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Quantum Coding Course

Ali Kookani
Yousef Mafi

www.quantumatlas.ir QuantumSTEM

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About us

Quantum Atlas is an educational group which aims to educate people in various fields of quantum, from hardware to software and quantum machine learning.

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QuantumSTEM

Quantum Atlas

Syllabus

Section 1	**Lecture 1**	Quantum Computation and Information (Theoretical lecture) – By Y. Mafi
	Lecture 2	Quantum Circuits (Coding lecture) – By A. Kookani
Section 2	**Lecture 3**	Quantum Simulation (Coding lecture) – By A. Kookani
	Lecture 4	IBMQ and Error Correction (Implementation and Theoretical lecture) – By Y. Mafi
Section 3	**Lecture 5**	Quantum Algorithm (Theoretical lecture) – By Y. Mafi
	Lecture 6	Quantum Algorithm Simulation (Coding lecture) – By A. Kookani

Today

01
Classic Computation

02
Quantum Bit (Qubit)

03
Quantum Operation

04
Quantum Circuit

01

Classic Computation

Logic Bit

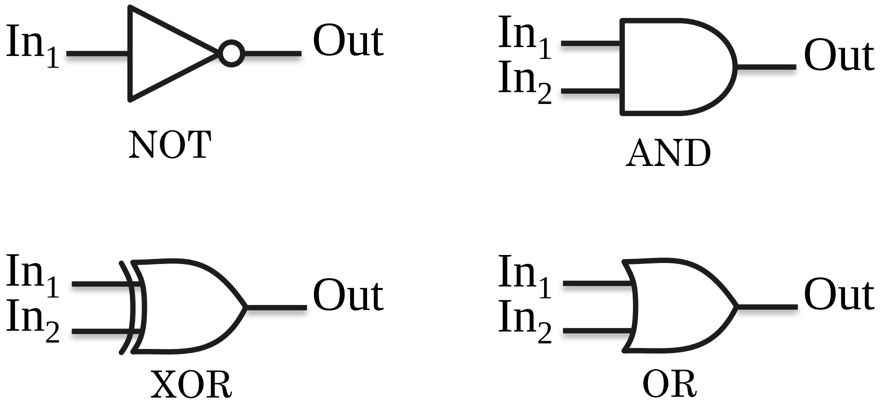
Logic bits are the fundamental units of information in digital systems, representing the smallest possible data value.

A logic bit can have one of two states:

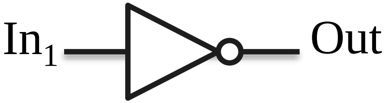
- 0 (Low): Typically represents the "off" state, false condition, or absence of a signal.
- 1 (High): Typically represents the "on" state, true condition, or presence of a signal.

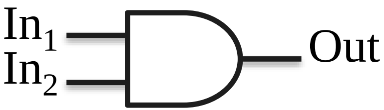
Logic gate

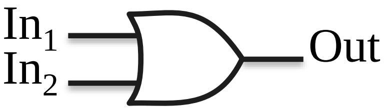
Logic gates are the building blocks of digital circuits. They are electronic devices that perform basic logical functions on one or more logic bits (binary inputs) and produce a single binary output. Each gate implements a specific logical operation.



Logic gate





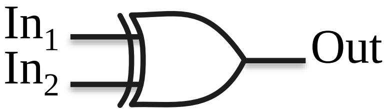


NOT	
In_1	Out
A	$\bar{A}$
0	1
1	0

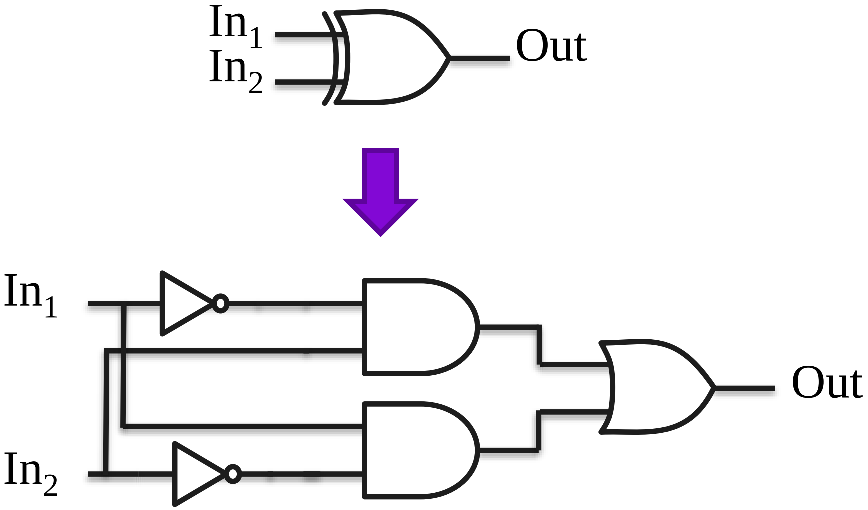
AND		
In_1	In_2	Out
A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

