Discrete Mathematics - Math349: Test 1 [20 marks] **Duration:** 1 hr **Sept. 8, 2021**

Surname and Initials: _____
Student No.: _____

Calculators are not allowed!!!

Stick your barcode here	Enter your **Department** and **Group**

Please Note: This assessment has **3 questions**, in **1** page

Question 1 [5]

Let $\mathcal{N} = \{1, 2, 3, 4...\}$ be the universal set. For each $n \in \mathcal{N}$, let $A_n = \{n, 2n, 3n, ...\}$. Compute each of the following sets:

(a) $A_3 \cap A_5$ (1)

(b) $A_4 \cap A_6$ (1)

(c) $A_4 \cap A_5 \cap A_{10}$ (1)

(d) $\bigcup_{i \in I} A_i$, where $I = \{2, 3, 5, 7, ...\}$ is the set of prime numbers. (2)

Question 2 [4]

Let A, B, C be any set and $\{A_\alpha\}_{\alpha \in I}$ be a family of sets. Prove that

(a) (2)

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

(b) (2)

$$\left(\bigcap_{\alpha \in I} A_\alpha \right)^c = \bigcup_{\alpha \in I} A_\alpha^c$$

Question 3 [11]

(a) Let R be a relation defined on Z by aRb if and only if $3|(a + 2b)$.

i. Show that R is an equivalence relation on Z. (6)

ii. Determine $[-1]$, the equivalence class of -1 if R is the equivalent relation defined on the set $A = \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$. (1)

(b) Let $A = \{1, 2, 3, \cdots, 13, 14, 15\}$ and R be the equivalence relation defined on A by $a \equiv b \ (mod\ 5)$, that is $a - b$ is divisible by 5. Find the partition of A induced by R, i.e. the quotient $A|R$. (4)