G1-10: Risk Indicators in Risk Parity Portfolio

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Abstract

Risk has always been existent in the financial markets and both academics and practitioners have sought after ways to minimize it. Risk Parity Portfolio is an asset allocation strategy which focuses on asset allocation based on risk contribution rather than the traditional methods of weighting assets based on type of security. In this research, we formulate thresholds along with risk indices from risk indicators that are used to stop trades during or prior to market downside using both non-machine learning and machine learning methodologies and analyze the results obtained from back-testing against a 3x levered portfolio of 4 securities futures.

1 Introduction

A risk parity portfolio is one which has equal risk or volatility contribution by each asset in the portfolio (Qian, E., 2011), giving it the advantage of not requiring forecasts of expected returns unlike the mean-variance portfolio (Fabozzi et al, 2021). Furthermore, it has a remarkable edge over the 60/40 model used in traditional asset allocation where up to 90% of the volatility of the entire portfolio can be attributed to equities, with the portfolio performance being mainly decided by equity returns. (Fabozzi et al, 2021) Such portfolios are thus more diversified due to the higher Sharpe ratio and consequently higher returns for a specific amount of risk. By leveraging the risk parity portfolio up along the 'Capital Market Line' of portfolio, it is possible to keep the equal contribution of risk between assets while experiencing higher returns than a 60/40 portfolio (Qian, E., 2011) Still, such portfolios employ leverage and may be vulnerable to market downturn, requiring the use of indicators to determine when to stop trading during risky periods when financial crisis looms.

The terms "Risk On" and "Risk Off" have been continuously used in financial markets to describe situations whereby investors are either adding on risk or deleveraging their existing risk (Lee, 2012). A leveraged risk parity portfolio can take advantage of appropriate risk indicators to determine risk-on/off situations and stop trades during risk-off while continuing trades during risk-on. This allows preservation of returns prior to market drawdowns during "Risk-Off" and maximizing of returns during "Risk-On".

The following sections will contain literature reviews which cover financial stress and risk indicator criteria and methods of creating risk indices. Subsequent sections cover the general methodology, non-machine learning and machine learning methodologies and analysis of back-testing results on a 3x leveraged portfolio of 4 equities futures.

2 Literature Review

2.1 Financial Stress and risk indicators criteria

The Kansas City Financial Stress Index (KCFSI) has several requirements that can be referred to when constructing a risk index. The variables represent one of more of the 5 features of financial stress, they are indicators of prices or yields which react most quickly to shifting financial conditions, they are available at least monthly and go back to 1990 to be able to determine past periods of financial stress. (Hakkio & Keeton, 2009)

The 5 features of financial stress are greater uncertainty about fundamental values of financial assets, increased uncertainty about investors behavior, increased asymmetry of information, unwillingness to hold risky assets and unwillingness to hold illiquid assets. (Hakkio & Keeton, 2009). Consequently, these features result in higher volatility of asset prices, increased cost of borrowing and higher expected returns on risky and illiquid assets.

The KCFSI consists of several risk indicators that meet the criteria mentioned above like Aaa/10-year Treasury spread, Implied volatility of overall stock prices (VIX). (Hakkio & Keeton, 2009). These indicators tend to spike during times of financial crisis and this property is referenced during the picking or creation of risk indicators to be used in risk indices in this paper.

2.2 Formation of risk index using Non-Machine Learning methods

Combining the risk indicators into a single risk index can be done using several methods. The financial stress index of the International Monetary Fund (IMF) was created using standardized variables which were equally weighted. (Hakkio & Keeton, 2009).

The KCFSI uses another method of combining the risk indicators and was created from a single factor made up of risk indicators. The coefficients of the indicators were chosen to describe the largest amount of change in the indicators and identified using principal components. (Hakkio & Keeton, 2009).

Similarly in the research by S.Baek et al (2020), Principal Component Analysis (PCA) was used and the top 3 principal components formed from the chosen 16 risk indicators cumulatively explained about 90% of the variance. The 3 principal factors are then examined to see if there are any relationships between them and the market patterns.

These methodologies are referenced during the formation of the risk index using the selected risk indicators for the Non-Machine Learning method presented in this paper.

2.3 Formation of risk index using Machine Learning methods

Research by S.Baek et al (2020) suggested that machine learning techniques such as support vector machine (SVM) could be used on trading days, labelled as either uptrend or downtrend, to predict breakout signals. Their trained SVM model was found to have good predictive power for both insample and out-sample and could provide signals to long/short the Exchange Traded Fund (ETF). Annualised Sharpe ratios on various actively managed ETFs coupled with SVM trading signal was 7 times higher than risk of holding ETFs passively on average. Referencing this method, this paper presents the prediction of signals using similar Machine Learning methods.

3 Data Extraction

For this project, daily data was extracted from Bloomberg, with dates ranging from 1 Jan 1988 to 30 Nov 2021. For some indices, data did not go back to 1 Jan 1988 so for each back-testing methodology, the start date was based on when all data was available. Two sets of data were extracted: One for the purpose of creating the risk parity portfolio and another for the formation of risk indices to be applied to the risk parity portfolio.

4 Risk Indicators

As mentioned previously, indicators used should spike during times of financial distress. While some of the securities obtained from Bloomberg were suitable for use as risk indicators, others required modifications to display this property. These include: NYSE Index, FTSE100 Index, SP500 Index, Philadelphia Gold and Silver Index, Gold/Silver Spot, High Yield bond index, NYSE New Highs Minus Lows Index and correlation of DJIA vs components. A list of specific modifications to the risk indicators can be found in Appendix A.

Risk indicators were then used to form risk indices and thresholds were used to determine risk-off/risk-on periods using the Non-Machine Learning and Machine Learning methodologies described in the following section.