OR		
In_1	In_2	Out
A	B	$A + B$
0	0	0
0	1	1
1	0	1
1	1	1

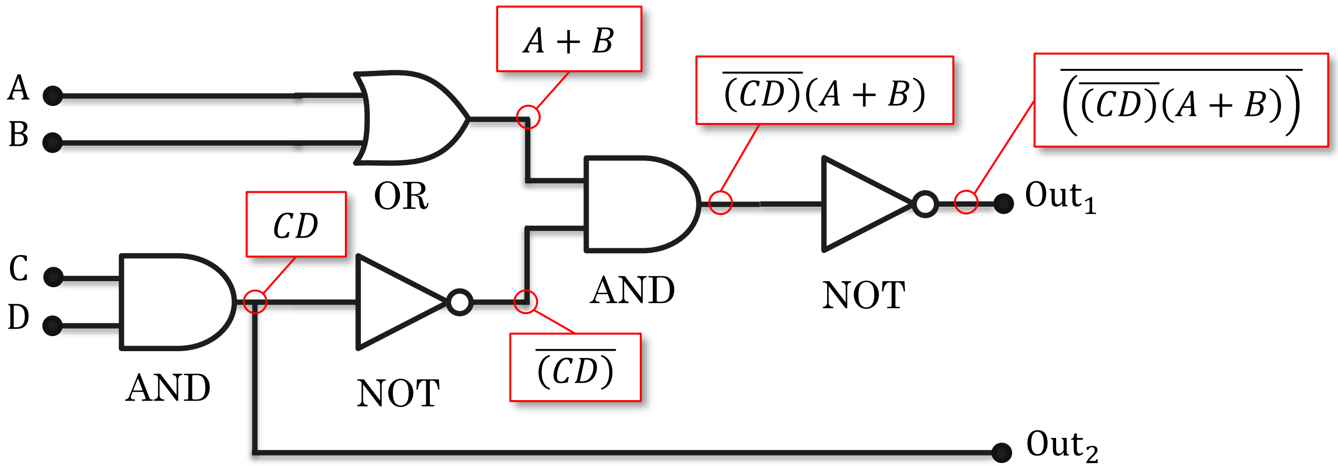
Logic gate



XOR		
In_1	In_2	Out
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

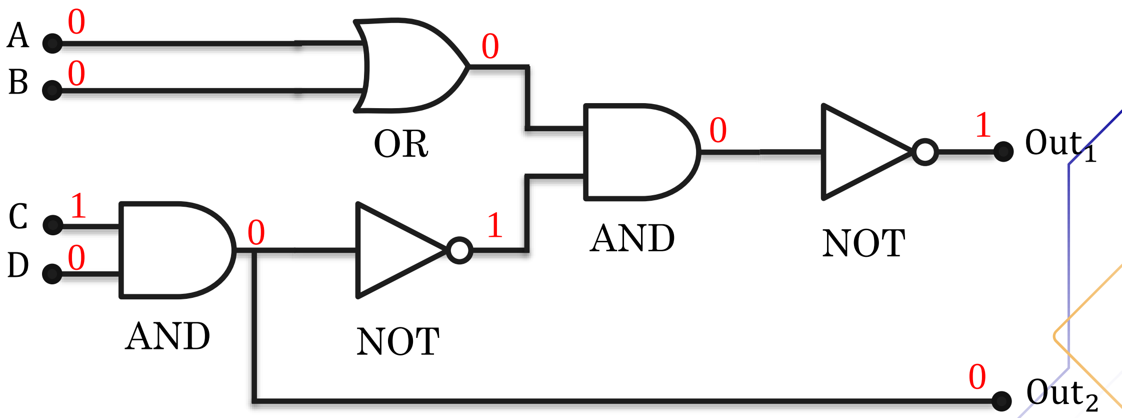


Logic circuit



Logic circuit

Inputs				Outputs	
A	B	C	D	Out_1	Out_2
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	1	1
…	…	…	…	…	…
1	1	1	1	1	1



Binary data

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Decimal representation:

$$463 = 400 + 60 + 3$$
$$= 4 \times 10^2 + 6 \times 10^1 + 3 \times 10^0$$

Binary representation:

$$13 = 1101$$
$$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 8 + 4 + 0 + 1$$

$$\begin{array}{r|l} 13 & 2 \\ -12 & 6 \\ 1 & \end{array} \quad \begin{array}{r|l} 6 & 2 \\ -6 & 3 \\ 0 & \end{array} \quad \begin{array}{r|l} 3 & 2 \\ -2 & 1 \\ 1 & \end{array}$$

$\Downarrow$

13 = 1101

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

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02

Quantum Bit (Qubit)

Qubit

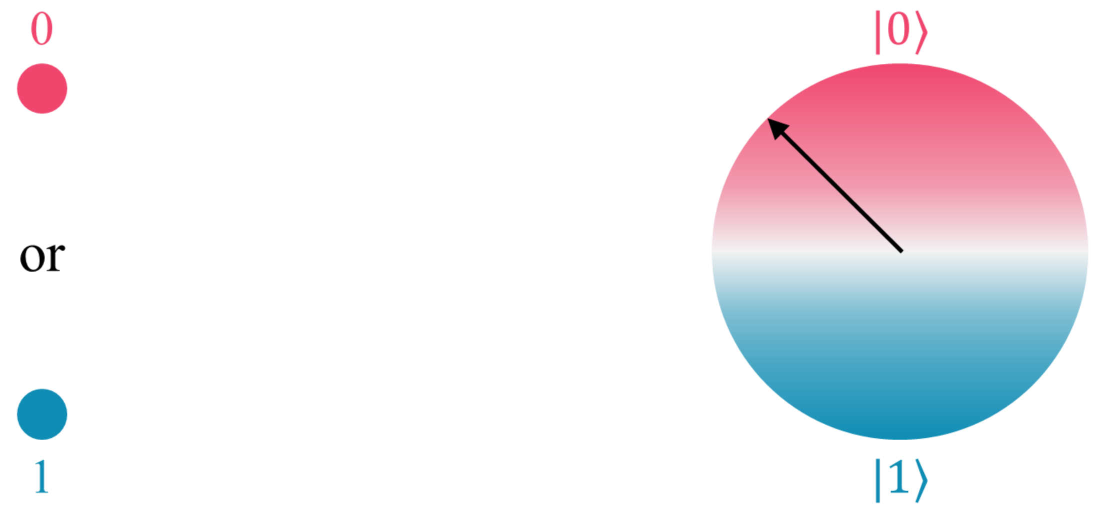
A qubit (quantum bit) is the fundamental unit of quantum information, analogous to the classical bit in traditional computing. Unlike a classical bit, which can be either 0 or 1, a qubit can exist in a superposition of both states simultaneously.

Types of Qubits in the Real World:

- Superconducting Qubits (Transmon qubits)
- Trapped Ion Qubits (Ions of elements like calcium or ytterbium)
- Topological Qubits (Based on anyons in 2D materials)
- Photonic Qubits (Single photons)
- Spin Qubits (Electrons in quantum dots)

Qubit

Classic Computer | Quantum Computer



Use Bits
(0's and 1's)

Use Qubits
(can be 0 and 1 at the same time)

Vector state

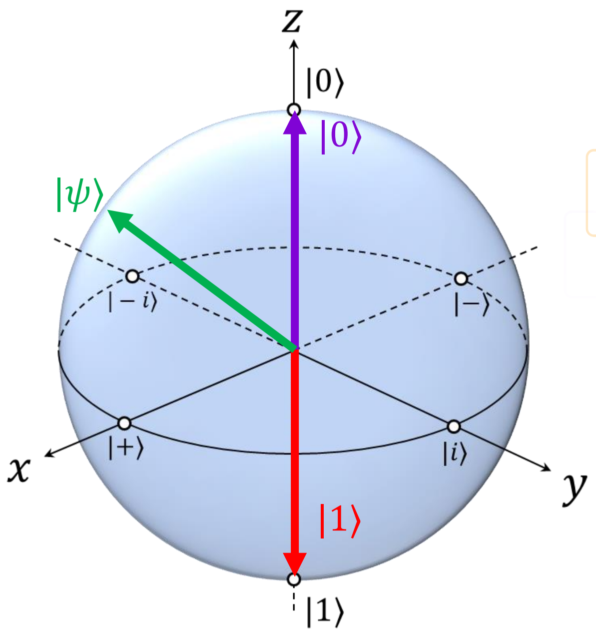
We still use 0's and 1's, but as vector states:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

A Qubit is in superposition if it is both $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow |\psi\rangle = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \alpha|0\rangle + \beta|1\rangle$$ Dirac Notation



Bloch sphere

Phase

Phase - Rotation around the z-axis by φ radians

$$|\psi\rangle = \alpha|0\rangle + e^{i\varphi}\beta|1\rangle$$

Phase type:

$$|\psi\rangle = e^{i\theta}(\alpha|0\rangle + e^{i\varphi}\beta|1\rangle)$$

Global Phase

Ignore!!!

