E 13: discretized Ekman equations

1 Numerical scheme.

$$\frac{\partial u_E}{\partial t} - f v_E = A_v \frac{\partial^2 u_E}{\partial z^2} \quad \therefore \quad \frac{\partial u_E}{\partial t} = A_v \frac{\partial^2 u_E}{\partial z^2} + f v_E$$

$$\frac{\partial v_E}{\partial t} + f u_E = A_v \frac{\partial^2 v_E}{\partial z^2} \quad \therefore \quad \frac{\partial v_E}{\partial t} = A_v \frac{\partial^2 v_E}{\partial z^2} - f u_E$$

Zonal components

$$\frac{\partial u_E}{\partial t} \approx \frac{(U_E + \Delta U_E)}{\Delta_t}$$

$$\frac{\partial^2 u_E}{\partial z^2} \approx \frac{u_E(z+\Delta z) - 2u_E(z) + u_E(z - \Delta z)}{\Delta z^2}$$

$$\left.\begin{aligned} t^n &= t_0 + n\Delta t \\ z_k &= z_0 + k\Delta z \end{aligned}\right\} \text{ so } U^n_{Ek} = (z_k ; t^n)$$

$$\left[\therefore \frac{U^{n+1}_{Ek} - U^n_{Ek}}{\Delta t} = A_v \frac{U^n_{Ek+1} - 2U^n_{Ek} + U^n_{Ek-1}}{\Delta z^2} + f v_E\right]$$

Meridional components.

$$\left[\frac{V^{n+1}_{Ek} - V^n_{Ek}}{\Delta t} = A_v \frac{V^n_{Ek+1} - 2V^n_{Ek} + V^n_{Ek-1}}{\Delta z^2} - f u_E\right]$$

(2) Courant number

Zonal:

$$U_{E_k}^{n+1} = U_{E_k}^{n} + \frac{A_v \Delta t}{\Delta z^2}\left(U_{E_{k+1}}^{n} - 2U_{E_k}^{n} + U_{E_{k-1}}^{n}\right) + \Delta t f v_E$$

$$\left.\begin{aligned} \therefore C_E &= \frac{A_v \Delta t}{\Delta z^2} \\ &= \frac{A_v}{\frac{\Delta z^2}{\Delta t}} \end{aligned}\right\} \text{non-dimensional Courant number}$$

→ this number is the same form for the meridional component below

Meridional:

$$V_{E_k}^{n+1} = V_{E_k}^{n} + \frac{A_v \Delta t}{\Delta z^2}\left(V_{E_{k+1}}^{n} - 2V_{E_k}^{n} + V_{E_{k-1}}^{n}\right) - f u_E \Delta t$$

(3) $C_E = \frac{A_v}{\frac{\Delta z^2}{\Delta t}}$

Vertical Eddy Viscosity (A_v) = $\underline{10^{-6}\ m^2\ s^{-1}}$

if $\Delta z = 5\ m$ and $\Delta t = 100$ seconds, then the maximum numerical vertical eddy viscosity is equal to $0.25\ m^2\ s^{-1}$.

But $C_E = \frac{10^{-4}\ m^2\ s^{-1}}{\frac{(5\ m^2)^2}{100\ s}}$

$= 4 \times 10^{-4}\ m^2\ s^{-1}$, which is close to zero.

$C_E \geq 0$