Engineering Parallel Bi-Criteria Shortest Path Search

Stephan Erb – *stephan.erb@student.kit.edu*

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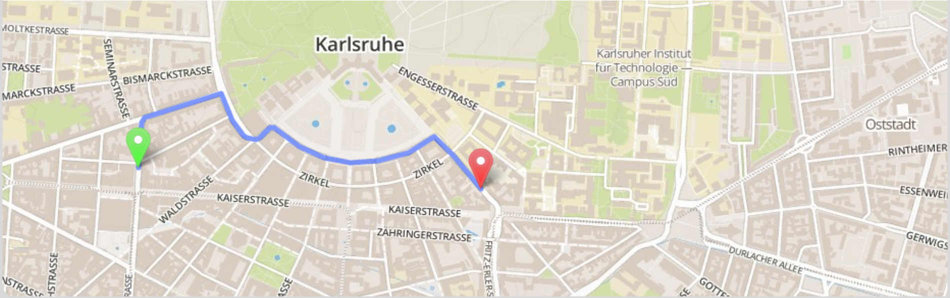
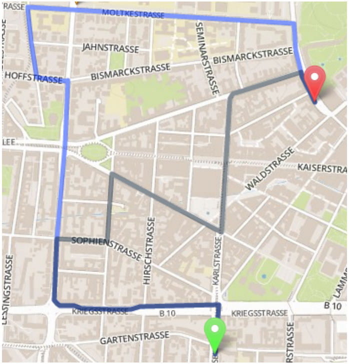
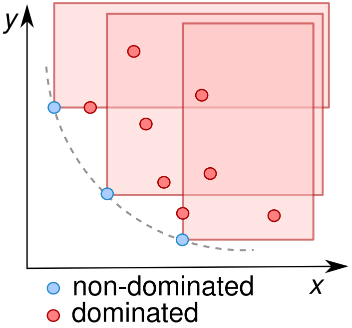


The Problem



Simultaneous optimization of **multiple** (possibly) conflicting **objectives**
(e.g., travel time and highway tolls)

Bi-Criteria Shortest Path Problem

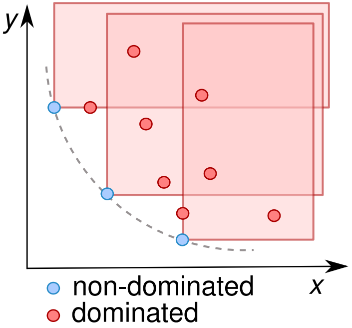


Graph $G = (V, E)$ with 2-dimensional edge weights $\vec{w} = (x, y)$

Pareto optimal path are non-dominated

A path p is dominated by a path q iff $(q \neq p) \wedge (q_x \leq p_x) \wedge (q_y \leq p_y)$

Bi-Criteria Shortest Path Problem



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Goal: Find all Pareto optimal paths from a start node to all other nodes **(one-to-all)**

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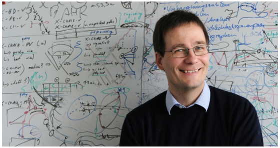
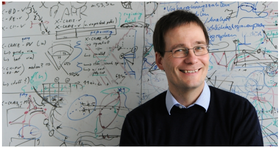
The Parallel Pareto Search Algorithm



Parallel Label-Setting Multi-Objective Shortest Path Search by P. Sanders and L. Mandow

IEEE International Parallel and Distributed Processing Symposium (IPDPS2013)

The Parallel Pareto Search Algorithm



Parallel Label-Setting Multi-Objective Shortest Path Search by P. Sanders and L. Mandow

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Provably efficient for two dimensions.
But is it also practical?

Dijkstra's Algorithm

Procedure DijkstraSearch(G,s)

