

**The surest way not to fail is to determine to succeed-Frank Lloyd Wright**

### IRRATIONAL NUMBERS (SURDS)

1. Express in the simplest form.

(a)  $\sqrt{98}$     (b)  $\sqrt[3]{375}$     (c)  $\sqrt[5]{160}$     (d)  $\sqrt{27m^2n^3}$     (e)  $\sqrt{2x^2 - 4xy + 2y^2}$

2. Find the value of    (a)  $\sqrt{50} + \sqrt{32} - \sqrt{18}$     (b)  $8\sqrt{63} + 5\sqrt{7} - 8\sqrt{28}$

(C)  $\sqrt[3]{54} + \sqrt[3]{128} - \sqrt[3]{432}$     (d)  $3\sqrt{\frac{1}{147}} - 7\sqrt{\frac{1}{27}} - \frac{11}{3}\sqrt{\frac{1}{3}}$

(e)  $\sqrt{18p^3q^3} + p\sqrt{8pq^3} - q\sqrt{50p^3q}$

3. If  $\sqrt{2} = 1.41$ ,  $\sqrt{3} = 1.7$ ,  $\sqrt{5} = 2.2$ ,  $\sqrt{7} = 2.6$  Find, correct to one decimal place.

(a)  $3\sqrt{500} - 2\sqrt{243}$     (b)  $4\sqrt{128} - 6\sqrt{108} + 5\sqrt{75}$

4. Simplify the following

(a)  $2\sqrt{10} \times \sqrt{15}$     (b)  $3\sqrt{33} \times 2\sqrt{77}$     (c)  $\sqrt{96} \times 2\sqrt{12}$

(d)  $\sqrt[3]{a+b} \times \sqrt[3]{a-b}$     (e)  $5\sqrt{288} \div 6\sqrt{28}$     (f)  $21\sqrt{588} \div 8\sqrt{98}$

(g)  $\frac{2\sqrt{15}}{3\sqrt{7}} \div \frac{4\sqrt{18}}{9\sqrt{35}}$     (h)  $(5 + 2\sqrt{3})^2$

(i)  $(5 + 3\sqrt{2})(5 - 3\sqrt{2})$     (j)  $(2\sqrt{7} + \sqrt{5})(3\sqrt{5} - \sqrt{7})$     (k)  $\frac{\sqrt{2}}{3-\sqrt{2}} \div \frac{2+3\sqrt{2}}{7}$

5. Express the following with rational denominators.

(a)  $\frac{1}{2\sqrt{2}-\sqrt{3}}$     (b)  $\frac{4\sqrt{75}}{25\sqrt{56}}$     (c)  $\frac{87}{7-2\sqrt{5}}$     (d)  $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$     (e)  $\frac{2\sqrt{a}}{5\sqrt{a}-3\sqrt{x}}$

ANSWERS: 1a  $7\sqrt{2}$ , (b)  $5\sqrt[3]{3}$  (c)  $2\sqrt[5]{5}$  (d)  $3mn\sqrt{3n}$  (e)  $(x-y)\sqrt{2}$

$$\begin{aligned}
 & 2a. 6\sqrt{2} \quad (b) 13\sqrt{7} \quad (c) \sqrt[3]{2} \quad (d) \frac{-39}{7} \sqrt{\frac{1}{3}} \quad (e) 0 \quad 3a. 35.4 \\
 & (b) 26.4 \quad 4a. 10\sqrt{6} \quad (b) 66\sqrt{21} \quad (c) 48\sqrt{2} \quad (d) \\
 & \sqrt[3]{a^2 - b^2} \quad (e) \frac{5\sqrt{14}}{7} \quad (f) \frac{21\sqrt{6}}{8} \quad (g) \frac{15}{2\sqrt{6}} \quad (h) 37 + 20\sqrt{3} \quad (i) 7 \quad (j) 1 + 5\sqrt{35} \quad (k) 1 \quad 5a. \frac{2\sqrt{2} + \sqrt{3}}{5} \\
 & (b) \frac{\sqrt{42}}{35} \quad (c) 21 + 6\sqrt{5} \quad (d) \frac{114 - 41\sqrt{6}}{30} \quad (e) \frac{10a + 6\sqrt{ax}}{25a - 9x}
 \end{aligned}$$

***There is no failure except in no longer trying-Elbert Hubbard***

### QUADRATIC EXPRESSIONS AND EQUATIONS

- *Factorisation*

Factorisation and expansion are opposite operations. For all the questions in this section, expand your answer. If you obtain the original expression then your factorisation is correct; otherwise keep trying.

#### (1) Common factor method

Factorize the following

$$\begin{aligned}
 & (a) a^2x^2 - 3a^2xy \quad (b) 2x(x - y) - 4y(y - x) \quad (c) x^2(a + b) + xy(b + a) \\
 & (d)(m + n)(2b - 3a) + 3a - 2b \quad \text{ANS 1d: } (2b - 3a)(m + n - 1)
 \end{aligned}$$

#### (2) Difference of two squares

Factorize the following

$$\begin{aligned}
 & (i) q^2 - 4 \quad (ii) 12x^2 - 3 \quad (iii) x^2 - 3 \quad (IV) 4a^2 - 9b^2 \quad (v) (2x - 5)^2 - 1 \\
 & (vi) 4x^2(x - 2) + y^2(2 - x)
 \end{aligned}$$

#### (3) Trinomials

Factorize the following

$$\begin{aligned}
 & (a) 6x^2 - 5x - 21 \quad (b) -2x^2 - 13x - 15 \quad (c) 4a^2 - 34a - 18 \\
 & (d) 4a^2 + 12ab + 9b^2 \quad (e) 3x^2y - 21xy^2 + 30y^3
 \end{aligned}$$

#### (4) Grouping

Factorize the following

- (a)  $6bx - 15cx + 10cy - 4by$       (b)  $x^2 - 2x - y^2 - 2y$   
 (c)  $a^2 - ab + 2a + b - 3$       (d)  $c^2 - b^2 - 2ab - 2ac$   
 (e)  $p^2 - 4m^2 + 4m - 1$

### MISCELLANEOUS

Factorize the following

- (a)  $3x^2 - 7x - 6$     (b)  $6x^2 - 9ax + 8cx - 12ac$     (c)  $4x^2 - 36$   
 (d)  $5x^2 + 8 + 14x$     (e)  $20x^2 + 24x - 9$     (f)  $5x^2 - 30xy + 45y^2$   
 (g)  $5 - \frac{45}{a^2}$     (h)  $4x^2 - 2(5x + 3)$     (i)  $5a^2b + 10ab^2$     (j)  $4x - 2 + y - 2xy$     (k)  $2 - 18x + 36x^2$   
 (l)  $2m^4 - m^3 + 4m - 2$     (m)  $3x^2y - 21xy^2 + 30y^3$   
 (n)  $-3y^2 + 9y + 30$     (o)  $81x^2 - 9$     (p)  $5a^2 + 20a - 60$     (q)  $30b^2 - b - 3$   
 (r)  $12y^2 + y - 1$     (s)  $2x^2 - 5x - 3$     (t)  $7u^2 + 11u - 6$     (u)  $2y^2 - 17y + 21$   
 (v)  $6y^2 - 7y - 20$

### • Algebraic Fractions

1. Simplify the following

$$(a) \frac{x^2 - 2xy}{xy - 2y^2} \quad (b) \frac{12x^4y^3(x+y)}{4x^2y^5(y+x)} \quad (c) \frac{(x-3)^3}{x^2-9} \quad (d) \frac{-2x+2y}{4y-4x} \quad (e) \frac{ax-b+x-ab}{ax^2-abx}$$

$$\text{Ans: a) } \frac{x}{y} \quad \text{b) } \frac{3x^2}{y^2} \quad \text{c) } \frac{(x-3)^2}{x+3} \quad \text{d) } \frac{1}{2} \quad \text{e) } \frac{a+1}{ax}$$

2. Simplify the following

$$(a) \frac{3a-3b}{5a+2b} \times \frac{25a^2-4b^2}{a^2+5ab-6b^2} \quad (b) \frac{ab+a}{b^2-1} \times \frac{a+3}{b-1} \times \frac{b^2-2b+1}{3a+9}$$

$$(c) \frac{a-2}{a^2-a-2} \times \frac{2a^2+3a+1}{6a^2+a-1} \quad (d) \frac{3a^2-7a+2}{a^2-a-2} \times \frac{a^2+2a+1}{3a^2+2a-1} \div \frac{9a^2-1}{9a^2-3a}$$

$$(e) \frac{x^2+x-6}{3x^2-12x} \div \frac{x^3-2x^2}{x^2-16} \times \frac{1}{x+4} \quad (f) \frac{5a-5ab}{2b^2+b-3} \times \frac{6-4b}{5b^2+10b+5} \div \frac{2a+4ab}{2b^2+3b+1}$$

$$\text{Ans: a) } \frac{15a-6b}{a+6b} \quad \text{b) } \frac{a}{3} \quad \text{c) } \frac{1}{3a-1} \quad \text{d) } \frac{3a}{3a+1} \quad \text{e) } \frac{x+3}{3x^2} \quad \text{f) } \frac{2b-3}{(2b+3)(b+1)}$$

3. Express the following as a single fraction

(a)  $1 - \frac{2x-5}{3}$  (b)  $\frac{2x-1}{x^2+2x-8} + \frac{x}{4-x^2}$  (c)  $\frac{x-2}{2} - \frac{x+1}{6} - \frac{x-3}{3}$

(d)  $\frac{6x}{(x+2)(x-2)} - \frac{2}{x+2} + \frac{3}{2-x}$  (e)  $\frac{x}{2x^2+x-1} + \frac{3}{3x^2+2x-1}$

Ans (a)  $\frac{8-2x}{3}$  b)  $\frac{x+1}{(x+4)(x+2)}$  c)  $-\frac{1}{6}$  d)  $\frac{1}{x+2}$  e)  $\frac{3x^2+5x-3}{(2x-1)(3x-1)(x+1)}$

4. Solve the following equations

(a)  $\frac{3}{x} = \frac{2}{x-2}$  (b)  $\frac{3}{2x-1} + \frac{4}{x-3} = \frac{7}{2x^2-7x+3}$  (c)  $\frac{3}{4x-3} + \frac{2}{2x-5} = 0$

(d)  $\frac{8}{2x+3} - \frac{1}{x-4} = \frac{3}{x+2}$  (e)  $\frac{x}{3x-1} - \frac{x-1}{3x+2} = 0$

(f)  $\frac{6}{2v^2+5v-12} - \frac{2}{4v^2-9} = \frac{4}{2v^2+11v+12}$  (g)  $4(x-7) = \frac{4x^2-5x+4}{x-4}$

Ans.a.6; b. $\frac{20}{11}$ ; c. $\frac{3}{2}$ ; d. $-\frac{17}{4}$ ; e. $\frac{1}{6}$ ; f.-11; g.2.77

5a) In a fraction, the denominator is 5 greater than the numerator .If 35 is added to the numerator; the value of the fraction will be equal to the reciprocal of the original fraction. What is the original fraction?

(b).A train runs 60km at a constant speed, if the speed had been 10km/hr more, it would have taken half an hour less for the journey; find the speed of the train.

(c). Divide 40 into two parts such that the sum of their reciprocals is equal to  $\frac{8}{75}$ .

(d). A tank can be filled with water by two pipes running together in 15 minutes. By the larger pipe alone, the tank can be filled 16 minutes sooner than by the small pipe alone; find the time in which each pipe alone would fill the tank.

(e). A boy bought a number of sweets for K280.If he had bought four less, each would have cost K8 more: how many sweets did he buy?

Ans) $\frac{1}{6}$ ; b) 30km/hr; c) 15, 25; d) 24,40min; e) 14;

- Quadratic Equations (equations where the highest power of the variable is 2)**

1. Solve the following equations by **factorizing**.

(a)  $9x^2 - 36 = 0$       (g)  $2x^2 - 11x = -15$       ans:3,5/2

(b)  $3x^2 - 10x + 3 = 0$       (h)  $3x^2 - 20x - 7 = 0$       ans:7,-1/3

(c)  $2x^2 - 7x = 39$       (i)  $3x^2 = 16x + 12$       ans:6,-2/3

(d)  $6x^2 - 13x + 6 = 0$       (j)  $2x^2 + 8x = 5x + 20$       ans:-4,5/2

(e)  $x^2 - \frac{3x}{4} + \frac{1}{8} = 0$       (k)  $(m + 3)^2 = 9$       ans:-6,0

(f)  $(3x + 7)(2x - 5) = 0$       (l)  $4(x - 2)^2 = 32$       ans: $2 \pm 2\sqrt{2}$

1b. Find the quadratic equations whose **roots** are: (i) 2,1    (ii) 1.5,3    (iii) -1,3/2

(iv)  $\sqrt{2}, -\sqrt{3}$     (v)  $-4 + 2\sqrt{5}, 4 - 2\sqrt{5}$     (vi)  $-1/2, 3/4$

2. Solve the following equations by **completing the square**

(a)  $x^2 + 16x - 57 = 0$     (b)  $13x = x^2 + 42$     (c)  $28 = 31x + 5x^2$     (d)  $38x - 357 = x^2$

(e)  $2x^2 - x - 15 = 0$     (f)  $12x^2 - 17x + 6 = 0$     (g)  $4x^2 - 10x + 5 = 0$     (h)  $x^2 - 12x + 5 = 0$     (i)  $5x^2 = 6x + 8$     (j)  $3x^2 - 10x + 5 = 0$

**Quadratic formula:**  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3. Solve the equations above (2) using the **quadratic formula**. (You should get the same answers as in 2)

4. Find two numbers differing by 7, such that the sum of their squares is 137.(ans:4,11)

5. Find a number which, when increased by 7 is equal to sixty times the reciprocal of the number.(ans:3)

6. Find two consecutive even integers whose product is 624.(ans:24,26)

7. Find three consecutive odd integers such that the square of the first, increased by the product of the other two, is 844.(ans:19,21,23)

8. The number of diagonals D of a polygon with n sides is given by the formula  $D = \frac{1}{2} n(n - 3)$

(a) Find  $D$  if  $n = 7$

(b) Find  $n$  if  $D = 20$  (ans: a14, b8)

9. The length of a room is 3m more than the breadth, and the area of the floor is  $270\text{m}^2$ . Find the dimensions of the room. (ans: 15m, 18m)

