اطلس کوانتوم

Quantum Coding Course

Ali Kookani
Yousef Mafi

Summer 2024

About us

Quantum Atlas is an educational group which aims to educate people in various fields of quantum, from hardware to software and quantum machine learning.

www.quantumatlas.ir

QuantumSTEM

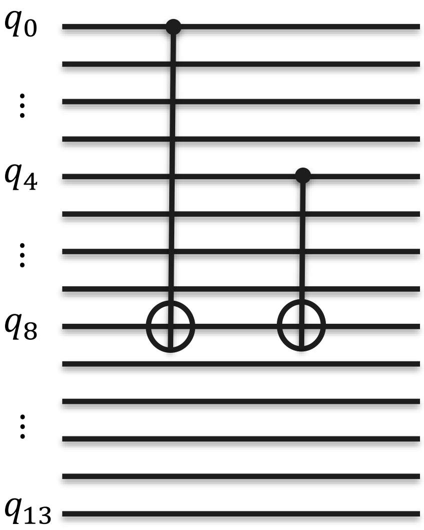
Quantum Atlas

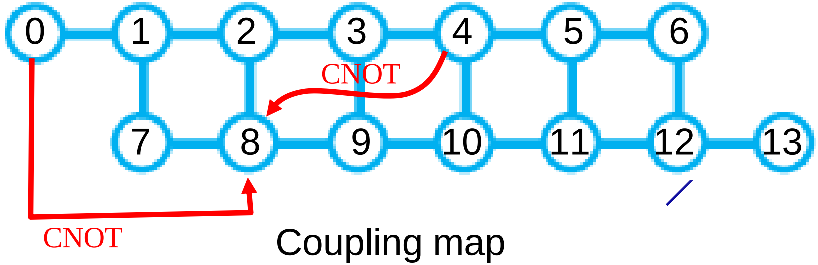
Syllabus

Section 1	**Lecture 1**	Quantum Computation and Information (Theoretical lecture) – By Y. Mafi
	Lecture 2	Quantum Circuits (Coding lecture) – By A. Kookani
Section 2	**Lecture 3**	Quantum Simulation (Coding lecture) – By A. Kookani
	Lecture 4	IBMQ and Error Correction (Implementation and Theoretical lecture) – By Y. Mafi
Section 3	**Lecture 5**	Quantum Algorithm (Theoretical lecture) – By Y. Mafi
	Lecture 6	Quantum Algorithm Simulation (Coding lecture) – By A. Kookani

Challenge 2

How can we implement the following circuit based on the coupling map? (only use CNOT gates)





Coupling map

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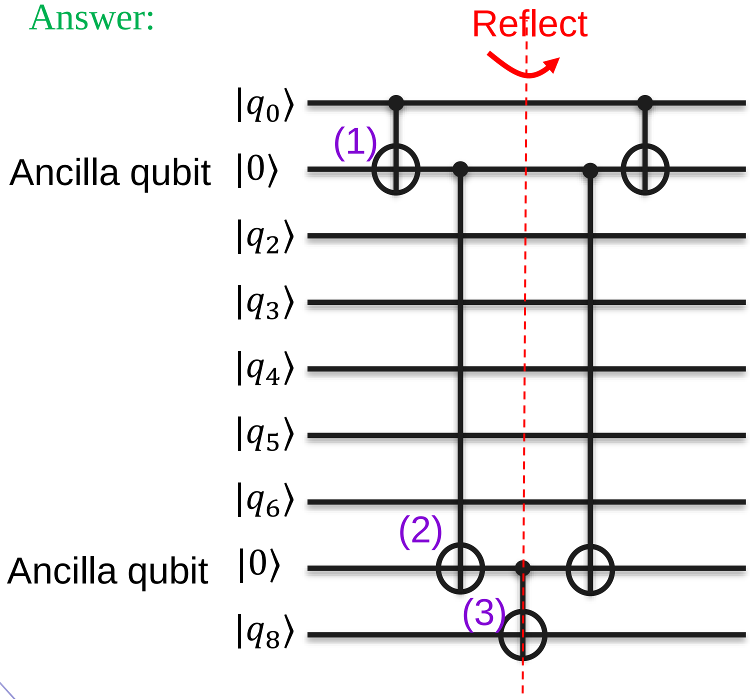

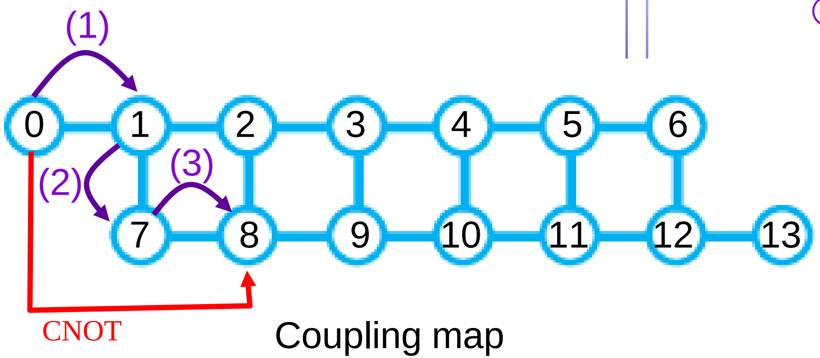

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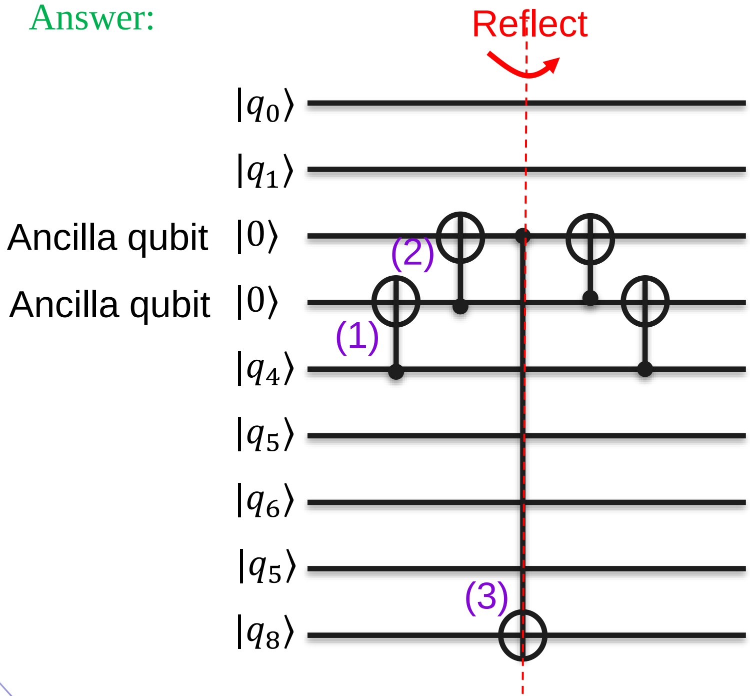
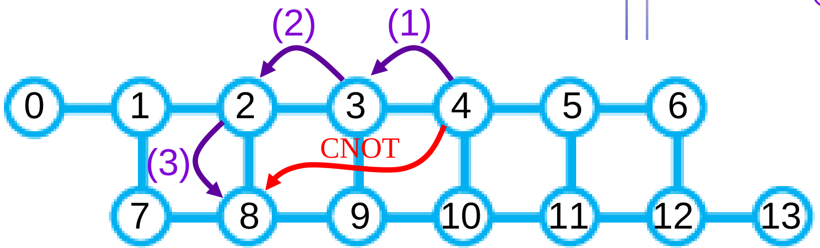


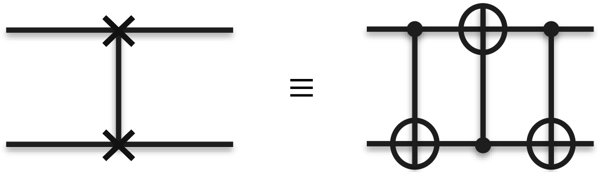


Coupling map

Swap gate

- **Purpose**: The SWAP gate exchanges the states of two qubits.
- **Operation**:
  - **Inputs**: Two qubits, Qubit q0 and Qubit q1.
  - **Outputs**: The states of Qubit q0 and Qubit q1 are swapped.

$$\text{Swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



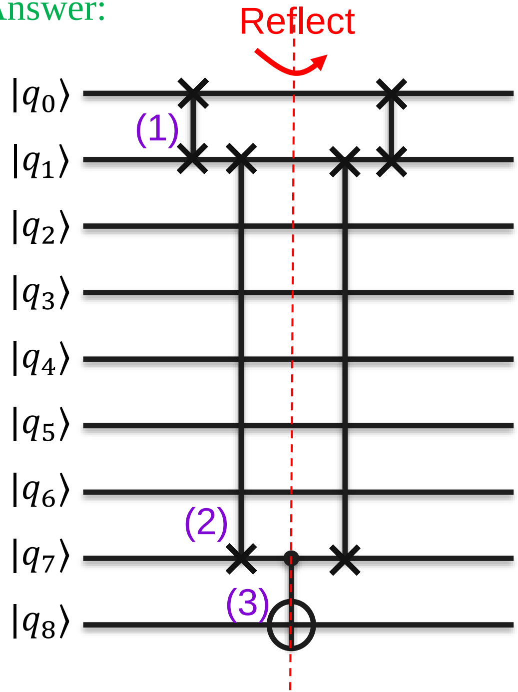
$$\begin{aligned} |\psi\rangle = |q_0 q_1\rangle &= (\alpha_0|0\rangle + \alpha_1|1\rangle) \otimes (\beta_0|0\rangle + \beta_1|1\rangle) \\ &= \alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \alpha_1\beta_1|11\rangle \end{aligned}$$