$L[v] = \infty$ for all $v \in V$; $L[s] = 0$

PriorityQueue $Q = \{(s, 0)\}$ // *tentative labels*

while *Q is not empty* **do**

(u, l) = $Q.removeMinimum()$

foreach $(u, v) \in E$ **do**

$l' = l + weight((u, v))$ // *candidate label*

if l' *represents new best path to v* **then**

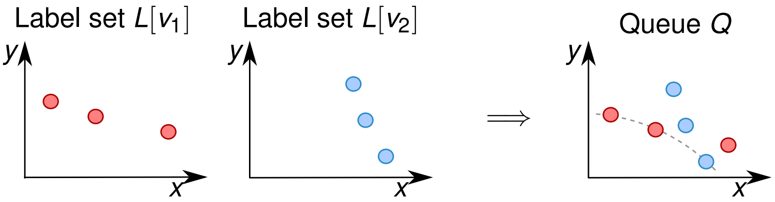
update $L[v]$

update Q

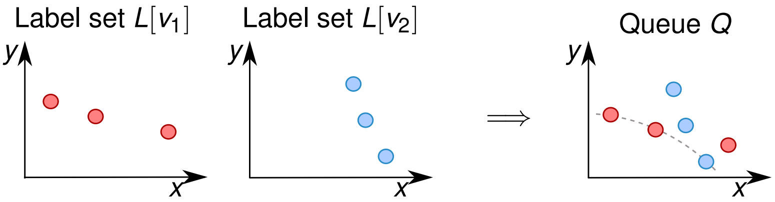
Label Setting Property:

Shortest path distance of node/labels extracted from Q is final

Bi-Criteria Label Setting



Bi-Criteria Label Setting



Classic Approaches [1]:

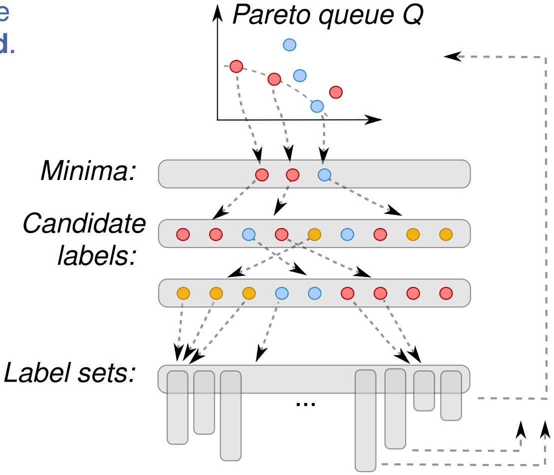
Extract **any** Pareto optimal label from Q

Sanders & Mandow:

Extract **all** Pareto optimal labels from Q

The Parallel Pareto Search Algorithm

All steps are **parallelized**.



Engineering Bulk Updates

Goal:

Apply sequence of k **sorted updates** (insertions & deletions)

Proposed Solution:

Parallel **red-black tree** of Frias and Singler [2]

Problem:

Binary trees can be multiple times slower than **cache-sensitive B-trees** [3]

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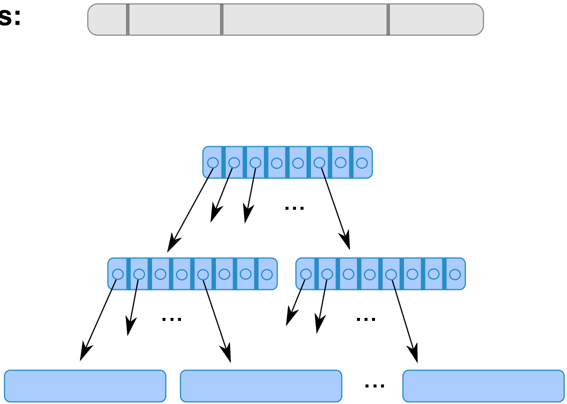
Parallel Bulk Updates for B-trees

Updates:
(sorted)

B-tree:



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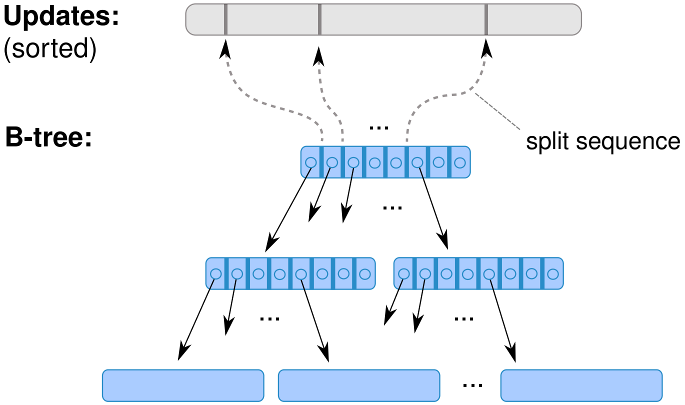
Parallel Bulk Updates for B-trees