10. The perimeter of a rectangle is 40 and its area is 96. Find its dimensions. (Ans: 8units, 12units)

### SUBJECT OF THE FORMULA

1. Make the letter in the brackets the subject of the formula.

$$(a) \frac{1}{a} = \frac{1}{b} + \frac{1}{c} \quad \dots \dots \dots (b) \text{ Ans: } \frac{ac}{c-a}$$

$$(b) r = \sqrt[3]{\frac{3v}{4\pi}} \quad \dots \dots \dots (v) \text{ Ans: } \frac{4\pi r^3}{3}$$

$$(c) t = \frac{m}{12} \sqrt{\frac{f}{g}} \quad \dots \dots \dots (f) \text{ Ans: } \frac{144gt^2}{m^2}$$

$$(d) S = \frac{n}{2}[2a + (n-1)d] \quad \dots \dots \dots (d) \text{ Ans: } \frac{2s-2an}{n^2-n}$$

$$(e) v = m\sqrt{a^2 - b} \quad \dots \dots \dots (a) \text{ Ans: } \sqrt{\frac{v^2+m^2b}{m^2}}$$

2. Express the given letter in terms of the other letters.

$$(a) k - \frac{m}{p} - t = 0 \quad \dots \dots \dots p \text{ Ans: } \frac{m}{k-t}$$

$$(b) v = \pi(R^2 - r^2)h \quad \dots \dots \dots r \text{ Ans: } \sqrt{\frac{\pi h R^2 - v}{\pi h}}$$

$$(c) n = \frac{ab}{b+f} \quad \dots \dots \dots f \text{ Ans: } \frac{ab-nb}{n}$$

$$(d) x = \sqrt{y^2 - z^2} \quad \dots \dots \dots y \text{ Ans: } \sqrt{x^2 + z^2}$$

$$(e) T = 2\pi\sqrt{\frac{l}{g}} \quad \dots \dots \dots g \text{ Ans: } \frac{4\pi^2 l}{T^2}$$

3. Given  $a = \sqrt{b^2 + c^2 - 2bcm}$

(a) Make  $m$  the subject of the formula. Ans:  $\frac{b^2 + c^2 - a^2}{2bc}$

(b) Hence calculate  $m$  if  $a = 14$ ,  $b = 12$ , and  $c = 5$  Ans:  $-\frac{9}{40}$

4. The time in seconds for a full swing or oscillation of a pendulum, length  $l$  metres, is given by

$$T = 2\pi \sqrt{\frac{l}{10}}$$

(a) Rearrange the formula to make  $l$  the subject. Ans:  $\frac{5T^2}{2\pi^2}$

(b) The pendulum of a clock is to make a complete oscillation every one second. What length to the nearest millimeter, must the pendulum be?

5. The focal length of a lens is given by the formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

(a) Calculate  $v$  if  $f = 3.0\text{cm}$  and  $u = 4.8\text{cm}$ . Ans: 8

(b) Rearrange the formula to give  $u$  in terms of  $f$  and  $v$ . Ans:  $\frac{fv}{v-f}$

(c) Find  $u$  if  $f = 2.6\text{cm}$  and  $v = 6.5\text{cm}$  Ans: 4.3cm

6. Given that  $y = \frac{2-5x}{2+3x}$ , express  $x$  in terms of the other letters

7. If  $x = (x - a)(x + b)$ , make  $a$  the subject of the formula

8. Make  $k$  the subject of the formula  $T = \sqrt[3]{\frac{k^4 - h^2}{gh}}$

### EXPONENTIAL AND LOGARITHMIC FUNCTIONS

1. Find the value of the following

- (a)  $16^{\frac{3}{4}}$  (b)  $4^{-\frac{5}{2}}$  (c)  $243^{\frac{2}{5}}$  (d)  $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$  (e)  $\left(\frac{81}{16}\right)^{\frac{3}{4}}$

$$(f) \left(\frac{32}{243}\right)^{-\frac{7}{5}} \quad (g) \quad 16^{\frac{1}{4}} + 27^{\frac{2}{3}} + 64^{\frac{5}{6}}$$

Ans. a)8 b) $\frac{1}{32}$  c)9 d) $\frac{3}{2}$  e) $\frac{27}{8}$  f) $\left(\frac{3}{2}\right)^7$  g)43

2. Simplify and express with positive indices

$$(a) \frac{2^n \times (2^{n-1})^n}{2^{n+1} \times 2^{n-1}} \times \frac{1}{4^{-n}} \quad (b) \frac{3 \cdot 2^n - 4 \cdot 2^{2n-2}}{2^n - 2^{n-1}} \quad (c) \frac{3^{n+4} - 6 \cdot 3^{n+1}}{3^{n+2} \times 7} \quad (d) \frac{2^{n+2} - 2^{n+1}}{2^n}$$

Ans. a) $2^{n^2}$  b)4 c)1 d)2

(3) Solve the following exponential equations

$$(a) 3^x = 81 \quad (b) 3^{2x-6} = 1 \quad (c) 5^x = \sqrt{125} \quad (d) 10^{1-x} = 0.01$$

$$(e) 4 \times 3^x = 36 \quad (f) 2^x + 1 = 9 \quad (g) 2^{2x} \times 2^{x+1} = 16$$

$$(h) 2^x \times 2^{x+2} = 64 \quad (i) 8^{2x+1} = 2^x \times 4^x \quad (j) 9^x = \frac{1}{3} \sqrt{3}$$

$$(k) 3^x + 3^{x+2} = 10 \quad (l) (3^x - 9)(3^x + 9) = 0$$

$$(m) 3^{x+1} - 3^{x-1} - 24 = 0 \quad (n) 2^{2x+3} + 1 = 9 \times 2^x$$

$$(o) 2^x + 2^{1-x} = 3$$

Ans. a)4 b)3 c) $\frac{3}{2}$  d)3 e)2 f)3 g)1 h)2 i)-1 j) $-\frac{1}{4}$  k)0 l)2 m)2 n)0,-3 o)0,1

4. Simplify the following without using tables or a calculator

$$(a) \frac{\log 343}{\log 49} \quad (b) \log_{10} 60 - \log_{10} 18 + \log_{10} 30$$

$$(c) \log \frac{11}{15} + \log \frac{490}{297} - 2 \log \frac{7}{9} \quad (d) \log_5 \frac{1}{25}$$

$$(e) \log_7 98 - \log_7 30 + \log_7 15 \quad (f) \frac{\log \frac{1}{9}}{\log \sqrt{3}} \quad (g) \frac{\log \sqrt{8}}{\log \sqrt[3]{2}}$$

$$(h) \log_{10} \frac{81}{32} + 3 \log_{10} \frac{5}{3} + \log_{10} \frac{1}{9} + \log_{10} 768$$

$$(i) 2 \log_{10} \frac{2}{9} + \frac{3}{2} \log_{10} \frac{9}{25} - 5 \log_{10} \frac{2}{3}$$

Ans. a.  $\frac{3}{2}$  b. 2 c.  $\log 2$  d. -2 e. 2 f. -4 g. 1 h. 3 i.  $-3 + 4 \log_{10} 3$

6. Express the following in exponential form.

a)  $\log x + \log y = 3$  b)  $x \log 2 = -1$  c)  $\log y = 1.6 \log x + 0.58$

Ans. a)  $xy = 1000$  b)  $2^x = 0.1$  c)  $y = 10^{0.58}x^{1.6}$

5. If  $2\log_a 2 + \log_a 10 - 3\log_a 3 = 3 + \log_a 5$ . Find the value of  $a$ . Ans.  $\frac{2}{3}$

6. Solve the following log equations

(a)  $\log(x + 3) = 2$       (b)  $\log_2(x + 3) + \log_2(x - 3) = 4$

(c)  $\log x + \log(x - 1) = \log(8x - 12) - \log 2$

(d)  $\log x - \log 3 = \log 6$

(e)  $\log_3 x + \log_3 5 = 1$

(f)  $\log x + \log 3 = \log(2 - 5x)$

(g)  $\log_4 8x - \log_4(x - 1) = 2$

(h)  $\log_6(x + 6) + \log_6(x - 3) = 2$

(i)  $3\log_2(x + 1) - 2 = 13$

Ans. a. 97 b. 5 c.  $\frac{2}{3}$  d. 18 e.  $\frac{3}{5}$  f.  $\frac{1}{4}$  g. none h. 6 i. 31

### VARIATION

- **Direct Variation**

1. If  $y = kx$  and if  $y = 5$  when  $x = 20$ , find  $y$  when  $x = 40$

2. If  $y = kx^2$  and if  $y = 48$  when  $x = 4$ , find  $y$  when  $x = 10$ .

3. The speed of a body falling under the influence of gravity varies directly as the time. At the end of one second, its speed is  $32.16\text{m/s}$ . Find a formula giving the velocity  $v$  in terms of the time  $t$

4. If  $y \propto x^2$  and if  $y = 10$  when  $x = 3$ , find  $x$  when  $y = 40$

5. The monthly rent for an office is directly proportional to the size of the office .If a  $420m^2$  Office rents for k1260 per month, and then what is the rent for a  $900m^2$  office?

6. If the cube of  $x$  varies as the square of  $y$  and if  $x = 3$  when  $y = 5$ , find the equation between  $x$  and  $y$

7. Fill the following table if  $y \propto x^2$

$x$	2	3	4	5	6
$y$			4.8		

8. If  $y$  varies directly as  $x^2$ , what is the effect on  $y$  when  $x$  is multiplied by 3? What is the effect on  $x$  when  $y$  is multiplied by 16?

Ans. 1)10 (2)300 (3) $v = 32.16t$  (4)6 (5)K2700 (6)

$$\sqrt[3]{\frac{27y^2}{25}}$$

- **Inverse Variation**

1. Suppose  $a$  is inversely proportional to  $b$  and when  $b = 5$ ,  $a = \frac{1}{2}$  find  $a$  when  $b = 12$

2. A quantity  $p$  is inversely proportional to the quantity  $q$ . When  $p = 7$ ,  $q = 6$ .Find  $p$  when  $q = 8$ .

3. Discover a law connecting  $y$  and  $x$  consistent with the table below

$x$	1	2	3	4
$y$	12	6	4	3

Plot the graph

4. If  $y \propto \frac{1}{x}$  and  $y = 1$ , when  $x = 1$ , find  $x$  when  $y = 5$

5.s varies inversely as the square root of  $v$  and  $s = 6$  when  $v = 9$ . Write a formula that describes this statement.

6. The volume of a gas in a cylinder at a fixed temperature is inversely proportional to the weight on the position. If the gas has a volume of  $6cm^3$  for a weight of  $30kg$ , then what would be the volume for a weight of  $20kg$ ?

7. If  $y = \frac{m}{x}$  and  $y = 12$  when  $x = 1.8$ , find  $m$ . Also find the value of  $y$  when  $x = 3.75$

Ans. 1)  $\frac{5}{24}$  (2)  $\frac{21}{4}$  (3)  $y = \frac{12}{x}$  (4)  $\frac{1}{5}$  (5)  $s = \frac{18}{w}$  (6) 9 (7) 21.6, 5.76

• **Joint Variation**

1. If  $x$  varies as  $y$  directly, and as  $z$  inversely and  $x = 14$  when  $y = 10, z = 14$ . Find  $z$  when  $x = 49, y = 45$ .

2. If  $A$  varies directly as the square root of  $B$  and inversely as the cube of  $c$  and if  $A = 24$  when  $B = 4$  and  $c = \frac{1}{2}$ , find  $B$  when  $A = 3$  and  $c = 2$

3. The cost for installing floor tiles varies jointly with the length and width of the room in which it is installed. If the labour cost for a 9m by 12m room is K3240, then what is the labour cost for a 6m by 8m room?

4. If  $D$  varies directly with  $t$  and inversely with the square of  $s$ , and  $D=12.35$  when  $t=2.8$  and  $s=2.48$ . Find  $D$  when  $t=5.63$  and  $s=6.81$

5. The volume of a gas in a gas absorber varies directly with the temperature and inversely with the pressure. The volume is  $10\text{cm}^3$  when the temperature is  $20^\circ\text{C}$  and the pressure is 40Kg. What is the volume when the temperature is  $30^\circ\text{c}$  and the pressure is 25kg?

6. The resistance of a wire varies directly with the length and inversely as the square of the diameter. If a wire of length 20cm and diameter 0.1cm has resistance of 2 ohms, then what is the resistance of a 30cm wire with a diameter of 0.2cm?

7. A boy estimates that the number of distinctions he scores in an examination varies directly as the number of subjects he takes up and inversely as the number of people in his class. If he scores 3 distinctions when there are 20 people in his class taking 6 subjects, how many additional subjects must he take to score 5 distinctions when there are 16 people in the class?

8. Given that  $y$  varies jointly as  $x$  and  $z^2$ , and that  $y = 6$  when  $x = 9, z = 3$ , find

(i) The value of  $y$  when  $x = 8, z = 2$

(ii) The value of  $x$  when  $y = 9, z = 4$ .

Ans.1) 18 (2)256 (3)K1440 (4)27.13 (5)24 (6)0.75 (7)2 (8) $\frac{64}{27}, 1944$

- **Partial Variation**

1.  $y$  is partly proportional to  $x$  and inversely proportional to  $x^2$ . If  $x = 1$  then  $y = 5$  and when  $x = -1$  then  $y = 1$ . Find the value of  $y$  when  $x = 20$
2. If  $y$  is equal to the sum of two quantities, one of which is constant and the other varies as  $x$  and if  $y = 17$  when  $x = \frac{1}{3}$  and  $y = 42$  when  $x = 2$ , find the relation between  $x$  and  $y$ . Hence find the values of  $y$  corresponding to  $x = \frac{1}{5}, 3$ , and  $5$ .
3. The expenses of a party are partly constant and partly proportional to the number of guests. For 80 guests the cost is K64000, and for 120 guests K88000. Find the cost for 200 guests. Also draw a graph to show the cost for any number of guests from 50 to 300.
4. The expenses of a school on transport are partly constant and partly vary as the number of boys. The expenses were K1000 000 for 150 boys and K840 000 for 120 boys. What will the expenses be when there are 330 boys?
5. The sum of the first  $n$  whole numbers 1, 2, 3, 4,..varies partly as  $n$  and partly as  $n^2$ . What is the sum for  $n = 2$  and for  $n = 3$ ? Hence find the general formula. Ans 3,6;  $\frac{1}{2}n(n + 1)$
6. On a certain building estate, the ground rents for plots of land vary partly as the frontage and partly as the frontage and depth jointly. The ground rent for a plot with 30cm frontage and 100cm depth is K38, and for a plot with 36cm frontage and 140cm depth is K60. Find the ground rent for a plot with 24cm frontage and 90cm depth. Ans. K28.