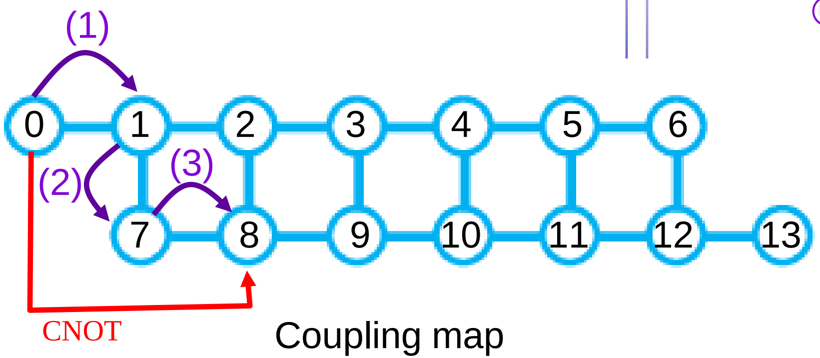
$$\begin{aligned} \text{Swap}|\psi\rangle = |\psi'\rangle &= \alpha_0\beta_0|00\rangle + \alpha_0\beta_1|10\rangle + \alpha_1\beta_0|01\rangle + \alpha_1\beta_1|11\rangle \\ &= (\beta_0|0\rangle + \beta_1|1\rangle) \otimes (\alpha_0|0\rangle + \alpha_1|1\rangle) \end{aligned}$$

Challenge 2

How can we implement the following circuit based on the coupling map? (only use CNOT gates)

Answer:





Coupling map

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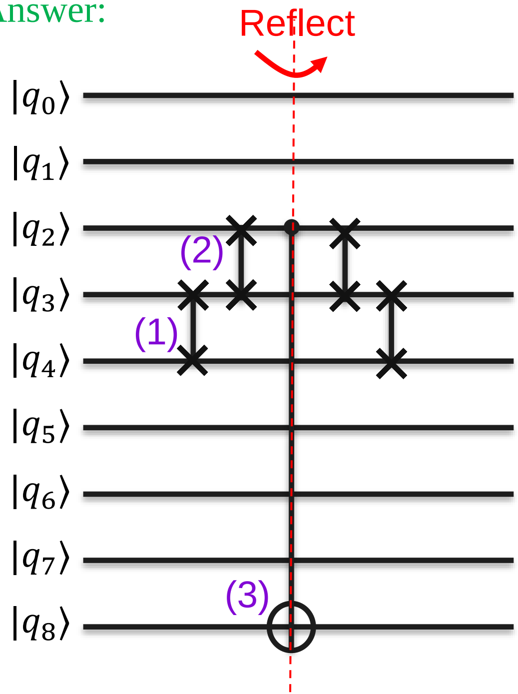

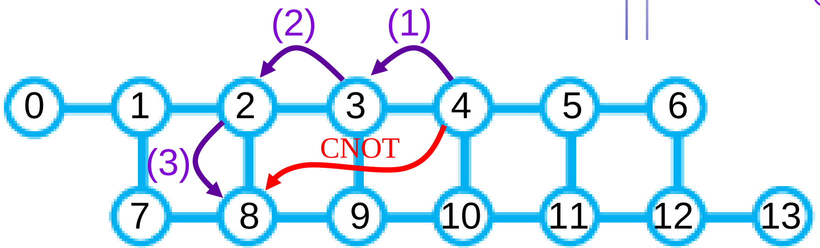

Coupling map

Today

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Quantum Properties

02

Deutsch-Jozsa Algorithm

(Quantum Phase Kick-Back)

03

Quantum Teleportation

(Quantum Entanglement)

04

Grover's Algorithm

(Quantum Superposition)

01

Quantum Properties

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Quantum Properties

❖ **Superposition:**
A quantum bit (qubit) can exist in multiple states simultaneously → parallel computations

❖ **Entanglement:**
The state of one qubit is directly related to the state of another, even when separated by large distances → process complex correlations between qubits

❖ **Quantum Interference:**
Quantum states can interfere with each other, leading to constructive or destructive interference → phase kickback effect

❖ **Quantum Tunneling**
Quantum particles can "tunnel" through barriers that would be insurmountable in classical physics → quantum annealing and optimization

02

Deutsch-Jozsa Algorithm

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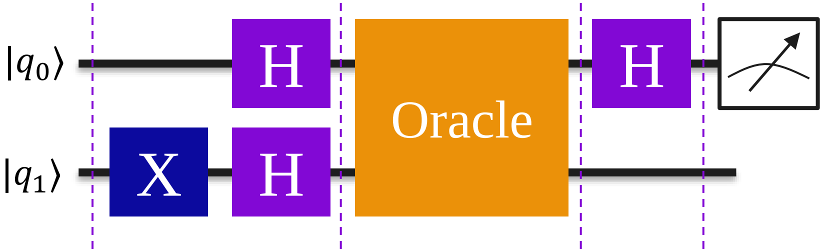
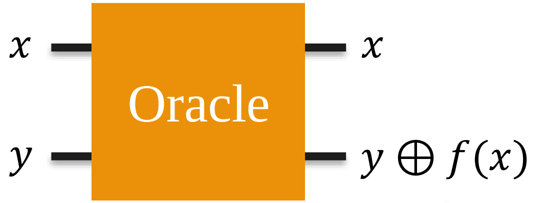
Deutsch Algorithm

- It solves the problem of determining whether a given function $f(x)$ is constant or balanced.
- Deutsch's Algorithm achieves this with only a single evaluation of the function by leveraging quantum superposition and interference.
- Time Complexity: Quantum $\mathcal{O}(1)$ Vs. Classic $\mathcal{O}(2^n)$





Deutsch Algorithm

1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$

2 $|\psi_1\rangle = (H \otimes HX)|\psi_0\rangle = (H \otimes HX)|00\rangle$

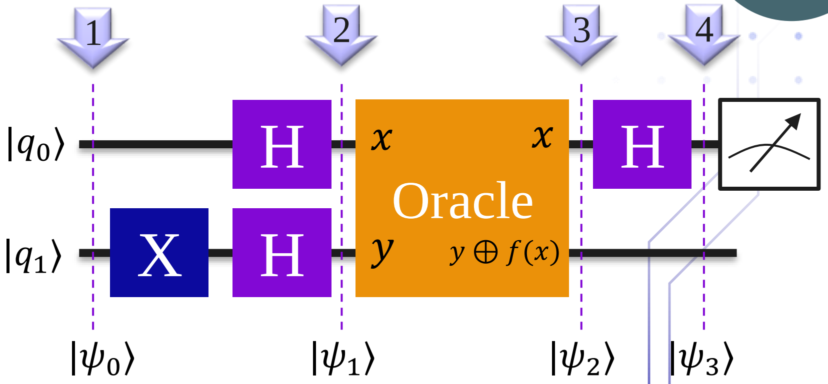
$$|\psi_1\rangle = |+-\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$|\psi_1\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

3 $|\psi_2\rangle = U_{oracle}|\psi_1\rangle$ $\rightarrow$ Assume $f(x)$ is zero-constant function:

$$|\psi_2\rangle = \frac{1}{2}(U_{oracle}\underbrace{|00\rangle}_{|xy\rangle} - U_{oracle}|01\rangle + U_{oracle}|10\rangle - U_{oracle}|11\rangle)$$

$$|\psi_2\rangle = \frac{1}{2}(\underbrace{|0\rangle|0 \oplus 0\rangle}_{|x\rangle|y \oplus f(x)\rangle} - |0\rangle|1 \oplus 0\rangle + |1\rangle|0 \oplus 0\rangle - |1\rangle|1 \oplus 0\rangle)$$



x	$f(x)$
0	0
1	0

Zero-Constant

Deutsch Algorithm

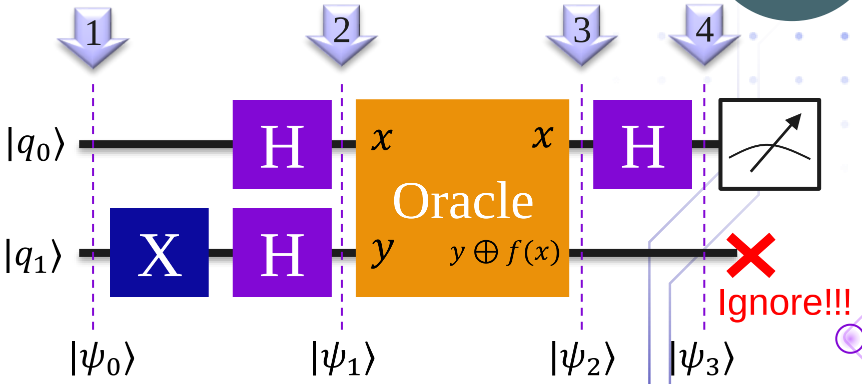
1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$

2 $|\psi_1\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

3 Assume $f(x)$ is zero-constant function:

$$|\psi_2\rangle = \frac{1}{2}(|0\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle)$$

$$|\psi_2\rangle = |+\ \ -\rangle$$



4 $|\psi_3\rangle = (H \otimes I)|\psi_2\rangle = (H \otimes I)|+\ \ -\rangle = |0-\rangle$

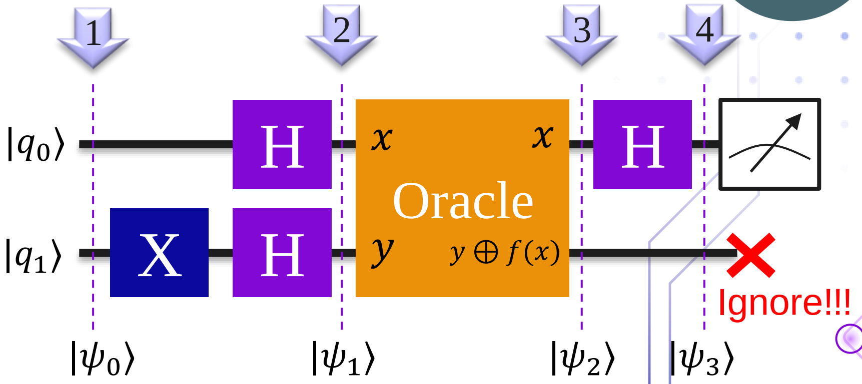
Ignore $|q_1\rangle$: $|\psi_3\rangle = |q_0\rangle = |0\rangle$ $\Longrightarrow$ $f(x)$ is constant if measurement of $|q_0\rangle$ become 0.

