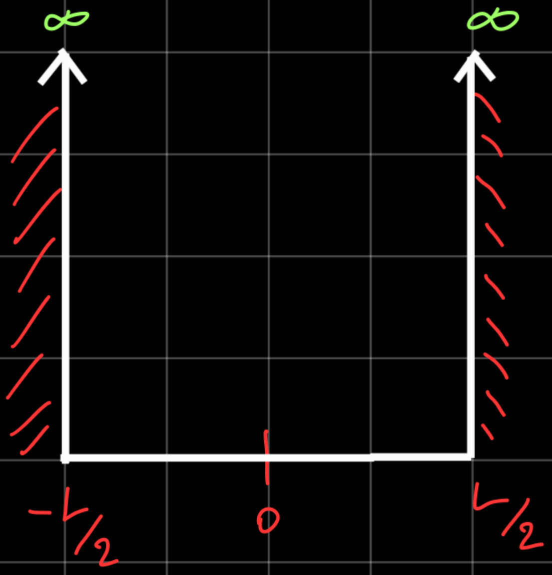
☐ تمرین 2: برای چاه کوانتومی بینهایت بالا که به صورت متقارن است توابع موج و انرژی ها
را بدست آورید و با نرم افزار متلب رسم نمایید.



حل: !!!

$$U(x) : \begin{cases} 0 & -L/2 < x < L/2 \\ \infty & O.W \end{cases}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \cancel{U(x)}^{\,0}\,\psi(x) = E\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} + \underbrace{\boxed{\frac{2mE}{\hbar^2}}}_{k^2}\psi(x) = 0 \longrightarrow s^2 + k^2 = 0 \Rightarrow s = \pm ik \;☑$$

$$* \quad \psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$* \quad e^{i\theta} = \cos\theta + i\sin\theta$$

$$\longrightarrow \psi_{(x)} = A[\cos kx + i\sin kx] + B[\cos kx - i\sin kx]$$

$$= \underbrace{(A+B)}_{A'}\cos kx + \underbrace{i(A-B)}_{B'}\sin kx$$

$$= A'\cos kx + B'\sin kx$$

اعمال شرایط مرزی = تابع موج و مشتق تابع موج در مرزها باید پیوسته باشد.

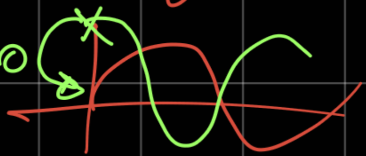
$$\psi(x = -L/2) = 0 \Longrightarrow A'\cos\frac{kL}{2} - B'\sin\frac{kL}{2} = 0$$

$$\psi(x = +L/2) = 0 \Longrightarrow A'\cos\frac{kL}{2} + B'\sin\frac{kL}{2} = 0$$

$$\begin{cases} 1)\quad 2A'\cos kL/2 = 0 \rightarrow kL/2 = n\,\pi/2 \Rightarrow k = \dfrac{n\pi}{L} \Rightarrow n \text{ فرد} \\ 2)\quad 2B'\sin kL/2 = 0 \rightarrow kL/2 = \dfrac{n\pi}{2} \Rightarrow k = \dfrac{n\pi}{L} \Rightarrow n \text{ زوج} \end{cases}$$

* زوج cos نمی‌تواند 0 باشد چون ? که دیگر هیچ معنا نخواهد.

0 پس فرد cos باید 0 باشد.



n زوج

1) $E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = \frac{\hbar^2 \pi^2}{2mL^2} n^2 = E_0 n^2$

n فرد

2) $E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 = \frac{\hbar^2 \pi^2}{2mL^2} n^2 = E_0 n^2$

ضریب A' = (نرمالیزاسیون) n زو

$$\int_{-L/2}^{L/2} \psi(x)^* \psi(x)\, dx = 1$$

$$\int_{-L/2}^{L/2} A'^2 \sin^2 \frac{n\pi}{L}x\, dx = 1 \rightarrow A'^2 \int_{-L/2}^{L/2} \frac{1-\cos\frac{2\pi n}{L}x}{2}\, dx = 1$$

$$\Rightarrow A'^2 \int_{-L/2}^{L/2} \frac{1}{2}\, dx = 1 \Rightarrow A'^2 \times \frac{1}{2} \times L = 1 \Rightarrow \boxed{A' = \sqrt{\frac{2}{L}}}$$

ضریب B' = (نرمالیزاسیون) n زوج

$$\int_{-L/2}^{L/2} \psi(x)^* \psi(x)\, dx = 1$$

$$\int_{-L/2}^{L/2} B'^2 \sin^2 \frac{n\pi}{L}x\, dx = 1 \rightarrow B'^2 \int_{-L/2}^{L/2} \frac{1-\cos\frac{2\pi n}{L}x}{2}\, dx = 1$$

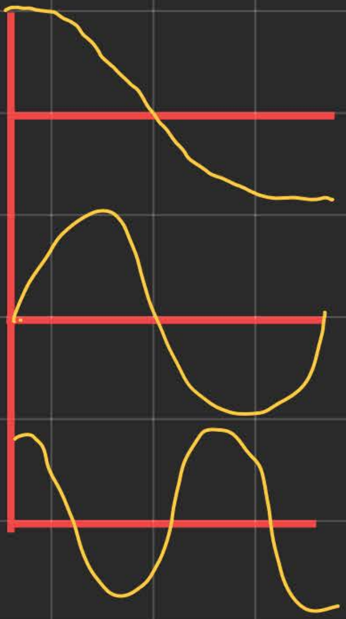
$$\Rightarrow B'^2 \int_{-L/2}^{L/2} \frac{1}{2}\, dx = 1 \Rightarrow B'^2 \times \frac{1}{2} \times L = 1 \Rightarrow \boxed{B' = \sqrt{\frac{2}{L}}}$$

$n = 1$ $\psi_1(x) = \sqrt{\frac{2}{L}} \cos \frac{\pi}{L} x$ E_0

$n = 2$ $\psi_2(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi}{L} x$ $4 E_0$

$n = 3$ $\psi_3(x) = \sqrt{\frac{2}{L}} \cos \frac{3\pi}{L} x$ $9 E_0$

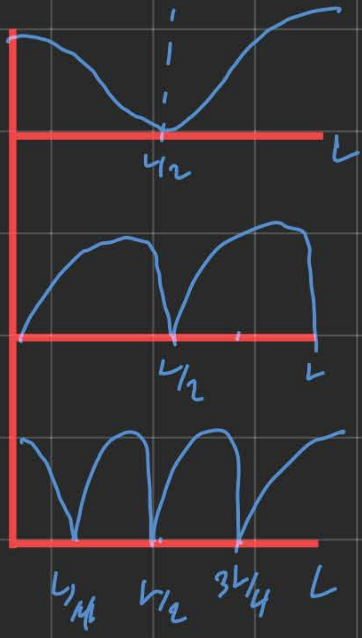
⋮ ⋮ ⋮



$\lambda = 2L$

$\lambda = L$

$\lambda = L$



$\psi^* \psi$