5 General Methodology

All methodologies were conducted on a long only risk parity portfolio of 4 equities index futures as seen in Appendix A. The weights of the assets were calculated based on equal contribution of risk using python code from QuantDare (fjrodriguez2, 2017) modified for the purpose of this project along with changes referencing marginal contribution and objective function formulas (Fabozzi et al, 2021). A pure equities futures risk parity portfolio was chosen to effectively display the difference in performance of a portfolio with the use of risk index to activate risk off versus a portfolio that was susceptible to sell down during financial distress.

Both non machine learning and machine learning methods of forming risk indicators and thresholds were used on the portfolio and then back-tested to measure the performance.

5.1 Non-Machine Learning (non-ML) Methodology

For the Non-ML methodology, all data was split into 2 time periods with 31 Jan 2003 as the divider. This was done to allow the 1st time period (~23 Nov 1990 - 30 Jan 2003) to capture the 1997 Asian financial crisis and the 2000 Dot-com bubble and the 2nd time period (~31 Jan 2003 -30 Nov 2021) to capture the 2008 Subprime mortgage crisis and the COVID-19 crisis. For the 1st period, the starting date was taken at a date where all tickers did not have missing values. For each period, there was a 260-business day buffer duration where all positions were 0 for the portfolio to allow the covariance matrix of returns to stabilize. After this buffer duration, the portfolio was rebalanced 4 times roughly every 260 or so business days, with weights of each asset calculated to provide equal risk contribution to the portfolio.

Min-max scaling was conducted on the risk indicators in the respective time periods to transform them into a continuous range of values 0 to 1 to ensure that the scale of the values of all the indicators were the same. This was to ensure no single risk indicator overshadowed the others. After that, the risk index was formed from the scaled risk indicators.

2 methods were used in the non-ML method to form the risk index. For the 1st method (Linear Combination), the scaled risk indicators were added up using equal weights to form the risk index. For the 2nd method (PCA), Principal Component Analysis (PCA) was conducted to obtain the top 4 principal components that explained ~86% of the total variation. Min-max scaling was further applied to the principal components and these were then added up using equal weights to form the risk index.

For each risk index, above a certain percentage of the maximum risk index value within each period, all positions of assets were set to 0 and all trades were stopped. For realism, an additional 19 business day smoothing was applied so that when positions were set to 0, they remained at 0 for a total of 20 business days or 1 actual month. This was done to prevent constant liquidation and make the risk index and thresholds practical.

To find the thresholds, the range of thresholds within each period where portfolio with risk indicators outperformed the one without was studied to find the intersection between these ranges for the 1st and 2nd periods. The threshold with the best performance in both time periods that allowed cumulative returns of portfolio with the risk index to outperform the one without while keeping everything constant would be taken as the final threshold.

5.2 Machine Learning (ML) Methodology

Taking inspiration from S.Baek et al's method of predicting turning points in price momentum using support vector machine (SVM) (S.Baek et al, 2020), risk prediction was structured as a ML supervised classification problem and a similar labelling technique was applied to the SPX index. The SPX index was chosen as the reference price for labelling because it is representative of the general market sentiment.

The suggested 3% up-down momentum labelling technique broke the reference price into segments, where each segment was labelled as up-trend(down-trend) if the price difference was 3% higher(lower) from beginning to end of segment. Another method of labelling was applied which used simple moving average (SMA) of 20 days crossover to determine risky periods where spot price of SPX Index was lower than its SMA of 20 days.

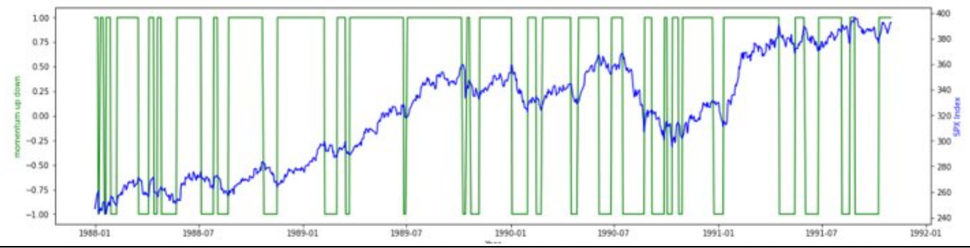
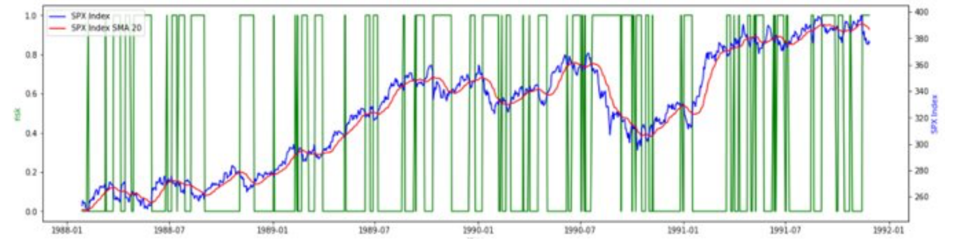
Labelling	SPX Index labelled with risk on/off segments across time
3% up-down momentum	
SMA 20 crossover with spot price	

Figure 5.1: Illustration of Risk-On/Risk-Off periods for both techniques

A 70:30 train-test split was conducted on the daily features and labels. ~23 Nov 1990 was taken as the start date for the training set because that was when all data was available and 1 Mar 2012 was the start date of the 1st quarter for the test set after 70% of training data. Essentially, the same basket of futures as the non-ML method was used in the portfolios, and the portfolio was rebalanced at exactly the first day of every quarter to ensure equal risk contribution from each asset.

The features were formed from the same pool of risk indicators discussed below in the non-ML approach section. Unlike the non-ML method, where both a risk index and a threshold were formed, the ML method uses the min-max scaled risk indicators to form input features for training to determine risk-on or risk off. The min-max scaler fitted on training set was used to transform the test set. To make the machine learn a 3-days forward-looking forecast, labels were matched to daily features of the past 3 days for training and testing.