Ans. 1)40.0075 (2)15,57,87 (3)K136000 (4)K196000

### FUNCTIONS

1. For the function  $f(x) = 3x - 1$ . Determine (i)  $f(-1)$  (ii)  $f(0)$  (iii)  $f(a)$  (iv)  $f(a - 3)$  (v)  $4f(4a)$  (vi)  $a$  if  $f(-3) = a$

Ans.i)-4 ii)-1 iii)3a-1 iv)3a-10 v)48a-4 vi)-10

2. Given  $f(x) = 5 - 3x$ , evaluate  $3f(2) + 4f(0)$  Ans.17

- 3(a) for the function  $g(x) = (x + 1)^2$ , Find the images of  $-3, -1, 0, 1, 2$ ,

- b. Represent the function above (i) graphically (ii) by tables (iii) as ordered pairs.

4. The domain of the function  $f(x) = \frac{2x+1}{x-1}$  is  $\{0, 2, 4\}$ . Find the range of the function.

Ans.  $\{-1, 5, 3\}$

5. If the range for the function  $g(x) = x^2 - 2$  is  $\{-2, -1, 7\}$  find the domain.

Ans.  $\{0, 1, -1, -3, 3\}$

6. The range of the function  $f(x) = 1 - \frac{2}{x}$  is  $\{-1, 2, 4\}$ . Find domain.

Ans.  $\left\{1, -2, -\frac{2}{3}\right\}$

7. For the linear function  $f(x) = ax + b$ , where  $a$  and  $b$  constants,  $f(-2) = 7$  and  $f(2) = -1$ . Find the values of  $a$  and  $b$

Ans.  $a = -2, b = 3$

### PROGRESSIONS

- **Arithmetic Progression**

Formulae:  $n^{\text{th}}$  term =  $a + (n - 1)d$

$$\text{Sum}, s_n = \frac{n}{2}(a + l) \text{ OR } s_n = \frac{n}{2}\{2a + (n - 1)d\}$$

1. Find the 7<sup>th</sup> and 24<sup>th</sup> terms of the series 21, 18, 15... Ans. 3; -48

2. The 5<sup>th</sup> and 22<sup>nd</sup> terms of an A.P are 39 and -114 respectively. Find the series. Ans. 75, 66, 57...

3. Find the 8<sup>th</sup> and 20<sup>th</sup> terms of the series

(a) 10, 4, -2.....

(b) -4.2, -3, -1.8.....

(c) a, 4a, 7a,.....

(d)  $x, -2x, -5x, \dots \dots \dots$

Ans. a) -32, -104; b) 4.2, 18.6; c) 22a, 58a; d)  $-20x, -56x$

4. The 1<sup>st</sup> and 3<sup>rd</sup> terms of an A.P are 60 and 32, find the  $n^{\text{th}}$  term. Ans.  $74 - 14n$

5. If  $p, 5p, 6p + 9$  are in A.P find  $p$  and continue for 4 terms. Ans. 3

6. Find the sum of 4.9, 5.6, 6.3..... to 20 terms. Ans.231

7. Given L=-88, n=16, s=-448, find a and d. Ans.32;-8

8. How many terms of the AP 42, 39, 36.....must be taken that the sum may be 312?

Ans.16

9. The sum of the third and seventh terms of an AP is 13. The sum of the terms is 104. Find the first three terms of the AP. Ans.3.5,4.25,5

10. In an AP, the sum of four terms is 28 and the sum of eight terms is 48, find the sum of twelve terms. Ans.60

11. A man has to travel 162km, he goes 30km the first day, 27 the second, 24 the third, and so on. How many days does he take for the journey? Ans.9

12. A clerk accepts a position with an initial salary of K14000 for the first year. He receives an increase of K500 for each of ten succeeding years. How much has he earned at the end of the ten years? Ans.K162500

- ***Geometric Progressions***

Formulae:  $n^{\text{th}}$  term =  $ar^{n-1}$

$$\text{Sum, } S_n = \frac{a(1-r^n)}{1-r}; r < 1 \text{ OR } S_n = \frac{a(r^n-1)}{r-1}; r > 1$$

1. Find the 8<sup>th</sup> term of the series  $\frac{-1}{3}, \frac{1}{2}, \frac{3}{4} \dots$  Ans.  $\frac{729}{128}$

2. Find the 5<sup>th</sup> and 8<sup>th</sup> terms of the following G.Ps. (i) 64, -32, 16... (ii)

3,6,12.... Ans. 4,  $-\frac{1}{2}$ ; 48,384;

3. Sum the following G.P

(a) 4, 12, 36...to 10 terms

(b) 108, 72, 48..... to 8 terms. Ans. 118096; 311

4. Find (i) the 12<sup>th</sup> term of the G.P 128, 64, 32...

(ii) a formula for the  $n^{\text{th}}$  term Ans.  $\frac{1}{16}, \frac{256}{2^n}$

5. Find three numbers in a G.P such that their sum is 42 and their product 512. Ans. 2, 8, 32

6. Given that  $x + 2, x + 3$  and  $x + 6$  are the first three terms of a G.P, find (a) the value of  $x$   
 (b) the 5<sup>th</sup> term of the G.P Ans. - $\frac{3}{2}; \frac{81}{2}$

7. The third term of a G.P is 40 and the sixth term is 625. Find the first term. Ans. 6.4

8. The sum of  $n$  terms of an A.P for all values of  $n$  is  $3n^2 - 5n$ . Find the first term and common difference. Ans.-2;6

9. The sum of  $n$  terms of a G.P is  $3^n - 1$  for all values of  $n$ . Find the first three terms. Ans.2,6,8

10. The first three consecutive numbers in a G.P are 18,  $x$  and 4.5. Find (a) the two possible values of  $x$ . Ans.9,-9;

(b) The sum of the first five terms. Ans.34.875, 12.375

11. In an A.P the ratio of the 3<sup>rd</sup> term to the 6<sup>th</sup> term is 9:4 and the sum of the first 5 terms is 60. Find the sum of the first 10 terms. Ans.  $64\frac{4}{9}$

12. The first three of the four numbers 12,  $x$ ,  $y$ , 4 are in A.P and the last three are in G.P. Find  $x$  and  $y$ . Ans.9, 4; 6,-

***Nothing can stop the man with the right mental attitude from achieving his goal; nothing on earth can help the man with the wrong mental attitude-Thomas Jefferson***

### SIMULTANEOUS EQUATIONS: ONE LINEAR, ONE QUADRATIC.

1. Solve the following pairs of equations.

(a)  $x = y$ ;  $2x^2 - y^2 = 1$  (b)  $x - 2y = 1$ ;  $xy = 3$  (c)  $x + y = 0$ ;  $xy - y^2 = -8$

(d)  $2x - y = 5$ ;  $xy = 0$  (e)  $3x + y = 4$ ;  $xy = 1$  (f)  $x + y = 1$ ;  $x^2 - xy = 15$

(g)  $3x - 4y = 4$ ;  $y^2 = x$  (h)  $x + y = 6$ ;  $x^2 + 2y^2 = 24$

(i)  $3x + y = 8$ ;  $3x^2 - xy + 9 = y^2$  (j)  $x + y = 11$ ;  $xy = 30$

Ans.a).1,1;-1,-1 (b)3,1;-2,- $\frac{3}{2}$  (c)2,-2;-2,2 (d) $\frac{5}{2}, 0; 0, -5$  (e)1,1; $\frac{1}{3}, 3$

(f) $3, -2; -\frac{5}{2}, \frac{7}{2}$  (g) $4, 2; \frac{4}{9}, -\frac{2}{3}$  (h) $4, 2$  (i) $5, 7; \frac{11}{9}, -\frac{13}{3}$  (j) $5, 6; 6, 5$ ;

2. Find two consecutive integers the sum of whose squares is 145. Ans. 8, 9
3. Find the area of a right triangle if the hypotenuse is 25cm and the perimeter is 60cm. Ans 150cm<sup>2</sup>
4. The base of a rectangle is twice its altitude. If 2 is added to the base and also to the altitude, the area is 60cm<sup>2</sup>. Find the original base and the altitude. Ans 8cm, 4cm
5. If the larger of certain two numbers be multiplied by the difference of the two numbers, the result is 44. The product of the two numbers is 77. What are the numbers? Ans 11, 7
6. The perimeter of a rectangle is 80cm and the area is 375cm<sup>2</sup>. Find the dimensions of the rectangle. Ans 15cm, 25cm
7. The sum of a certain fraction and its reciprocal is  $\frac{10}{3}$ . If three times the numerator is decreased by twice the denominator; the result is equal to -12. Find the fraction. Ans.  $\frac{4}{12}$
8. The length of a rectangle exceeds its width by 4cm. If the length were halved and the width decreased by 5cm, the area would be decreased by 55cm<sup>2</sup>. Find the length of the rectangle.
9. A rectangle is 72m<sup>2</sup> in area and its perimeter is 34m. Find its dimensions.
10. Two squares have a total area of 274cm<sup>2</sup> and the sum of their perimeters is 88cm. Find the side of the larger square.
11. Solve the following equations:
- (a)  $x + y = 3$ ;  $xy = 2$    (b)  $x - y = 3$ ;  $xy + 10x + y = 150$
- (c)  $2x + y = 6$ ;  $2x^2 - 3y^2 = 20$    (d)  $3x + 2y = 13$ ;  $xy = 2$
- (e)  $-3x + y + 15 = 0$ ;  $2x^2 + 4x + y = 0$

### POLYNOMIALS

1. Use long division to evaluate

(a)  $(x^3 + 2x^2 - x + 6) \div (x - 3)$

(b)  $(3x^4 - 2 - 5x) \div (x^2 - 3x)$

(c)  $(4x^3 - x - 9) \div (2x - 3)$

(Check your answer: multiply the quotient by the divisor, add the remainder to get the dividend)

2. If  $x^3 - 2ax + 15$  is divisible by  $x + 5$ , find the value of  $a$

3. Factorise  $x^3 + x^2 - 10x + 8$ , hence solve the equation  $x^3 + x^2 - 10x + 8 = 0$

4. If the expressions  $x^3 + 2x^2 + 3a + a$  and  $x^3 + x^2 + 9$  leave the same remainder when divided by  $x + 2$ , find the value of  $a$

5. Determine the values of  $p$  and  $q$  in order that the expression  $px^3 + qx^2 - 58x - 15$  may be divisible by  $x^2 + 2x - 15$

6. Show that

(i)  $x - 1$  is a factor of  $x^{29} - 13x + 12$

(ii)  $x - a$  is a factor of  $x^3 - 4ax^2 - a^3$

(Hint: use the factor theorem)

7. Factorise

(a)  $x^3 - 4x^2 + x + 6$       (b)  $2x^3 - 3x^2 - 8x + 12$

(c)  $6x^3 - 13x^2 + 9x - 2$       (d)  $x^3 - 2x^2 - 9x + 18$

(Check your answer by expanding to get the original question)

Ans:2) 11; 3)  $x = -4, 1, 2$ ; 4)  $\frac{5}{4}$

8. If  $ax^3 + bx^2 + cx + 2 \equiv (2x - 1)(x^2 + x - 2)$ , find the values of  $a, b, c$ .

9. If  $3x^3 + 8x^2 + 4x - 5 = x(Ax + B)(x - 2) + C$ , find the values of the letters.

10. The expression  $2x^3 + ax^2 + bx - 2$  is exactly divisible by  $x - 2$  and  $2x + 1$ . Find the values of  $a$  and  $b$  and hence find the third factor. (-1, -2)

11. The expression  $ax^3 - x^2 + bx - 1$  leaves remainders of -33 and 77 when divided by  $x + 2$  and  $x - 3$  respectively. Find the value of  $a$  and  $b$  and the remainder when divided by  $x - 2$ . Ans:3, 2, 23.

### PROBABILITY

1. Out of 20 lamp bulbs, three are faulty. You select a bulb at random. What is the probability that it works?
2. A die, numbered 1 to 6 is thrown. What is the probability of getting 6?
3. A bag contains 7 red balls, 8 blue balls, 3 yellow balls and the remainders of the 20 balls are white. What is the probability of drawing;  
(a) a red ball?      (b) a white ball?
4. Two tetrahedral dice with faces marked 2, 4, 6, and 8 are thrown together and the score is found by adding the scores on the two dice. What is the probability of obtaining a score of:  
(a) 10,    (b) 12
5. A die is thrown and a coin is tossed. What is the probability of getting a 5 and a head?
6. A man throws a die numbered 1 to 6. What is the probability that he gets a multiple of 5 or a multiple of 3?
7. In a game of bingo, there are 20 red, 3 black, 15 blue and 12 white balls left. What is the probability that the ball picked is either a red ball or a blue ball?
8. A bag contains 5 blue and 7 white balls. A ball is selected at random, its colour noted and it is returned to the bag. The bag is shaken and a second random selection is made. What is the probability that two white balls are selected? Use a tree diagram.
9. A bag contains 5 blue and 7 white balls. A ball is selected at random, its colour is noted, and it is not returned to the bag. What are the probabilities that:  
(a) Both are blue?    (b) Both are white?

(c) One of each colour is chosen?

10. On mother's day Mrs. Phiri is given a box of chocolates. Half of them are dark chocolates and half are milk chocolates. The box contains 10 chocolates and Mrs. Phiri eats two immediately, taking them out of the box at random.

(a) Draw a tree diagram to show the possible outcomes of the type of chocolate eaten.

(b) What is the probability that Mrs. Phiri eats two milk chocolates?

(c) What is the probability that Mrs. Phiri eats one of each kind of chocolate?

(d) Mrs. Phiri eats a third chocolate. What is the probability that she has eaten three dark chocolates?