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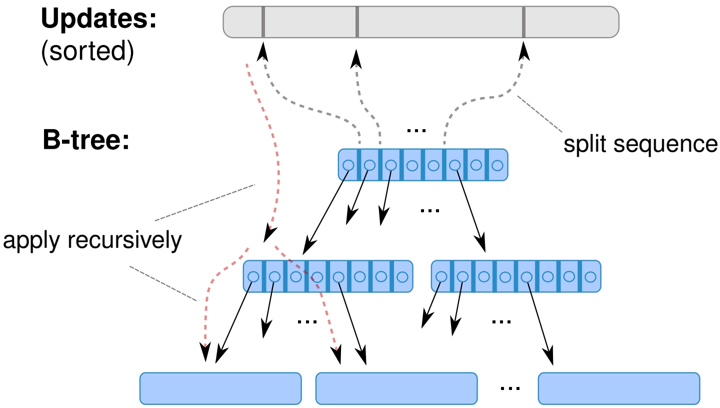
Parallel Bulk Updates for B-trees



Subtrees updated **independently in parallel**.

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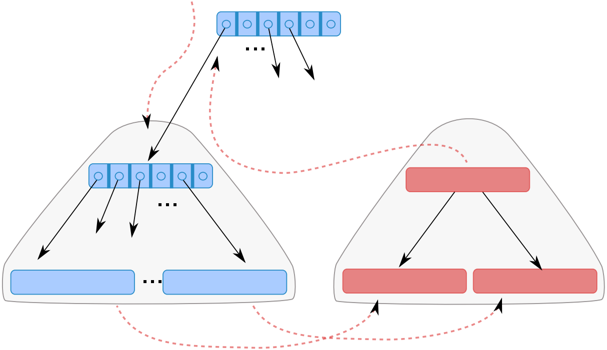
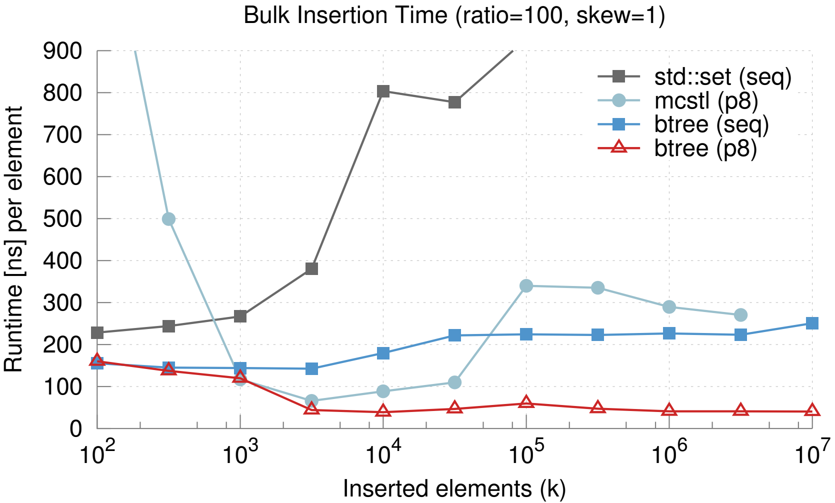
Parallel Rebalancing of B-trees

Parallel **partial-rebuilding** [4] of unbalanced subtrees:



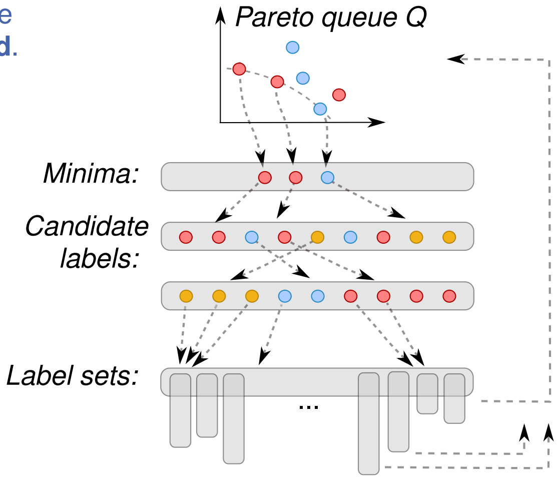
Amortized update bound of $\mathcal{O}(k/p \cdot \log N)$ for the application of k updates using p threads on a tree of size N.

