1. know how to hedge

2. $\mathbb{Q}$: all assets earn risk-free rate

$\Downarrow$

$\mathbb{Q}^*$ $dS_t = \mu S_t dt + \sigma S_t dW_t$

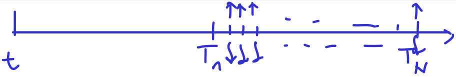
3. $\mathbb{Q}^*$: $\dfrac{V_0}{B_0} = \mathbb{E}\left[\dfrac{V_T}{B_T}\right]$

$d\left(\dfrac{S_t}{B_t}\right) = \underline{0}\, dt + \_\, dW_t^*$

$dS_t = r S_t dt + \sigma S_t dW_t^*$

$$\frac{S_0}{B_0} = \mathbb{E}^*\left[\frac{S_T}{B_T}\right]$$

Swap Market Model



Let us denote the **par swap rate** for the $[T_n, T_N]$ swap as $S_{n,N}$:

$$S(0) = \frac{1 - D(0,T)}{\sum_i \Delta_i D_i(0)}$$

$$S_{n,N}(t) = \frac{D_n(t) - D_N(t)}{\sum_{i=n+1}^{N} \Delta_{i-1} D_i(t)}. = \frac{D_n(t) - D_N(t)}{P_{n+1,N}(t)}$$

The term in the denominator is also called the **present value of a basis point** (PVBP)

PV01

$$P_{n+1,N}(t) = \sum_{i=n+1}^{N} \Delta_{i-1} D_i(t).$$

Note that a one-period swap rate $S_{i,i+1}$ is equal to the LIBOR rate. We can now write the value of a payer and receiver swap as

$$\text{Payer Swap} = P_{n+1,N}(t)(S_{n,N}(t) - K)$$
$$\text{Receiver Swap} = P_{n+1,N}(t)(K - S_{n,N}(t))$$

$$\text{Payer Swap} = PV_{flt} - PV_{fix}$$

$$= \left[D_n(t) - D_N(t) \right] - K \sum_{i=n+1}^{N} \Delta_{i-1} \cdot D_i(t)$$

$$= \left[D_n(t) - D_N(t) \right] - K \, P_{n+1,N}(t)$$

$$= P_{n+1,N}(t) \left[\frac{D_n(t) - D_N(t)}{P_{n+1,N}(t)} - K \right]$$

Pricing a Swaption

The PVBP is a portfolio of traded assets and has strictly positive value. It can therefore be used as a numeraire.

If we use $P_{n+1,N}(t)$ as a numeraire, then under the measure $\mathbb{Q}^{n+1,N}$ associated to the numeraire $P_{n+1,N}(t)$, all $P_{n+1,N}$ rebased values must be martingales in an arbitrage-free world.

In particular, the par swap rate $S_{n,N}$ must be a martingale under $\mathbb{Q}^{n+1,N}$. The swap market model makes the assumption that $S_{n,N}$ is a lognormal martingale under $\mathbb{Q}^{n+1,N}$. We write down the process

$dF_t = \sigma F_t dW_t^*$

$$dS_{n,N}(t) = \sigma_{n,N} S_{n,N}(t) dW^{n+1,N}(t),$$

where $W^{n+1,N}(t)$ is a Brownian motion under $\mathbb{Q}^{n+1,N}$.

A **swaption** (short for swap option) gives the right to enter at time T_n into a swap with fixed rate K. A **receiver swaption** gives the right to enter into a receiver swap, and a **payer swaption** gives the right to enter into a payer swap.

Pricing a Swaption

expiry × tenor 10×10

Swaptions are often denoted as $T_n \times (T_N - T_n)$, where T_n is the option expiry date (and also the start of the underlying swap), and $T_N - T_n$ is the tenor of the underlying swap.

The payoff of a payer swaption is given by

$$[P_{n+1,N}(T)(S_{n,N}(T) - K)]^+ .$$

$$P_{n+1,N}(T)\left[S_{n,N}(T)-K\right]^+$$

Using $P_{n+1,N}$ as a numeraire, we can value the payer swaption under the measure $\mathbb{Q}^{n+1,N}$

$$\frac{V^{\text{payer}}_{n,N}(0)}{P_{n+1,N}(0)} = \mathbb{E}^{n+1,N}\left[\frac{V^{\text{payer}}_{n,N}(T_n)}{P_{n+1,N}(T_n)}\right] = \mathbb{E}^{n+1,N}\left[\frac{\cancel{P_{n+1,N}(T)}\left(S_{n,N}(T)-K\right)^+}{\cancel{P_{n+1,N}(T)}}\right]$$

$$\Rightarrow \quad V^{\text{payer}}_{n,N}(0) = P_{n+1,N}(0)\mathbb{E}^{n+1,N}[(S_{n,N}(T) - K)^+].$$

The remaining steps required to derive a formula for a swaption is identical to how we would handle a vanilla European option.