*Success is the sum of small efforts, repeated day in and day out-Robert Collier*

### GEOOMETRY

- ***Summary of theorems required for MSCE.***

1. A straight line from the center of a circle bisecting a chord is perpendicular to the chord.
2. A straight line from the center of a circle perpendicular to a chord bisects the chord.
3. Equal chords are equidistant from the centre of the circle
4. Chords which are equidistant from the center of the circle are equal.
5. The angle subtended by an arc at the center of the circle is twice the angle subtended by the same arc at the circumference.
6. Angles in the same segment are equal.
7. An angle in a semi-circle is a right angle.
8. The opposite angles of a cyclic quadrilateral are supplementary.
9. The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.
10. A tangent is perpendicular to a radius at the point of contact.
11. Tangents from an external point to a circle are equal.
12. If a straight line touches a circle and from the point of contact a chord is drawn, the angles which the chord makes with the tangent are equal to the angles in the alternate segment of the circle.

13. If two circles touch one another, the line joining their centers passes through the point of contact.

### CHORD PROPERTIES

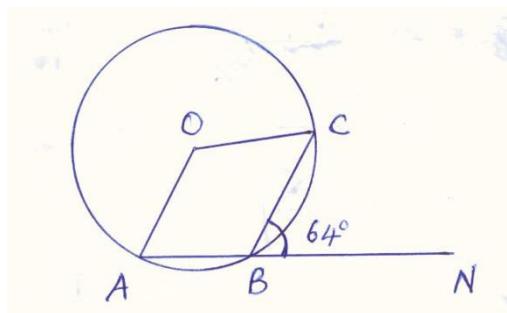
**NOTE: It is a must that you make a sketch of each question below to help in your solution.**

1. AB is a chord of a circle of radius 10cm. AB is 8cm. Find the distance of the center of the circle from AB.(Ans.9.17cm)
2. A chord of length 10cm is at a distance of 12cm from the center of the circle .Find the radius.(Ans.13cm)
3. A chord of a circle of radius 7cm is at a distance of 4cm from the centre. Find its length.(Ans.11.5cm)
4. In a circle of a radius 5cm, there are two parallel chords of length 4cm and 6cm. Find the distance between them.(Ans.8.58;0.583cm)
5. A chord of a circle is 10cm long and is 4cm from the centre. Find the length of a chord which is 3cm from the center.(Ans.11.3cm)
6. AB, CD are two chords of a circle at a distances  $d$  cm,  $7d$  cm from the center. Find the length of CD in terms of  $d$ .(Ans.8d)
7. AB, CD are two chords of a circle center O, radius 7cm which meet at right angles. If AB=6cm, CD=10cm, find the length of OP.(Ans.8cm)
8. The length of the common of two intersecting circles is 10cm, and the distance between the two centers is 6cm. Find the radius of each circle.(Ans.5.83cm)
9. AB is a chord of a circle, center O, T is any point equidistant from A and B, prove OT bisects  $\angle ATB$
10. Two circles, center A, B intersect at XY; prove that AB bisects XY at right angles.
11. Prove that a straight line drawn from the center of a circle perpendicular to a chord bisects the chord.
12. If AB and CD are equal chords of a circle, center O prove that  $\angle AOB = \angle COD$

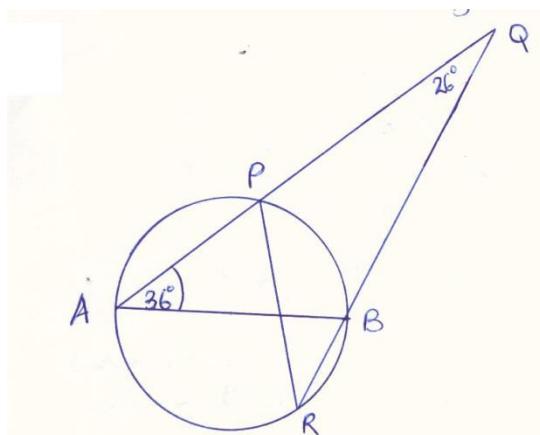
### ANGLE PROPERTIES

**NOTE:** where answers have been provided, give the reasons behind those answers.

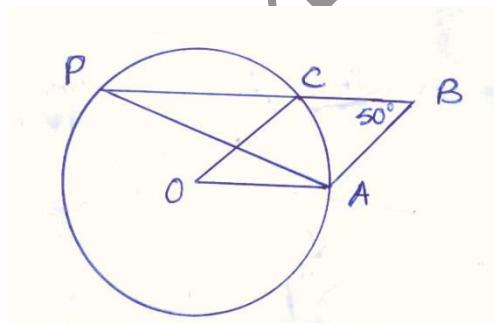
1a. In the figure below, O is the center of the circle; ABN is a straight line. Find  $\angle AOC$



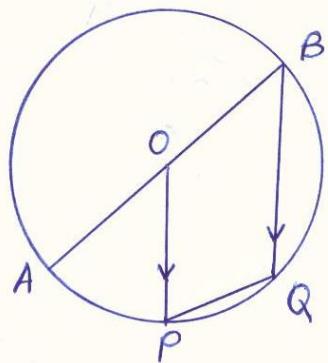
1b. In the figure below, AB is a diameter of the circle APBR and RBQ and APQ are straight lines.  $\angle BAQ = 36^\circ$ ,  $\angle AQR = 26^\circ$  Find  $\angle BPR$



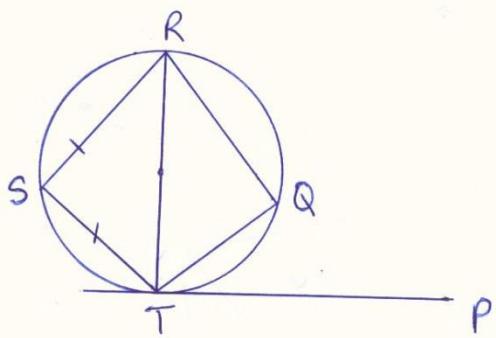
2. In the figure below, O is the center of the circle; OABC is a parallelogram, and BCP is a straight line. Find  $\angle OAP$  (Ans.  $25^\circ$ )



3. AB is a diameter and O is a center of the circle below.  $\angle ABQ = 40^\circ$  Find  $\angle OPQ$

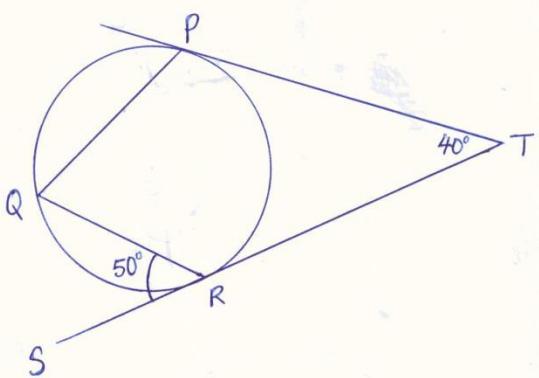


4. Angle  $\angle PTQ = 25^\circ$ , RT is diameter and  $TS = SR$



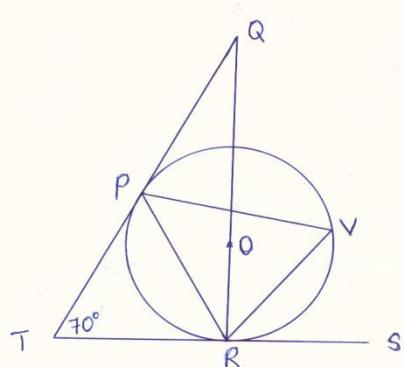
Find angles (i)  $\angle QTR$ , (ii)  $\angle TRQ$  (iii)  $\angle SRQ$

5. In the figure below TP and TR are tangents to a circle at points P and R respectively.  $\angle PTR = 40^\circ$  and  $\angle QRS = 50^\circ$



Find (i)  $\angle PQR$  (ii)  $\angle PRQ$  (Ans.  $70^\circ$ ;  $60^\circ$ )

6. In the figure below, O is the center of the circle. TPQ and TRS are tangents to the circle. Angle PTR=70°



Find angles

- (i) TPR      (ii) PVR      (iii) PQR      (iv) OPR

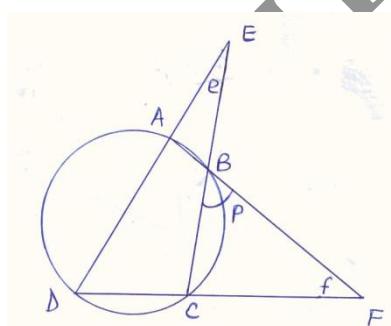
7. ABCD is a cyclic quadrilateral. If  $\angle ADC = 70^\circ$  and  $\angle ACD = 50^\circ$ , find  $\angle CBD$

8. Two chords  $AB, CD$  of a circle meet when produced at K, if  $\angle KAD = 31^\circ$  and  $\angle AKC = 42^\circ$ , Find  $\angle KBC$ . (Ans.  $107^\circ$ )

9. D is a point on the minor arc AB of a circle, centre O. If  $\angle ADB = x^\circ$  and  $\angle AOB = y^\circ$ , find  $x$  in terms of  $y$ .

10. P, Q, R are points on a circle, centre O,  $\angle POQ = 54^\circ$ ,  $\angle OQR = 36^\circ$  and P, R are on opposite sides of OQ. Find  $\angle QPR$  and  $\angle PQR$ . (Ans.  $63^\circ; 81^\circ$ )

11.

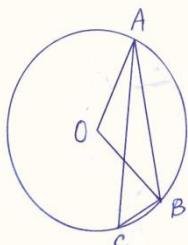


(i) If  $e = 32^\circ$  and  $f = 40^\circ$ , find P

(ii) Prove that  $e+f=180-2p$

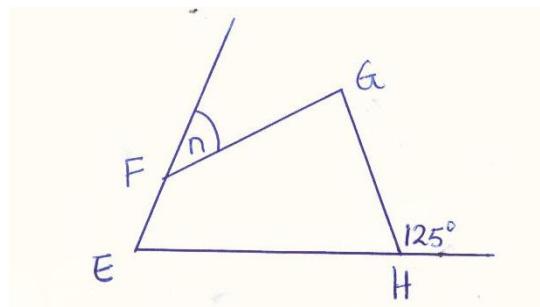
(iii) If AC cuts BD at K, and  $e=40^\circ$ ,  $f=20^\circ$ ,  $\angle BKC = 100^\circ$ , prove that  $\angle BAC = 2\angle BCA$

12. ABCD is a cyclic quadrilateral .If AC bisects the angles BAD, BCD, prove that  $\angle ABC$  is a right angle.



13. In the figure above, O is the center of the circle ABC. If  $\angle ACB = \angle OAB$ , prove that  $\angle AOB$  is a right angle.

### CONCYCLIC POINTS.

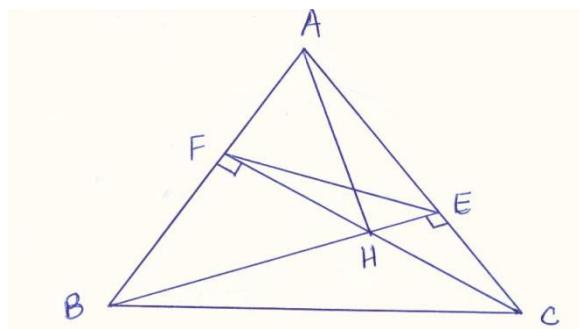


1. If  $n=55^\circ$ , prove that E, F, G, H are concyclic.

2. PQRS is a quadrilateral with diagonals PR and QS. If  $\angle PQR = 70^\circ$ ,  $\angle PRQ = 35^\circ$ ,  $\angle QSR = 75^\circ$ . Prove that P, Q, R, S, are concyclic.

3 ABCD is a quadrilateral in which AB=AD and DB=DC. If  $\angle DBA = x^\circ$  and  $\angle DBC = 2x^\circ$ , Prove that A, B, C, D are concyclic.

4. ABCD is parallelogram, any circle through A and D cuts AB, DC, at P and Q. Prove B, C, Q, and P are concyclic.

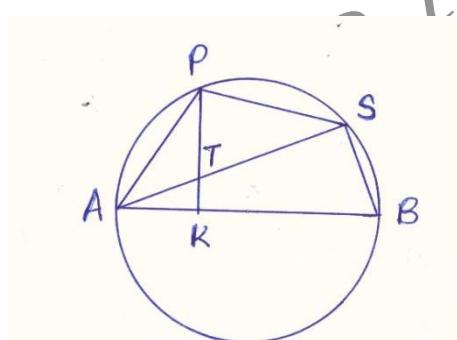
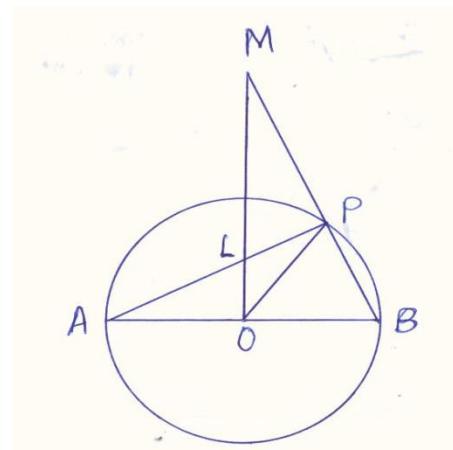


5. In the figure above,

(a) Prove that (i) B, F, E, C are concyclic. (ii)  $\angle AEF = \angle ABC$

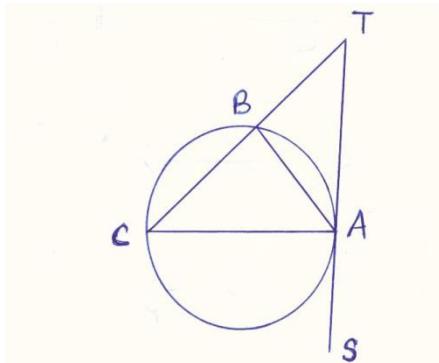
(b) Prove that (i) A, E, H, F are concyclic. (ii)  $\angle AHE = \angle ACB$

6. O is the center of the circle below and OLM is perpendicular to AOB. Prove that (i) A, O, M, P are concyclic (ii)  $\angle OPA = \angle OMB$



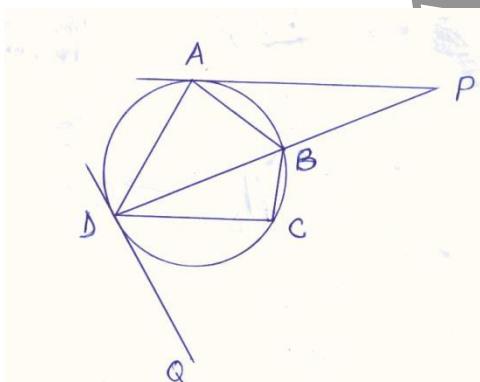
7. In the figure AB is a diameter of the circle. If  $\angle APK = \angle ASP$ , prove that (i)  $\angle PKB = 90^\circ$  (ii) STKB is a cyclic quadrilateral.