Deutsch Algorithm

1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$

2 $|\psi_1\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

3 Assume $f(x)$ is balanced function:



$$|\psi_2\rangle = \frac{1}{2}(U_{oracle}|00\rangle - U_{oracle}|01\rangle + U_{oracle}|10\rangle - U_{oracle}|11\rangle)$$

$$|\psi_2\rangle = \frac{1}{2}(|0\rangle|0 \oplus 1\rangle - |0\rangle|1 \oplus 1\rangle + |1\rangle|0 \oplus 0\rangle - |1\rangle|1 \oplus 0\rangle)$$

$$|\psi_2\rangle = \frac{1}{2}(|0\rangle|1\rangle - |0\rangle|0\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle) = |--\rangle$$

x	$f(x)$
0	1
1	0

Balanced

4 $|\psi_3\rangle = (H \otimes I)|\psi_2\rangle = (H \otimes I)|- \ -\rangle = |1-\rangle$

Ignore $|q_1\rangle$: $|\psi_3\rangle = |q_0\rangle = |1\rangle$ ⟹ $f(x)$ is balanced if measurement of $|q_0\rangle$ become 1.

Deutsch-Jozsa Algorithm

1 $|\psi_0\rangle = |q_0 q_1 q_2\rangle = |000\rangle$

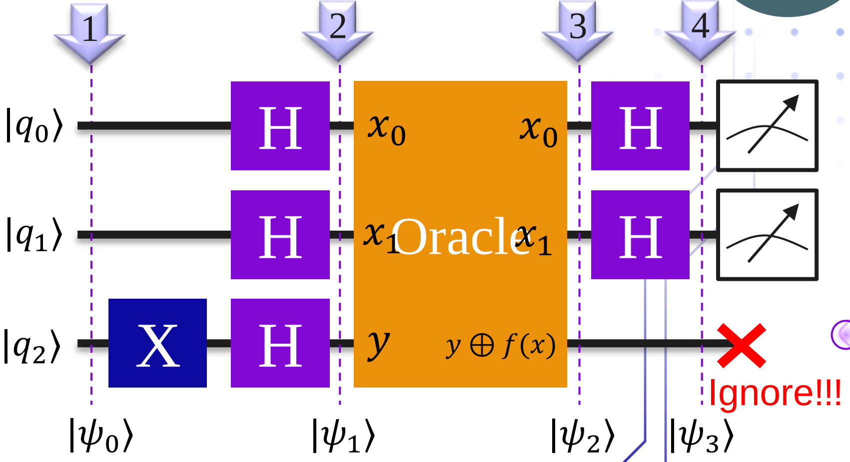
2 $$|\psi_1\rangle = \frac{1}{2\sqrt{2}}(|000\rangle - |001\rangle + |010\rangle - |011\rangle + |100\rangle - |101\rangle + |110\rangle - |111\rangle)$$

3 1) Assume $f(x)$ is constant function:

$$|\psi_2\rangle = \frac{1}{2\sqrt{2}}(|001\rangle - |000\rangle + |011\rangle - |010\rangle + |101\rangle - |100\rangle + |111\rangle - |110\rangle)$$

2) Assume $f(x)$ is balanced function:

$$|\psi_2\rangle = \frac{1}{2\sqrt{2}}(|001\rangle - |000\rangle + |010\rangle - |011\rangle + |100\rangle - |101\rangle + |111\rangle - |110\rangle)$$



x_1	x_1	$f(x)$
0	0	1
0	1	1
1	0	1
1	1	1

One-constant

x_1	x_1	$f(x)$
0	0	1
0	1	0
1	0	0
1	1	1

Balanced

Deutsch-Jozsa Algorithm

1 $|\psi_0\rangle = |q_0q_1q_2\rangle = |000\rangle$

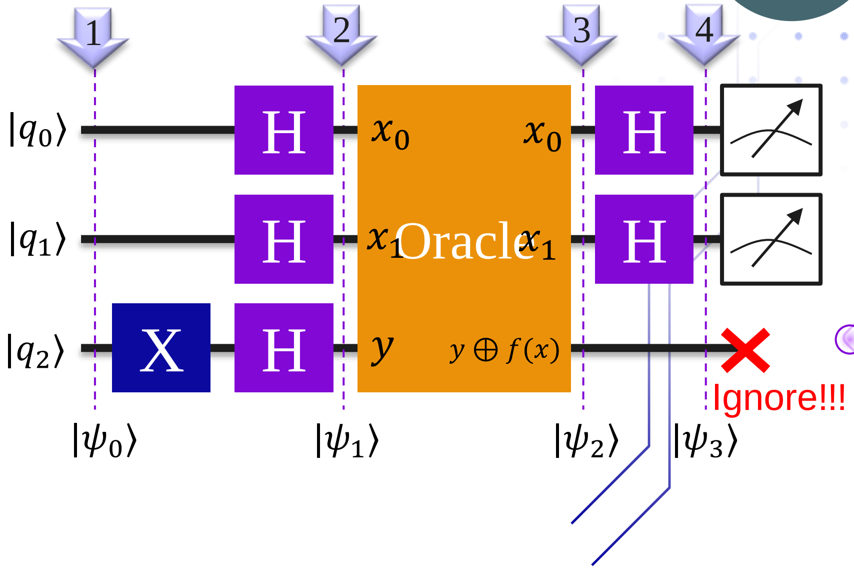
2 $|\psi_1\rangle = \frac{1}{2\sqrt{2}}(|000\rangle - |001\rangle + |010\rangle - |011\rangle + |100\rangle - |101\rangle + |110\rangle - |111\rangle)$

3 1) Assume $f(x)$ is constant function:

$|\psi_2\rangle = |++-\rangle$

2) Assume $f(x)$ is balanced function:

$|\psi_2\rangle = |---\rangle$

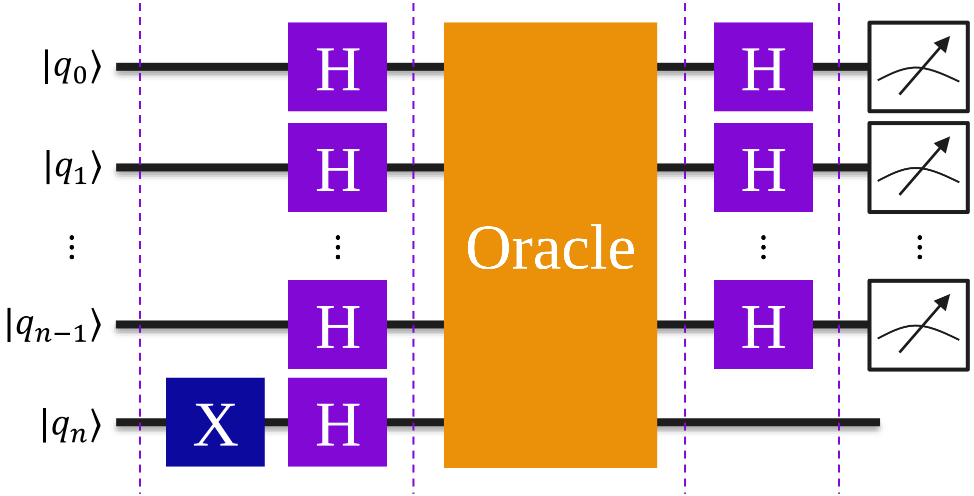


4 1) $|\psi_3\rangle = (H \otimes H \otimes I)|\psi_2\rangle = (H \otimes H \otimes I)|---\rangle = |00-\rangle$ ➡ Ignore $|q_2\rangle$: $|\psi_3\rangle = |q_0q_1\rangle = |00\rangle$ Constant.

2) $|\psi_3\rangle = (H \otimes H \otimes I)|\psi_2\rangle = (H \otimes H \otimes I)|---\rangle = |11-\rangle$ ➡ Ignore $|q_2\rangle$: $|\psi_3\rangle = |q_0q_1\rangle = |11\rangle$ Balanced.

Deutsch-Jozsa Algorithm

- An extension of Deutsch's Algorithm.
- Classical Algorithm: $2^{n-1} + 1$ queries $\rightarrow \mathcal{O}(2^n)$
- The Deutsch-Jozsa Algorithm: **just one query** $\rightarrow \mathcal{O}(\mathbf{1})$



03

Quantum Teleportation

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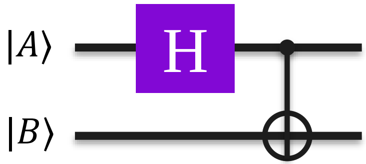
Quantum Entanglement

- Two or more qubits become intertwined such that the state of one qubit instantaneously influences the state of the other, no matter the distance separating them.
- It is a key resource for quantum computing, quantum communication, and quantum cryptography.
- When two qubits are entangled, their individual quantum states cannot be described independently; they must be considered as part of a combined system.