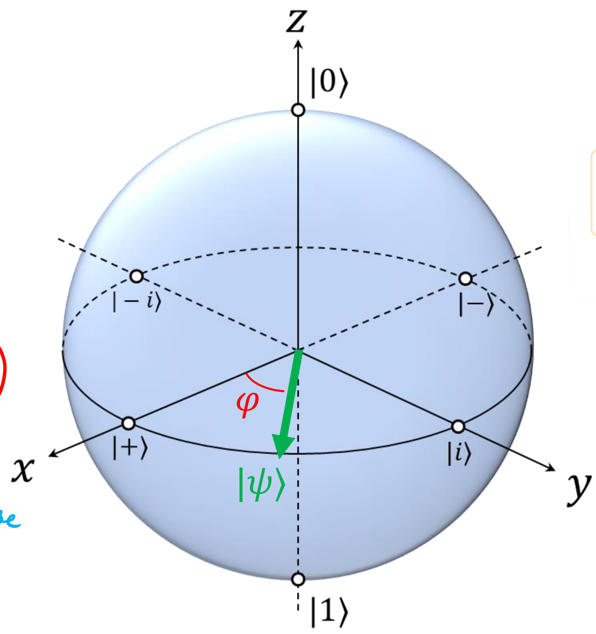
Relative Phase

$|\psi\rangle = e^{i\theta}\alpha|0\rangle + e^{i\phi}\beta|1\rangle$

$\Rightarrow |\psi\rangle = e^{i\theta}(\alpha|0\rangle + e^{i(\phi-\theta)}\beta|1\rangle)$

global phase

relative phase



Bloch sphere

Multi-qubit system

What is tensor product?

$$A = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$A \otimes B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}_{2\times1} \otimes \begin{bmatrix} 5 \\ 4 \end{bmatrix}_{2\times1} = \begin{bmatrix} 3 \begin{bmatrix} 5 \\ 4 \end{bmatrix} \\ 2 \begin{bmatrix} 5 \\ 4 \end{bmatrix} \end{bmatrix}_{4\times1} = \begin{bmatrix} 15 \\ 12 \\ 10 \\ 8 \end{bmatrix}_{4\times1}$$

Multi-qubit system

Vector state of two-qubit quantum system:

$|q_0\rangle$: First qubit $\quad$ $|q_1\rangle$: Second qubit

$$|\psi\rangle = |q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$$

$$|q_0\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \begin{bmatrix}\alpha_0\\ \alpha_1\end{bmatrix}$$

$$|q_1\rangle = \alpha_2|0\rangle + \alpha_3|1\rangle = \begin{bmatrix}\alpha_2\\ \alpha_3\end{bmatrix}$$

$$\Rightarrow \quad |\psi\rangle = \begin{bmatrix}\alpha_0\\ \alpha_1\end{bmatrix}_{2\times 1} \otimes \begin{bmatrix}\alpha_2\\ \alpha_3\end{bmatrix}_{2\times 1} = \begin{bmatrix}\alpha_0\begin{bmatrix}\alpha_2\\ \alpha_3\end{bmatrix}\\ \alpha_1\begin{bmatrix}\alpha_2\\ \alpha_3\end{bmatrix}\end{bmatrix}_{4\times 1} = \begin{bmatrix}\alpha_0\alpha_2\\ \alpha_0\alpha_3\\ \alpha_1\alpha_2\\ \alpha_1\alpha_3\end{bmatrix}_{4\times 1}$$

Multi-qubit system

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Vector state of two-qubit quantum system:

$|q_0\rangle$: First qubit $\quad$ $|q_1\rangle$: Second qubit

$$|\psi\rangle = |q_0\rangle \otimes |q_1\rangle = |q_0 q_1\rangle$$

$$|q_0\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \begin{bmatrix}\alpha_0\\ \alpha_1\end{bmatrix}$$

$$|q_1\rangle = \alpha_2|0\rangle + \alpha_3|1\rangle = \begin{bmatrix}\alpha_2\\ \alpha_3\end{bmatrix}$$

$$|\psi\rangle = (\alpha_0|0\rangle + \alpha_1|1\rangle) \otimes (\alpha_2|0\rangle + \alpha_3|1\rangle)$$

$$= \alpha_0\alpha_2|\overset{q_0q_1}{00}\rangle + \alpha_0\alpha_3|\overset{q_0q_1}{01}\rangle + \alpha_1\alpha_2|\overset{q_0q_1}{10}\rangle + \alpha_1\alpha_2|\overset{q_0q_1}{11}\rangle$$

Multi-qubit system

Vector state of multi-qubit quantum system:

$|q_0\rangle$: First qubit

$|q_1\rangle$: First qubit

$\vdots$

$|q_{n-1}\rangle$: n$^{\text{th}}$ qubit

$$|\psi\rangle = |q_0\rangle \otimes |q_1\rangle \otimes \cdots \otimes |q_{n-1}\rangle = |q_0 q_1 \dots q_{n-1}\rangle$$

Example: $|\psi\rangle = |0\rangle \otimes |1\rangle \otimes \cdots \otimes |0\rangle = |01 \dots 0\rangle$

Example: $|\psi\rangle = \sum_{x=0}^{2^n-1} \alpha_x |x\rangle$??? Superposition form

Multi-qubit system

$$|\psi\rangle = \sum_{x=0}^{2^n-1} \alpha_x |x\rangle$$

Assume $n = 3 \rightarrow$ $|\psi\rangle = \sum_{x=0}^{2^3-1} \alpha_x |x\rangle \rightarrow \sum_{x=0}^{7} \alpha_x |x\rangle$

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle + \alpha_4|4\rangle + \alpha_5|5\rangle + \alpha_6|6\rangle + \alpha_7|7\rangle$$

3-bit binary representation ➡ $|\psi\rangle = \alpha_0|000\rangle + \alpha_1|001\rangle + \alpha_2|010\rangle + \alpha_3|011\rangle + \alpha_4|100\rangle + \alpha_5|101\rangle + \alpha_6|110\rangle + \alpha_7|111\rangle$