TANGENTS AND ALTERNATE SEGMENTS.



1. and CBT is a straight line.

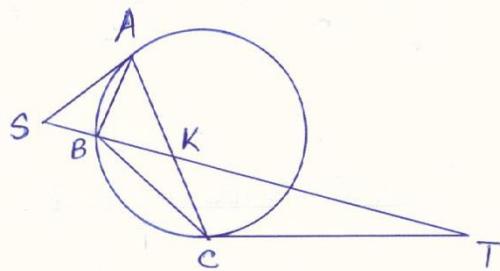
- (i) If  $\angle CAT = 124^\circ$ , find  $\angle ABC$
- (ii) If  $\angle TAC = 118^\circ$ ,  $\angle ATC = 26^\circ$ , Find  $\angle BAT$
- (iii) If  $\angle CAS=65^\circ$ ,  $\angle BAT=47^\circ$ , Find the angles of ABC
- (v) If  $\angle ABC = 80^\circ$  and  $\angle ATC = 40^\circ$ , prove that  $AC=AT$ .



2. In the figure above AP, DQ are the tangents. If  $\angle PAB = 42^\circ$ ,  $\angle QDC=55^\circ$ ,  $\angle BDC=24^\circ$ , Find the angles of the quadrilateral ABCD. (Ans.  $79^\circ, 114^\circ, 101^\circ, 66^\circ$ )

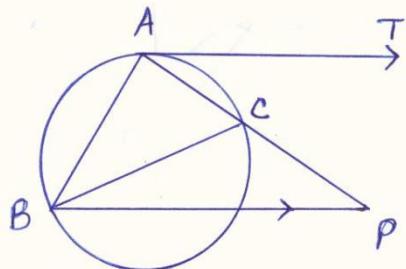
3. In the figure above (question 2) AP, DQ, are tangents and BD is a diameter. If  $\angle BAP=x^\circ$ ,  $\angle BPA=y^\circ$ , Find a relation between x and y.

(Ans:  $2x + y = 90$ )



4.

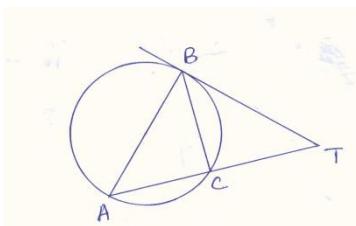
In the figure above AS, CT are tangents and SBKT is a straight line .If  $\angle AST=42^\circ$ ,  $\angle CKT=55^\circ$ ,  $\angle SBC=155^\circ$ , find  $\angle BAC$  and  $\angle CTS$ . (Ans. $53^\circ, 28^\circ$ )



5.

In the figure above BP is a parallel to the tangent AT; ACP is a straight line. Prove that  $\angle ABP=\angle ACB$

6. ABC is a triangle inscribed in a circle; the tangents at B and C meet at T. If  $\angle BAC=60^\circ$ , Prove that triangle TBC is equilateral.



7.

In the figure above BT is a tangent and ACT is a straight line .If  $\angle ABC=\angle T$ , prove that AB is a diameter.

### CONSTRUCTIONS.

1. Construct a circle of radius 4cm and take a point 6cm from the centre. Construct the tangents from this point to the circle and measure their lengths.
2. Construct a circle of radius 3cm and take a point 5cm from the center. Construct a chord of length 4cm which when produced, passes through this point.
3. Draw a line AB of length 5cm and describe a circle with AB as diameter. Construct a point on AB produced such that the tangent from it to the circle is of length 3cm.

***They fail, and they alone, who have not striven-Thomas Aldrich***

## CO-ORDINATE GEOMETRY

- Standard form of straight line graph:  $y = mx + c$
- Gradient( $m$ ) =  $\frac{y_2 - y_1}{x_2 - x_1}$
- Determine:  $y$ -intercept; let  $x = 0$ ;  $x$ -intercept; let  $y = 0$ ;
- Parallel lines have the same gradient
- Distance formula:  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Coordinates of midpoint:  $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

1. Draw sketch graphs of each of the following on separate sets of axes.

(a)  $y = -3x + 4$       (b)  $y = \frac{2}{3}x - 2$       (c)  $2x + 3y = 12$

(d)  $y = -6x$       (e)  $y = 5$       (f)  $x = -3$

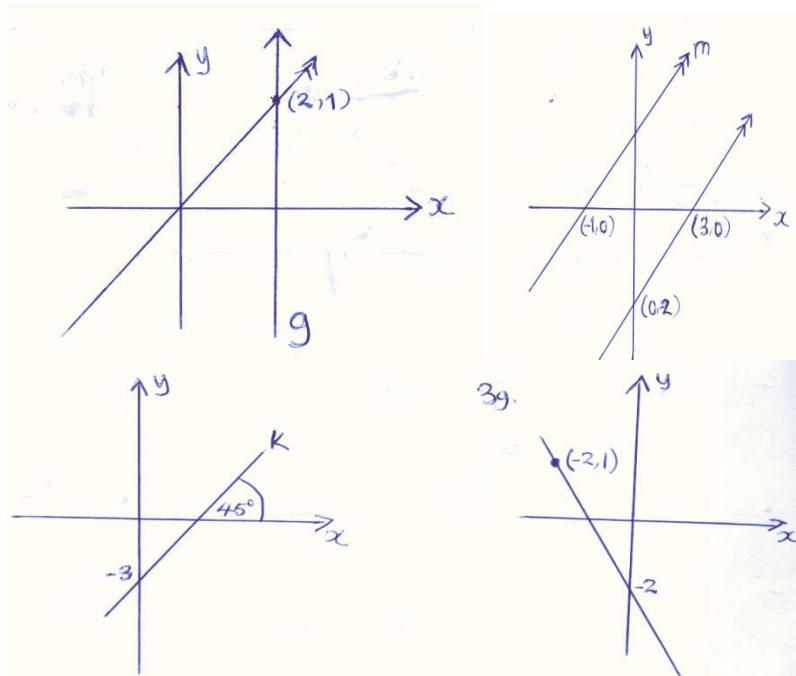
2. Given  $x - y + 1 = 0$  and  $\frac{1}{6}y + \frac{1}{3}x = \frac{2}{3}$ . Without drawing the two graphs determine

(a) The co-ordinates of the intercepts of the two graphs.

(b) The co-ordinates of the point of intersection.

Ans.a.(0,1),(-1,0);(0,4),(2,0); b.(1,2)

3. Determine the equations of the marked graphs.



$$(\text{Ans. } x = 2; y = \frac{2}{3}x + \frac{2}{3}; y = x - 3; y = -\frac{3x}{2} - 2;)$$

4. Given  $A(3, -2)$  and  $B(2, 5)$

- Determine the gradient of AB
- Determine the y-intercept of line AB
- Determine the equation of AB
- Write down the equation for the line parallel to AB which passes through the origin.
- Find the equation for the line parallel to AB and passing through  $(1, 5)$  (Ans.  $-7; 19; y = -7x + 19; y = -7x; y = -7x + 12;$ )

5. If the points  $(-2, -3), (3, 5)$ , and  $(13, p)$  are collinear, find the value of  $p$ .

6. Find the distance between the following pairs of points.

- $(-1, -3); (-2, -5)$
- $(0, -3); (4, 0)$
- $(-5, 0); (-7, -4)$
- $(a, 0); (0, a)$
- $(a, a+b); (a-b, b)$

$$(\text{Ans. } 2.2; 5; 4.5; \sqrt{2}a; \sqrt{a^2 + b^2};)$$

7. The vertices of triangle ABC are  $A(-1,3)$   $B(2,7)$  and  $C(6,4)$

(i) Find the squares of the lengths of the sides.

(ii) What type of triangle is it? (Ans. Right-angled)

(iii) Find the area of a triangle. (12.5 units<sup>2</sup>)

8. Find the midpoint and length of the line segment  $(1,7)$  and  $(5,4)$

9. A circle has center at  $(1,2)$  one point on its circumference is  $(-3,-1)$  what is the radius of the circle?

10. State the co-ordinates of the midpoints of

(a)  $(p, 2p)$  and  $(3p, -4p)$  (b)  $(a, a - 4)$  and  $(a + 2, 6 + a)$

11. The midpoint of PQ is  $(2,3)$ . If the co-ordinates of P are  $(-1,4)$ , find the co-ordinates of Q

12. The points A and B are  $(a, -4)$  and  $(-3, b)$  respectively. If the midpoint of AB is  $(-2,3)$  find the values of  $a$  and  $b$ .

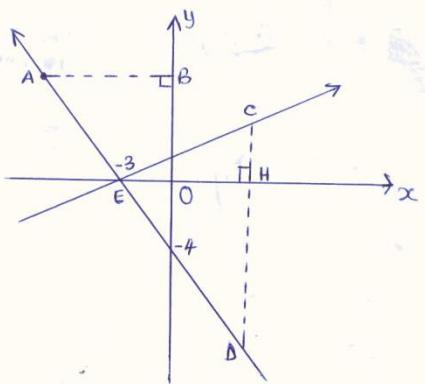
13. The lines AD and CE are represented in the accompanying figure. The y-intercept of CE is 1.

A) Determine the equations of AD and CE

B) Determine the length of AB if OB=2

c) Determine the length of CD if H  $(3, 0)$

D) Determine the length of ED



(Ans. $y = -\frac{4x}{3} - 4$ ;  $y = \frac{1}{3}x + 1$ ; 4.5; 10; 10 units;)

### INEQUALITIES AND LINEAR PROGRAMMING

1. Illustrate with a graph the following inequalities.

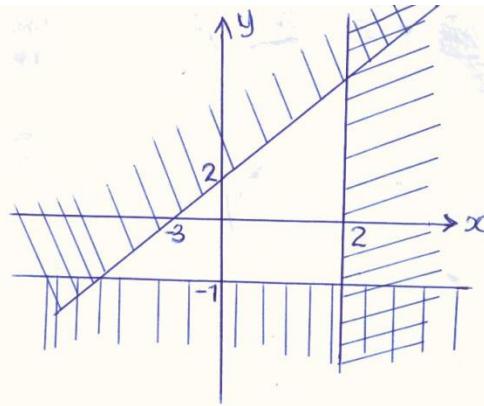
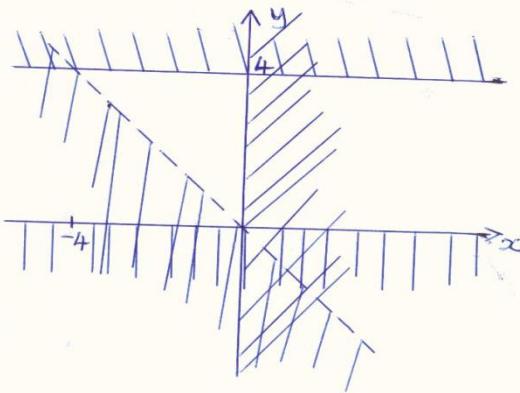
- (a)  $x \geq 5$    (b)  $y < 4$    (c)  $y > -2$    (d)  $x \leq 0$    (e)  $-1 \leq x < 4$
- (f)  $4 \leq y \leq 7$    (g)  $x < 3$  and  $x \geq 6$    (h)  $y < -4$  and  $y \geq -1$

2. Show on a graph the solution region for the given set of inequalities.

- (a)  $x \geq -1, y \geq 0, 2x + y \leq 5$
- (b)  $y < x - 2, x + y > 2, x \geq 0$
- (c)  $x + y \geq 3, 2y \leq x - 4, y < 3$

*Energy and perseverance conquer all things-Benjamin Franklin*

3. Write down the set of inequalities represented by the graphs below



3a. Draw a graph to illustrate the following inequalities, shading out the unwanted regions  
 $2x + y < 7$ ,  $x \geq 0$ ,  $y \geq 1$

(b) Given that  $x$  and  $y$  are both integers, list the co-ordinates of all possible solutions to the above inequalities.(3a)

4. Find the maximum and minimum values of  $z = 3x + 4y$  subject to the following constraints.

$$x + 2y \leq 14, \quad 3x - y \geq 0 \text{ and}$$

$$x - y \leq 2 \quad \text{Ans.} 34, -15$$

5. Maximize and minimize

$$Z = 0.4x + 3.2y \text{ for the constraints } x \geq 0, y \geq 0, x \leq 5, x + y \leq 7, x + 2y \geq 4 \text{ and } y \leq x + 5 \quad \text{Ans.} 188, -2$$

6 .A furniture manufacturer can make from 30 to 60 tables a day and from 40 to 100 chairs a day. It can make at most 120 units in one day. The profit on a table is K150, and the profit on the chair is K65. How many tables and chairs should they make per day to maximize profit? How much is the maximum profit? Ans:60tables,60 chairs; K12900.

8. One serving of food A contains 2 grams of protein and 6 grams of carbohydrates. One serving of food B contains 4 grams of protein and 3 grams of carbohydrates. A dietitian wants a meal that contains at least 12 grams of protein and at least 18 grams of carbohydrates. If the cost of food A is K90 per serving and the cost of food B is K200 per serving then how many servings of each food would minimize the cost and satisfy the constraints? Ans.6 of A; none of B

9. A small dog house requires  $8m^2$  of plywood and  $6m^2$  of insulation. A large dog house requires  $16m^2$  of plywood and  $3m^2$  of insulation. Only  $48m^2$  of plywood and  $18m^2$  of insulation are available. If a small dog house sells for K15 thousand and a large dog house sells for K20 thousand, then how many of each type should be built to maximize the revenue?

Ans.2 of each

10. A tailoring business makes two types of garments, A and B. Garment A requires 3m of material while garment B requires 2.5m of material. The business uses not more than 600m of material daily in making both garments. It must make not more than 100 garments of type A and not less than 80 of type B each day.

(a) Write down four inequalities from this information.