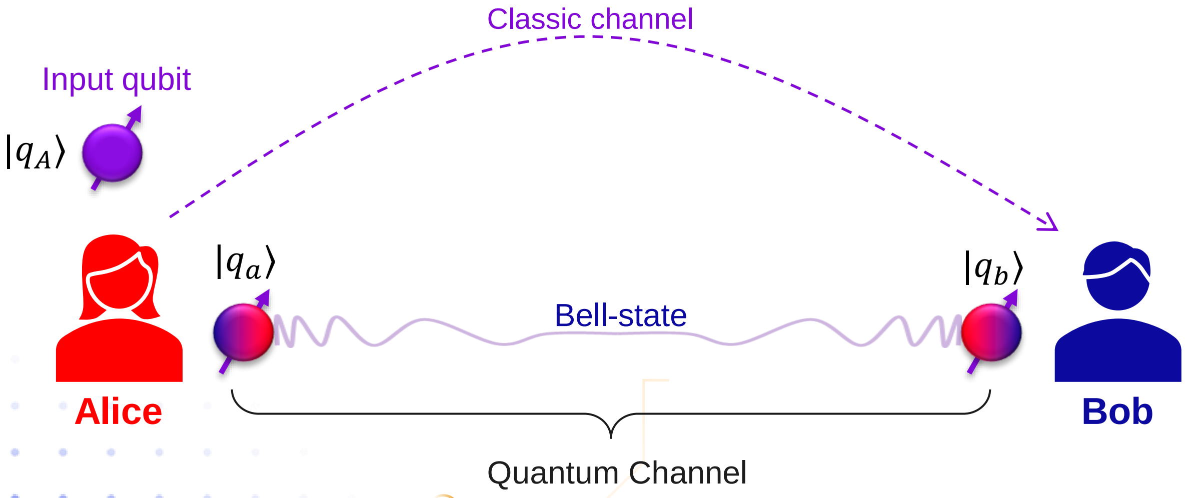


Bell-state

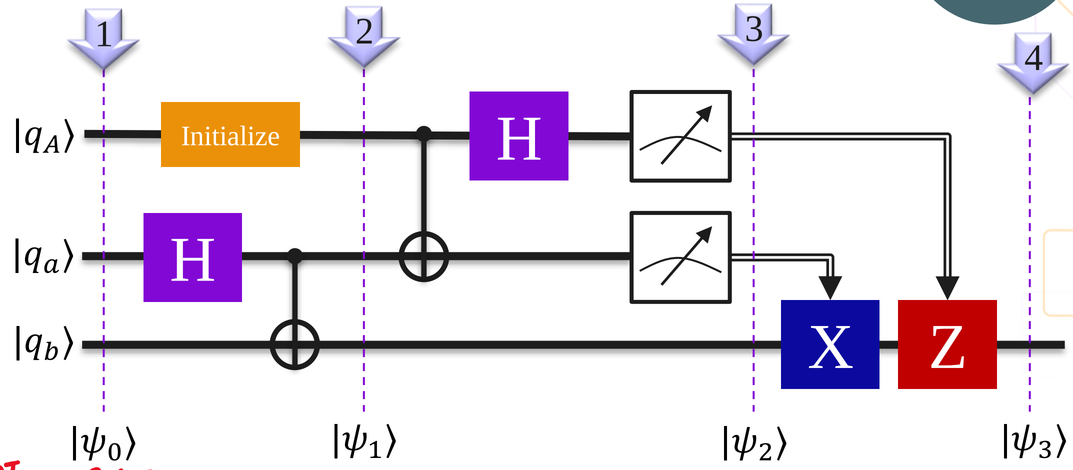
$$|AB\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Quantum Teleportation

- Quantum teleportation is a process by which the quantum state of a particle (such as a qubit) is transmitted from one location to another,
- without physically moving the particle itself.
- This process relies on quantum entanglement and classical communication.



Quantum Teleportation



1 $|\psi_0\rangle = |q_A q_a q_b\rangle = |000\rangle$

2 $|\psi_1\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle)$$

3 $|\psi_2\rangle = \frac{1}{\sqrt{2}}(CNOT \otimes I)(\alpha|\overset{CNOT}{000}\rangle + \alpha|\overset{CNOT}{011}\rangle + \beta|\overset{CNOT}{100}\rangle + \beta|\overset{CNOT}{111}\rangle)$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(H \otimes I \otimes I)(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \alpha|100\rangle + \alpha|111\rangle - \beta|110\rangle + \beta|001\rangle - \beta|110\rangle + \beta|001\rangle)$$

4 $|\psi_2\rangle = \frac{1}{\sqrt{2}}(\alpha|\overset{CZ}{\underset{CX}{000}}\rangle + \alpha|\overset{CZ}{\underset{CX}{011}}\rangle + \alpha|\overset{CZ}{\underset{CX}{100}}\rangle + \alpha|\overset{CZ}{\underset{CX}{111}}\rangle - \beta|\overset{CZ}{\underset{CX}{110}}\rangle + \beta|\overset{CZ}{\underset{CX}{001}}\rangle - \beta|\overset{CZ}{\underset{CX}{110}}\rangle + \beta|\overset{CZ}{\underset{CX}{001}}\rangle)$

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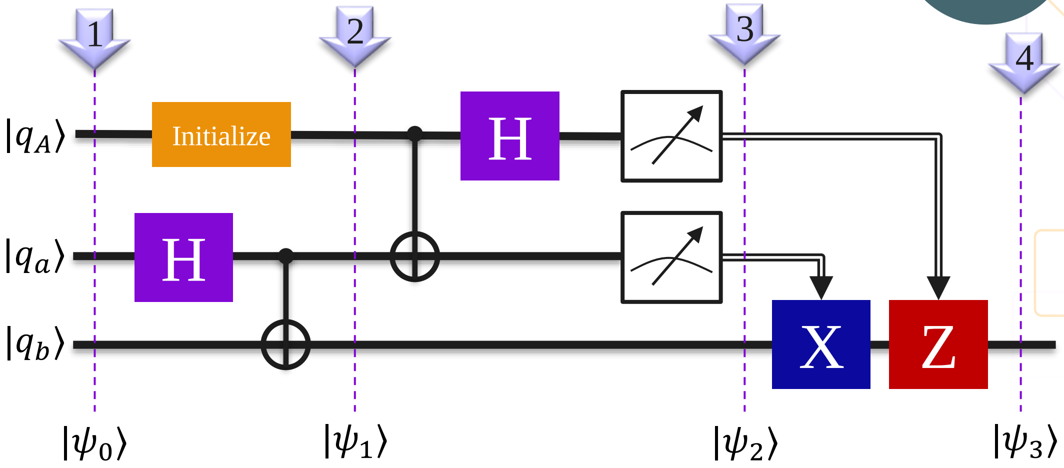
Quantum Teleportation



1 $|\psi_0\rangle = |q_A q_a q_b\rangle = |000\rangle$

2 $|\psi_1\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)$

$|\psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle)$

3 $|\psi_2\rangle = \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \alpha|100\rangle + \alpha|111\rangle - \beta|110\rangle + \beta|001\rangle - \beta|110\rangle + \beta|001\rangle)$

4 $|\psi_3\rangle = \frac{1}{4}(\alpha|0\rangle + \alpha|1\rangle + \alpha|0\rangle + \alpha|0\rangle + \beta|1\rangle + \beta|1\rangle + \beta|1\rangle + \beta|1\rangle)$

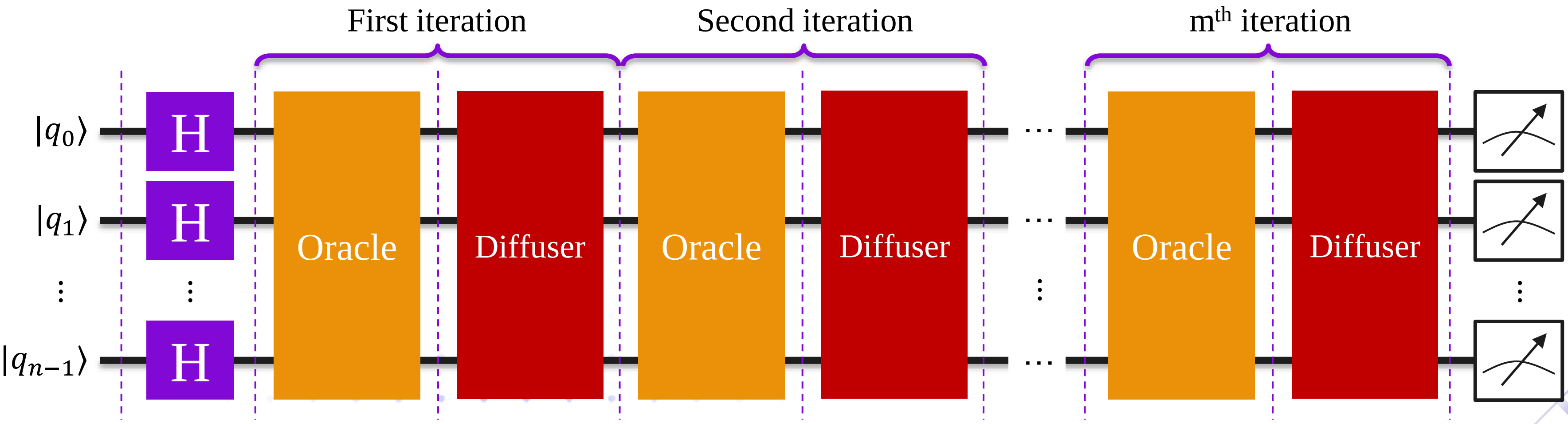
$|\psi_3\rangle = |q_b\rangle = \alpha|0\rangle + \beta|1\rangle$ Teleported state!!!