03

Quantum Operation

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Quantum single-qubit gate

X Gate

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Hadamard Gate

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Y Gate

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

S Gate

$$S = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix}$$

Z Gate

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

T Gate

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

Quantum single-qubit gate

The X-gate flips the qubit π radians around the x-axis on the Bloch Sphere

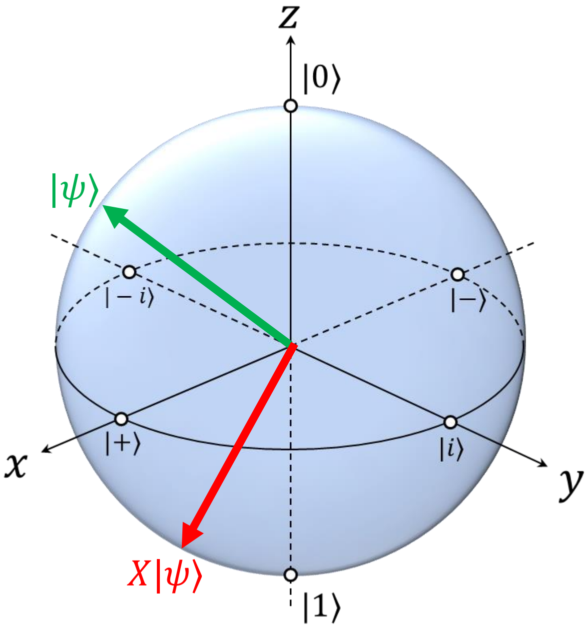
X Gate

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \qquad |0\rangle \xrightarrow{X} |1\rangle$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \qquad |1\rangle \xrightarrow{X} |0\rangle$$

$$\begin{aligned} X|\psi\rangle = X(\alpha|0\rangle + \beta|1\rangle) &= \alpha X|0\rangle + \beta X|1\rangle \\ &= \alpha|1\rangle + \beta|0\rangle \end{aligned}$$



Quantum single-qubit gate

The H-gate changes superposition basis.

Hadamard Gate

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = |+\rangle$$

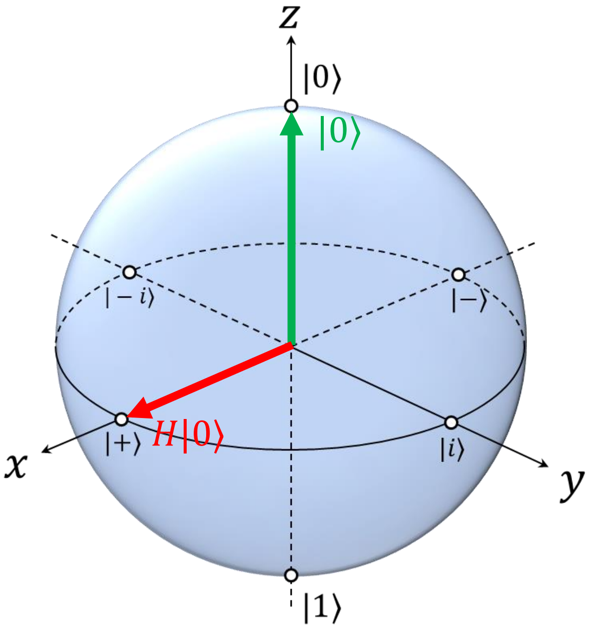
$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = |-\rangle$$

$$|0\rangle \xrightarrow{H} |+\rangle \qquad |1\rangle \xrightarrow{H} |-\rangle$$

$$H|\psi\rangle = H(\alpha|0\rangle + \beta|1\rangle) = \alpha H|0\rangle + \beta H|1\rangle$$

$$= \alpha|+\rangle + \beta|-\rangle$$

X-basis: $\{|+\rangle, |-\rangle\}$



Quantum single-qubit gate

The S-gate Adds a relative phase of $e^{\frac{i\pi}{2}}$

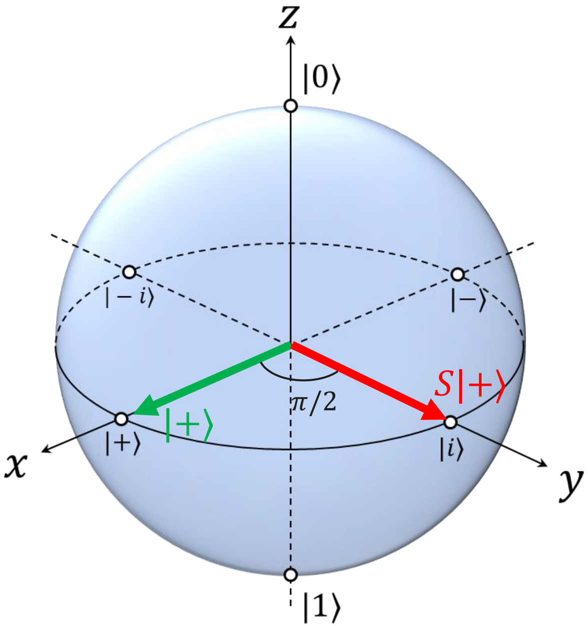
S Gate

$$S = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix}$$

$$S|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ e^{i\pi/2}\beta \end{bmatrix} = \alpha|0\rangle + e^{i\pi/2}\beta|1\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{S} \alpha|0\rangle + e^{i\pi/2}\beta|1\rangle$$

$$S|\psi\rangle = S(\alpha|0\rangle + \beta|1\rangle) = \alpha S|0\rangle + \beta S|1\rangle$$
$$= \alpha|1\rangle + e^{i\pi/2}\beta|0\rangle$$



Unitary operation

A linear transformation U is unitary if

$$U^\dagger U = I$$

where $U^\dagger$ is the conjugate transpose and I is the identity matrix.

- Key Properties:
  1. Norm Preservation: $||U\boldsymbol{v}|| = ||\boldsymbol{v}||$ (lengths of vectors remain unchanged).
  2. Inner Product Preservation: $\langle U\boldsymbol{u}, U\boldsymbol{v}\rangle = \langle \boldsymbol{u}, \boldsymbol{v}\rangle$ (angles between vectors are preserved).
  3. Reversibility: $U^{-1} = U^\dagger$ (unitary transformations are reversible).
  4. Determinant: $|\det(U)| = 1$ (volume preservation).