(b) Graph these inequalities

(c) If the business makes a profit of K80 on garment A and a profit of K60 on garment B, how many garments of each type must it make to maximize its profit? Ans. 100 of A; 120 of B

### GRAPHS OF QUADRATIC FUNCTIONS.

1. Draw a sketch graph for each of the following on separate sets of axes.

(a)  $y = x^2 - 9$     (b)  $y = -x^2 + 4$     (c)  $y = \frac{1}{2}x^2$

(d)  $y = -2x^2 - 8$     (e)  $y = 4x^2 - 8x - 5$     (f)  $y = -4x^2 - 8x - 5$

2. Find the co-ordinates of the turning point of the curve

$y = (2x - 3)^2 + 6$  and sketch the curve

3. The function  $1 + bx + ax^2$  has a maximum value of 4 where  $x = -1$ . Find the values of  $a$  and  $b$ . (Ans.  $-3, -6$ .)

4. The quadratic equation  $x^2 + px + q = 0$  has roots -2 and 6. Find the values of P and Q

5. Determine the equation for each of the following.

(i) turning point; (1, -1) and passes through (0, -2)

(ii) turning point;  $(\frac{3}{4}, -\frac{41}{8})$  and passes through  $(-\frac{4}{5}, 0)$

(Ans. $y = -2 + 2x - x^2$ ;  $y = 2x^2 - 3x - 4$ ;

6(a). Draw the graph of  $y = x^2 + 2x - 3$  for  $-4 \leq x \leq 2$

(b) On the same grid, draw the graph  $y = -3$ , hence solve the equation  $x^2 + x - 2 = 0$

7. Draw  $y = x^2 - 2x$  for values of  $x$  from -1 to +3, and  $y = x - 1$ , hence solve the equation  $x^2 - 2x = x - 1$  (Ans.0.4,2.6)

8. Draw the graphs of  $y = x^3$  and  $y = 3x^2 + 2x - 5$  between the values of  $x = -2$  and

4. Solve  $x^3 - 3x^2 - 2x + 5 = 0$ . (Ans.3.1,1.2,-1.3)

9. using the same axes draw the graphs of  $y = x^3 + 3x^2 - 2x - 4$  and  $y = 2x + 2$  for  $-4 \leq x \leq 2$ . use your graph to estimate the solution of the equation (i)  $x^3 + 3x^2 - 2x - 4 = 0$ ; (ii)  $x^3 + 3x^2 - 4x - 6 = 0$

(Ans.-3.3,-1 or 1.3; -3.7,-1,1.7;)

10. Romeo throws a pebble at Juliet's window. The path followed by the pebble is given by the equation  $y = 2x - \frac{x^2}{100}$  where  $x$  is the horizontal distance and  $y$  is the vertical distance from Romeo.

- Draw up a table of values, taking  $x$  from 0 to 5.
- Draw the graph which shows the path the pebble takes.
- Juliet's window is 1 metre high and the window sill is 4 metres above the ground. Will the pebble hit the window if it is thrown from a distance of: (i) 3m; (ii) 2.5m from the house?
- If Romeo is to hit the window, between what distances from the house should he be when throwing the pebble? (Ans.2m,2.5m)

### SIMILARITY

- The quadrilaterals ABCD and PQRS are similar. The area of quadrilateral ABCD is  $7\text{cm}^2$ . If  $PQ=2AB$ , calculate the area of quadrilateral PQRS.
- The area of a triangle is  $24\text{cm}^2$ . after an enlargement, the area is  $384\text{cm}^2$ . find the scale factor of the enlargement.
- The radii of two circles are in the ratio 1:6. the area of the larger circle is  $306\text{cm}^2$ . Calculate the area of the smaller circle.

4. A semicircle has an area of  $108\text{cm}^2$ . If it is enlarged to a semicircle of area  $192\text{cm}^2$ ; in what ratio are the diameters of the two circles?

### STATISTICS

1. Below is a table showing the number of seats not booked on Air Malawi's daily flight between Lilongwe and London over 10 weeks.

19	1	8	11	15	19	21	17	1	23	19	11	12	15
21	11	8	4	15	27	21	20	14	18	7	11	23	21
8	17	1	19	12	16	21	25	28	29	17	15	11	8
16	8	2	8	6	10	11	15	9	8	6	2	3	8
21	18	27	32	37	4	11	19	21	34	21	15	12	11

By means of a tally chart, find the frequencies in the class intervals 0-4, 5-9, 10-14 ...30-34, 35-39.

Represent the grouped data by means of a histogram. Find the mode. Construct a frequency polygon for the data.

2. The acreage of 39 farms around Mtendere Campus is shown below.

41	66	233	775	169
36	338	233	236	64
183	61	13	308	77
520	77	27	217	5
650	462	106	52	52
505	94	75	265	402
196	70	132	28	220
760	143	46	539	

(a) Construct a frequency distribution for the data using eight classes.

(b) Construct a frequency polygon and draw a histogram for the data as well

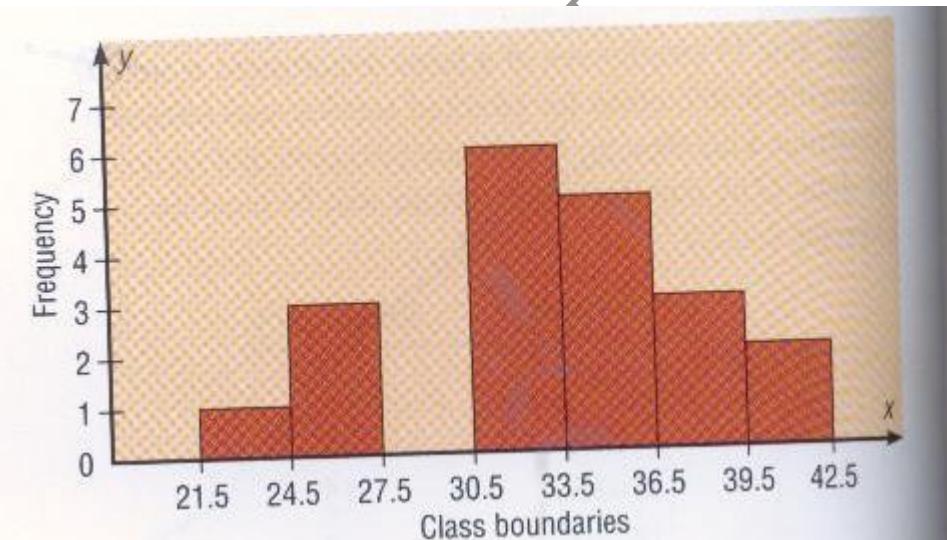
3. Thirty cars were tested for fuel efficiency, in km/gallon. The following frequency distribution was obtained:

Class boundaries	Frequency
------------------	-----------

7.5-12.5	3
12.5-17.5	5
17.5-22.5	15
22.5-27.5	5
27.5-32.5	2

Construct a histogram and frequency polygon for the data.

4 (a) Using the following histogram (i) Construct a frequency distribution; include class limits,



frequencies and midpoints.

(ii) Construct a frequency polygon

(b) Using the results above answer the following questions

(i) How many values are in the class 27.5-30.5?

(ii) How many values fall between 24.5 and 36.5?

(iii) How many values are below 33.5?

(iv) How many values are above 30.5?

5. Find the mean, variance and standard deviation of the data

below; 6.5, 6.5, 9.5, 8.0, 14.0, 8.5, 3.0, 7.5, 16.5, 7.0, 8.0

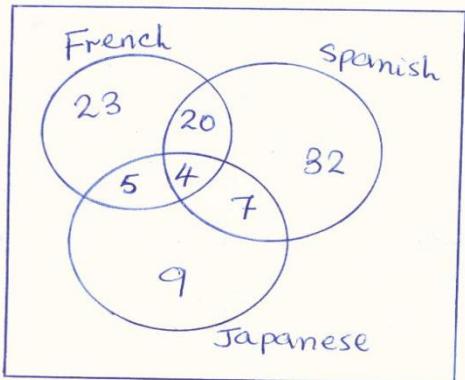
6. Find the mean, variance and standard deviation of the data in questions 1, 2 and 3.

***Impatience never commanded success-Edwin Chapin***

### **SET THEORY**

- 1) Let  $A = \{1,2,4,5\}$ ,  $B = \{2,3,5,6\}$ ,  $C = \{3,5,7,8\}$  and  $D = \{5,6,8,9\}$ .
- a) List the elements of the following sets
- i)  $A \cup B \cup C$    ii)  $B \cup C \cup D$
- b) Find (i)  $n(A \cup B) \cup (C \cup D)$    (ii)  $n(A \cap B) \cup (C \cap D)$
- 2) If the universal set  $\xi = \{1,2,3, \dots, 12\}$ .  $A = \{1,3,5,7\}$ ,  $B = \{2,7,10\}$  and  $C = \{1,2,3,5\}$ . Find the following:
- (i)  $C \cap (A \cap B)$    (ii)  $A \cap (B' \cap C')$    (iii)  $(B \cap C \cap A)'$
- 3) Use Venn diagrams to prove that:
- i)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- ii)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- iii)  $(A \cup B)' = A' \cap B'$
- 4) If  $\xi = \{\text{all integers less than } 20\}$   $X = \{\text{odd numbers}\}$   $C = \{\text{prime numbers}\}$
- a) List the following (i)  $X \cup Y$    (ii)  $Y \cap C$
- b) Find (i)  $n(X' \cap C)$    (ii)  $n(X \cup Y \cup C)'$

5) John asked five adults whether they had studied French, Spanish, or Japanese in a school.



According to the diagram, how many had studied

- a) Japanese but not French?
- b) French and Spanish?
- c) French or Spanish?
- d) French and Spanish but not Japanese?

6) In a class of 50 students, 18 take French, 26 take B/K and 2 take both French and B/K. How many students are not enrolled in either French or B/K?

7) A veterinarian surveys 26 of his patrons. He discovers that 14 have dogs, 10 have cats and 5 have fish. Four have dogs and cats, 3 have dogs and fish, and 1 has a cat and fish. If no one has all three kinds of pets, how many patrons have none of the pets?

8) A counselor is planning schedules for 30 students. Sixteen students say they want to take French, 16 want to take Spanish, and 11 want Latin. Five want both French and Latin, and of these, 3 wanted to take Spanish as well. 5 want only Latin, and 8 want only Spanish. How many students want French only?

9) In a class of 40 students, 7 take Math and biology, 5 take Biology and Science, 6 Science and Maths. 17 take Biology, and the number taking Math only is twice that taking Biology only. 5 take Science only. How many take all three subjects?

10) 100 students study at least one of the following subjects everyday: Math, Biology and Commerce. 2 students study Biology only, 17 students study Math and Biology and 23 students study Biology and Commerce. If 71 students do not study Biology, how many study all three?

### **TRIGONOMETRY**

1a) From a point 40m away from the wall of a house, the angles of elevation of the top and bottom of a window are  $29^\circ$  and  $24^\circ$ . Calculate the height of the window. (Ans.4.36m)

1b) From a point, the angle of elevation of a tower is  $30^\circ$ . If the tower is 20m distant from the point, what is the height of the tower?

1c) A man 1.8m tall observes the angle of elevation of a tree to be  $26^\circ$ . If he is standing 16m from the tree, find the height of the tree.

1d) A man, lying down on top of a cliff 40m high, observes the angle of depression of a boat to be  $20^\circ$ . If he is in line with the boat, calculate the distance between the boat and the foot of the cliff.

1e) A tree is 20m tall. When the altitude of the sun is  $62^\circ$ , what length of shadow will it cast?

1f) A man standing on top of a mountain 1200m high observes the angle of depression of a steeple to be  $43^\circ$ . How far is the steeple from the mountain?

1g) A girl standing on top of a cliff 80m high is in line with two cars whose angles of depression are  $17^\circ$  and  $21^\circ$ . Calculate the distance between the two cars.

2) The legs of a pair of compasses are 12cm long. What is the distance between the tips if the legs contain an angle of  $154^\circ$ ? (Ans.23.4cm)

3) P is 3.5km east of Q; R is due north of P and is 4.5km from Q. What is the bearing of R from Q? (Ans. $51.1^\circ$ )

4) The diagonals of a rectangle contain an angle of  $68^\circ$ ; the longer side is 7cm. What is the length of a diagonal?

5) A chord of length 10cm subtends an angle of  $84^\circ$  at the centre of a circle. What is the radius? (7.47cm)

6) A man walks 1000m on a bearing of  $025^\circ$  and then 800m on a bearing of  $035^\circ$ . How far north is he of his starting point? (1560m)

- 7) A ladder 6.5m long stands against a vertical wall and its foot is 4.5m from the wall. What angle does the ladder make with the ground?
- 8) A man standing on a cliff 140m high observes two boats in a vertical plane with him. The angles of depression are  $37^\circ$  and  $53^\circ$ . What is the distance between the boats?
- 9) Tangents are drawn to a circle of radius 6cm from a point 9.3cm from the centre. Find the angle between them.

10a). Given that  $\sin \theta = \frac{2}{\sqrt{13}}$ , find without using a calculator,  $\cos \theta$ .

b) Simplify the following without a calculator.

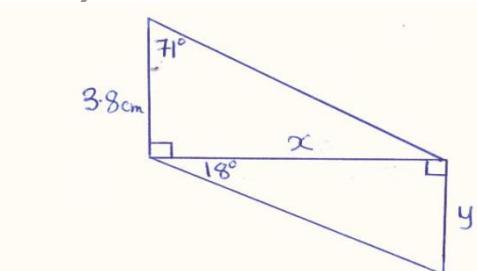
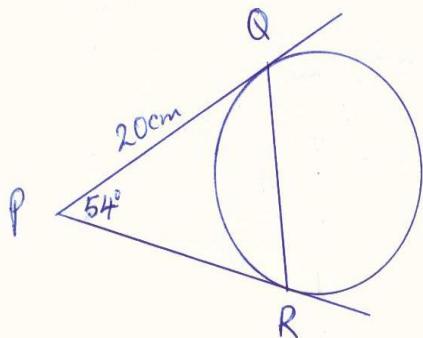
(i)  $\sin 90^\circ + \sqrt{3} \tan 30^\circ - \tan 0^\circ$  (Ans.1)

(ii)  $2 \cos 30^\circ \sin 30^\circ + \sin 60^\circ$  (Ans. $\sqrt{3}$ )

(iii)  $x + \sin 45^\circ = \cos 45^\circ$  (Ans.0)

(iv)  $x \cos 60^\circ = \sin 30^\circ$  (Ans.1)

- 10) In the figures below, find QR, x and y.



