IOM Exercise 9

Part 1

$EQ_{29}: \frac{\partial^2 u}{\partial x^2} = \frac{u(x+\Delta x) - 2u(x) + u(x-\Delta x)}{\Delta x^2} + \mathcal{O}(\Delta x)$

Discretization

discrete

$$\left.\begin{aligned} t^n &= t_0 + n\Delta t \\ x_i &= x_0 + i\Delta x \end{aligned}\right\} \text{ so } U_i^n = (x_i, t^n)$$

$u(x+\Delta x) = U_{i+1}^n \; ; \; u(x) = u_i^n \; ; \; u(x-\Delta x) = U_{i-1}^n$

$$\therefore \frac{\partial^2 u}{\partial x^2} \approx \frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{\Delta x^2}$$

Part 2

$EQ_{33}: \frac{\partial u}{\partial z} = \frac{U_{(z+\Delta z)} - U_{(z-\Delta z)}}{2\Delta z} + \mathcal{O}(\Delta z^2)$

$U_{ijk}^n = U(x_i, y_j, \underbrace{z_k, t^n})$

$\therefore U_k^n = U(z_k, t^n)$

$\therefore U_{(z+\Delta z)} = U_{k+1}^n \; ; \; U_{(z-\Delta z)} = U_{k-1}^n$

$$\therefore \frac{\partial u}{\partial z} \approx \frac{U_{k+1}^n - U_{k-1}^n}{2\Delta z}$$