Table 5.2 shows the accuracy of predicting risk versus no risk on daily basis on the test set for each model and each labelling method applied.

Models / Labelling Method	SVM	Random Forest	Gradient Boosting
SMA Crossover	0.753	0.753	0.753
3% Momentum rise/drop	0.720	0.734	0.606

Table 5.2: Table of accuracy for predicting risk versus no risk, for each model and labelling method

The SMA Crossover labelling technique yielded better accuracy for the 3 models compared to the 3% Momentum rise/drop labelling technique. The SVM model using SMA Crossover labelling was chosen to produce the back-testing result on the test set since its accuracy was the highest and comparable to the other 2 models with the same labelling methods.

In SVM model, each trading day with n number of features (n = 10) corresponds to a point in n-dimensional space. The SVM model we deployed uses radial basis function kernel to transform this input space to a higher dimensional space, and then it learns the hyperplane to separate the binary labelled data. The learnt hyperplane was applied on test data to predict if each trading day is risk-on risk off.

6 Results and Analysis

To measure the effectiveness of our indicator, the positions and number of trades and stop trades were identified for both time periods and portfolios (with and without risk indicators). The breakdown of trades, cumulative returns, Sharpe ratio and maximum drawdown were compared and analysed to provide insights for the differences between the performances of the portfolios in the two time periods. In this section, the results of said methodologies will be discussed in more detail.

6.1 Non-ML: Linear Combination 2 (LC2)

2 Linear Combinations of risk indicators were developed, LC1 and LC2. As LC2 outperformed LC1 and both were roughly similar in terms of risk indicators used, the results for LC1 were placed in Appendix B. The following 4 risk indicators were used for LC2: US Corp BAA 10 Yr Spread, SP500 Index, FTSE100 Index and Philadelphia Gold and Silver Index, with modifications made to SP500 Index, FTSE100 Index and Philadelphia Gold and Silver Index as shown in Appendix A. These risk indicators were subjected to min-max scaling and a linear combination was formed from the equally weighted risk indicators. The threshold value used was 0.5865 of max value to indicate stop trades. The formation of the risk index is given by the equation:

$$\text{Risk Index} = \sum_{1}^{N} \mathrm{R}_n \text{ , where } \mathrm{R}_n \text{ represents the min max scaled risk indicator}$$

The LC2 method identified 9 and 4 stop-trade periods over the 2 periods respectively. For the 1st period, the stop-trade periods appeared repeatedly from 1999 to the end of the period while for the 2nd period, the 4 periods were distinctly from 2008 to 2009, from 2015 to 2016 and in 2020.

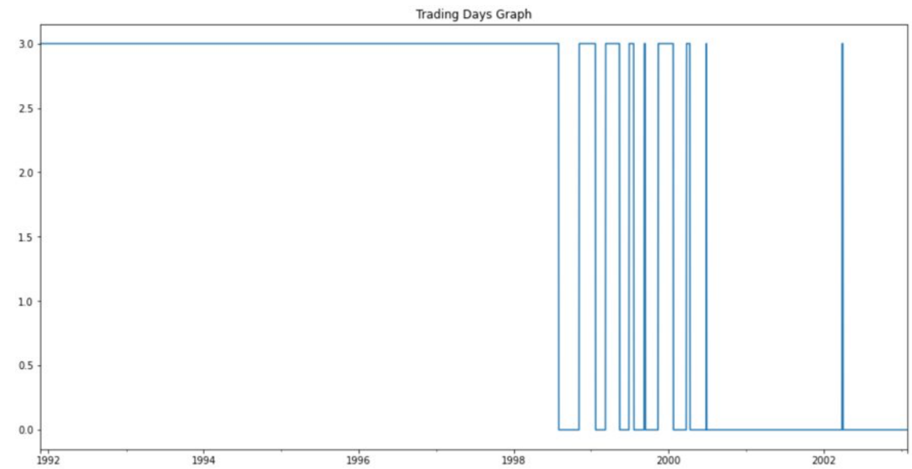


Figure 6.1: Illustration of trading positions for 1st time period. Values on the y axis of 3 represent trading positions while 0 represents stop-trade positions, notice that stop trades periods are clearly defined due to the additional 19 business day smoothing which prevents 0 positions from changing back to 1 for a total of 20 business days

Figure 6.2: Illustration of log cumulative returns for 1st time period of portfolio with risk index vs without, red dots signify stop-trade periods

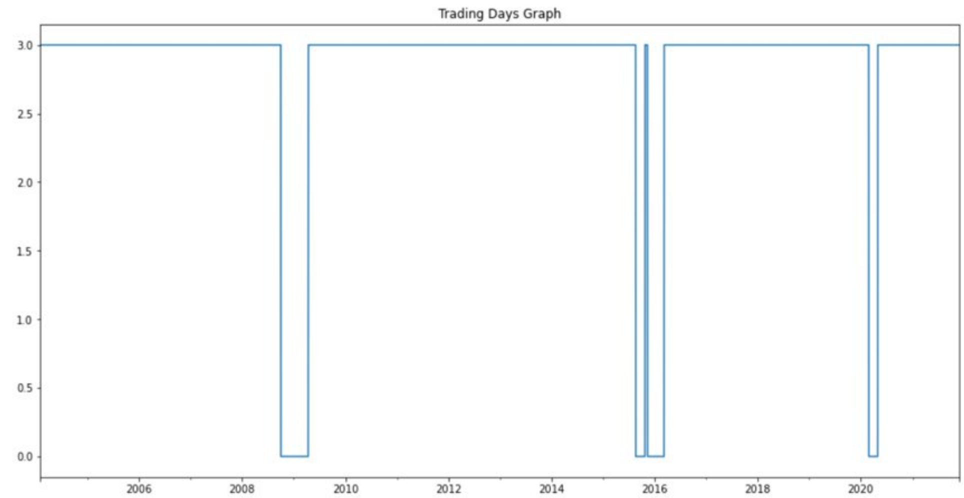


Figure 6.3: Illustration of trading positions for 2nd time period

Figure 6.4: Illustration of log cumulative returns for 2nd time period of portfolio with risk index vs without