Experimental Results



The Parallel Pareto Search Algorithm

All steps are **parallelized**.



Evaluation Setup

Karlsruhe Institute of Technology

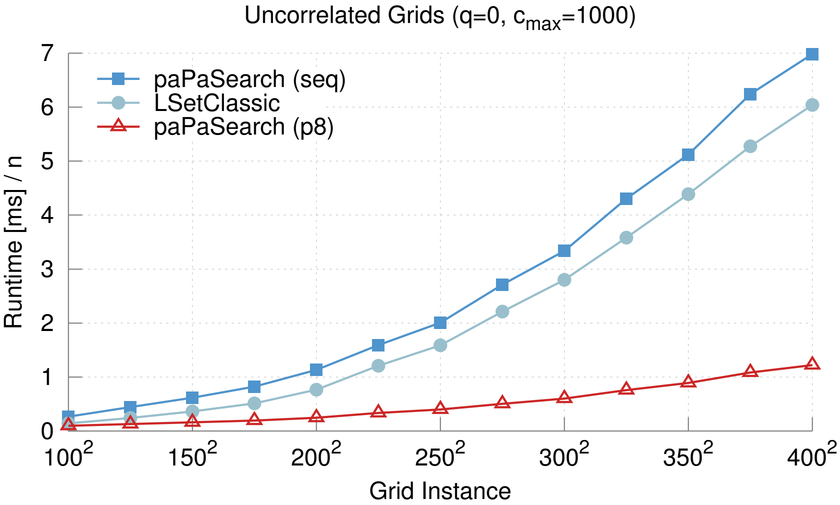
System:

- 2× Intel Xeon E5-2670 with **16 cores** in total, **2.6 GHz**, 2× 20MB L3 Cache
- 64 GB main memory

Tuned Competitor:

- highly tuned binary heap [5]
- fast dominance checks

Experimental Results



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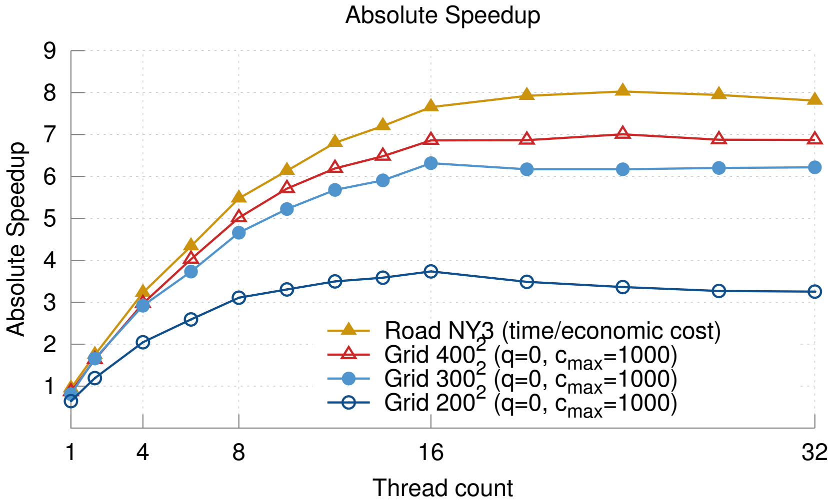