04

Grover's Algorithm

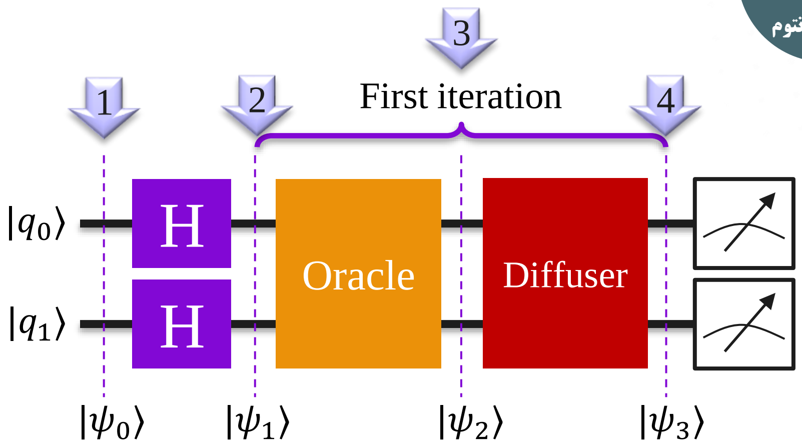
Grover's Algorithm

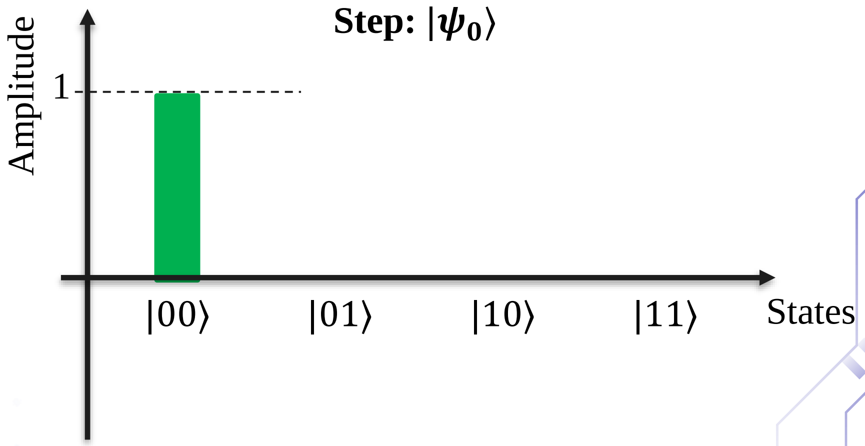
- A quadratic speedup for unstructured search problems (unsorted database).
- classical algorithms: N operations to search through N items $\rightarrow \mathcal{O}(N)$
- Grover's Algorithm: $\sqrt{N}$ operations to search through N items $\rightarrow \mathcal{O}(\sqrt{N})$
- We need $\approx \sqrt{N}$ iteration



Grover's Algorithm

1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$



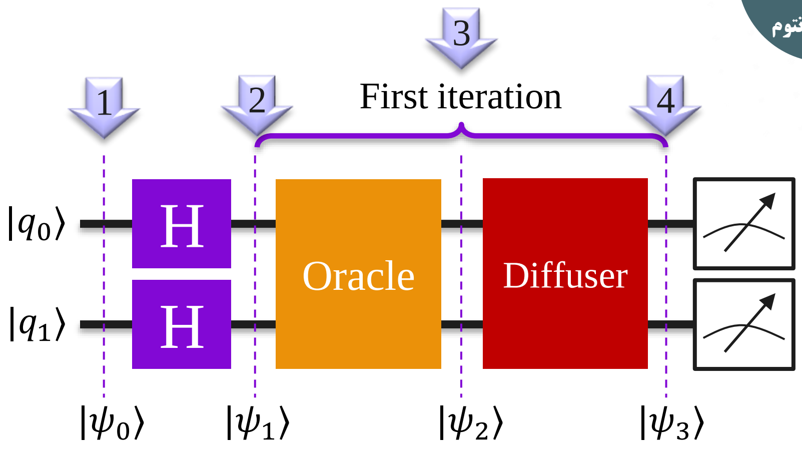


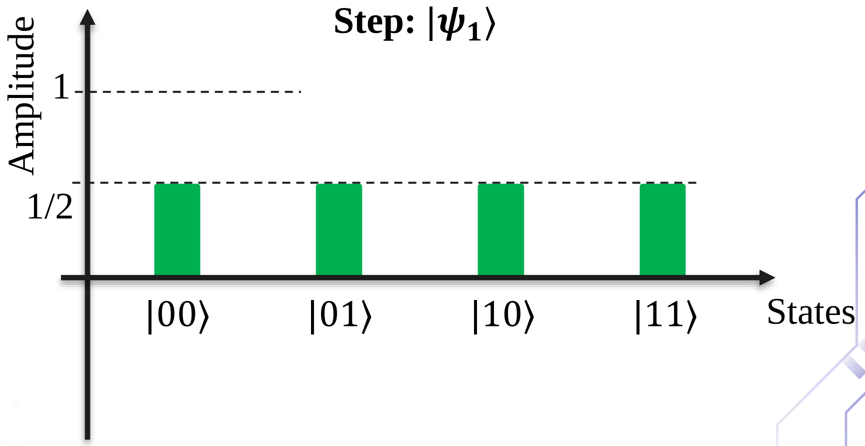
Grover's Algorithm

1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$

2 $|\psi_1\rangle = (H \otimes H)|00\rangle$

$$|\psi_1\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$





Grover's Algorithm

1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$

2 $|\psi_1\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

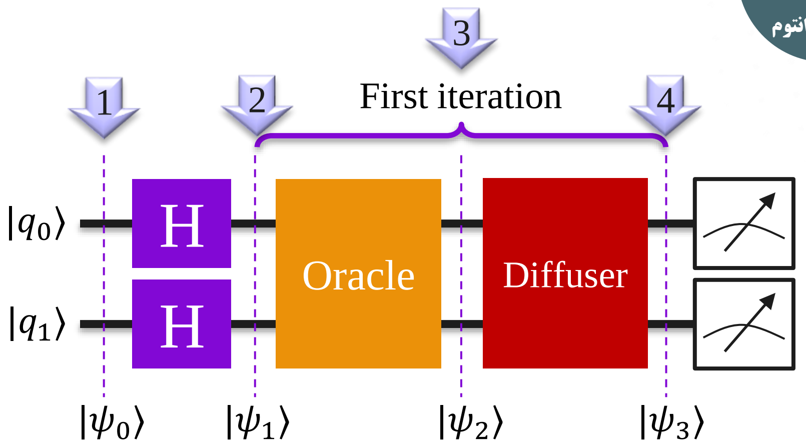
3 $|\psi_2\rangle = \frac{1}{2}U_o(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

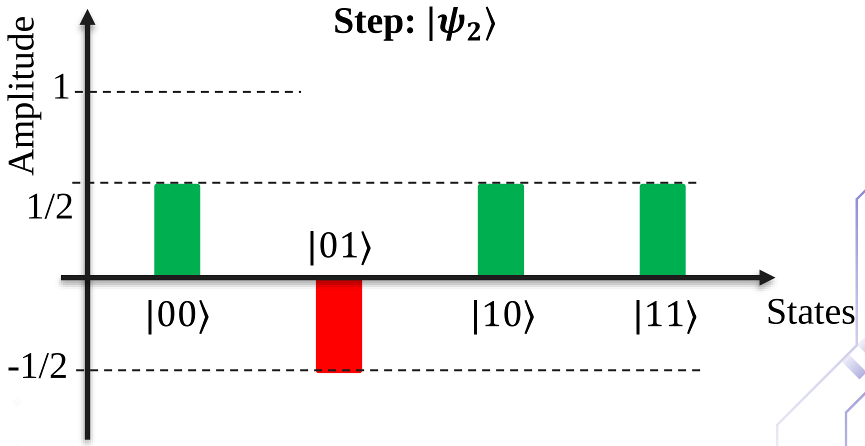
$U_o = I - 2|s\rangle\langle s|$ $\quad$ $|s\rangle$: the desired state

Assume $|s\rangle = |01\rangle$:

$$U_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

$$U_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





Grover's Algorithm

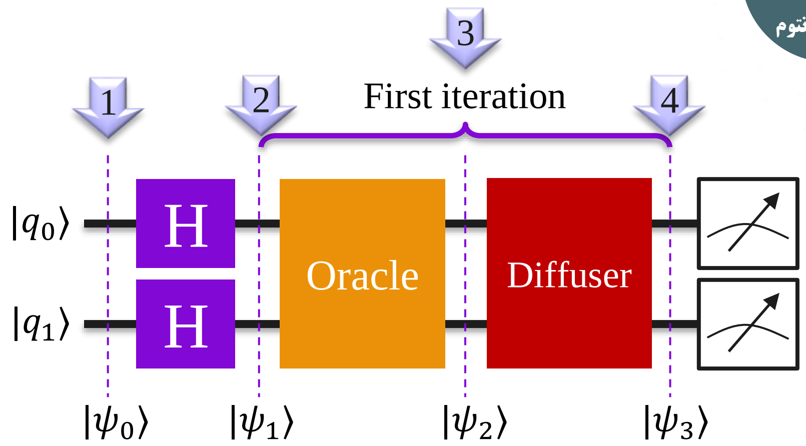
1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$

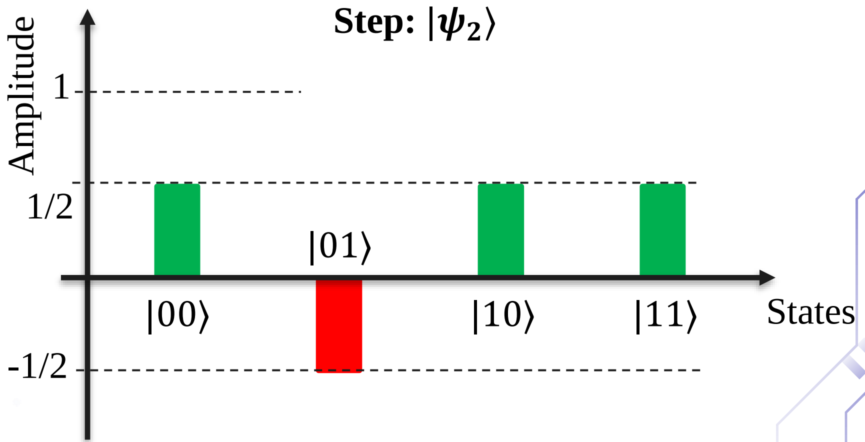
2 $|\psi_1\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

3 $|\psi_2\rangle = \frac{1}{2}U_o(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

$$|\psi_2\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$|\psi_2\rangle = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$$





Grover's Algorithm

1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$

2 $|\psi_1\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \Rightarrow |\phi\rangle$

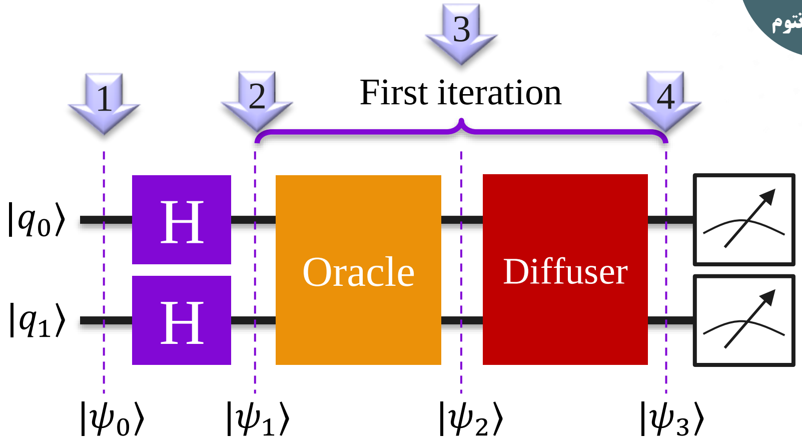
3 $|\psi_2\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

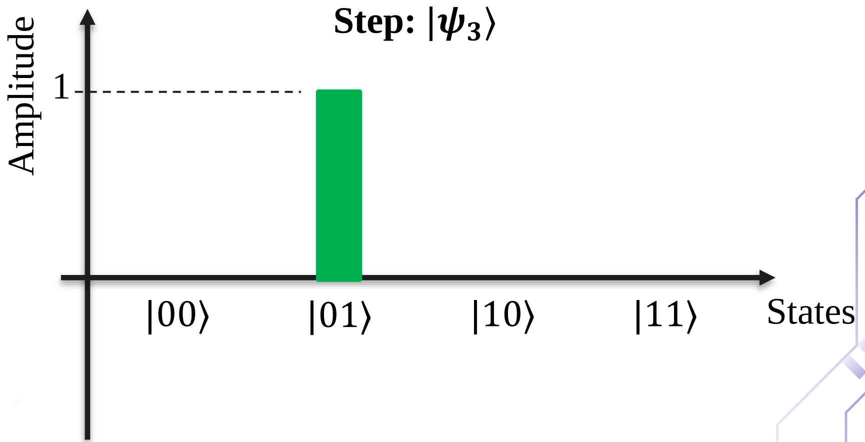
4 $|\psi_3\rangle = \frac{1}{2}U_D(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

$U_D = 2|\phi\rangle\langle\phi| - I$ $\quad$ $|\phi\rangle$: the basic state

$$U_D = 2\begin{bmatrix}0.5\\0.5\\0.5\\0.5\end{bmatrix}\begin{bmatrix}0.5 & 0.5 & 0.5 & 0.5\end{bmatrix} - \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$U_D = \begin{bmatrix}-0.5 & +0.5 & +0.5 & +0.5\\+0.5 & -0.5 & +0.5 & +0.5\\+0.5 & +0.5 & -0.5 & +0.5\\+0.5 & +0.5 & +0.5 & -0.5\end{bmatrix}$$





Grover's Algorithm

1 $|\psi_0\rangle = |q_0 q_1\rangle = |00\rangle$

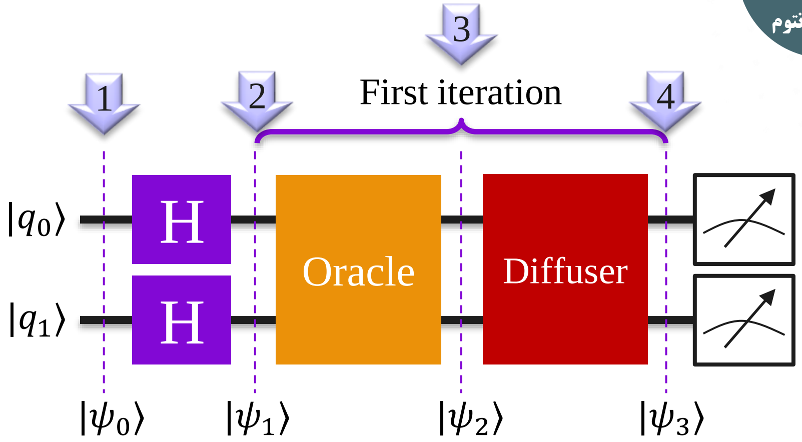
2 $|\psi_1\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \Rightarrow |\phi\rangle$

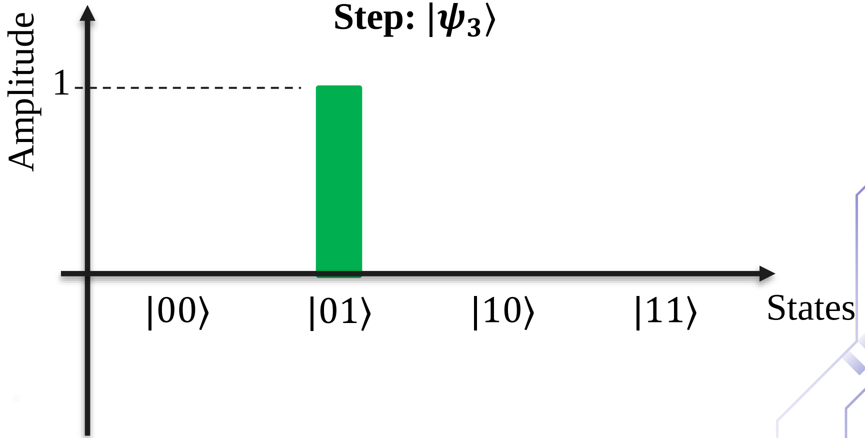
3 $|\psi_2\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

4 $|\psi_3\rangle = \frac{1}{2}U_D(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

$$|\psi_3\rangle = \frac{1}{2}\begin{bmatrix} -0.5 & +0.5 & +0.5 & +0.5 \\ +0.5 & -0.5 & +0.5 & +0.5 \\ +0.5 & +0.5 & -0.5 & +0.5 \\ +0.5 & +0.5 & +0.5 & -0.5 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

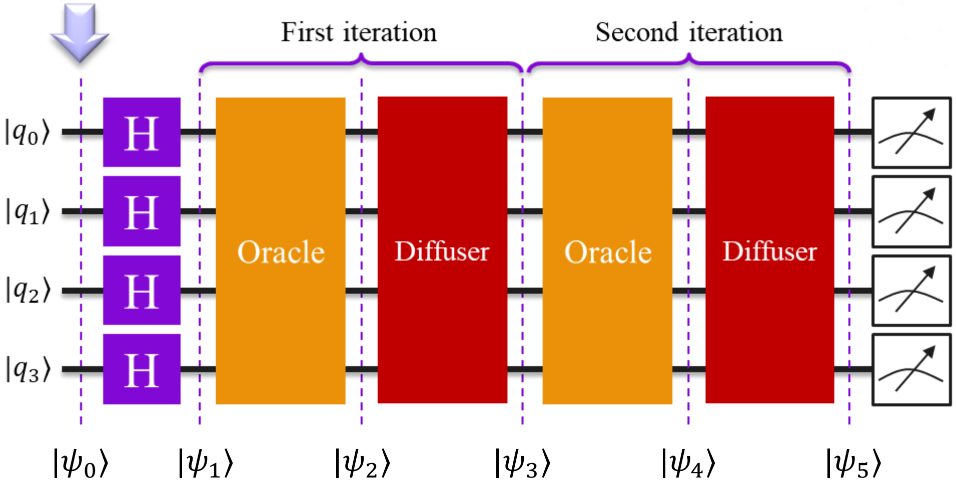
$$|\psi_3\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle \Rightarrow |s\rangle$$

