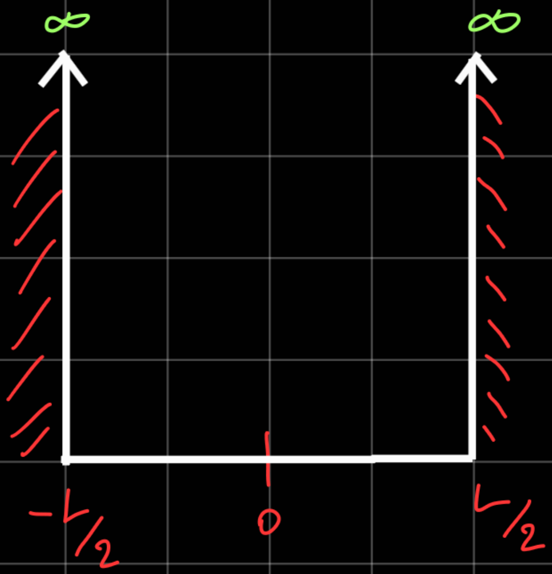
تمرین 2: برای چاه کوانتومی بی‌نهایت بالا که به صورت متقارن است توابع موج و انرژی ها

را بدست آورید و با نرم افزار متلب / رسم نمائید.



حل: !!!

$$U(x) = \begin{cases} 0 & -L/2 < x < L/2 \\ \infty & O.W \end{cases}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \cancelto{0}{U(x)}\,\psi(x) = E\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} + \underbrace{\boxed{\frac{2mE}{\hbar^2}}}_{k^2}\psi(x) = 0 \longrightarrow s^2 + k^2 = 0 \Rightarrow s = \pm ik$$

$* \quad \psi(x) = Ae^{ikx} + Be^{-ikx}$

$$* \quad e^{i\theta} = \cos\theta + i\sin\theta$$

$$\Longrightarrow \psi_{(x)} = A[\cos kx + i\sin kx] + B[\cos kx - i\sin kx]$$

$$= \underbrace{(A+B)}_{A'}\cos kx + i\underbrace{(A-B)}_{B'}\sin kx$$

$$= A'\cos kx + B'\sin kx$$

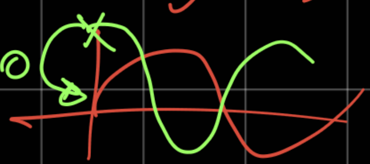
اعمال شرایط مرزی : تابع موج و مشتق تابع موج در مرزها باید پیوسته باشد.

$$\psi(x = -L/2) = 0 \Longrightarrow A'\cos\frac{kL}{2} - B'\sin\frac{kL}{2} = 0$$

$$\psi(x = +L/2) = 0 \Longrightarrow A'\cos\frac{kL}{2} + B'\sin\frac{kL}{2} = 0$$

$$\begin{cases} 1) \quad 2A'\cos kL/2 = 0 \rightarrow kL/2 = n\pi/2 \Rightarrow k = \dfrac{n\pi}{L} \Rightarrow n \text{ فرد} \\ 2) \quad 2B'\sin kL/2 = 0 \rightarrow kL/2 = \dfrac{n\pi}{2} \Rightarrow k = \dfrac{n\pi}{L} \Rightarrow n \text{ زوج} \end{cases}$$

* زوج cos نمی‌تواند 0 باشد چون اگر کسینوس بگیریم خطا خواهد شد.



0 یعنی خود cos باید 0 باشد.

n فرد

$$1)\quad E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = \frac{\hbar^2\pi^2}{2mL^2}n^2 = E_0 n^2$$

n زوج

$$2)\quad E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = \frac{\hbar^2\pi^2}{2mL^2}n^2 = E_0 n^2$$

ضریب A' = (نرمالیزاسیون) n فرد

$$\int_{-L/2}^{L/2} \psi_{(x)}^{*}\,\psi_{(x)}\,dx = 1$$

$$\int_{-L/2}^{L/2} A'^2 \sin^2\frac{n\pi}{L}x\,dx = 1 \rightarrow A'^2\int_{-L/2}^{L/2}\frac{1-\cos\frac{2\pi n}{L}x}{2}\,dx = 1$$

$$\Rightarrow A'^2\int_{-L/2}^{L/2}\frac{1}{2}\,dx = 1 \Rightarrow A'^2\times\frac{1}{2}\times L = 1 \Rightarrow \boxed{A' = \sqrt{\frac{2}{L}}}$$

ضریب B' = (نرمالیزاسیون) n زوج

$$\int_{-L/2}^{L/2} \psi_{(x)}^{*}\,\psi_{(x)}\,dx = 1$$

$$\int_{-L/2}^{L/2} B'^2 \sin^2\frac{n\pi}{L}x\,dx = 1 \rightarrow B'^2\int_{-L/2}^{L/2}\frac{1-\cos\frac{2\pi n}{L}x}{2}\,dx = 1$$