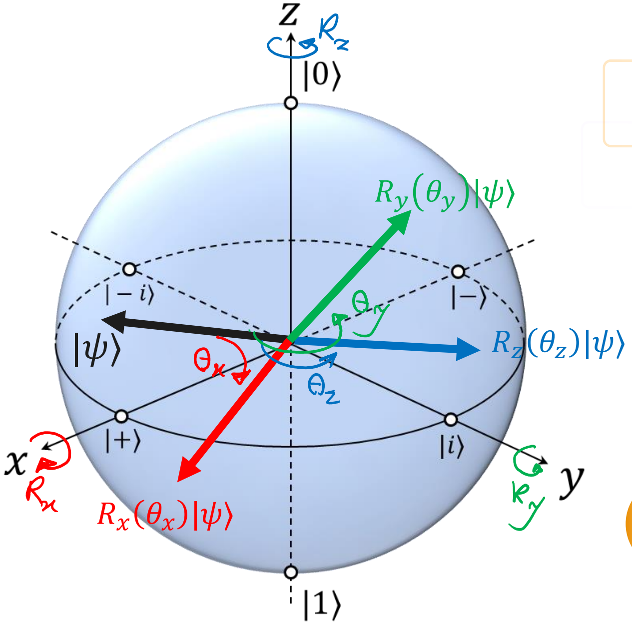
Rotation gate

The rotation operators about the x, y, and z axes, defined by the equations:

$$R_x(\theta_x) \equiv e^{-\frac{i\theta_x X}{2}} = \cos\frac{\theta_x}{2} I - i\sin\frac{\theta_x}{2} X = \begin{bmatrix} \cos\frac{\theta_x}{2} & -i\sin\frac{\theta_x}{2} \\ -i\sin\frac{\theta_x}{2} & \cos\frac{\theta_x}{2} \end{bmatrix}$$

$$R_y(\theta_y) \equiv e^{-\frac{i\theta_y Y}{2}} = \cos\frac{\theta_y}{2} I - i\sin\frac{\theta_y}{2} Y = \begin{bmatrix} \cos\frac{\theta_y}{2} & -\sin\frac{\theta_y}{2} \\ -i\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{bmatrix}$$

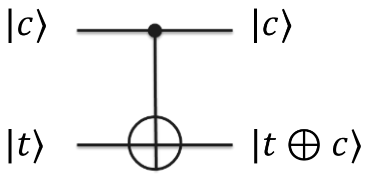
$$R_z(\theta_z) \equiv e^{-\frac{i\theta_z Z}{2}} = \cos\frac{\theta_z}{2} I - i\sin\frac{\theta_z}{2} Z = \begin{bmatrix} e^{-\frac{i\theta_z}{2}} & 0 \\ 0 & e^{+\frac{i\theta_z}{2}} \end{bmatrix}$$



Controlled gate

The main controlled operation is the controlled-NOT (CNOT).

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$CNOT|q_0\rangle|q_1\rangle = |q_0\rangle|q_1 \oplus q_0\rangle \quad \rightarrow$$

$$CNOT|0\rangle|0\rangle = |0\rangle|0 \oplus 0\rangle = |0\rangle|0\rangle$$

$$CNOT|0\rangle|1\rangle = |0\rangle|1 \oplus 0\rangle = |0\rangle|1\rangle$$

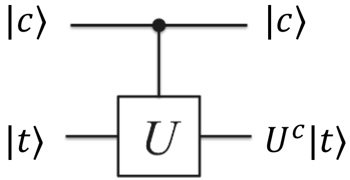
$$CNOT|1\rangle|0\rangle = |1\rangle|0 \oplus 1\rangle = |1\rangle|1\rangle$$

$$CNOT|1\rangle|1\rangle = |1\rangle|1 \oplus 1\rangle = |1\rangle|0\rangle$$

Controlled gate

A controlled U operation is a two qubit operation with a control and a target qubit.

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{U^c}} \\ 0 & 0 & & \end{bmatrix}$$

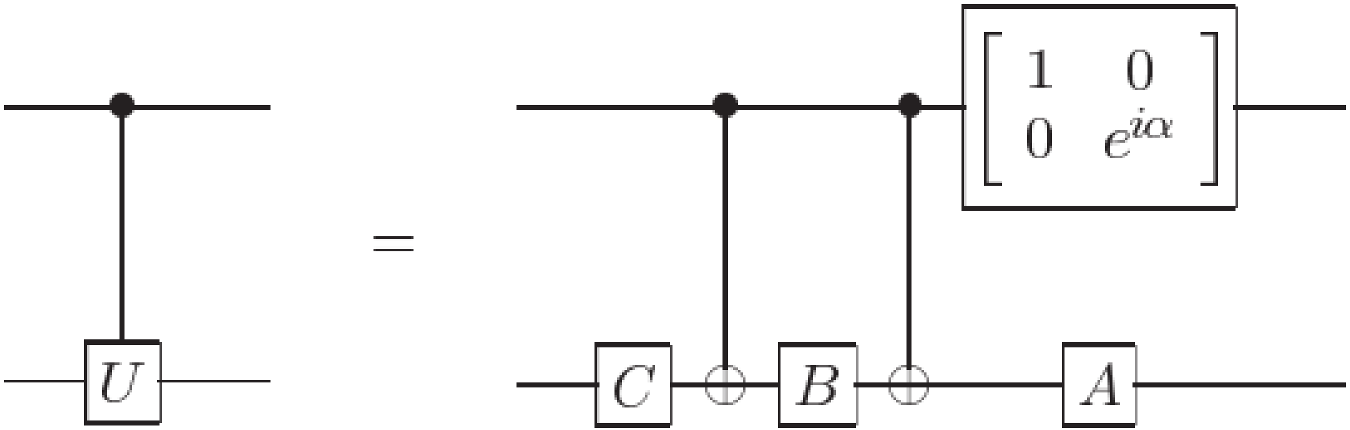


$$U^c = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

For example: $U^c = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = CNOT$

Controlled gate

Circuit implementing the controlled-U operation.