	1st Period		2nd Period	
	With Risk Indicator	Without Risk Indicator	With Risk Indicator	Without Risk Indicator
Winning Days (Returns>1)	1044	1517	2381	2536
Stop Days (Returns = 1)	981	0	313	0
Losing Days (Returns<1)	885	1393	1946	2104
Cumulative Returns	14.655	2.756	33.725	6.977
Annualized Sharpe Ratio	0.830	0.412	0.711	0.469
Max Drawdown	4.451	27.299	10.309	4.243

Table 6.5: Table of results for both portfolios with LC2 index and without across both time periods. Values are rounded off to 3 D.P.

As seen from Table 6.5, for both periods, the portfolio with LC2 risk index had higher cumulative returns (5.32 and 4.83 times higher than without risk indicator for the 1st period and 2nd period respectively) and higher annualized Sharpe ratio (2.01 and 1.52 times higher than without risk indicator for the 1st period and 2nd period respectively), meaning the risk index made the risk parity portfolio both safer and much more profitable. For the 1st period, the max drawdown (4.451) was a lower value of 0.304 times of the final cumulative returns (14.655) for the portfolio with the risk index unlike the portfolio without where the max drawdown (27.299) was an extremely high value of 9.905 times of final cumulative returns (2.756).

While the max drawdown in the 2nd period was higher for the portfolio with the risk index, this is acceptable since in the 2nd period, the proportion of the max drawdown (10.309) relative to final cumulative returns (33.725) was a lower value of 0.306 unlike the portfolio without risk index where the proportion of max drawdown (4.243) relative to final cumulative returns (6.977) was 0.608.

Overall, this meant that the portfolio with LC2 risk index reduces the proportion of the max drawdown relative to the final cumulative returns compared to the portfolio without risk index for both time periods (9.601 lower for 1st period and 0.302 lower for 2nd period), reducing the amount of risk.

6.2 Non-ML: Principal Component Analysis (PCA)

The following 12 risk indicators were used: NYSE Index, FTSE100 Index, SP500 Index, Philadelphia Gold and Silver Index, VIX Index, US Corp BAA 10 Year Spread, US Corp AAA 10 Year Spread, NYSE New Highs Minus New Lows Index, High Yield Bond Index, DXY Index, Gold/Silver Spot, Correlation of DJIA vs components. Please refer to Appendix A for a list of risk indicators and modifications.

These risk indicators were first subjected to min-max scaling, and then PCA is done to obtain the top 4 Principal Components (PC) that explained approximately 86% of the total variance for both time periods. Min-max scaling was further applied to the principal components and these were then added up using equal weights to form the risk index, similar to the LC2 method. The threshold value of 0.703 of the max value to indicate stop trades was used. The formation of the risk index is given by the equation:

$$\text{Risk Index} = \sum_{1}^{N} \mathrm{PC}_n \text{, where } \mathrm{PC}_n \text{ represents the min max scaled PC}$$

The PCA method identified 7 and 12 stop-trade periods over the 2 periods, more than LC2. Notably difference is that it has identified periods the 1994 and 2010-2011, and although identified the periods 2015-2016 as potential stop-trade periods, the stop-trade algorithm was not triggered.

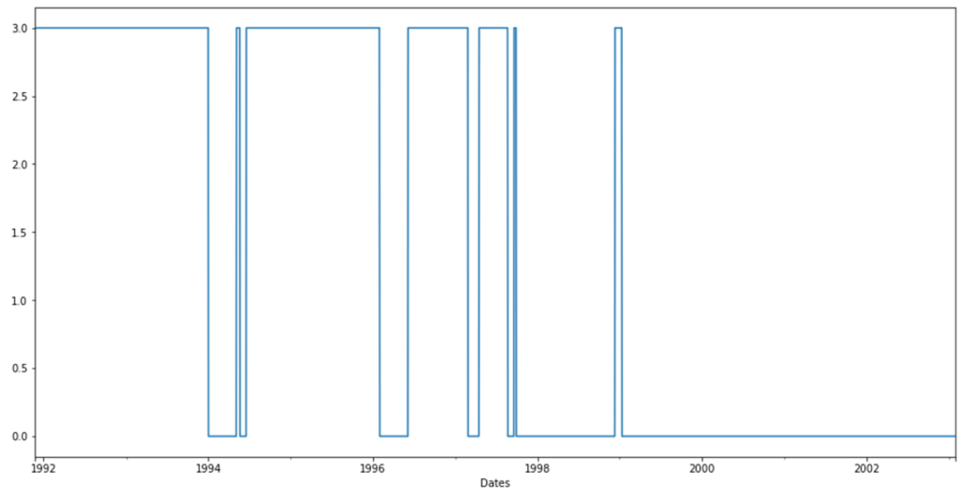


Figure 6.6: Illustration of trading positions for 1st time period

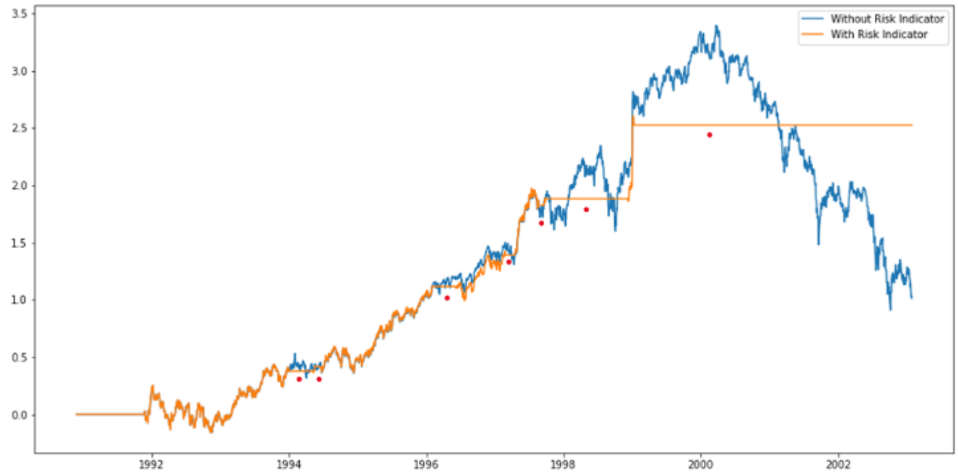


Figure 6.7: Illustration of log cumulative returns for 1st time period of portfolio with risk index vs without

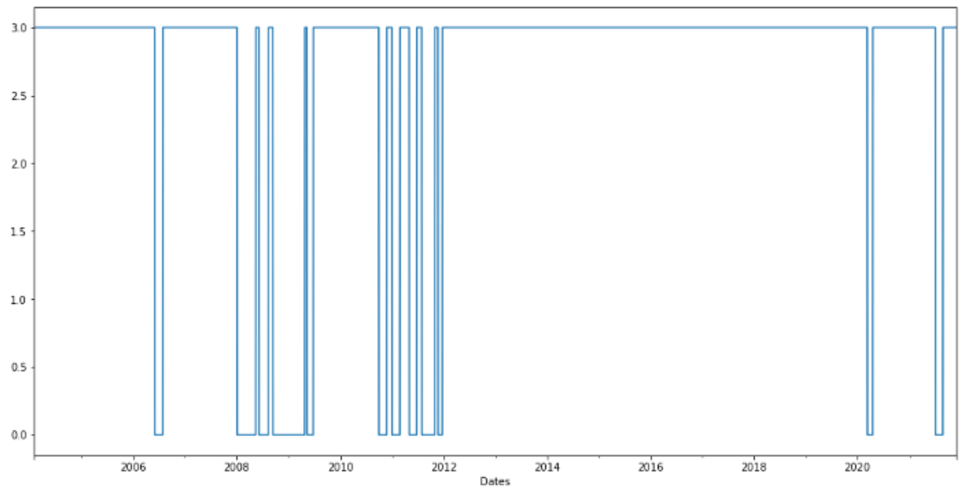


Figure 6.8: Illustration of trading positions for 2nd time period

Figure 6.9: Illustration of log cumulative returns for 2nd time period of portfolio with risk index vs without

	1st Period		2nd Period	
	With Risk Indicator	Without Risk Indicator	With Risk Indicator	Without Risk Indicator
Winning Days (Returns>1	701	1517	2209	2536
Stop Days (Returns = 1)	1620	0	644	0
Losing Days (Returns<1)	589	1393	1787	2104
Cumulative Returns	12.480	2.756	28.985	6.977
Annualized Sharpe Ratio	0.839	0.412	0.695	0.469
Max Drawdown	1.158	27.299	20.333	4.243

Table 6.10: Table of results for both portfolios with the PCA method risk index and without across both time periods. Values are rounded off to 3 D.P.

As seen from Table 6.10, for both periods, the portfolio with risk index had higher cumulative returns (4.53 and 4.15 times higher than without risk indicator for the 1st period and 2nd period respectively) and higher annualized Sharpe ratio (2.04 and 1.48 times higher than without risk indicator for the 1st period and 2nd period respectively), indicating the effectiveness of the risk index, similar to LC2. For the 1st period, the max drawdown (1.158) was 0.0928 times of the final cumulative returns (12.489) for the portfolio with the risk index unlike the portfolio without where the max drawdown (27.299) was 9.905 times of final cumulative returns (2.756).

However, the huge max drawdown for 2nd period for portfolio with risk indicator was less than ideal despite being superior in other areas. As seen in Figure 6.9, this could be attributed to 2 factors. Firstly, in the periods leading up to the 2020 Covid crisis, the portfolio with risk indicator accumulated much larger returns and thus during the crisis, it appeared to have experienced huge losses despite the proportion of drawdown being quite close (0.702 times for risk indicator portfolio vs 0.608 times for risk indicator). Secondly, although it managed to activate the stop-trade mechanism during the crisis, it activated too late in the crisis and hence unable to avoid the initial sell-off in the market. Nevertheless, despite the setback, the portfolio with the risk indicators was still able to achieve better performance stated above, as well as reducing the proportion of max drawdown relative to the final cumulative returns for the 1st time period (9.813 lower).

6.3 Analysis of results for Non-ML methodology

Generally, LC2 had higher cumulative returns for both periods compared to the PCA method despite the lower number of stop days triggered. This could suggest that LC2 was more sensitive in detecting periods where markets experience more severe financial distress. Despite this, it was not able to avert the 2008 financial crisis as effectively as the PCA method. This can be seen from comparing Figure 6.4 and 6.9 where the stop-trade period for LC2 only activated when a sell-down was already underway unlike the PCA method.

Compared to LC2, the PCA method identified more stop-trade periods for both time periods, suggesting it to be more sensitive in detecting periods where markets experience less severe sell-offs. An example to note is the 2006 period whereby LC2 did not identify it as a "Risk-Off" period (Figure 6.3), while PCA identified and triggered the stop-trade mechanism (Figure 6.8). Furthermore, the risk index formed from PCA makes periods of financial stress more apparent, seen in Figure 6.11.