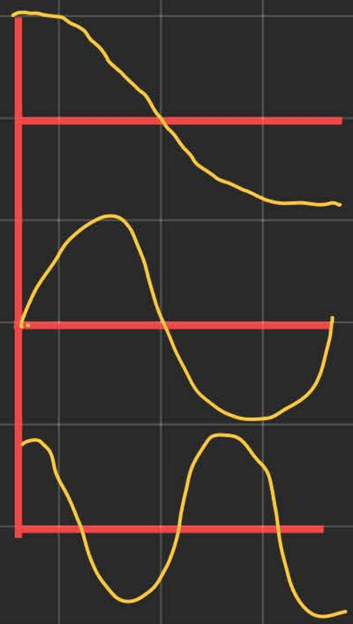
$$\Rightarrow B'^2\int_{-L/2}^{L/2}\frac{1}{2}\,dx = 1 \Rightarrow B'^2\times\frac{1}{2}\times L = 1 \Rightarrow \boxed{B' = \sqrt{\frac{2}{L}}}$$

$n = 1$ $\quad \psi_1(x) = \sqrt{\frac{2}{L}} \cos \frac{\pi}{L} x$ $\quad E_0$ $\quad \lambda = 2L$

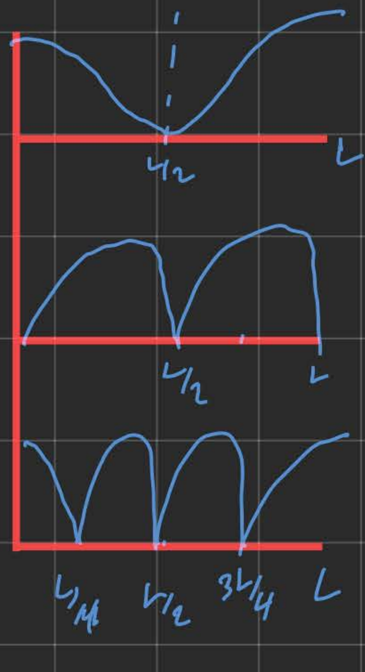
$n = 2$ $\quad \psi_2(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi}{L} x$ $\quad 4E_0$ $\quad \lambda = L$

$n = 3$ $\quad \psi_3(x) = \sqrt{\frac{2}{L}} \cos \frac{3\pi}{L} x$ $\quad 9E_0$ $\quad \lambda = L$

$\vdots \qquad \vdots \qquad \vdots$



$\psi^* \psi$



In this assignment, we have to draw the wave functions and the probability of presence, of course we print the energy values in each level. In $(-\frac{L}{2}, \frac{L}{2})$

It is noteworthy that the results and calculation items are stated in the previous part.

First, we observe the energy values for 4 levels:

```
% Calculate and print the expression E_n = E0 * userNumber^2
En_odd  = E0 * n^2;
En_even = E0 * n^2;
```

```
Enter a number of energy level: 4
En_odd_1 = E0
En_even_2 = 4*E0
En_odd_3 = 9*E0
En_even_4 = 16*E0
```

Now, based on what was discussed in the class, we drew the wave functions (of course, the code is also given below).

```
% Sine function with changing formula based on userNumber
y_odd  = sqrt(2/L)*cos(n*pi*x/L);
y_even = sqrt(2/L)*sin(n*pi*x/L);
```

```
% Create subplots
subplot(userNumber, 2, (n-1)*2 + 1);
fplot(y_even, [-L/2, L/2]);
title(['Wave Function, n = ' num2str(n)]);
xlabel('x');
ylabel(['sin(' num2str(n) '\pix/' num2str(L) ')']);
grid on;

subplot(userNumber, 2, (n-1)*2 + 2);
fplot(y_even^2, [-L/2, L/2]); % Plot y^2 for the product with the conjugate
title(['Probability of Attendance, n = ' num2str(n)]);
xlabel('x');
ylabel(['sin(' num2str(n) '\pix/' num2str(L) ')^2']);
grid on;
```

```
% Create subplots
subplot(userNumber, 2, (n-1)*2 + 1);
fplot(y_odd, [-L/2, L/2]);
title(['Wave Function, n = ' num2str(n)]);
xlabel('x');
ylabel(['cos(' num2str(n) '\pix/' num2str(L) ')']);
grid on;

subplot(userNumber, 2, (n-1)*2 + 2);
fplot(y_odd^2, [-L/2, L/2]); % Plot y^2 for the product with the conjugate
title(['Probability of Attendance, n = ' num2str(n)]);
xlabel('x');
ylabel(['cos(' num2str(n) '\pix/' num2str(L) ')^2']);
grid on;
```

As it is clear, since we have to get the probability, we have to use the $\int_0^L \psi\psi^*$ formula: