Consider the following:

$$\phi = \min_{x \in X} \left\{ c^T x + \max_{P \in \mathbb{P}} \mathbb{E}_P \{ Q(x, \tilde{w}) \} := f(x) \right\} \qquad (M)$$

$$\phi^k = \min_{x \in X} \left\{ f^k(x) = c^T x + \max_{P \in \mathbb{P}} \mathbb{E}_P \{ Q^k(x, \tilde{w}) \} \right\} \qquad (M^k)$$

$\{Q^k(x,w)\}$ uniformly converges, and satisfies $Q^k(x,w) \leq Q^{k+1}(x,w) \leq Q(x,w)$

Define by $\mathcal{Q}(x) = \max_{P \in \mathbb{P}} \mathbb{E}_P \{ Q(x, \tilde{w}) \}$, and

$$\mathcal{Q}^k(x) = \max_{P \in \mathbb{P}} \mathbb{E}_P \{ Q^k(x, \tilde{w}) \}$$

To prove : Does $\{\mathcal{Q}^k(x)\}$ converge uniformly on X.

$$Q^k(x,w) \leq Q^{k+1}(x,w) \qquad \forall w \in \Omega$$

$$\mathbb{E}_P \{ Q^k(x,\tilde{w}) \} \leq \mathbb{E}_P \{ Q^{k+1}(x,\tilde{w}) \} \qquad \forall P \in \mathbb{P}$$

$$\Rightarrow \quad \mathcal{Q}^k(x) \leq \mathcal{Q}^{k+1}(x) \leq \mathcal{Q}(x)$$

Therefore $\{\mathcal{Q}^k(x)\}$ converges uniformly on X.

Let $x \in X$ and suppose $x^k \to x$. We have

$$f(x) = \lim_{k \to \infty} f^k(x^k) = \limsup_k f^k(x^k)$$

$$\geq \limsup_k \phi^k$$

$$\Rightarrow \quad \min_{x \in X} f(x) = \phi \geq \limsup_{k \to \infty} \phi^k \qquad \text{—— } (1)$$

In the above, the first equality follows from equivalence between uniform and continuous convergence.

Suppose $\hat{x}^k$ is a solution of M^k, $k=1,\dots$ and $\hat{x}$ is the accumulation point of $\{\hat{x}^k\}$.

$$
\begin{aligned}
f(\hat{x}) &= \lim_{k\to\infty} f^k(\hat{x}^k) \qquad \text{(Continuous convergence)} \\
&= \liminf_{k\to\infty} f^k(\hat{x}^k) \\
&\leq \liminf_{k\in K} f^k(x^k) \qquad \text{Looking only at a subsequence } K, \\
&= \liminf_{k\in K} \phi^k \qquad f^k(x^k) = \phi^k \\
&\leq \limsup_{k\in K} \phi^k \qquad \text{Trivial inequality} \\
&= \limsup_{k\to\infty} \phi^k \qquad \{\phi^k\} \text{ is a bounded sequence in } \mathbb{R}, \text{ therefore the subsequence converges, and converges to the original sequence's limit point.} \\
&\leq \phi \qquad \text{Follows from (1)}
\end{aligned}
$$

Since ϕ is the optimal value of a minimization problem. We have $\hat{x} \in \operatorname{argmin} f(x)$.

This leads us to conclude the following.

Theorem: If $\hat{x}^k$ is a solution to M^k, and if $\hat{x}$ is the accumulation point of $\{\hat{x}^k\}$ then $\hat{x}$ is a solution to M.