However, it being too sensitive makes it vulnerable to noises in the market, and thus not able trigger the stop-trade mechanism effectively in some periods. This can be seen from the 2015-2016 period where the index has identified as a period of potential “Risk-Off” period as seen in the spikes shown in Figure 6.11, yet did not trigger a Stop-Trade as seen in Figure 6.8. This could be due to the higher threshold value used in PCA method, resulted from the more sensitive signals from the indicators.

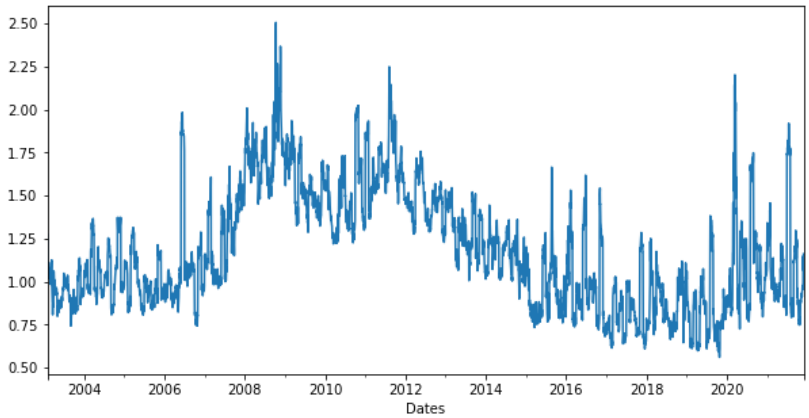


Figure 6.11: Illustration of risk index derived from risk indicators using PCA, for 2nd period.

6.4 ML: Support Vector Machine (SVM)

The following 10 risk indicators were used for SVM: NYSE Index, FTSE100 Index, SP500 Index, Philadelphia Gold and Silver Index, VIX Index, US Corp BAA 10 Yr Spread, US Corp AAA 10 Yr Spread, NYSE New Highs Minus New Lows Index, High Yield Bond Index, DXY Index, with modifications to NYSE Index, FTSE100 Index, SP500 Index, Philadelphia Gold and Silver Index as seen in Appendix A. These risk indicators were subjected to min-max scaled to form input features for training using ML SVM model to determine risk-on or risk off.

The SVM model trained on the train set (23 Nov 1990 – 29 Feb 2012) with risk binary label was run against the test set (1 Mar 2012 – 25 Nov 2021) to produce a predicted risk score of 1 or 0 daily. Due to the nature of the daily prediction, thresholding was applied to sum of rolling 10 days predicted risk to smoothen the signal. This was to prevent liquidation and buying assets at high frequency.

Figure 6.12 below showed the trading position after the above-mentioned smoothing was applied. It is notable that during the covid-induced market drawdown in Mar 2020, the model predicted a prolonged risky period shown by the dip in figure 6.12 during the corresponding period. Thus, the model had liquidated and stayed out of market for a substantial period of time.

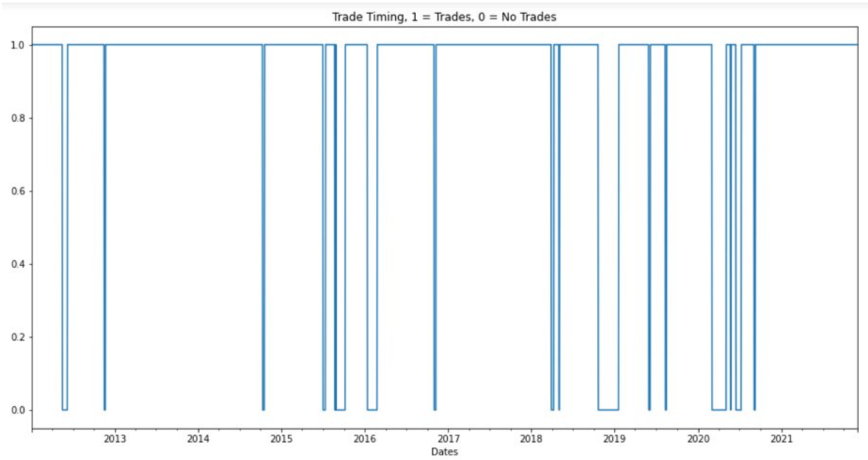


Figure 6.12: Illustration of trading positions for test period

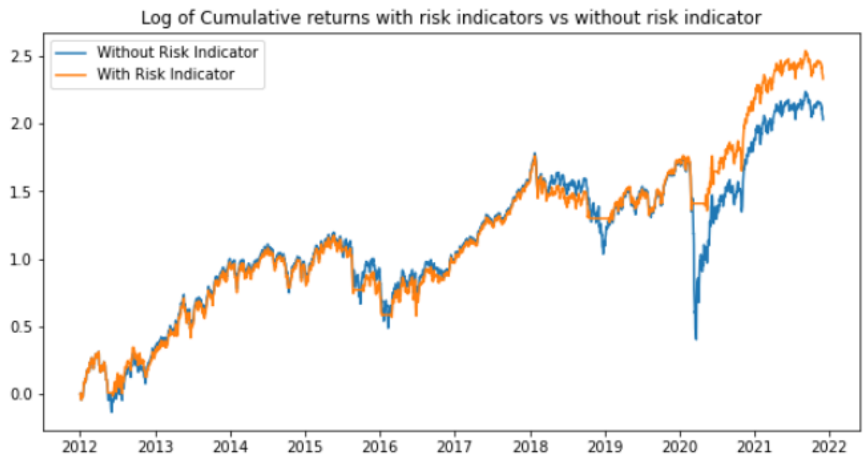


Figure 6.13: Illustration of trading position for back-test period of portfolio with risk index vs without

Due to the accurate prediction of risk-off periods, it resulted in timely liquidation of portfolios to preserve gains. Figure 6.13 shows that with risk indicators (orange curve), drawdown was smaller compared to without risk indicators (blue curve) especially during Mar 2020 sell-off. The performance vis-à-vis without risk indicators is summarised in table 6.14.