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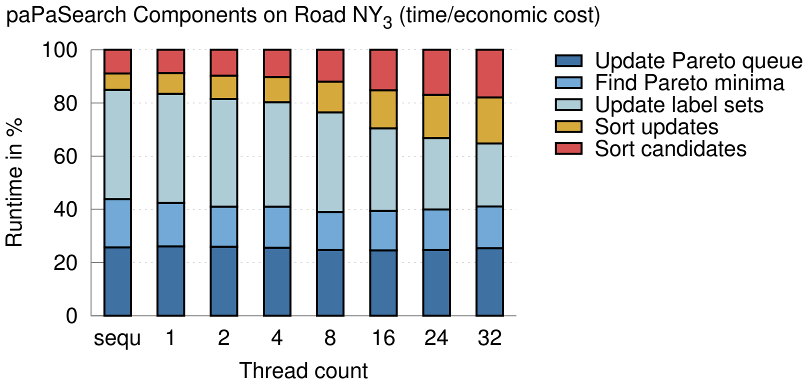
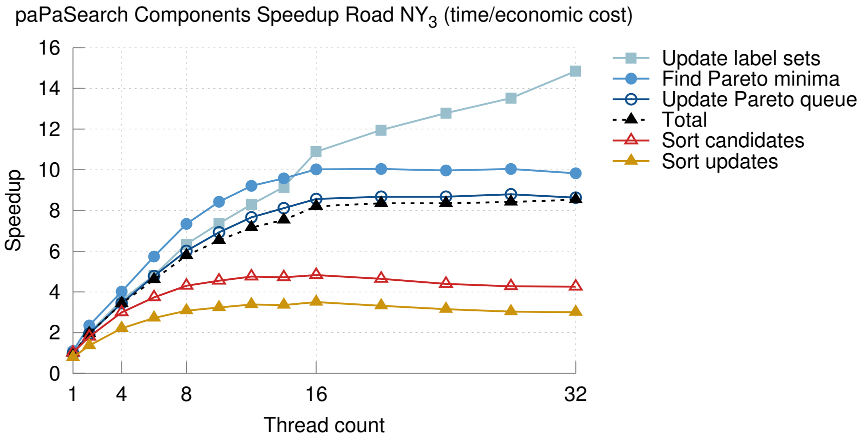
Component Analysis



Component Analysis



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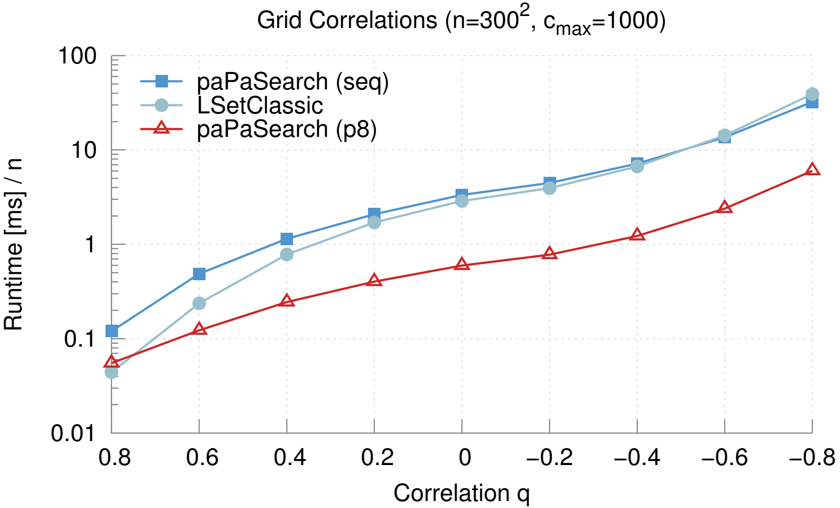

Conclusion

Summary:

- Parallel label-setting is **practical** on modern **multiprocessors**
- It excels at **large & difficult instances**

Future Work:

- Delta-stepping [6] to increase work per iteration
- scalable sorting of small collections?
- Point-to-point search

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Thank You!

Results will be published on `http://algo2.iti.kit.edu`.

References I

[1] P. Hansen, “Bicriterion path problems,” in *Multiple Criteria Decision Making Theory and Application*, ser. Lecture Notes in Economics and Mathematical Systems. Springer Berlin Heidelberg, 1980, vol. 177, pp. 109–127. [Online]. Available: http://dx.doi.org/10.1007/978-3-642-48782-8_9

[2] L. Frias and J. Singler, “Parallelization of bulk operations for stl dictionaries,” in *Euro-Par 2007 Workshops: Parallel Processing*, ser. Lecture Notes in Computer Science, L. Bougé, M. Forsell, J. Träff, A. Streit, W. Ziegler, M. Alexander, and S. Childs, Eds. Springer Berlin Heidelberg, 2008, vol. 4854, pp. 49–58. [Online]. Available: http://dx.doi.org/10.1007/978-3-540-78474-6_8

References II

[3] T. M. Chilimbi, M. D. Hill, and J. R. Larus, “Cache-conscious structure layout,” in *Proceedings of the ACM SIGPLAN 1999 conference on Programming language design and implementation*, ser. PLDI ’99. New York, NY, USA: ACM, 1999, pp. 1–12. [Online]. Available: http://doi.acm.org/10.1145/301618.301633

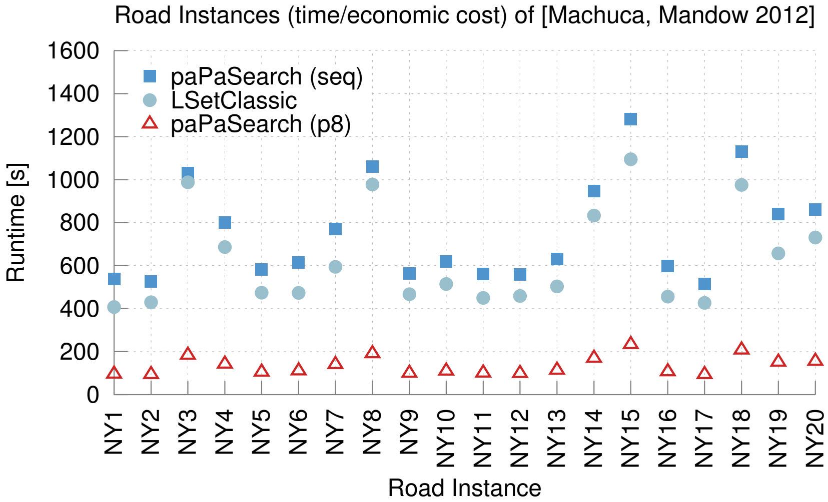
[4] M. Overmars, *The design of dynamic data structures*. Springer Verlag, 1983.

[5] P. Sanders, “Fast priority queues for cached memory,” *Journal of Experimental Algorithmics (JEA)*, vol. 5, Dec. 2000. [Online]. Available: http://doi.acm.org/10.1145/351827.384249

[6] U. Meyer and P. Sanders, “δ-stepping: a parallelizable shortest path algorithm,” *Journal of Algorithms*, vol. 49, no. 1, pp. 114 – 152, 2003. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0196677403000762

Backup

Experimental Results



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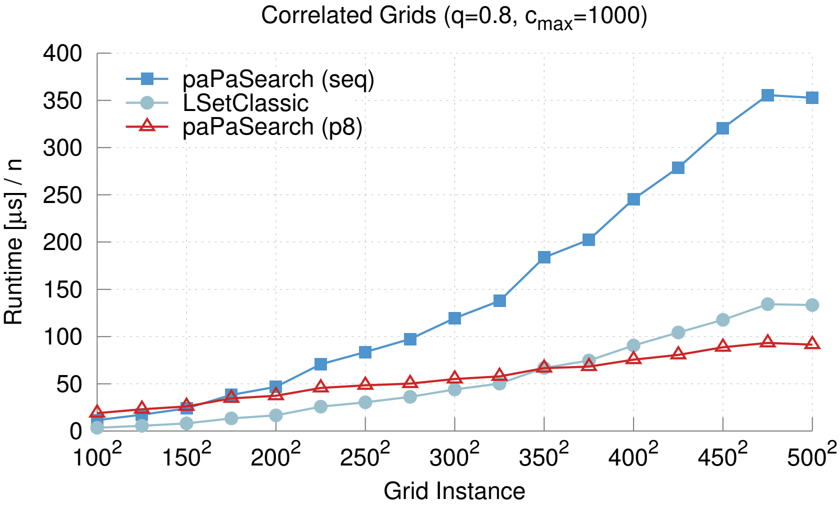

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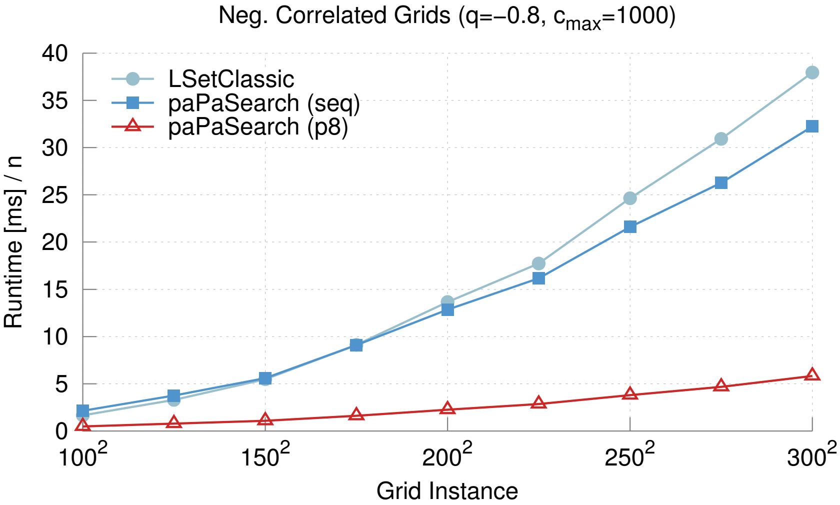