Pricing a Swaption

The swap rate follows the stochastic differential equation

$$dS_{n,N}(t) = \sigma_{n,N} S_{n,N}(t) dW^{n+1,N}(t),$$

where $W^{n+1,N}(t)$ is a Brownian motion under $\mathbb{Q}^{n+1,N}$. The solution is given by

$F_T = F_0 e^{-\frac{1}{2}\sigma^2 T + \sigma W_T^*}$

$$S_{n,N}(T) = S_{n,N}(0) e^{-\frac{1}{2}\sigma^2_{n,N} T + \sigma_{n,N} W^{n+1,N}(T)}.$$

Evaluating the expectation, we obtain

$$\begin{aligned} V^{payer}_{n,N}(0) &= P_{n+1,N}(0) \mathbb{E}^{n+1,N}[(S_{n,N}(T) - K)^+] \\ &= P_{n+1,N}(0)[S_{n,N}(0)\Phi(d_1) - K\Phi(d_2)], \end{aligned}$$

where

$$d_1 = \frac{\log \frac{S_{n,N}(0)}{K} + \frac{1}{2}\sigma^2_{n,N} T}{\sigma_{n,N}\sqrt{T}}, \quad d_2 = d_1 - \sigma_{n,N}\sqrt{T}. \quad \triangleleft$$

Swaption Vols – ATM Vols

$$\text{Black}\big(\overset{F}{S_{9,10}(0)},\ \overset{K}{S_{9,10}(0)},\ \overset{\sigma}{28.72\%},\ \overset{T}{10}\big)$$

tenor

expiry

91) Asset | 92) Actions | 93) Settings | Interest Rate Volatility Cube

USD | USD BVOL Cube (Default) | Bid | Date 10/29/15

1) Analyze Cube | 2) Market Data

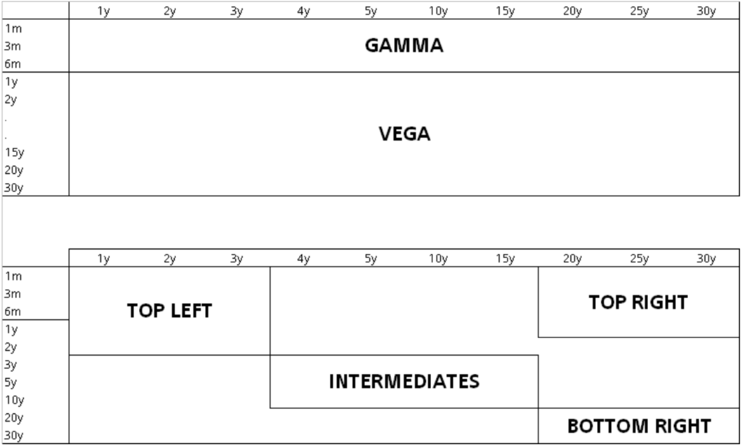
Swap Curve (23) US Dollar | Index Tenor 3M | Show Vol Black

View Strike ATM | Discounting IBOR | Show Strikes

Table | Charts

Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	6Yr	7Yr	8Yr	9Yr	10Yr	12Yr	15Yr
1Mo	66.10	62.21	55.17	49.96	46.57	42.67	40.01	37.95	36.32	34.93	33.42	31.65
3Mo	70.23	63.36	57.30	52.53	48.96	44.96	42.20	40.23	38.56	37.12	35.62	33.86
6Mo	64.49	59.82	55.44	50.78	47.56	44.22	41.86	40.16	38.75	37.52	36.11	34.49
9Mo	61.24	56.74	52.60	48.62	45.77	43.00	40.94	39.50	38.25	37.09	35.75	34.21
1Yr	58.49	54.16	50.10	46.94	44.14	41.76	39.93	38.66	37.61	36.68	35.47	34.08
2Yr	52.60	48.17	44.87	42.30	40.20	38.71	37.53	36.65	35.84	35.19	34.21	32.99
3Yr	47.94	44.17	41.45	39.51	37.94	36.84	35.88	35.12	34.48	33.94	33.11	32.00
4Yr	43.43	40.52	38.55	37.09	35.81	34.97	34.30	33.72	33.27	32.92	32.17	31.12
5Yr	39.96	37.89	36.69	35.71	34.78	34.01	33.41	32.99	32.61	32.31	31.59	30.56
6Yr	37.44	36.00	35.03	34.21	33.40	32.80	32.32	31.97	31.66	31.40	30.73	29.77
7Yr	35.22	34.29	33.51	32.84	32.18	31.75	31.38	31.06	30.80	30.57	29.95	29.04
8Yr	33.81	32.87	32.23	31.74	31.25	30.91	30.60	30.33	30.09	29.90	29.32	28.48
9Yr	32.43	31.53	31.13	30.79	30.45	30.18	29.89	29.67	29.46	29.29	28.77	27.95
10Yr	31.21	30.41	30.19	29.98	29.75	29.52	29.26	29.07	28.88	28.72	28.24	27.45
12Yr	30.02	29.26	28.90	28.89	28.88	28.62	28.37	28.20	28.16	28.12	27.62	26.86
15Yr	28.25	27.56	27.43	27.33	27.36	27.26	27.19	27.20	27.23	27.23	26.76	25.99