	Back Test Period	
	With Risk Indicator	Without Risk Indicator
Winning Days (Returns>1	1286	1414
Stop Days (Returns = 1)	253	0
Losing Days (Returns<1)	1038	1163
Cumulative Returns	10.494	7.620
Annualized Sharpe Ratio	0.901	0.715
Max Drawdown	2.420	4.450

Table 6.14: Table of results for both portfolios with and without Indicators for back-test period. Values are rounded off to 3 D.P.

As seen from Table 6.14, during the Back Test period, the portfolio based on ML SVM generated risk signals had higher cumulative returns (1.38 times higher than without risk indicator) and higher annualized Sharpe ratio (1.26 times higher than without risk indicator). The max drawdown (2.420) was a lower value of 0.231 times of the final cumulative returns (10.494) for the portfolio with the risk signal compared to the portfolio without where the max drawdown (4.450) was 0.584 times of final cumulative returns (7.620), showing an overall reduction of 0.353.

7 Conclusion and further work

7.1 Conclusion

Risk parity portfolios aims to achieve balanced risk exposure portfolios that deliver higher risk-adjusted returns than the market. By creating various risk indexes based on carefully selected risk indicators and different methodologies, the risk parity portfolios presented in this paper were able to obtain higher risk-adjusted returns. Although there are periods where the various risk indexes did not identify the entire periods of financial crisis, they were effective to a certain degree, allowing the risk parity portfolios to switch positions thereby avoiding huge downsides experienced in the market.

	Non-ML : LC2	Non-ML: PCA	ML : SVM
Final Cumulative Returns (Times higher than without risk indicator)	5.32 (first period) 4.83 (second period)	4.53 (first period) 4.15 (second period)	1.38 (test period)
Annualized Sharpe Ratio (Times higher than without risk indicator)	2.01 (first period) 1.52 (second period)	2.04 (first period) 1.48 (second period)	1.26 (test period)
Max Drawdown relative to final cumulative returns (Reduction with risk indicator)	9.601 (1st period) 0.302 (2nd period)	9.813 (1st period) -0.093(2nd period)	0.353 (test period)

Table 7.1: Table of summarized performances of the 3 risk indicator methods.

Overall, from Table 7.1, it is evident that the risk parity portfolios using each method of employing risk indicators vastly outperformed the ones without, showing that all 3 methodologies add value in determining risk on and risk off periods. Despite PCA method not being able to reduce the proportion of drawdown for the 2nd time period, the portfolio with the risk indicators was still able to achieve better performance than the one without risk indicators.

7.2 Further work

While methodologies of employing the risk indicators were able to successfully identify and avoid trades, there are times where it did not work as intended, and at times identifying risk-off periods too late or risk-on periods too soon, resulting in losses as presented in the paper. As such, modifications to improve the sensitivity of the risk index and reduce noise could address such issues. This could be done by exploring risk indicators that target specific areas of the market. One example is the Put-Call ratio used by many investors to assess the sentiment of the market. However, one limiting factor is many such indicators lack data prior to 2000. Although we have explored many such indicators, this limitation prevented us from using these indicators to form the risk index. A possible solution is to shift the time periods to later periods whereby data is available, but at a trade-off that earlier financial crises will not be captured in the research. Nonetheless, the financial market during the pre 2003 period was subjected to a different set of financial regulations and trading technicalities, compared to that of the post 2003 period. As such, the omission of earlier data could perhaps not have much of an impact on post 2003 analysis.

8 References

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APPENDIX A

Bloomberg Ticker	Index Name	Modifications
Portfolio securities		
SP1 Index	S&P 500 Generic Futures	NIL
GX1 Index	Generic 1st DAX Futures	NIL
Z 1 Index	Generic 1st FTSE 100 Futures	NIL
NK1 Index	Generic 1st Nikkei 225 Futures	NIL
Risk Indicators		
NYA Index	NYSE Index	Negative NYA Index Momentum - -1* (Index price – 80 day mean price)
SPX Index	SP500 Index	Negative SPX Index Momentum - -1* (Index price – 30 day mean price)
UKX Index	FTSE100 Index	Negative UKX Index Momentum - -1* (Index price – 30 day mean price)
NWHLNYHL Index	BBG New Highs Minus New Lows NYSE	Negative NWHLNYHL Index
LG30TRUU Index	High Yield bond Index	-1 * Percentage change
XAU, XAG Curncy	Gold and Silver Spot	Ratio of XAU/XAG
VIX Index	CBOE Volatility Index	NIL
BASPCAAA Index	US Corp AAA 10 Year Spread	NIL
BICLB10Y Index	US Corp BAA 10 Year Spread	NIL
DXY Curncy	US Dollar Index Spot	NIL
XAU Index	Philadelphia Stock Exchange Gold and Silver Index	Negative XAU Index
INDU Index	DJIA Index	Calculated month-to-month change in R-Square value based on the regression between DJIA and it's components
MMM US, KO US, PG US, IBM US, MCD US, MRK US, DIS US	3M, Coca Cola, Procter & Gamble, IBM, Mcdonald, Merck & Co, Disney	NIL

Table A: Bloomberg Tickers and Index Names. Data obtained from Bloomberg

APPENDIX B

B.1 Non-ML: Linear Combination 1 (LC1)

The following 5 risk indicators were used for LC1: VIX Index, US Corp BAA 10 Yr Spread, SP500 Index, FTSE100 Index, Philadelphia Gold and Silver Index, with modifications made to SP500 Index, FTSE100 Index, Philadelphia Gold and Silver Index as seen in Table A in the Appendix. The threshold value used was 0.606% of the max value to indicate stop trades. The LC1 method identified 10 and 3 stop-trade periods over the 2 periods respectively. Overall, LC1 did not perform as well as LC2.