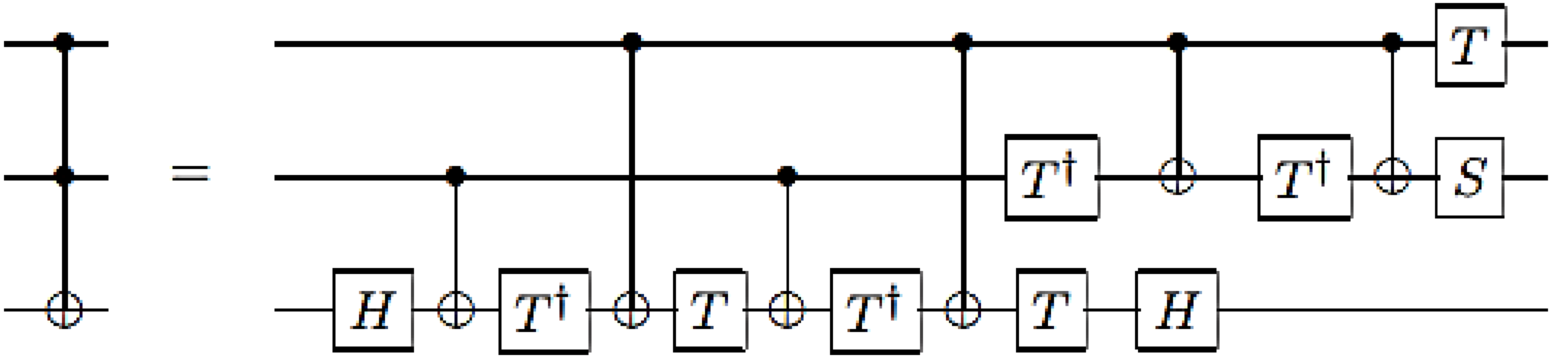


$$U = e^{i\alpha} AXBXC \qquad \text{where } ABC = I$$

Multi-control gate

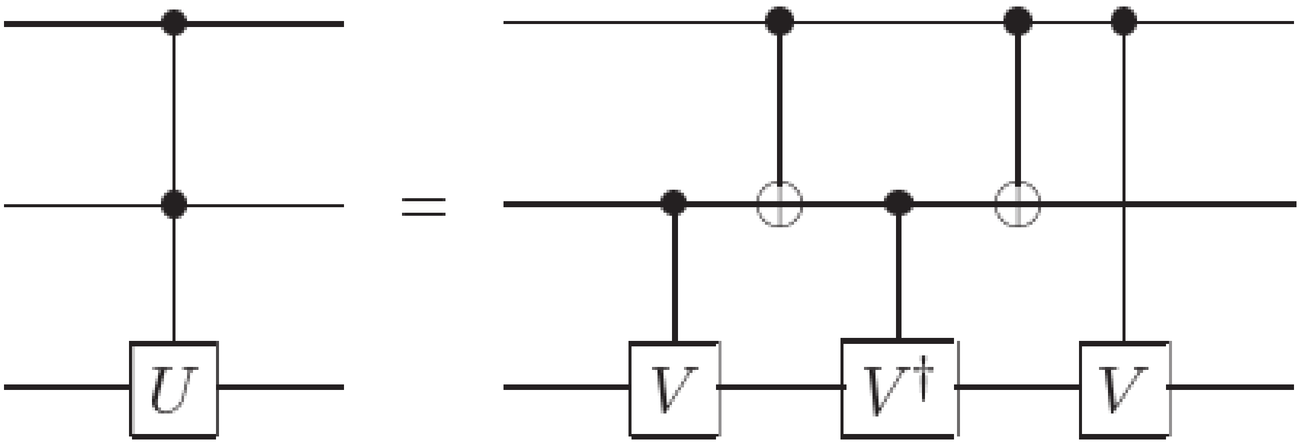
Like logic operations, we can build multi-controlled quantum gates from just gate set.

Two-controlled X gate (Toffoli gate):



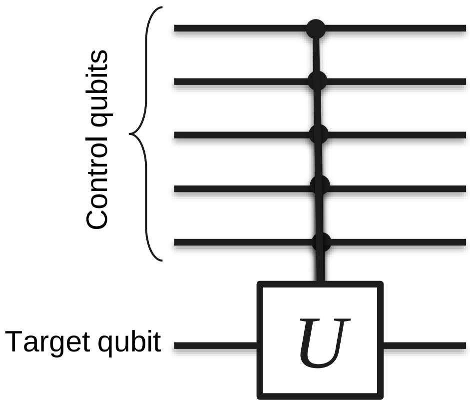
Multi-control gate

Two-controlled U gate:

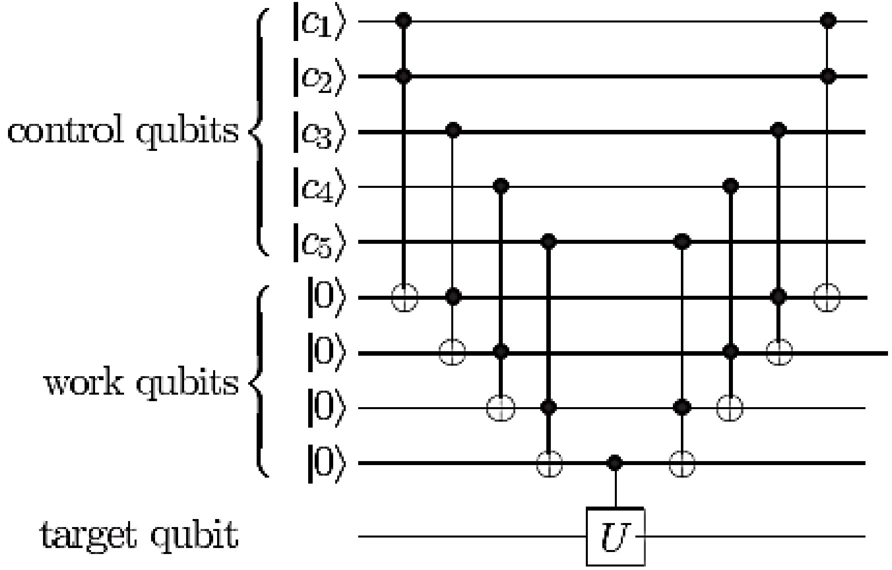


Multi-control gate

m-controlled U gate:



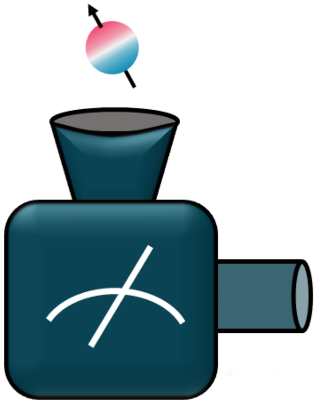
Using ancilla qubits



Measurement

A final element used in quantum circuits is measurement.

If we measure the qubit $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ for several times.



Measurement

Probability of measuring $|\psi\rangle$ as 0 is: $|\alpha|^2$

Probability of measuring $|\psi\rangle$ as 1 is: $|\beta|^2$

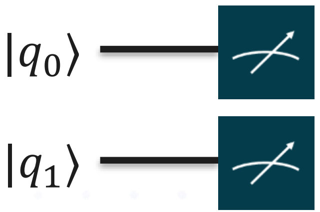
Measurement

$$|\psi\rangle = \alpha|0\rangle + e^{i\varphi}\beta|1\rangle$$

Probability of measuring of $|\psi\rangle$ as 0 is: $|\alpha|^2$

Probability of measuring of $|\psi\rangle$ as 1 is: $\left|e^{i\varphi}\beta\right|^2 = |\beta|^2$

$$|\psi\rangle = |q_0 q_1\rangle = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + e^{i\phi}\beta\gamma|10\rangle + \beta\delta|11\rangle$$