Swaption ATM Vols



Swaption ATM Vols

Term	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y
1) 1M	12.70	13.60	15.90	18.40	20.60	22.60	24.80	26.50	28.00
2) 2M	13.30	14.20	16.70	19.40	21.90	24.20	26.40	28.30	29.90
3) 3M	14.10	15.00	17.20	20.00	22.50	25.10	27.60	29.50	31.10
4) 6M	14.60	16.10	18.90	22.20	24.70	27.20	29.40	31.60	33.50
5) 9M	15.80	17.70	20.50	23.60	[illegible]	[illegible]	30.90	32.80	34.70
6) 1Y	16.80	19.00	22.10	24.90	27.50	30.00	32.20	34.10	35.80
7) 18Y	18.80	21.90	24.90	27.40	29.90	32.40	34.40	36.30	38.00
8) 2Y	21.70	24.50	27.80	30.40	32.40	34.40	36.60	38.20	39.90
9) 3Y	28.00	30.50	33.10	35.00	36.80	38.60	40.30	41.80	43.10
10) 4Y	33.40	35.50	37.50	39.10	40.40	41.60	43.00	44.20	45.50
11) 5Y	37.80	39.70	41.10	42.20	43.30	44.40	45.40	46.40	47.40
12) 6Y	41.90	43.00	43.90	45.20	45.70	46.70	47.50	48.40	49.10
13) 7Y	44.70	45.60	46.30	47.10	47.70	48.40	49.00	49.60	50.20
14) 10Y	49.70	49.80	50.30	50.80	51.10	51.40	51.70	51.80	52.10
15) 12Y	51.30	50.90	51.00	51.40	51.60	52.00	52.30	52.50	52.50
16) 15Y	51.70	51.50	51.90	52.00	52.00	52.10	52.20	52.10	52.40
17) 20Y	51.20	51.10	51.40	51.30	51.30	51.50	51.30	51.20	51.20
18) 25Y	50.30	50.30	50.50	50.40	50.30	50.10	49.90	49.30	49.30
19) 30Y	49.30	49.40	49.60	49.60	49.70	49.30	48.60	48.00	47.40

Swaption Vols – Smile/Skew

$K = S_{1,1}(0)$

Global Swaption Skews
Tullett Prebon
Last Update 08:

SMKR412 (c) 2020 Tullett Prebon Information 16-Dec-2020 08:47 LDN

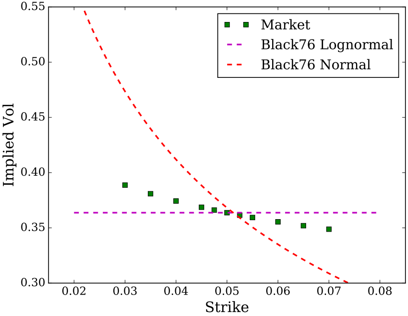
EUR Swaption Volatility Smile based on Spot Premium and IBOR curve

(Normal Volatility)

OPTION/ TENOR	-200	-100	-50	-25	ATM	25	50	100	200	ATM STRIKE
1Y1Y	51.9	36.2	24.4	18.5	16.8	22.4	29.1	42.1	65.6	-0.57
3M2Y	74.2	48.9	31.4	21.3	15.0	25.3	36.2	56.1	91.8	-0.54
2Y2Y	46.5	34.5	26.7	24.1	24.4	27.9	32.5	42.4	61.5	-0.47
1Y5Y	57.5	42.2	32.0	27.4	26.9	30.8	36.6	48.7	71.7	-0.43
5Y5Y	46.0	42.4	41.3	41.7	42.4	43.4	44.7	48.0	56.0	-0.08
3M10Y	88.3	61.5	43.7	35.3	32.2	39.6	50.1	70.9	109.3	-0.26
1Y10Y	66.0	50.7	41.0	37.6	36.8	39.3	43.8	54.8	77.0	-0.21
2Y10Y	58.4	48.7	42.9	41.2	40.8	41.9	44.1	50.4	65.0	-0.13
5Y10Y	52.5	49.2	47.6	47.2	47.4	47.9	48.7	51.0	57.5	0.087
10Y10Y	52.4	51.9	51.7	51.7	52.3	52.9	53.4	54.9	59.1	0.236
15Y15Y	49.9	49.3	49.0	49.1	49.7	50.4	50.8	51.9	55.0	0.010
10Y20Y	51.9	49.9	48.9	48.7	49.3	49.9	50.2	51.3	55.1	0.073
5Y30Y	54.3	50.0	48.5	48.1	48.2	48.5	49.1	50.8	56.6	-0.00
	-200	-100	-50	-25	ATM	25	50	100	200	

