IOM Exercise 8 → backward in space

$$U(x - \Delta x) = U(x) + \frac{\partial U}{\partial x}(-\Delta x) + \mathcal{O}(\Delta x^2)$$

$$\therefore U(x - \Delta x) = U(x) - \frac{\partial U}{\partial x}(\Delta x) + \mathcal{O}(\Delta x^2)$$

$$\text{so } \frac{\partial U}{\partial x} = \frac{U(x) - U(x - \Delta x)}{\Delta x} + \mathcal{O}\Delta x$$

and $\frac{\partial U}{\partial x} \approx$ Discretization

$$x_i = x_0 + i\Delta x$$

$$t_n = t_0 + n\Delta t$$

$$\text{so } U_i^n = U(x_i, t_n)$$

$$\therefore U(x - \Delta x) = U_{i-1}^n$$

$$\therefore \frac{\partial U}{\partial x} \approx \frac{U_i^n - U_{i-1}^n}{\Delta x}$$