QF603 Quantitative Analysis of Financial Markets

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Bayesian Revision HW

Solution

Consider Bayes' theorem $P(C|A) = \frac{P(A|C)\cdot P(C)}{P(A)}$, where: $P(C|A)$ is the posterior probability (updated belief about the manager after observing the data), $P(A|C)$ is the likelihood (probability of observing the manager beat the market 2 years in a row given they are a particular type of manager), $P(C)$ is the prior probability (our belief before observing the data) and $P(A)$ is the unconditional probability (the probability of observing the manager beat the market 2 years in a row given the prior beliefs).

1. Likelihood of each type of manager beating the market 2 years in a row:
   - Underperformers: $P(A|C = 0.05) = 0.05^2 = 0.0025$
   - In-line performers: $P(A|C = 0.5) = 0.5^2 = 0.25$
   - Outperformers: $P(A|C = 0.7) = 0.7^2 = 0.49$

2. Unconditional probability of observing the manager beat the market 2 years in a row:

   We compute $P(A)$, which is the total probability of observing the event across all types of managers, weighted by their prior probabilities:

   $$P(A) = P(A|C = 0.05) \cdot P(C = 0.05) + P(A|C = 0.5) \cdot P(C = 0.5) + P(A|C = 0.7) \cdot P(C = 0.7).$$

   Substitute the values:

   $$P(A) = (0.0025 \cdot 0.1) + (0.25 \cdot 0.8) + (0.49 \cdot 0.1) = 0.00025 + 0.2 + 0.049 = 0.24925.$$

3. Updated beliefs about the manager (posterior probabilities)

   Now, we use Bayes' theorem to update the probabilities for each type of manager.

   - Underperformers:
   $$P(C = 0.05|A) = \frac{P(A|C = 0.05) \cdot P(C = 0.05)}{P(A)} = \frac{0.0025 \cdot 0.1}{0.24925} = \frac{0.00025}{0.24925} \approx 0.001003$$
   - In-line performers:
   $$P(C = 0.5|A) = \frac{P(A|C = 0.5) \cdot P(C = 0.5)}{P(A)} = \frac{0.25 \cdot 0.8}{0.24925} = \frac{0.2}{0.24925} \approx 0.8024$$
   - Outperformers:
   $$P(C = 0.7|A) = \frac{P(A|C = 0.7) \cdot P(C = 0.7)}{P(A)} = \frac{0.49 \cdot 0.1}{0.24925} = \frac{0.049}{0.24925} \approx 0.1966$$