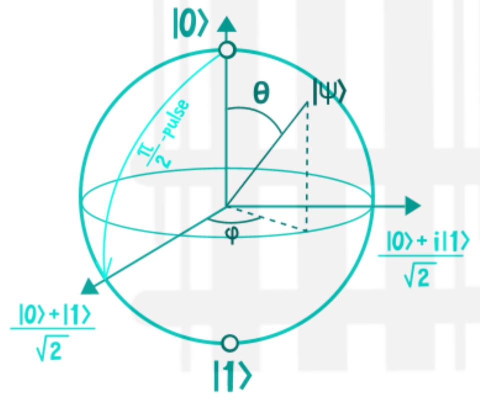
VECTORS AND INTRO TO MATRICES

INTRO TO MATRICES

WHAT DO MATRICES MEAN FOR Q.COMP?



As we saw, we represent q-states as vectors

Our goal is to manipulate these states

↓

q.algorithms

↓

q.gates ⟶ Matrices

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

MATRIX

We can think of a matrix as a collection of row vectors or a collection of a column vectors

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}$$

Geometrically, they are transformations that allow us to both rotate and scale vectors

MATRIX NOTATION AND SHAPE

An (n x m) matrix is written as

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}$$

Matrix Shape: (#rows x #cols)

SOLVING LINEAR SYSTEMS OF EQUATIONS

The identity matrix

It's defined as

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

$a*1=a$

$AI=IA=A$

$\vec{v}^T I = \vec{v}^T \quad \vec{v}I = \vec{v}$

Matrix inversion

$$XX^{-1} = X^{-1}X = I$$

X^{-1} is thus the inverse of Matrix X

Many matrices do not have inverses

MATRIX ADDITION

General equation (only add matrices of the same shape)

$$A+B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1m}+b_{1m} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2m}+b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nm}+b_{nm} \end{pmatrix}$$

MATRIX-VECTOR MULTIPLICATION

General equation (the vector height must match the matrix width)

$$A\vec{x} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} a_{11}x_1 & a_{12}x_2 & \cdots & a_{1m}x_m \\ a_{21}x_1 & a_{22}x_2 & \cdots & a_{2m}x_m \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}x_1 & a_{n2}x_2 & \cdots & a_{nm}x_m \end{pmatrix} = \begin{pmatrix} \langle \vec{a}_1, \vec{x} \rangle \\ \langle \vec{a}_2, \vec{x} \rangle \\ \vdots \\ \langle \vec{a}_n, \vec{x} \rangle \end{pmatrix}$$

MATRIX TRAPOSE

$$\text{If } \mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}, \text{ then } \mathbf{X}^T = \begin{pmatrix} x_{11} & x_{21} & \cdots & x_{n1} \\ x_{12} & x_{22} & \cdots & x_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1m} & x_{2m} & \cdots & x_{nm} \end{pmatrix}$$

Matrix conjugate trapose

$$\text{If } \mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}, \text{ then } \mathbf{X}^\dagger = \begin{pmatrix} x^*_{11} & x^*_{21} & \cdots & x^*_{n1} \\ x^*_{12} & x^*_{22} & \cdots & x^*_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ x^*_{1m} & x^*_{2m} & \cdots & x^*_{nm} \end{pmatrix}$$

MATRIX-SCALAR MULTIPLICATION

General equation

$$c \cdot A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} = \begin{pmatrix} c \cdot a_{11} & c \cdot a_{12} & \cdots & c \cdot a_{1m} \\ c \cdot a_{21} & c \cdot a_{22} & \cdots & c \cdot a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c \cdot a_{n1} & c \cdot a_{n2} & \cdots & c \cdot a_{nm} \end{pmatrix}$$

MATRIX-MATRIX MULTIPLICATION

General equation (the 1st matrix width must match the second matrix height)

$$AB = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ b_{21} & b_{22} & \cdots & b_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mk} \end{pmatrix} = \begin{pmatrix} \langle \vec{a}_1, \vec{b}_1 \rangle & \langle \vec{a}_1, \vec{b}_2 \rangle & \cdots & \langle \vec{a}_1, \vec{b}_k \rangle \\ \langle \vec{a}_2, \vec{b}_1 \rangle & \langle \vec{a}_2, \vec{b}_2 \rangle & \cdots & \langle \vec{a}_2, \vec{b}_k \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \vec{a}_n, \vec{b}_1 \rangle & \langle \vec{a}_n, \vec{b}_2 \rangle & \cdots & \langle \vec{a}_n, \vec{b}_k \rangle \end{pmatrix}$$