Swaption Vol Calibration

Suppose the implied volatility across strike for a given swaption maturity and tenor is given by the green markers in the following figure:



The at-the-money volatility is 0.36, and the forward swap rate is 0.05.

Extension to the Black Model

An immediate and straightforward extension is the Black Normal model:

$\mathbb{Q}^{n+1,N}$:

$$dS_{n,N}(t) = \sigma_{n,N} dW^{n+1,N}(t).$$

This is an arithmetic Brownian motion.

If the implied volatility skew we observed in the market is between normal and lognormal, then we can make use of the displaced-diffusion (shifted lognormal) model:

$$dS_{n,N}(t) = \sigma_{n,N}[\beta S_{n,N}(t) + (1-\beta)S_{n,N}(0)]dW^{n+1,N}(t).$$

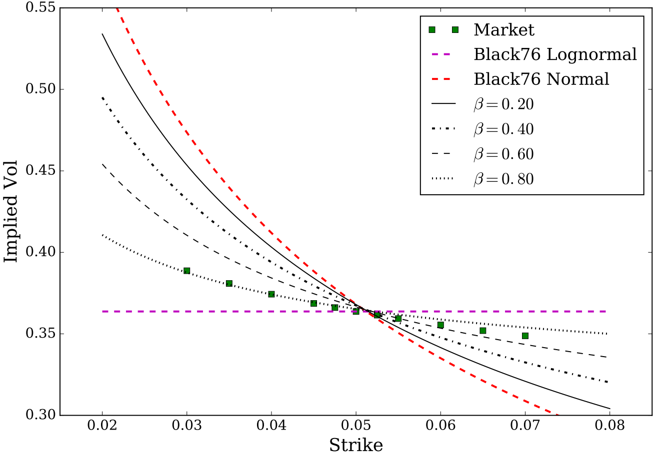
Recall that the solution is given by

$$S_{n,N}(T) = \frac{S_{n,N}(0)}{\beta} e^{\sigma_{n,N}\beta W^{n+1,N}(T) - \frac{\sigma^2_{n,N}\beta^2 T}{2}} - \frac{1-\beta}{\beta} S_{n,N}(0)$$

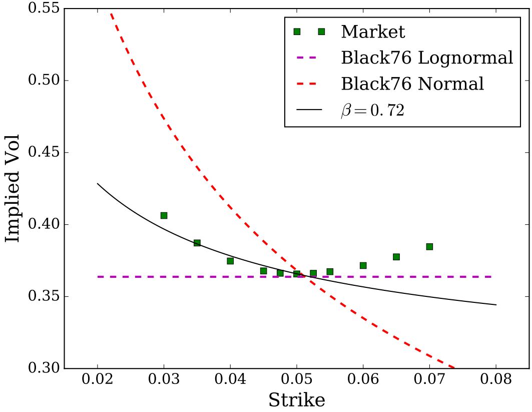
The swaption price under the displaced-diffusion model is

$$V_{n,N}(0) = P_{n+1,N}(0)\text{Black}\left(\frac{S_{n,N}(0)}{\beta},\ K + \frac{1-\beta}{\beta}S_{n,N}(0),\ \sigma\beta,\ T\right)$$

Swaption Vol Calibration – Displaced Diffusion

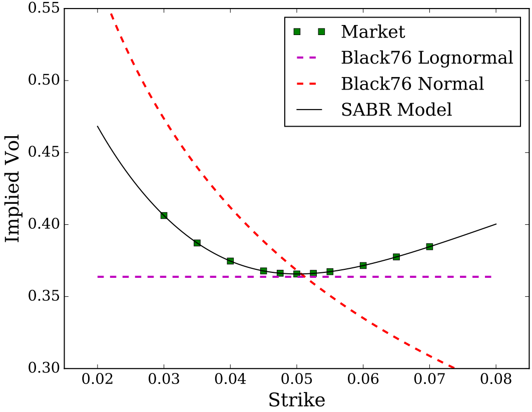


SABR Model



Displaced-diffusion model can only fit to implied volatility skew – there will be mismatch if the implied volatility surface also exhibit "smile" characteristic.