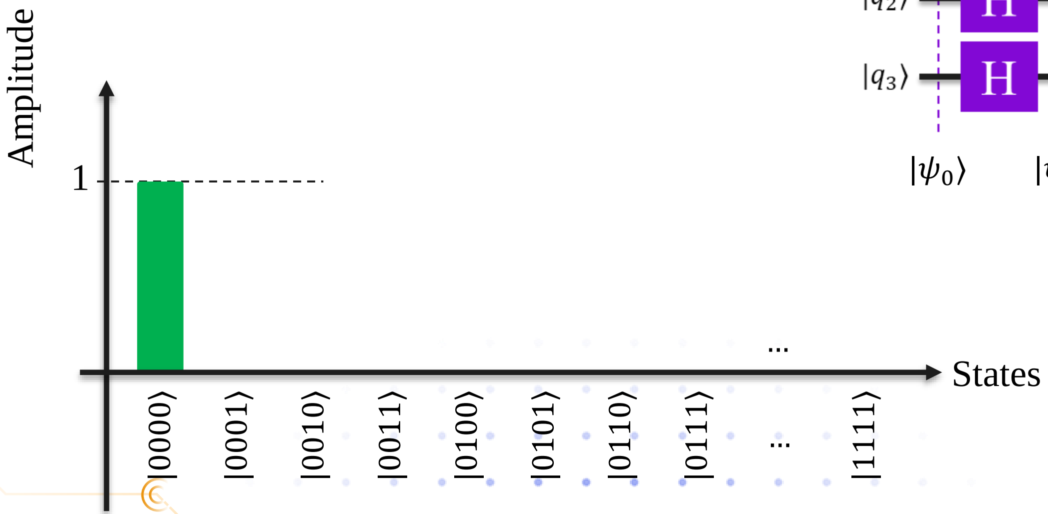




Grover's Algorithm

Step 0: Initialization

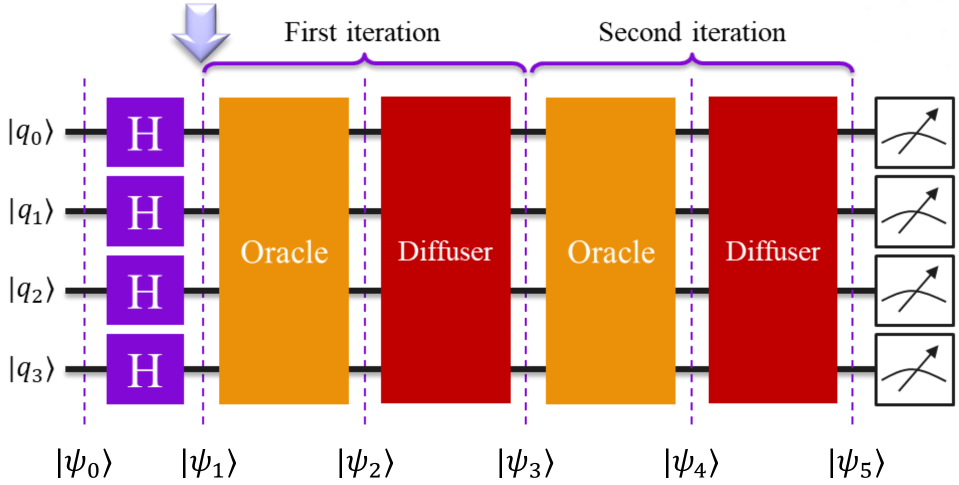


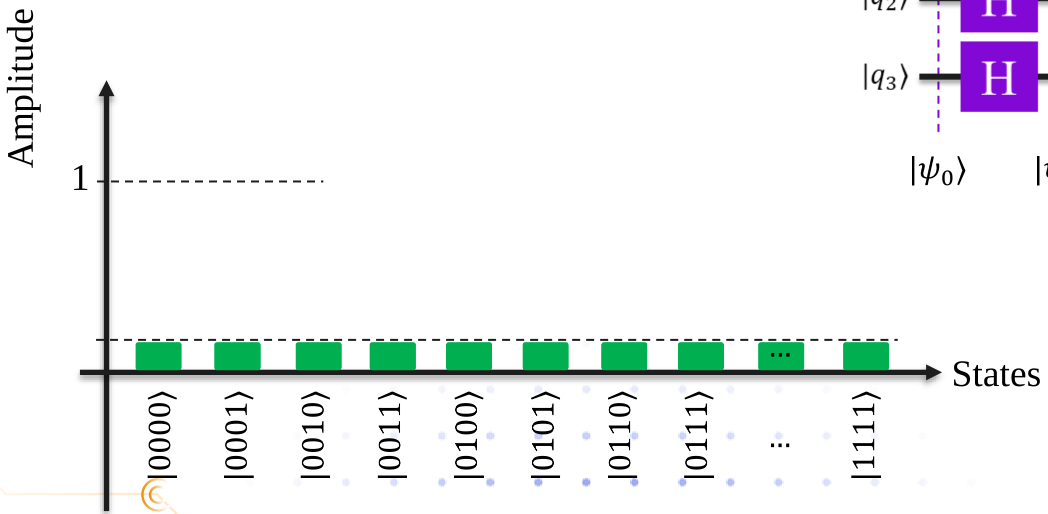


Grover's Algorithm

Step 1: Superposition

We need $\approx \sqrt{4} = 2$ iteration.

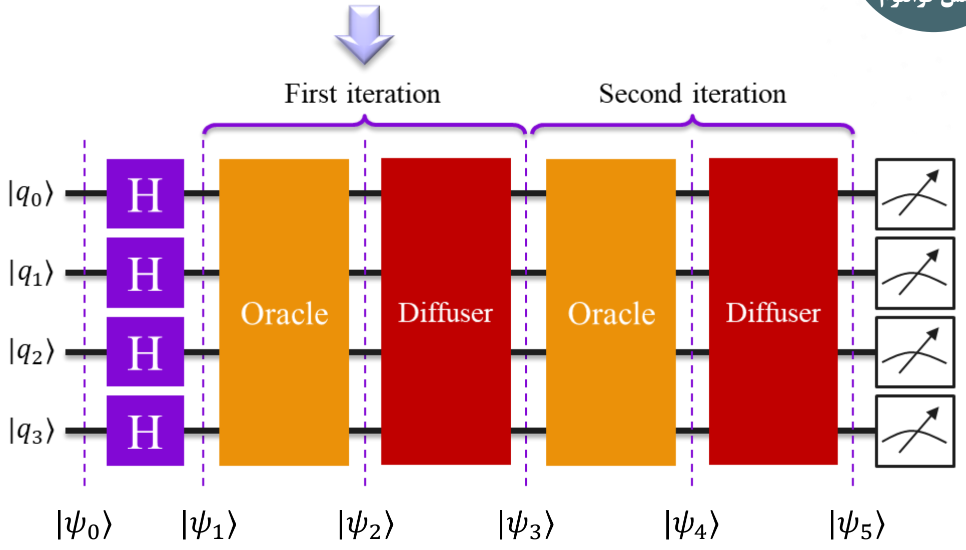


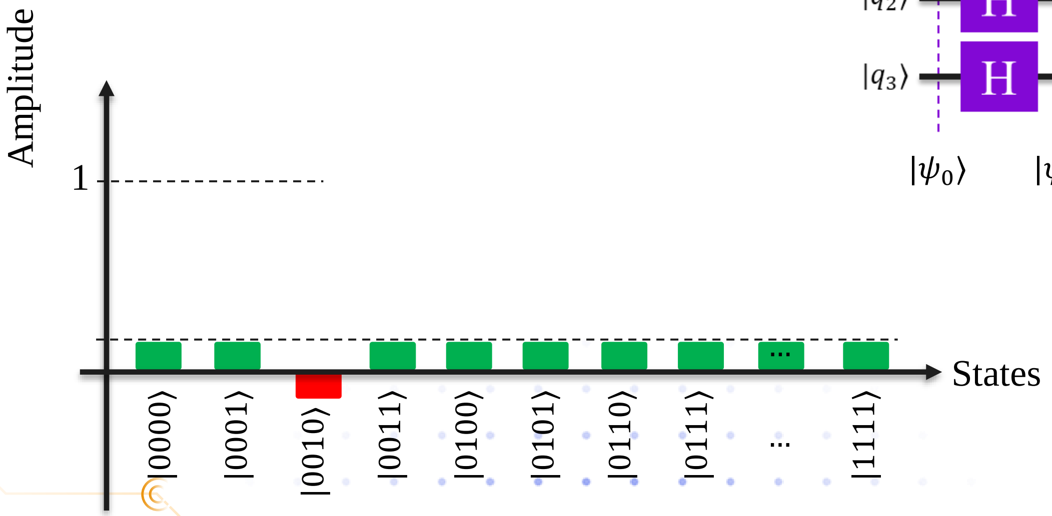


Grover's Algorithm

Step 2: Apply oracle (iteration 1)