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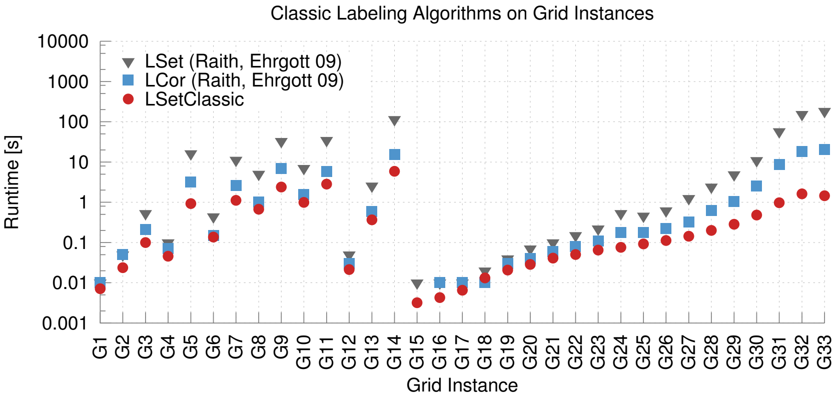
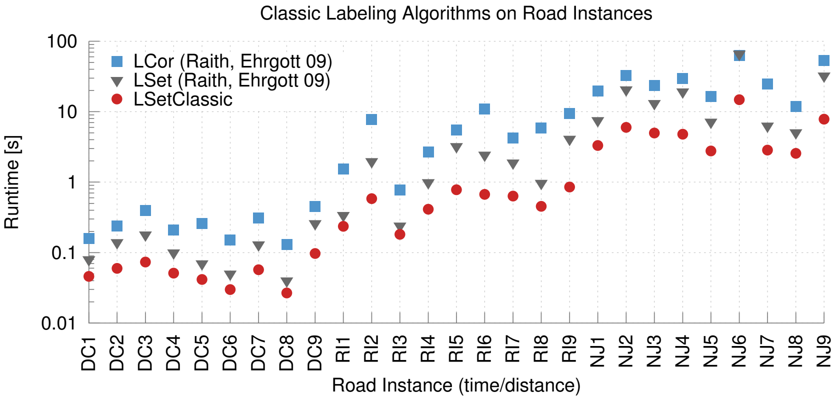
Classic Competitor

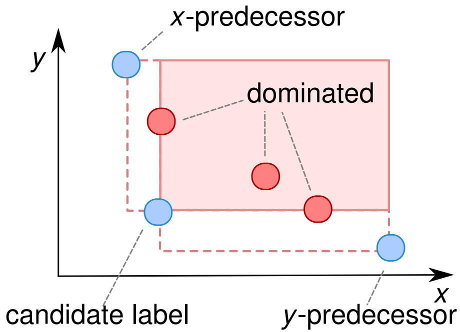


Classic Competitor



Dominance checks

Fast dominance checks on lexicographically sorted label sets:



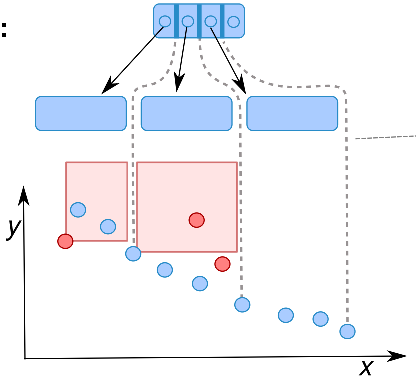
Predecessor queries as **tree traversals**.
Running in $\mathcal{O}(\log N)$ for a label set of size N.

Tuned Dominance checks

Batched dominance check & candidates filtering:

B-tree Label Set:

Logical View:



Router keys are copies of the largest label in each subtree.

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