SABR Model



SABR model is able to fit both skew and smile in the implied volatility surface – this is the standard volatility model used in fixed-income market.

Session 5
Constant Maturity Swap Payoffs

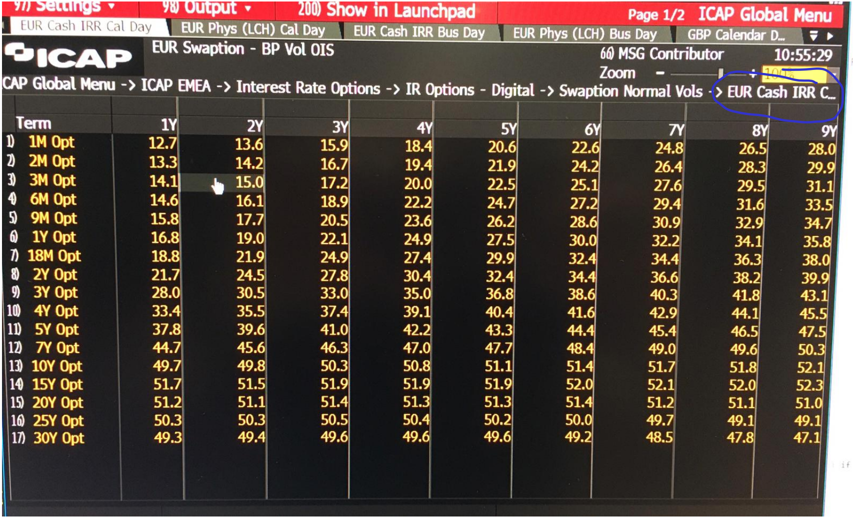
Tee Chyng Wen

QF605 Fixed Income Securities

Swap-Settled Swaptions

Term	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y
1) 1M	12.70	13.60	15.90	18.40	20.60	22.60	24.80	26.50	28.00
2) 2M	13.30	14.20	16.70	19.40	21.90	24.20	26.40	28.30	29.90
3) 3M	14.10	15.00	17.20	20.00	22.50	25.10	27.60	29.50	31.10
4) 6M	14.60	16.10	18.90	22.20	24.70	27.20	29.40	31.60	33.50
5) 9M	15.80	17.70	20.50	23.60	[illegible]	[illegible]	30.90	32.80	34.70
6) 1Y	16.80	19.00	22.10	24.90	27.50	30.00	32.20	34.10	35.80
7) 18Y	18.80	21.90	24.90	27.40	29.90	32.40	34.40	36.30	38.00
8) 2Y	21.70	24.50	27.80	30.40	32.40	34.40	36.60	38.20	39.90
9) 3Y	28.00	30.50	33.10	35.00	36.80	38.60	40.30	41.80	43.10
10) 4Y	33.40	35.50	37.50	39.10	40.40	41.60	43.00	44.20	45.50
11) 5Y	37.80	39.70	41.10	42.20	43.30	44.40	45.40	46.40	47.40
12) 6Y	41.90	43.00	43.90	45.20	45.70	46.70	47.50	48.40	49.10
13) 7Y	44.70	45.60	46.30	47.10	47.70	48.40	49.00	49.60	50.20
14) 10Y	49.70	49.80	50.30	50.80	51.10	51.40	51.70	51.80	52.10
15) 12Y	51.30	50.90	51.00	51.40	51.60	52.00	52.30	52.50	52.50
16) 15Y	51.70	51.50	51.90	52.00	52.00	52.10	52.20	52.10	52.40
17) 20Y	51.20	51.10	51.40	51.30	51.30	51.50	51.30	51.20	51.20
18) 25Y	50.30	50.30	50.50	50.40	50.30	50.10	49.90	49.30	49.30
19) 30Y	49.30	49.40	49.60	49.60	49.70	49.30	48.60	48.00	47.40