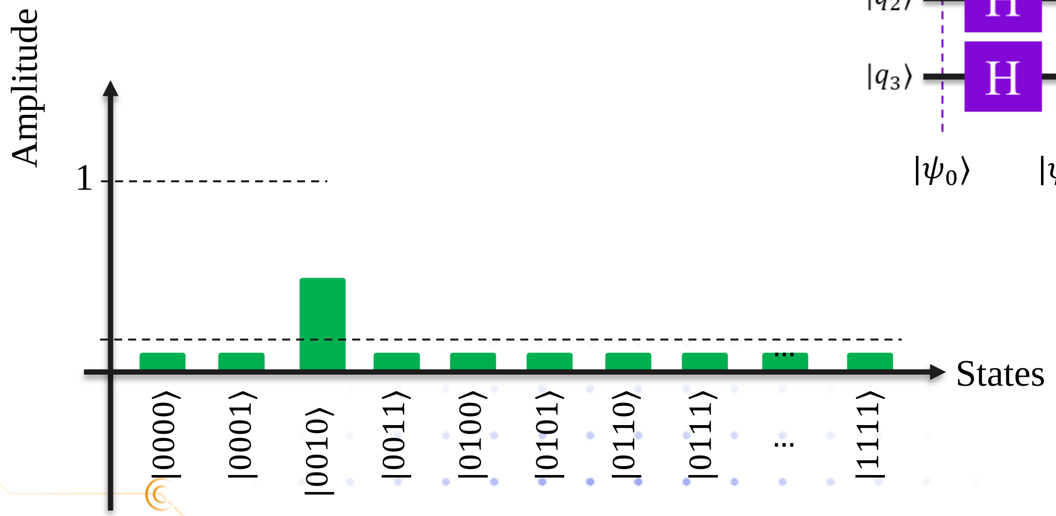
Assume $|0010\rangle$ is $|s\rangle$

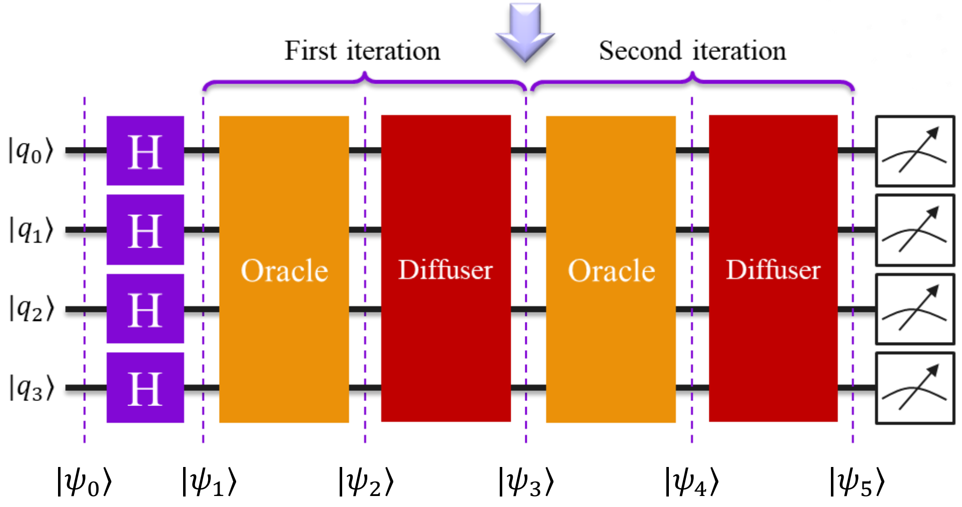




Grover's Algorithm

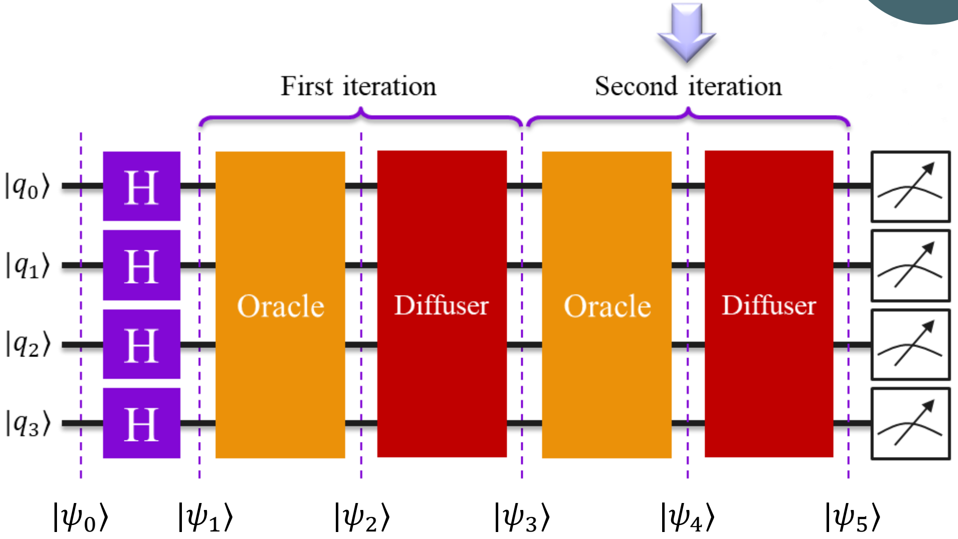
Step 3: Apply Diffuser (iteration 1)

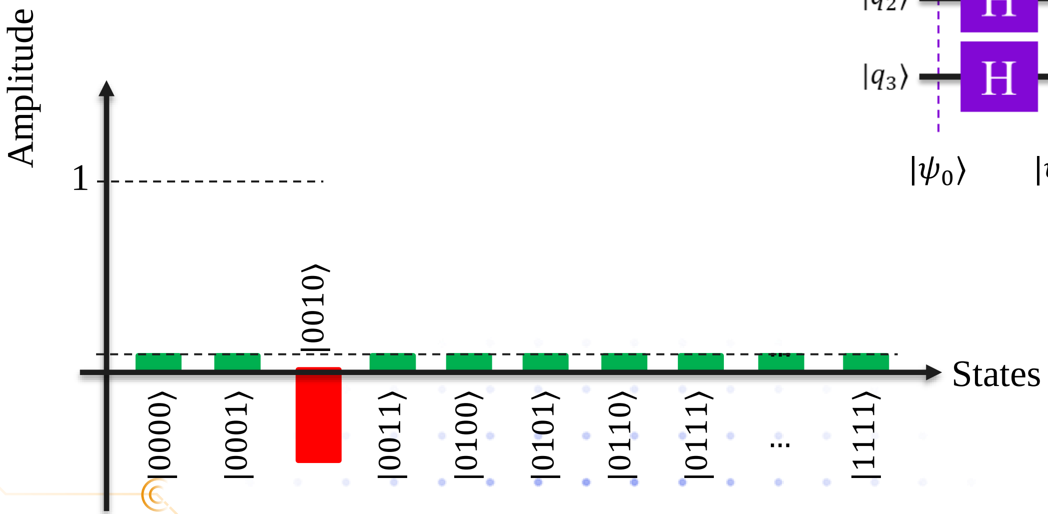




Grover's Algorithm

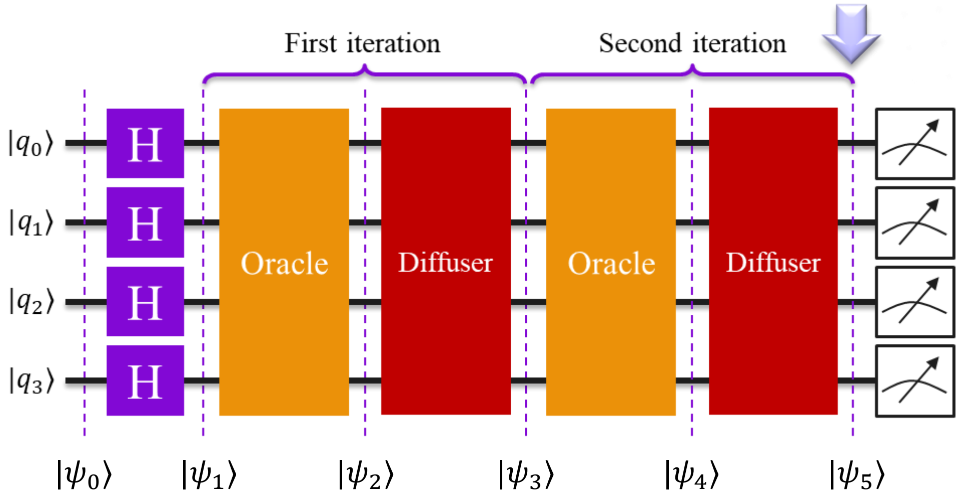
Step 4: Apply Oracle (iteration 2)

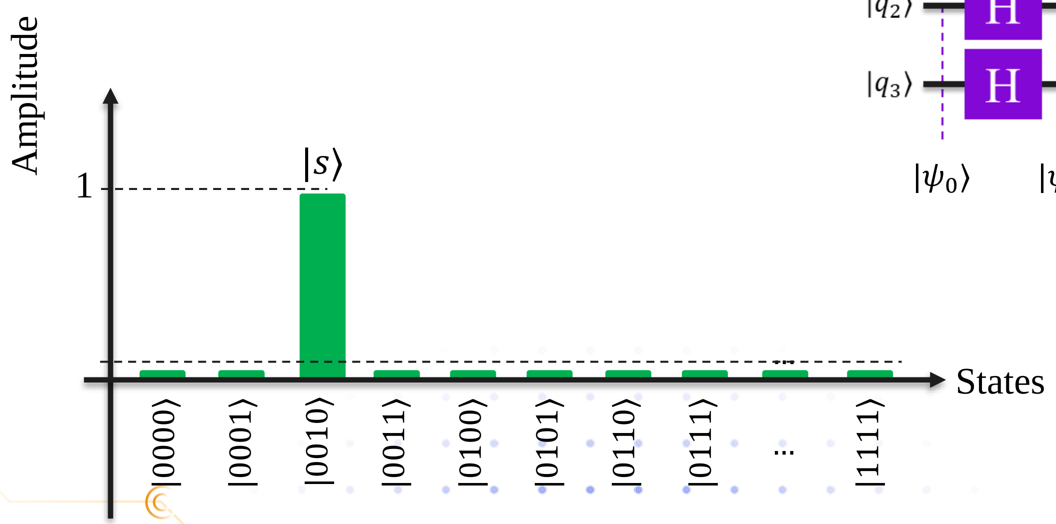




Grover's Algorithm

Step 4: Apply Diffuser (iteration 2)





Coding time ;)

اطلس کوانتوم