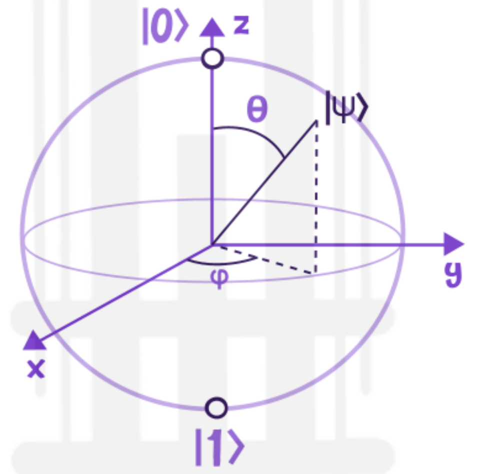
VECTORS AND INTRO TO MATRICES

MORE VECTORS

WHAT DO VECTORS MEAN FOR Q.COMP?



Qubits are two-level quantum systems that lie in the Bloch Sphere and their states can be represented as vectors

$$\vec{\psi} = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix}$$

SHAPES(VECTORS)

ROWS

COLUMNS

Vector shape

(#rows x #columns)

★ column vector: (n x 1)

★ row vector: (1 x m)

VECTOR TRASPOSE

The traspose is an operation which flips the shape of a vector

It does not change anything about the vector geometrically, just changes the shape

If $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$ then its traspose is $\vec{v}^T = (v_1 \ v_2 \cdots v_n)$

If $\vec{\omega} = (\omega_1 \, \omega_2 \cdots \omega_n)$ then its traspose is $\vec{\omega}^T = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix}$

THE INNER PRODUCT

The traspose is an operation which flips the shape of a vector

$$\langle \vec{v}, \vec{\omega} \rangle = \vec{v}\,\vec{\omega}^T = \sum_{i=1}^{n} v_i \omega_i$$

where $\vec{v}, \vec{\omega} \in R^n$ are row vectors (scalar product/dot product)

It is a:

VECTOR TO SCALAR MAPPING

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \ \vec{\omega} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

$$\langle \vec{v}, \vec{\omega} \rangle = \vec{v}^T \vec{\omega}$$

$$= (v_1 \ v_2 \ v_3) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = v_1\omega_1 + v_2\omega_2 + v_3\omega_3$$

$$= \sum_{i=1}^{3} v_i \omega_i$$

CALCULATING VECTOR MAGNITUDE

$$\langle \vec{v}, \vec{v} \rangle = \vec{v}\,\vec{v}^T = \sum_{i=1}^{n} v_i v_i = \sum_{i=1}^{n} v_i^2 = \|\vec{v}\|^2, \ \vec{v} \in R^n$$

$$\therefore \|\vec{v}\| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

The inner product of a vector with itself gives us the magnitude squared of the vector

VECTOR ORTHOGONALITY

Let's consider some possibilites when $\vec{x}$ and $\vec{y}$ are unit vectors

PARALLEL | ORTHOGONAL | ANTI-PARALLEL

$\theta = 0^\circ$, $\langle \vec{x}, \vec{y} \rangle = 1$

$\theta = 90^\circ$, $\langle \vec{x}, \vec{y} \rangle = 0$

$\theta = 180^\circ$, $\langle \vec{x}, \vec{y} \rangle = -1$

VECTOR NORMALIZATION

$$\frac{\vec{v}}{\sqrt{\langle \vec{v}, \vec{v} \rangle}} = \frac{\vec{v}}{\|\vec{v}\|}$$

A vector is normalized if it has a magnitude of 1 (unit vector)

GEOMETRICALLY COMPARING VECTORS

$$\theta = \cos^{-1}\left(\frac{\langle \vec{x}, \vec{y} \rangle}{\|\vec{x}\|\|\vec{y}\|} \right)$$

$$\langle \vec{x}, \vec{y} \rangle = \|\vec{x}\|\|\vec{y}\|\cos\theta$$

where $\theta = \angle(\vec{x}, \vec{y})$ is the angle between $\vec{x}$ and $\vec{y}$

CONJUGATE TRASPOSE

$$\vec{v}^\dagger = \left(\vec{v}^T\right)^* = \left(\vec{v}^*\right)^T$$

THE COMPLEX INNER PRODUCT

$$\langle \vec{v}, \vec{\omega} \rangle = \vec{v}^\dagger \vec{\omega} = \sum_{i=1}^{n} v_i^* \omega_i$$, where $\vec{v}, \vec{\omega} \in C^n$

LINEAR COMBINATIONS

A linear combination of a set terms is simply the addition of those terms multiplied by scalar coefficients

In the case of vectors, a linear combination is simply a weighted sum of vectors

$$\vec{v} = a_1\vec{v}_1 + a_2\vec{v}_2 + a_n\vec{v}_n = \sum_{i=1}^{n} a_i \vec{v}_i$$

In the case of quantum states, a superposition is simply a linear combination of quantum states

$$\frac{1}{\sqrt{2}}\left|\text{ALIVE}\right\rangle + \frac{1}{\sqrt{2}}\left|\text{DEAD}\right\rangle$$