IRR-Settled Swaptions



Term	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y
1) 1M Opt	12.7	13.6	15.9	18.4	20.6	22.6	24.8	26.5	28.0
2) 2M Opt	13.3	14.2	16.7	19.4	21.9	24.2	26.4	28.3	29.9
3) 3M Opt	14.1	15.0	17.2	20.0	22.5	25.1	27.6	29.5	31.1
4) 6M Opt	14.6	16.1	18.9	22.2	24.7	27.2	29.4	31.6	33.5
5) 9M Opt	15.8	17.7	20.5	23.6	26.2	28.6	30.9	32.9	34.7
6) 1Y Opt	16.8	19.0	22.1	24.9	27.5	30.0	32.2	34.1	35.8
7) 18M Opt	18.8	21.9	24.9	27.4	29.9	32.4	34.4	36.3	38.0
8) 2Y Opt	21.7	24.5	27.8	30.4	32.4	34.4	36.6	38.2	39.9
9) 3Y Opt	28.0	30.5	33.0	35.0	36.8	38.6	40.3	41.8	43.1
10) 4Y Opt	33.4	35.5	37.4	39.1	40.4	41.6	42.9	44.1	45.5
11) 5Y Opt	37.8	39.6	41.0	42.2	43.3	44.4	45.4	46.5	47.5
12) 7Y Opt	44.7	45.6	46.3	47.0	47.7	48.4	49.0	49.6	50.3
13) 10Y Opt	49.7	49.8	50.3	50.8	51.1	51.4	51.7	51.8	52.1
14) 15Y Opt	51.7	51.5	51.9	51.9	51.9	52.0	52.1	52.0	52.3
15) 20Y Opt	51.2	51.1	51.4	51.3	51.3	51.4	51.2	51.1	51.0
16) 25Y Opt	50.3	50.3	50.5	50.4	50.2	50.0	49.7	49.1	49.1
17) 30Y Opt	49.3	49.4	49.6	49.6	49.6	49.2	48.5	47.8	47.1

Swap-Settled Swaptions

The swaptions we have covered so far in our Market Model discussion are **swap-settled swaptions** — when you exercise, you enter into a swap contract with your counterparty.

The payoff of the swaptions are

$$\text{Payer Swaption} = \Big[P_{n+1,N}(T)(S_{n,N}(T) - K)\Big]^+$$
$$\text{Receiver Swaption} = \Big[P_{n+1,N}(T)(K - S_{n,N}(T))\Big]^+$$

where

$$P_{n+1,N}(T) = \sum_{i=n+1}^{N} \Delta_{i-1} D_i(T).$$

Upon exercising, we get

$$\text{Payer Swaption} = V^{flt}(T) - V^{fix}(T)$$
$$\text{Receiver Swaption} = V^{fix}(T) - V^{flt}(T)$$

IRR-Settled Swaptions

An **Internal-Rate-of-Return (IRR)-settled swaption** has the following payoff:

$$\text{Payer Swaption} = \Big[\text{IRR}(S_{n,N}(T))(S_{n,N}(T) - K)\Big]^+$$

$$\text{Receiver Swaption} = \Big[\text{IRR}(S_{n,N}(T))(K - S_{n,N}(T))\Big]^+$$

where

$$\text{IRR}(S) = \sum_{i=1}^{(T_N - T_n)\times m} \frac{\frac{1}{m}}{\left(1 + \frac{S}{m}\right)^i}$$

and $\frac{1}{m} = \Delta$ is the day count fraction corresponding to the payment frequency (m) of the swap.

IRR-settled swaptions are <u>settled in cash</u> based on the value of the payoff observed on the maturity date.

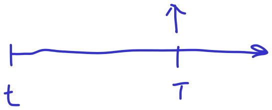
Swap-settled swaptions are common in the USD market, while IRR-settled swaptions are common in the European (EUR & GBP) markets.

$\mathbb{Q}^{n+1,N}$: $dS_{n,N}(t) = \sigma_{n,N} S_{n,N}(t)\, dW^{n+1,N}(t)$

$$\frac{V^{pay}_{swap}(t)}{P_{n+1,N}(t)} = \mathbb{E}^{n+1,N}\left[\frac{V^{pay}_{swap}(T)}{P_{n+1,N}(T)}\right] = \mathbb{E}^{n+1,N}\left[\frac{\cancel{P_{n+1,N}(T)}\left(S_{n,N}(T)-K\right)^+}{\cancel{P_{n+1,N}(T)}}\right]$$

$$\frac{V^{pay}_{IRR}(t)}{P_{n+1,N}(t)} = \mathbb{E}^{n+1,N}\left[\frac{V^{pay}_{IRR}(T)}{P_{n+1,N}(T)}\right] = \mathbb{E}^{n+1,N}\left[\frac{IRR\left(S_{n,N}(T)\right)\left(S_{n,N}(T)-K\right)^+}{P_{n+1,N}(T)}\right]$$

$\mathbb{Q}^T$:



$$\frac{V^{pay}_{IRR}(t)}{D(t,T)} = \mathbb{E}^T\left[\frac{V^{pay}_{IRR}(T)}{\cancelto{1}{D(T,T)}}\right]$$

$$= \mathbb{E}^T\left[IRR\big(S_{n,N}(T)\big)\cdot\big(S_{n,N}(T)-K\big)^+\right]$$

$$\simeq IRR\big(S_{n,N}(0)\big)\,\mathbb{E}^T\left[\big(S_{n,N}(T)-K\big)^+\right]$$

$$\simeq IRR\big(S_{n,N}(0)\big)\,\mathbb{E}^{n+1,N}\left[\big(S_{n,N}(T)-K\big)^+\right]$$