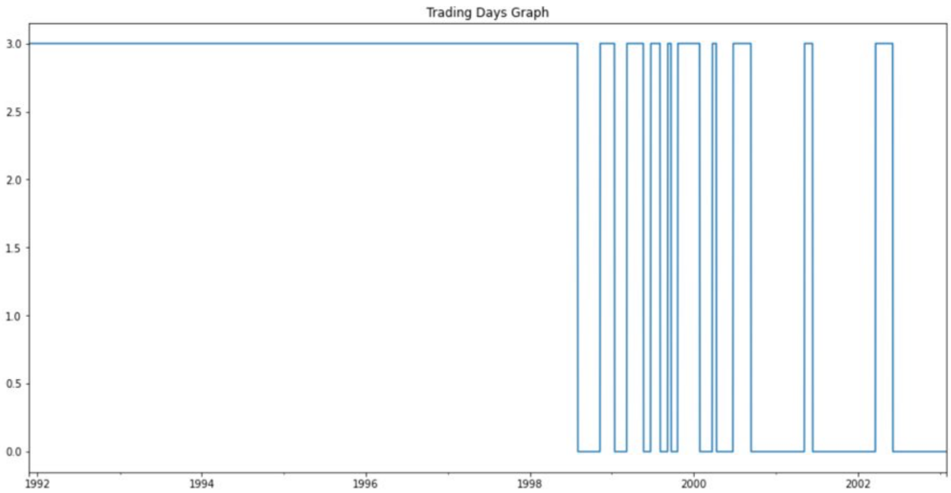

Figure B.1: Illustration of trading positions for 1st time period.

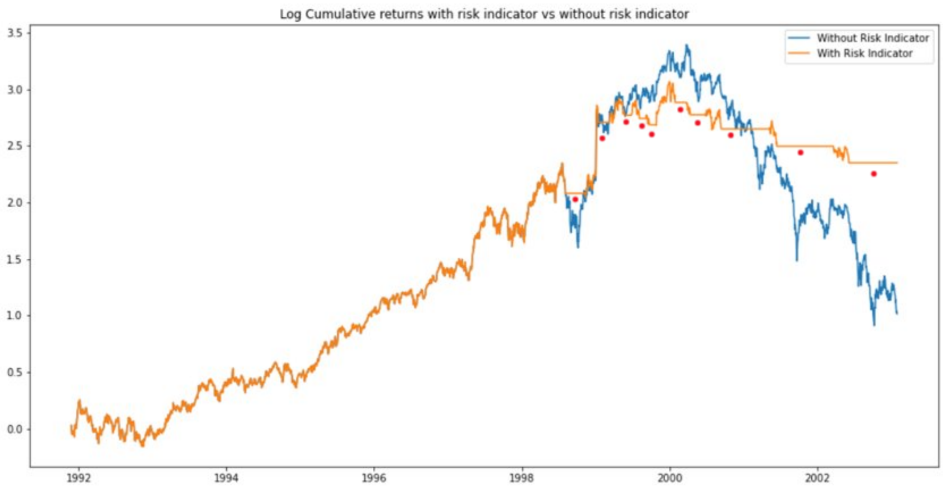

Figure B.2: Illustration of log cumulative returns for 1st time period of portfolio with risk index vs without

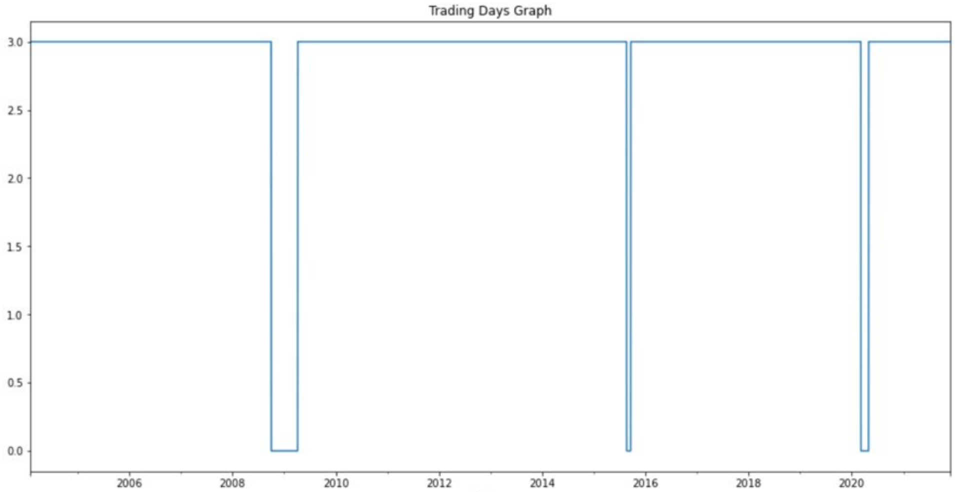

Figure B.3: Illustration of trading positions for 2nd time period.

Figure B.4: Illustration of log cumulative returns for 2nd time period of portfolio with risk index vs without

	1st Period		2nd Period	
	With Risk Indicator	Without Risk Indicator	With Risk Indicator	Without Risk Indicator
Winning Days (Returns>1	1122	1517	2442	2536
Stop Days (Returns = 1)	808	0	193	0
Losing Days (Returns<1)	980	1393	2005	2104
Cumulative Returns	10.471	2.756	27.583	6.977
Annualized Sharpe Ratio	0.735	0.412	0.672	0.469
Max Drawdown	10.999	27.299	10.030	4.243

Table B.5: Table of results for both portfolios with LC1 risk index and without across both time periods