Probability of measuring of $|\psi\rangle$ as 00 is: $|\alpha\gamma|^2$

Probability of measuring of $|\psi\rangle$ as 01 is: $|\alpha\delta|^2$

Probability of measuring of $|\psi\rangle$ as 10 is: $\left|e^{i\phi}\beta\gamma\right|^2 = |\beta\gamma|^2$

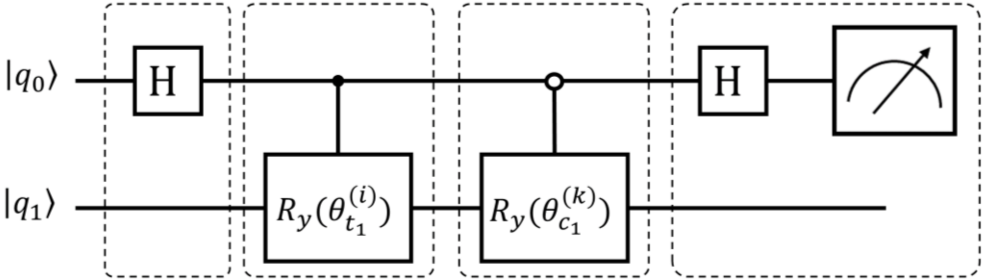
Probability of measuring of $|\psi\rangle$ as 11 is: $|\beta\delta|^2$

04

Quantum Circuit

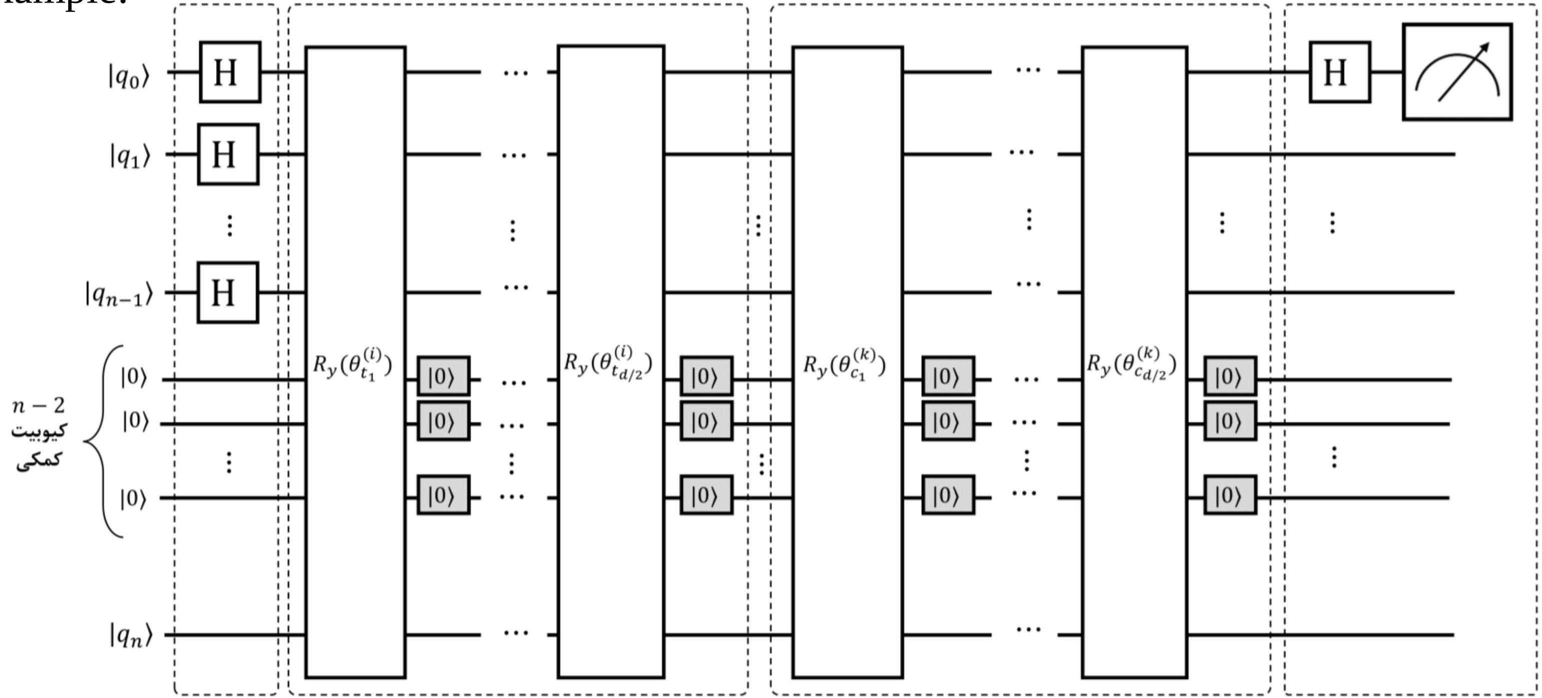
Quantum Circuit

Example:



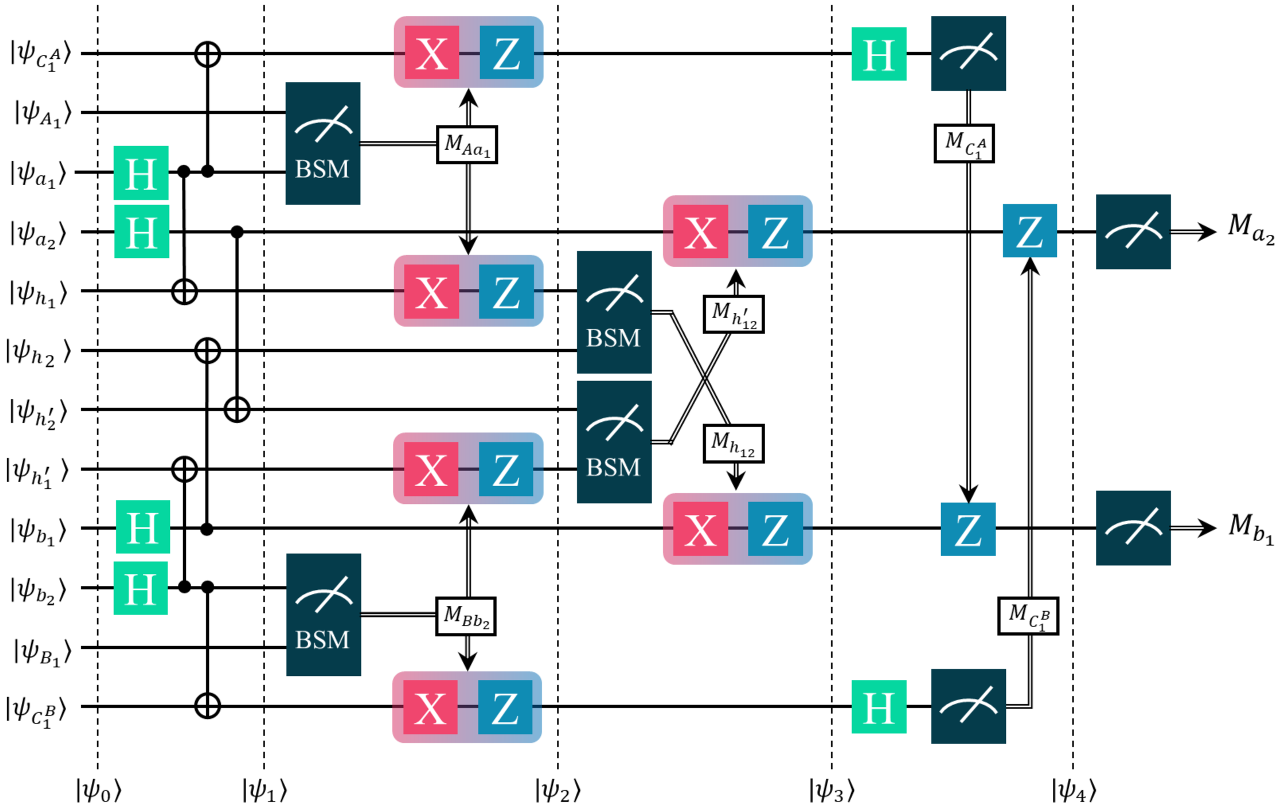
Quantum Circuit

Example:



Quantum Circuit

Example:



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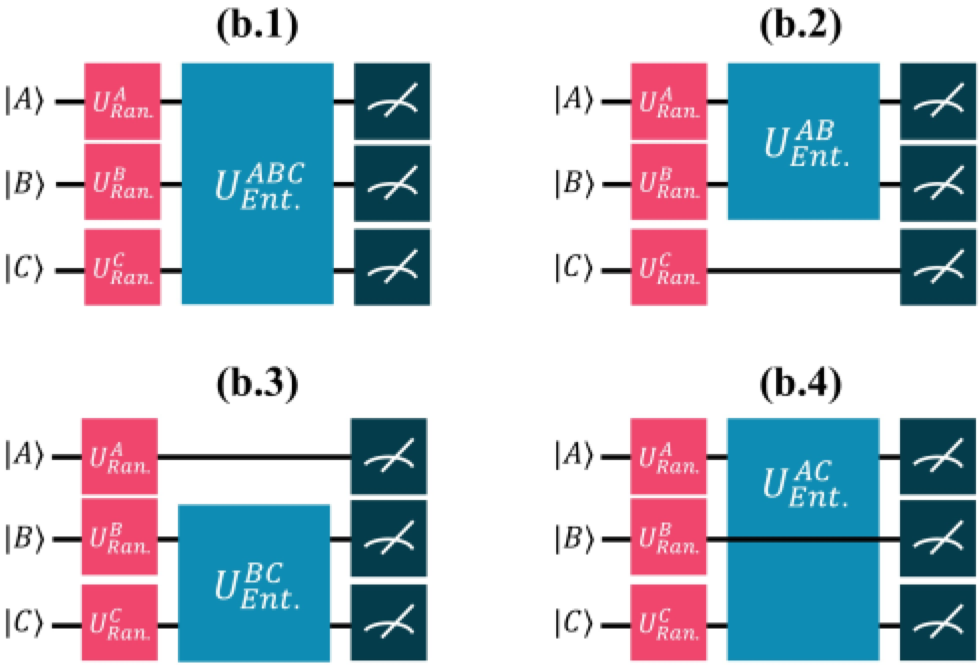
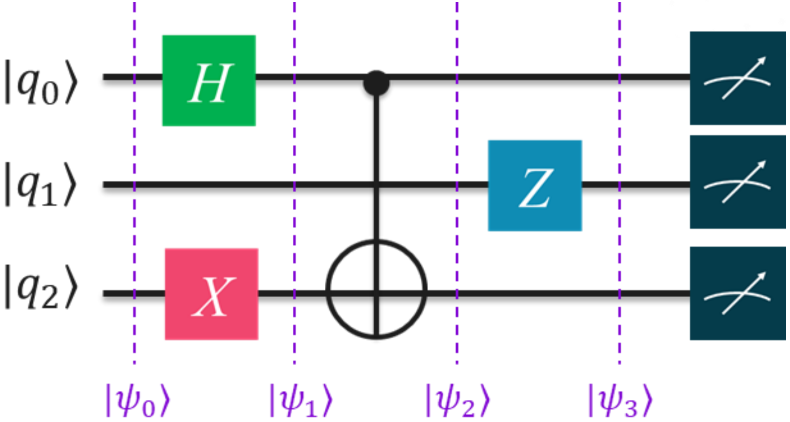
Quantum Circuit

Example:



Quantum Circuit



Step 0: $|\psi_0\rangle = |000\rangle$

Step 1: $|\psi_1\rangle = (H \otimes I \otimes Z)|000\rangle$

$$\rightarrow |\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle)$$

CNOT CNOT

Step 2: $|\psi_2\rangle = \frac{1}{\sqrt{2}}(\underline{|000\rangle} + \underline{|100\rangle})$

$$\rightarrow |\psi_2\rangle = \frac{1}{\sqrt{2}}(\underline{|000\rangle} + \underline{|101\rangle})$$

Step 3: $|\psi_3\rangle = (I \otimes Z \otimes I)\frac{1}{\sqrt{2}}(|000\rangle + |101\rangle) = \frac{1}{\sqrt{2}}\left((I \otimes Z \otimes I)|000\rangle + (I \otimes Z \otimes I)|101\rangle\right)$

$$\rightarrow |\psi_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |101\rangle)$$

Probability of measuring of $|\psi\rangle$ as 000 is: $\left|\frac{1}{\sqrt{2}}\right|^2 = 0.5$

Probability of measuring of $|\psi\rangle$ as 101 is: $\left|\frac{1}{\sqrt{2}}\right|^2 = 0.5$

Cheat Sheet

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$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \rightarrow \quad X|0\rangle = |1\rangle, \qquad X|1\rangle = |0\rangle$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \rightarrow \quad Y|0\rangle = i|1\rangle, \qquad Y|1\rangle = -i|0\rangle$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow \quad Z|0\rangle = |0\rangle, \qquad Z|1\rangle = -|1\rangle$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \rightarrow \quad H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle, \qquad H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{2}} \end{pmatrix} \rightarrow \quad S|0\rangle = |0\rangle, \qquad S|1\rangle = e^{\frac{i\pi}{2}}|1\rangle$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix} \rightarrow \quad T|0\rangle = |0\rangle, \qquad T|1\rangle = e^{\frac{i\pi}{4}}|1\rangle$$

Coding time ;)

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