IRR-Settled Swaptions

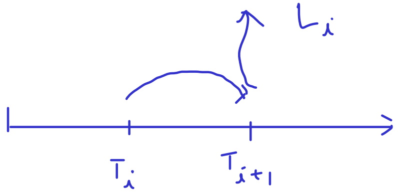
The Market Model used to value IRR-settled swaptions is:

$$V_{n,N}(0) \approx D(0,T) \cdot \mathsf{IRR}(S_{n,N}(0)) \cdot \mathsf{Black}(S_{n,N}(0), K, \sigma_{n,N}, T)$$

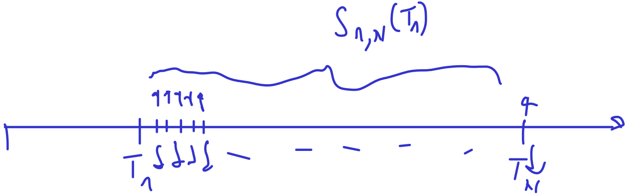
Historical Note:

- In the USD market, participants agree on the value of the PV01 $P_{n+1,N}$, i.e. there is no dispute on the discount factors.
- In the earlier days, market participants disagree on the PV01 value in the Euro and Sterling market.
- To avoid ambiguity, market participants agree to use the IRR formula to discount cashflows in the EUR and GBP market.
- The rational was that since $D(0,T) = \frac{1}{(1+r)^T}$, a good approximation would be to use the observed swap rate $S_{n,N}(T)$ for discounting.

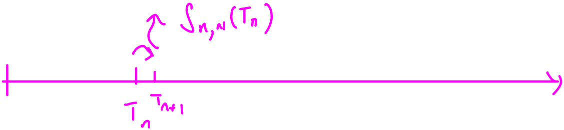
Libor



swap



CMS



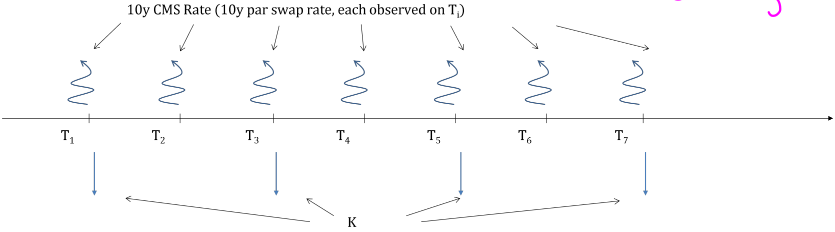
Constant Maturity Swap

A **constant maturity swap** (CMS) pays a swap rate rather than a LIBOR rate on its floating leg.

⇒ Can be either quoted **in arrears** or **in advance**.

⇒ The payment can be **capped** or **floored**.

CMS leg



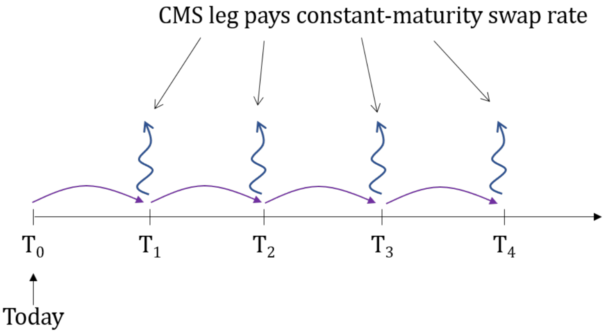
CMS is an instrument having cashflows "paid at the wrong timing":

⇒ A $10y$ CMS rate to be paid one year later is not exactly equal to the forward swap rate $S_{1y,10y}$.

⇒ **Convexity correction** is required to obtain the right price.

CMS Leg

A CMS leg pays the constant-maturity swap rate periodically over time:

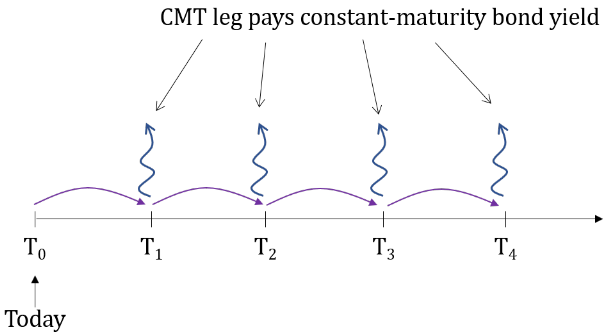


The CMS rate you receive at time T_{i+1} the par swap rate in the market at T_i.