- 12) In the triangle ABC,  $\angle A = 62^\circ$ ;  $\angle B = 70^\circ$  and  $AC=3.8\text{cm}$ . Find angle C and the sides AB and BC. (Ans.  $3.01\text{cm}; 3.57\text{cm}$ )

- 13) In triangle LMN,  $LM=7.3\text{cm}$ , angle L is  $41.1^\circ$ , and  $MN=5.1\text{cm}$ . Find: (i) angle LNM (ii) angle LMN; hence find the two possible values for LN. (ans.  $70.2^\circ, 109.8^\circ$ ;  $68.7^\circ, 29.1^\circ$ ;  $7.23, 3.77\text{cm}$ )

- 14) In triangle XYZ,  $XY=3.4\text{cm}$ ,  $YZ=5.2\text{cm}$  and  $XZ=4.9\text{cm}$ . Find the three angles of the triangle. (Ans.  $75.2^\circ; 65.6^\circ; 39.2^\circ$ )

15) In quadrilateral ABCD, AB=4.7cm, AC=8.1cm, BD=9.1cm and CD=7.5cm.  $\angle CAB = 72^\circ$ . Find:  
(i) BC (ii) Angle BDC (Ans.8.01cm, 56.7°)

16a) Three posts A,B and C are such that B is 20m south of A and C is on a bearing of 290° from B.Calculate the angle ACB given that AC=32m.

16b) An aircraft flies 50km from an aerodrome A on a bearing of 065° and then flies 80km on a bearing of 040°.find the distance of the aircraft from A and also its bearing from A.

16c) A boat sails 10km from a harbor H on bearing of S30°E.It then sails 15km on a bearing of N20°E.How far is the boat from H? What is its bearing from H?

16d) A and B are two points on a coast,B being 2000m due east of A.A man sails from A on a bearing of 060° for 800m to a ship P,where he changes course to 070° and sails for1000m to another ship Q.Calculate:

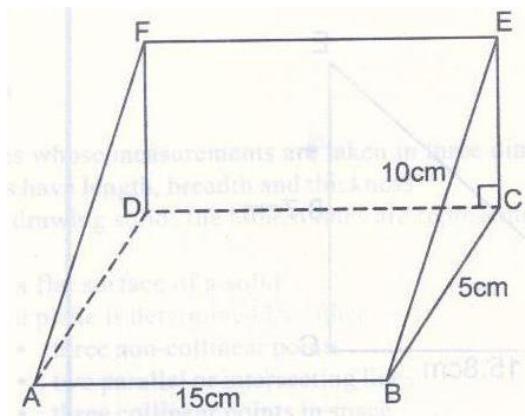
- (i) the distances of P and Q from the line AB
- (ii) the distance QB
- (iii) the course he must set to sail directly from Q to B

17) ABC is a triangular plot enclosed by three straight fences.AB=100m and has a bearing of N25°E. AC=50m and its bearing is N74°E.Find the area of the plot to the nearest square-metre. (Ans.1900m<sup>2</sup>)

18) ABC is triangle with AB=28cm, AC=35cm and BC=25cm.Find the size of angle A and hence find the area of the triangle.

***If you do not hope, you will not find what is beyond your hopes-St Clement***

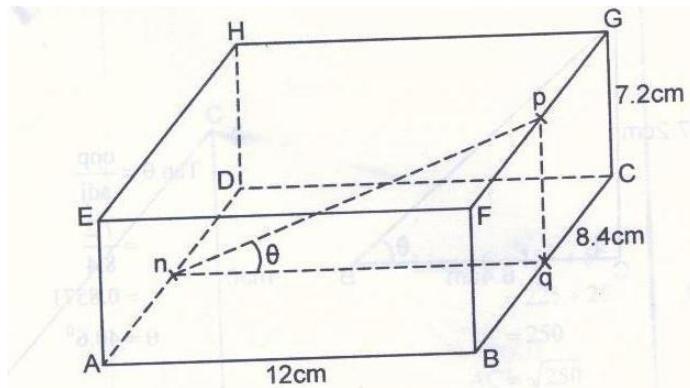
### **THREE DIMENSIONAL FIGURES**



The figure above shows a wedge in which  $AB=15\text{cm}$ ,  $BC=5\text{cm}$  and  $BE=10\text{cm}$ .  $\angle BCE = 90^\circ$ .

Calculate (a) the angle line AE makes with the plane ABCD (Ans:  $28.8^\circ$ )

(b) The angle line BE makes with line AD (Ans:  $60^\circ$ )



The figure above shows a cuboid ABCDEFGH with a rectangular base ABCD in which  $AB=12\text{cm}$ ,  $BC=8.4\text{cm}$  and  $CG=7.2\text{cm}$ . p, q and n are the mid-points of FG, BC and AD respectively.

Calculate (i) the length AF (ii) the length AC (iii)  $\angle CBG$ ;  $\angle pnq$ .

Ans(i)  $14.0\text{cm}$ ; (ii)  $14.6\text{cm}$ ; (iii)  $40.6^\circ$  (iv)  $31.0^\circ$

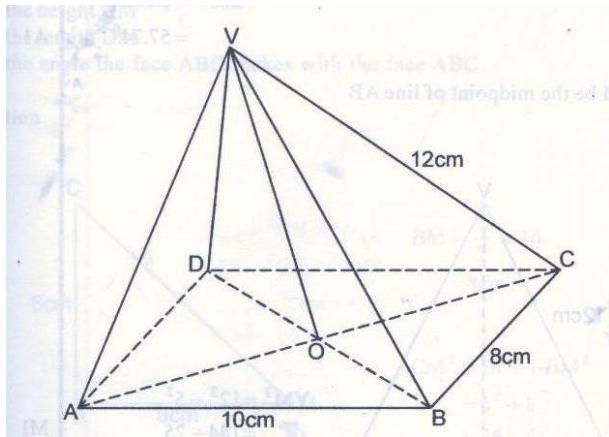
The figure below shows a right pyramid with a rectangular base ABCD. VO is the height of the pyramid. VA=VB=VC=VD=12cm. AB=10cm, BC=8cm.

Calculate (i) the length of the projection of VA on the base ABCD

(ii) the angle line VA makes with the base ABCD

(iii) the angle the face VAB makes with the base ABCD

Ans: (i)  $12.81\text{cm}$ ; (ii)  $57.7^\circ$  (iii)  $68.5^\circ$



## MATRICES

Example. A triangle with vertices  $A(1,3)$ ,  $B(2,1)$  and  $C(3,1)$  is transformed into another triangle with vertices  $A'(-3,1)$ ,  $B'(-1,2)$  and  $C'(-1,3)$ . Find the transformation matrix.

Solution: Let the transformation matrix be  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\text{Then } \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -3 & -1 & -1 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} a + 3b & 2a + b & 3a + b \\ c + 3d & 2c + d & 3c + d \end{pmatrix} = \begin{pmatrix} -3 & -1 & -1 \\ 1 & 2 & 3 \end{pmatrix}$$

$$a + 3b = -3; 2a + b = -1; c + 3d = 1; 2c + d = 2;$$

Solve the equations above simultaneously:  $a = 0; b = -1; c = 1; d = 0$

$$\text{Thus matrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

1) Given that  $A = \begin{pmatrix} -6 & 4 \\ 1 & 5 \end{pmatrix}$   $B = \begin{pmatrix} 1 & 4 \\ -2 & -3 \end{pmatrix}$  evaluate the following:

- a)  $A + B$
- b)  $A - B$
- c)  $A + 4B$
- d)  $6A - 2B$
- e)  $AB$

f) BA

2a) Find the matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  such that  $\begin{pmatrix} 2 & 3 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b) Find the values of the unknowns in each of the following cases:

i)  $\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix}$  ii)  $\begin{pmatrix} 4 & 0 \\ 1 & a \end{pmatrix} \begin{pmatrix} b & c \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 20 & 12 \\ -1 & d \end{pmatrix}$

c) A triangle with vertices  $A(-3,4), B(-3,1), C(-1,1)$  is mapped onto triangle  $A'(-3,-4), B'(-3,-1)$  and  $C'(-1,-1)$ . Find the transformation matrix.

4) On a particular day a trader sold four crates of eggs and six bales of sugar. The next day, she sold five crates of eggs and seven bales of sugar. On the third day, she sold six crates of eggs and nine bales of sugar. Represent this information in a matrix and hence state the order of the matrix.

5)  $\begin{pmatrix} 2 & 6 \\ 7 & -3 \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 0 & 28 \\ 24 & 2 \end{pmatrix}$ . Find  $p, q, r, s$ .

### VECTORS

1. Example. The coordinates of a point  $P$  are  $(2,3)$  and the column vector of  $PQ$  is  $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ , find;

(i) the coordinates of point  $Q$

(ii) The coordinates of  $M$ , the midpoint of the line segment  $PQ$

(iii) The length of the line segment  $PM$

Solution: (i)  $PQ = OQ - OP$

$$\begin{pmatrix} 6 \\ 4 \end{pmatrix} = OQ - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$OQ = \begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \end{pmatrix}$$

$$\therefore Q = (8,7)$$

$$(ii) M = \left( \frac{8+2}{2}, \frac{7+3}{2} \right) = (5,5)$$

$$(iii) |PM| = \sqrt{(5-2)^2 + (5-3)^2} = \sqrt{3^2 + 2^2} = \sqrt{13} \text{ Units}$$

2. Find the displacement vector  $\overrightarrow{PQ}$  which joins  $P(3, -2)$  and  $Q(7, -4)$

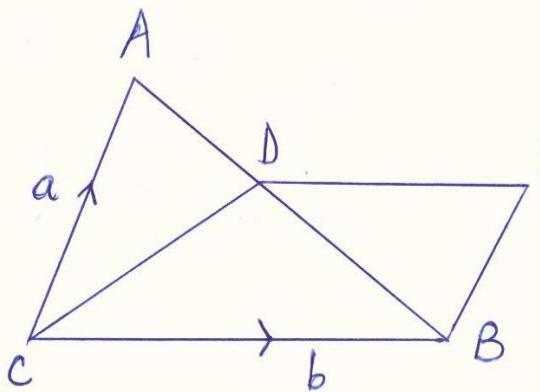
3. Calculate the modulus(magnitude) of the vectors (i)  $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$  (ii)  $\begin{pmatrix} -8 \\ 15 \end{pmatrix}$

3) Given that  $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} 6 \\ k \end{pmatrix}$ , find the value of  $k$  if the vectors are parallel

4) Given that the coordinates of points  $P$ ,  $Q$  and  $R$  are  $P(2,1)$ ,  $Q(4,2)$  and  $R(x,3)$ , find the value of  $x$  if  $P, Q, R$  are collinear. Ans: 6

5) If  $\vec{a} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$ ,  $\vec{b} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ , calculate  $|4b - 3a|$  Ans: 50.2

6a) In the figure below,  $\overrightarrow{CA} = a$  and  $\overrightarrow{CB} = b$ .  $BE$  is parallel to  $CA$ ,  $BE = \frac{1}{2}CA$  and  $AD = \frac{2}{3}AB$ . Find in terms of  $a$  and  $b$ ;  $\overrightarrow{AB}$ ,  $\overrightarrow{AD}$ ,  $\overrightarrow{BE}$ ,  $\overrightarrow{DB}$ ,  $\overrightarrow{DE}$ .



7) If  $\vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ ,  $\vec{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ ,  $\vec{c} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$ , find the numbers  $p$  and  $q$  such that  $p\vec{a} + q\vec{b} = \vec{c}$  Ans: 2, -3.

8) If  $\vec{p} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ,  $\vec{q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ,  $\vec{r} = \begin{pmatrix} 1 \\ -16 \end{pmatrix}$ , find numbers  $x$  and  $y$  such that  $x\vec{p} + y\vec{q} = \vec{r}$  Ans: -2, 5.

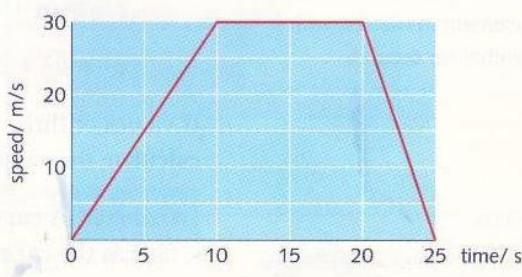
10) The position vectors of three points  $A$ ,  $B$  and  $C$  are  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ ,  $\begin{pmatrix} 5 \\ 11 \end{pmatrix}$  and  $\begin{pmatrix} 11 \\ 9 \end{pmatrix}$  respectively. If  $\overrightarrow{OB} = m\overrightarrow{OA} + n\overrightarrow{OC}$ , where  $m$  and  $n$  are scalars; find the values of  $m$  and  $n$ .

## TRANSFORMATIONS

1. Draw the rectangle  $ABCD$  with vertices  $A(3,1)$ ,  $B(5,1)$ ,  $C(5,4)$  and  $D(3,4)$ . The rectangle is transformed by the translation  $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$  into rectangle  $A'B'C'D'$ . Draw rectangle  $A'B'C'D'$
2. Pentagon  $ABCDE$  has vertices  $A(-2, -1)$ ,  $B(0, -1)$ ,  $C(1,1)$ ,  $D(-1,3)$  and  $E(-3,1)$ . After a translation, the coordinates of  $C'$  are  $(-1,2)$ 
  - a) Describe the translation using an ordered pair
  - b) Graph the two pentagons
3. A triangle is translated by the vector  $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ . Then the result is translated by  $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ . what is the final position of the triangle?
4. Draw triangle  $ABC$  with vertices  $A(4,6)$ ,  $B(10,2)$ , and  $C(14,10)$ .Draw triangle  $A'B'C'$  which is an enlargement of  $ABC$  with a scale factor of 1.5, centre  $(0, 0)$ .
5. Triangle  $ABC$  has vertices  $A(1,3)$ ,  $B(3,0)$  and  $C(1,0)$ .Enlarge the triangle; (i) by a scale factor of 2 with centre  $(-2,-3)$ 
  - (ii) By scale factor -2 with centre  $(-2,-3)$
6. Rectangle  $ABCD$  is enlarged into  $A'B'C'D'$ . $ABCD$  has vertices  $A(6,2)$ ,  $B(8,2)$ ,  $C(8,1)$ ,  $D(6,1)$  and  $A'B'C'D'$  has vertices  $A'(-1,4)$ ,  $B'(5,4)$ ,  $C'(5,2)$ ,  $D'(-1,2)$ .Define the enlargement.(i.e. find the center of enlargement and scale factor)
7. Triangle  $ABC$  has vertices  $A(5,6)$ ,  $B(6,4)$ ,  $C(8,5)$ .It is enlarged into triangle  $A'B'C'$  with vertices  $A'\left(2\frac{1}{2}, 2\right)$ ,  $B'(3,1)$ ,  $C'\left(4,1\frac{1}{2}\right)$ .Define the enlargement.

