will be as below, which is correct based on our analysis. (Because when the mass of the second medium increases, it will be difficult to reach more transmission mode)

```
Enter the U0: 1
Enter the m1: 1
Enter the m2: 10
```



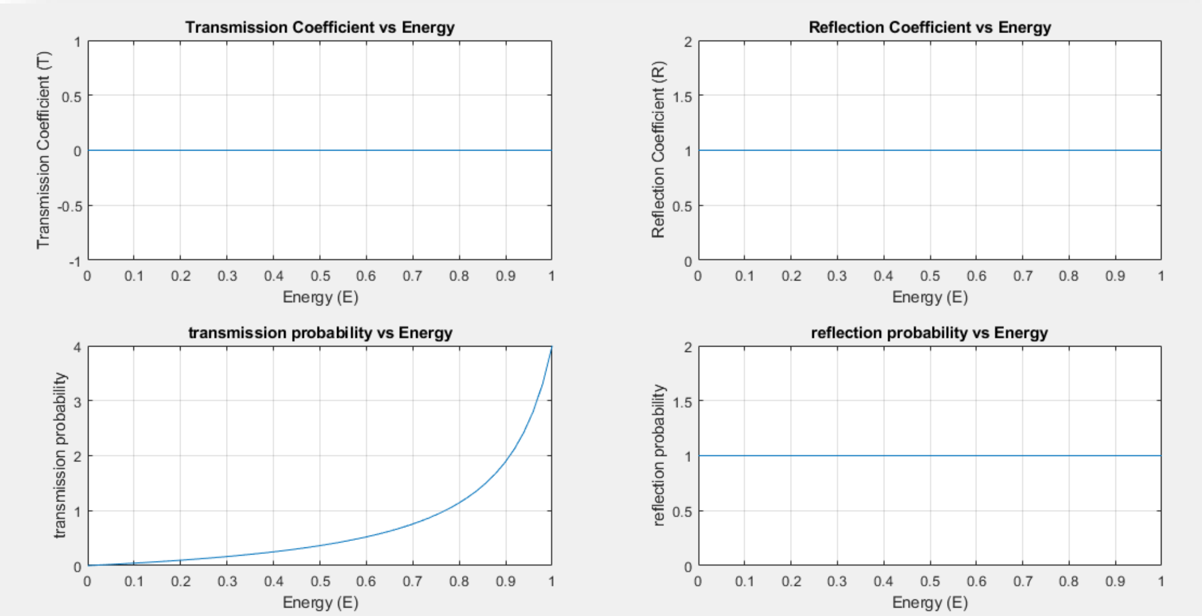
2) $E < U_0$

```
k1 = sqrt(2 * m1 * E / H^2);
k2 = sqrt(2 * m2 * (U0 - E) / H^2);

% Formula transmission, reflection coefficient
Transmission = 0;
Reflection = 1;

T = 4 ./ (1 + (k2 .* m1 ./ (k1 .* m2)).^2);
R = 1;
```

Enter the following data:

```
Enter the U0: 1
Enter the m1: 10
Enter the m2: 1
```

Now the results will be as follows:



As we checked it in the analytical part, because the flow of current will not be possible due to less energy and we will have a reverse flow.

As it is known, for the Reflection function, it is its inverse because it follows the relationship

$$\boldsymbol{Transmission + Reflection = 1}$$

Two other graphs are displayed based on its formulas.

Now, if we put the mass values as below, the output will be as below, which is correct based on our analysis. (Because when the mass of the second medium increases, it will be difficult to reach more transmission mode)

```
Enter the U0: 1
Enter the m1: 1
Enter the m2: 10
```