FYI

CMT Leg

A closely related product is CMT, which pays the constant-maturity bond yield periodically over time:



The CMT bond yield you receive at time T_{i+1} the bond yield in the market at T_i.

CMS (or CMT) products give you an easy way to gain exposure to fixed-length longer-term interest rates.

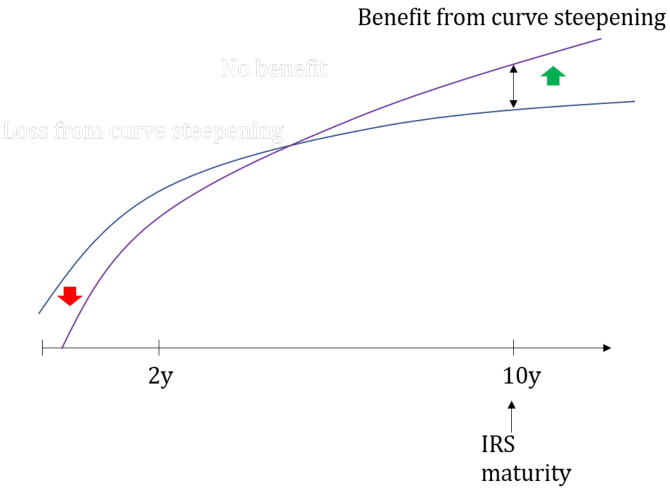
$\Rightarrow$ You can use it to express a view on a fixed point on the yield curve.

In contrast, if you use an IRS, then your exposure will progressively become shorter-term over time.

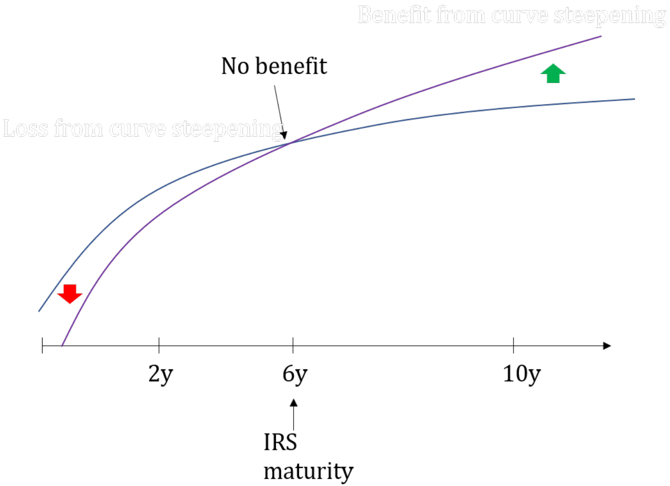
For example, suppose you think that the yield curve will steepen, so that $10y$ swap rate will increase, while $2y$ swap rate will decrease.

To this end, you long a $10y$ payer IRS. If the yield curve steepens, you benefit. If the yield curve flattens, you lose.

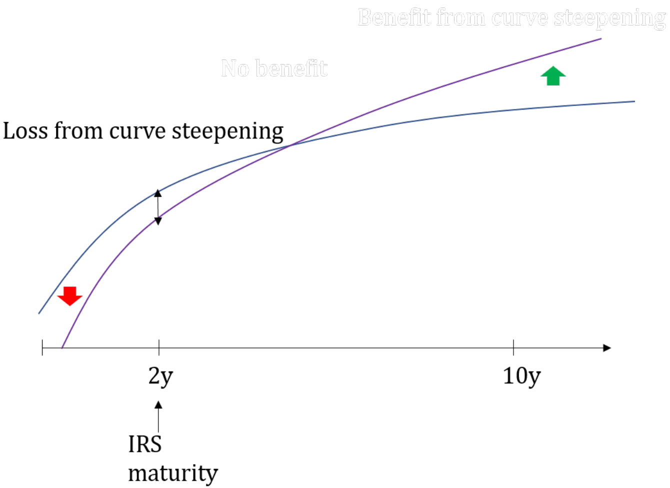
Trade day (initial)



4 years later



8 years later



Insurance companies or pension funds have long dated obligations — generally speaking, the exposure does not age time.

Year	Exposure
2y	-
5y	-
10y	-
15y	-
20y	-
30y	-
40y	-
50y	-

If they use IRS to hedge their exposure, the IRS sensitivity will progressively become shorter-term over time.

For hedge funds and other institutional clients, they use CMS products to speculate on the movement of the yield curve.

- Receive long-maturity CMS rate if they think yield will steepen
  $\Rightarrow$ Spread trade: Receive 10y pay 2y CMS

- Pay long-maturity CMS rate if they think yield will flatten
  $\Rightarrow$ Spread trade: Pay 10y receive 2y CMS

- CMS spread options