1. To estimate an ARIMA model without using `auto.arima`, the relevant functions must be called in an order that allows for data preprocessing, transformation, differencing (if needed), model estimation, and residual analysis. Here's a possible sequence of function calls with a brief explanation:

   - `monthdays`: This function is used if the time series data requires adjustment for monthly lengths, which may be necessary to avoid seasonal bias.
   - `BoxCox`: Apply Box-Cox transformation to stabilize variance if the data exhibits heteroscedasticity.
   - `ndiffs` and `nsdiffs`: These functions determine the required levels of differencing to achieve stationarity in the data. Use `ndiffs` for regular differencing and `nsdiffs` for seasonal differencing.
   - `diff`: Perform the differencing as indicated by `ndiffs` and `nsdiffs` results. You can use this function multiple times, depending on the number of differencing steps required.
   - `tsdisplay`: Display time series diagnostics, including ACF and PACF plots, to help in identifying appropriate AR and MA terms.
   - `arima`: Estimate the ARIMA model using the selected parameters.
   - `autoplot`: Visualize the fitted model against the original data to assess the fit.
   - `checkresiduals`: Evaluate the residuals of the model to confirm that they resemble white noise, indicating a well-fitted model.

   This order ensures that the data is prepared, transformed, differenced, and then modeled correctly, followed by diagnostic checks.

2. a) To propose a possible model based on the first ACF plot, we observe the following characteristics:
      - The ACF plot shows significant spikes at several lags but does not decay gradually, indicating it might be a MA model.
      - Based on the significant spikes at around lag 2 or 3, a reasonable model could be **MA(2/3)**.

   b) For the second ACF plot, we observe:
      - The ACF plot has a strong spike at lag 1 and then decays, which is characteristic of AR.
      - Given the initial significant spike, a model such as **AR(1)** would be appropriate.

   c) For the third ACF plot, the characteristics are as follows:
      - The ACF has a mix of decaying and significant spikes, suggesting the possibility of both AR and MA components.
      - A possible combination could be an **ARMA(1, 1)** or **AR(1)** with **MA(1)** to capture both the autoregressive and moving average characteristics.

3. To determine the number of cointegrating relationships, we examine the Johansen test output provided. The test checks the null hypothesis of the number of such relationships against the alternative hypothesis.

   - In the Johansen test output, we see the `lambda max` test statistics for $r = 0$ and $r \leq 1$.
   - For $r = 0$, the test statistic is 36.63, which is greater than the critical values (14.90, 19.19), suggesting we reject the null hypothesis of no cointegrating relationships.
   - For $r \leq 1$, the test statistic is 2.71, which is less than the critical values (6.50, 8.18, 11.65), so we do not reject the null hypothesis at this level.

   This indicates there is exactly **one** cointegrating relationship.

4. If there are no cointegrating relationships, then a Vector Error Correction Model (VECM) is not appropriate, as it relies on the existence of cointegration. Instead, we should proceed with a Vector Autoregressive (VAR) model, which is suitable for non-cointegrated, stationary time series. To build a VAR model, we would follow these steps:

   1. Render each variable stationary by differencing if necessary (check stationarity using ADF/KPSS).
   2. Determine the appropriate lag order for the VAR model using criteria such as AIC, BIC, or HQIC.
   3. Estimate the VAR model using the selected lag order.
   4. Evaluate residuals using diagnostics like the Ljung-Box test to ensure no autocorrelation remains.
   5. Check for model stability by verifying that all eigenvalues lie within the unit circle.

   Hence, the answer is to build a VAR model after ensuring all variables are stationary, and then use information criteria to select the lag order, estimate the model, and perform residual diagnostics to confirm the model's adequacy.

5. a) If there is exactly 1 cointegrated relationship, then we can build a VECM, as the existence of a single cointegrating relationship allows for such a model. The steps are:
      1. Identify the cointegrating vector and normalize it if needed.
      2. Set up the VECM with the appropriate lag order, which can be determined using information criteria like BIC or AIC.
      3. Estimate the VECM parameters.
      4. Check model diagnostics, such as residual autocorrelation and stability.

   b) If we cannot reject the hypothesis that there is more than 1 cointegrating relationship, then this suggests that there might be multiple cointegrating relationships, which still allows for a VECM, but we may need to account for these multiple relationships in the model setup. We should thus:
      1. Use Johansen's test results to determine the rank of cointegration.
      2. Set up a VECM considering the multiple cointegration ranks.
      3. Proceed with parameter estimation and diagnostics as usual.

6. a) The time series exhibits a clear upward trend in both its mean and variance, which indicates non-stationarity. Covariance stationarity requires that the mean and variance remain constant over time, which is not the case here.

   b) The time series has a non-constant mean and variance. It appears to wander without reverting to a mean, suggesting a random walk behavior, which is typically non-stationary.

   c) The time series shows some level of mean fluctuation over time without any evident mean reversion, and the variance also appears unstable. This behavior does not satisfy the requirements for covariance stationarity.

   d) The time series appears to have a constant mean and variance, with values fluctuating around a fixed mean level without noticeable trends or shifts in variance. This pattern suggests stationarity.

   e) The time series shows a mostly constant mean and variance, with one large spike (an outlier). Apart from this spike, the series maintains stability in both its mean and variance, which aligns with covariance stationarity.