### TRAVEL GRAPHS

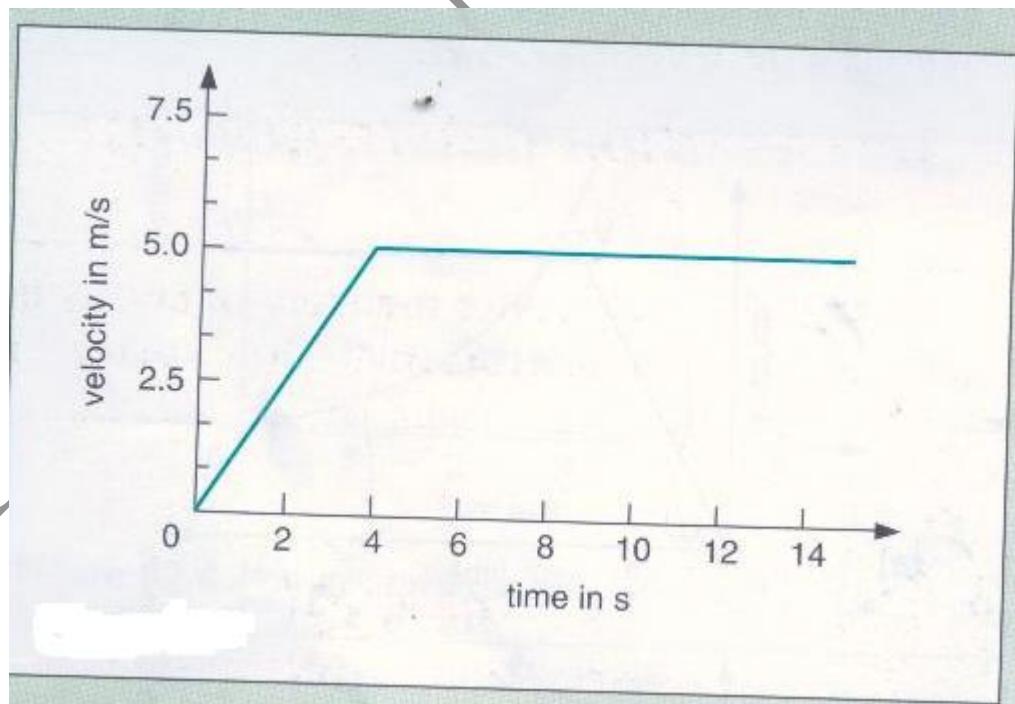
1



The speed-time graph above is for a motorcycle travelling along a road.

- What is the motorcycle's maximum speed?
- What is the acceleration during the first 10s?
- What is the deceleration during the last 5s?
- What distance is travelled during the first 10s?
- What is the total distance travelled?
- What is the time taken for the whole journey?
- What is the average speed for the whole journey?

2. The figure below shows an incomplete velocity-time graph for a boy running a distance of

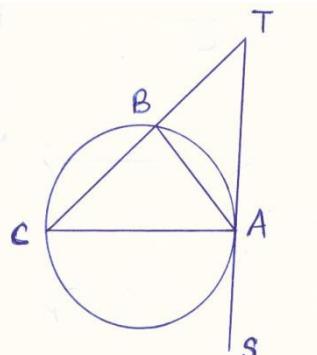
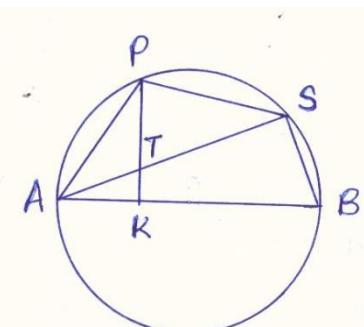
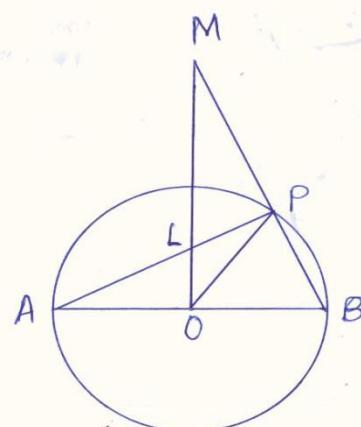
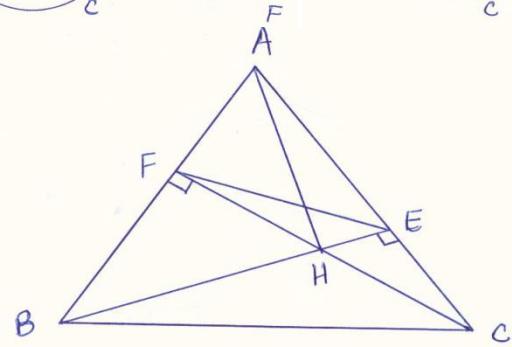
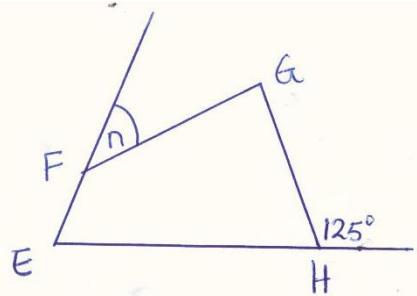
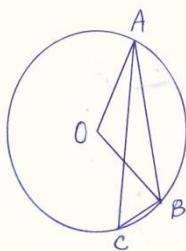
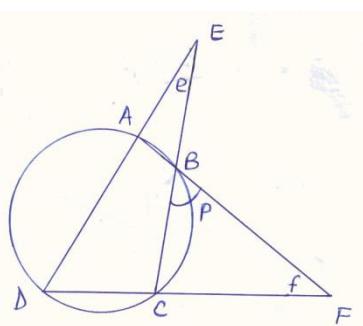
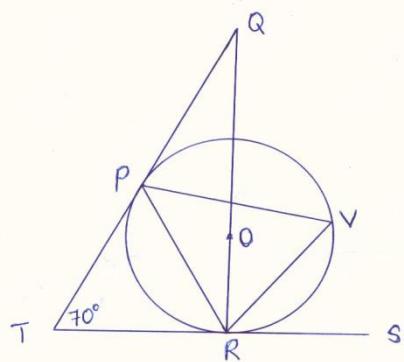
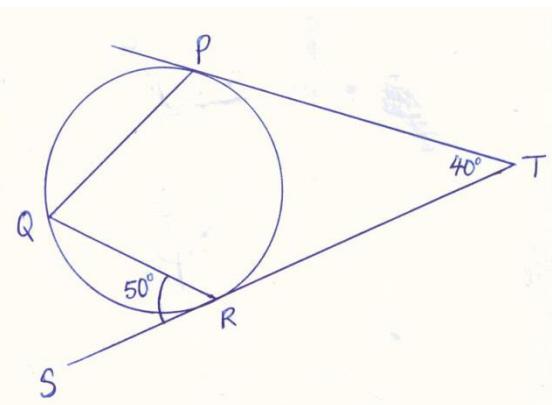
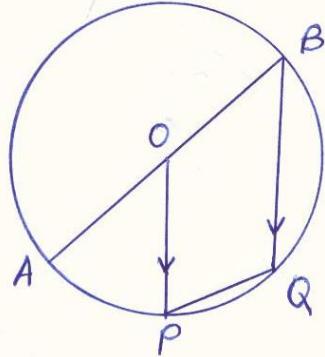


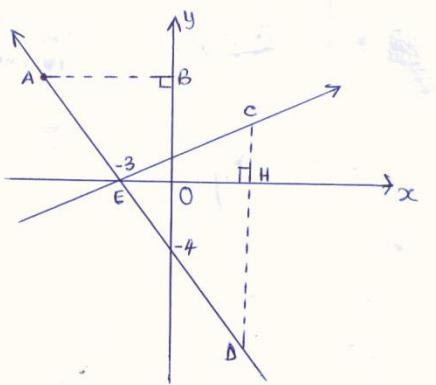
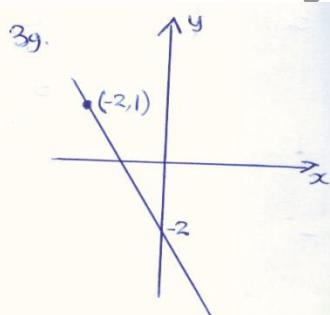
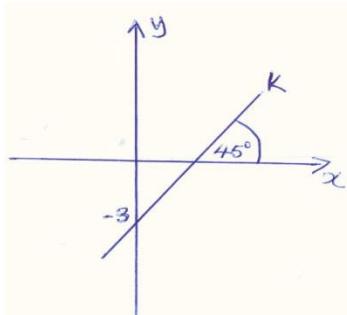
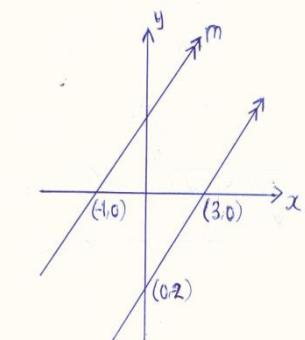
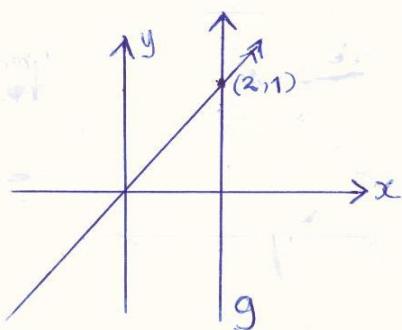
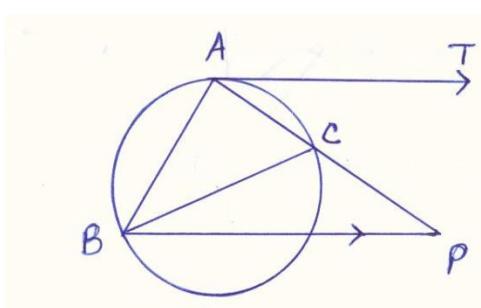
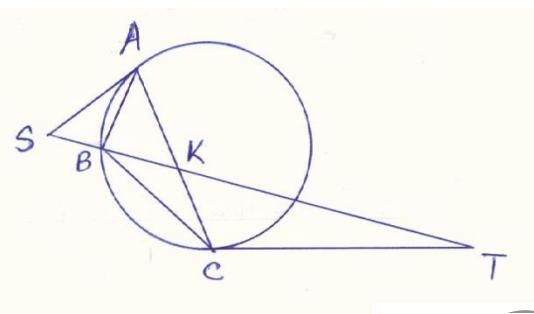
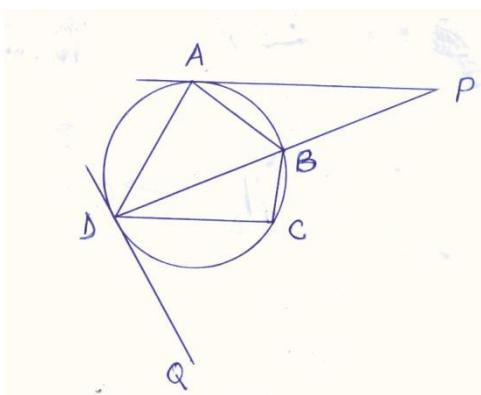
- a) What is the acceleration during the first 4 seconds?
- b) How far does the boy travel during the (i) first 4s? (ii) Next 9s?
- c) Copy and complete the graph showing clearly at what time he has covered a distance of 100m. Assume his speed remains constant at the value shown by the horizontal portion of the graph.
3. In a motorcycle test, the speed from rest was recorded at intervals
- | Time(s)     | 0 | 5  | 10 | 15 | 20 | 25 | 30 |
|-------------|---|----|----|----|----|----|----|
| Speed (m/s) | 0 | 10 | 20 | 30 | 40 | 40 | 40 |
- a) Plot a speed-time graph of the results
- b) What was the initial acceleration?
- c) How far did it move in (i) the first 20s? (ii) The next 10s?
4. A rocket under test reaches a speed of 210m/s from rest in 30s. It then travels at a constant speed for another 20s then decelerates uniformly until it comes to rest. Sketch a speed-time graph of its motion. How far has it travelled?

**We are what we repeatedly do. Excellence, therefore, is not an act but a habit-Aristotle**

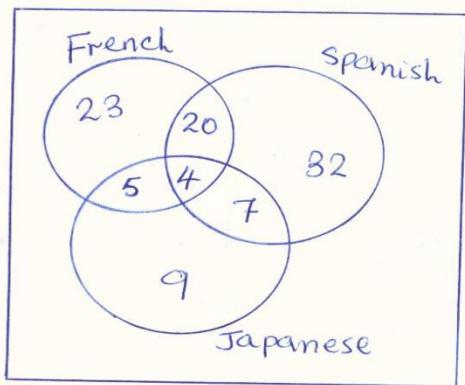
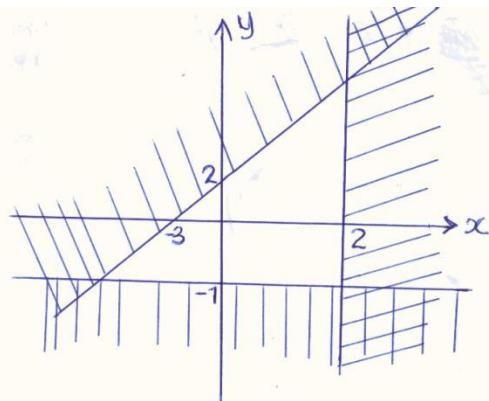
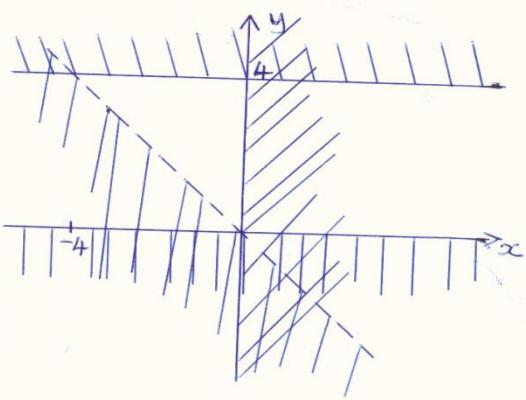
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