Discrete cmpd: return $= \left(1+\frac{R}{m}\right)^{mn}$ (R: rate, m: no. of paym/yr, n: no. of yrs); cts cmpd: return $= e^{RT}$
eff annual rate: $1+r_{\text{EAR}} = \left(1+\frac{R}{m}\right)^m$; bond equiv rate: $1+\frac{r_{\text{BEY}}}{2} = \left(1+\frac{R}{m}\right)^{\frac{m}{2}}$
Bond B (fn of yield y, cash flow c_i at time t_i): mdf $D = -\frac{1}{B}\frac{\partial B}{\partial y} = \frac{1}{B}D_\$$; Macaulay $D_{\text{Mac}} = \frac{1}{B}\sum_{i=1}^{n} t_i c_i \text{Disc}(t_i, y)$
Convexity: $C = \frac{1}{B}\frac{\partial^2 B}{\partial y^2} = \frac{1}{B}C_\$$; $\Delta B = B(y+\Delta y) - B(y)$; $\frac{\Delta B}{B} \approx -D\Delta y + \frac{1}{2}C(\Delta y)^2$; DV01 $= \frac{D_\$}{10^4}$
PVBP: $P_{1,n} = \sum_{i=1}^{n} \Delta_{i-1} D(0,T_i)$; par IRS $[T_0, T_n]$: $S = \frac{D(0,T_0)-D(0,T_n)}{P_{1,n}}$; fix rate payment/quote at time T_n: K
Flt rate quote at time T_i: $L(0,T_i)$; DF: $D(T_{i-1},T_i) = \frac{D(0,T_i)}{D(0,T_{i-1})}$, $D(0,0)=1$; LI: $D(0,T) = \frac{D(0,T-\Delta)+D(0,T+\Delta)}{2}$
Day count convt: $\frac{30}{360}$; forward LIBOR (uc) rate: $D(T_0,T_{i-1}) = (1+\Delta_{i-1}L(T_{i-1},T_i))D(T_0,T_i)$ (simp cpmd)
LIBOR market: $dL_i(t) = \sigma_i L_i(t)\, dW_t^{i+1}$, $L_i(t) = \frac{1}{\Delta_i}\frac{D_i(t)-D_{i+1}(t)}{D_{i+1}(t)}$; OIS (c): $D_o(0,T_i) = \left(1+\frac{f_o(0,T_i)}{360}\right)^{-360T_i}$
$PV_{fix} = KP_{1,n} = \sum_{i=1}^{n} \Delta_{i-1}D(0,T_i)L(T_{i-1},T_i) = 1 - D(0,T_n) = PV_{flt}$; $V_{rec} = PV_{fix} - PV_{flt}$ (-ve for V_{pay})

Futures contract: settlement price $= 100 - \frac{\sum_{i=1}^{n} f_i}{n}$ (contract price; n: no. of calendar days in that month)

Futures price $=$ Settlement price $-\frac{\sum_{i=k+1}^{n}\mathbb{E}[f_i]}{n}$ (k: no. of days to date); $FX_T = FX_t e^{(r^D - r^F)(T-t)}$
1-unit FX cur x = FX-unit domestic cur y: $FX_T = FX_t\frac{D_f(t,T)}{D_d(t,T)}$, $D_{x,y}(t,T) = D_y(t,T)\frac{FX_{x,y}(t,T)}{FX_{x,y}(t,t)}$
Binomial tree: $u = \frac{1}{d}$; non-Rn: $\mathbb{P}[u] = p$, $\mathbb{P}[d] = q = 1-p$; Rn: $\mathbb{Q}[u] = \mathbb{Q}[d] = \frac{1}{2}$; Radon-Nkd: $\frac{d\mathbb{Q}}{d\mathbb{P}}(\omega) = \frac{\mathbb{Q}(\omega)}{\mathbb{P}(\omega)}$
$\mathbb{E}^{\mathbb{P}}[S_n] = \mathbb{E}^{\mathbb{Q}}[S_n\frac{d\mathbb{P}}{d\mathbb{Q}}]$; $V_n = (S_n - K)^+$; call: $S_n > K$; put: $S_n < K$; $p^* = \frac{1+r-d}{u-d}$; mtg X_t: $\mathbb{E}^s[X_t] = X_s$ $(s<t)$
Rn expec: $V_n^E = \frac{1}{1+r}\mathbb{E}_n^*[V_{n+1}] = \frac{1}{1+r}[p^*V_{n+1}^u + q^*V_{n+1}^d]$; put $V_n^A = \max\left\{\frac{1}{1+r}[p^*V_{n+1}^u + q^*V_{n+1}^d], (K-S_n)^+\right\}$
MGF of $X \sim N(\mu,\sigma^2)$: $\mathbb{E}[e^{tX}] = e^{\mu t + \frac{\sigma^2}{2}t^2}$; Itô: $df = f_t\, dt + f_{X_t}\, dX_t + \frac{1}{2}f_{X_tX_t}\,(dX_t)^2 + f_{tX_t}dt\, dX_t$, $(dW_t)^2 = dt$
$\mathbb{E}^{i+1}[L_i(T)(L_i(T)-K)^+] = L_i(0)^2 e^{\sigma_i^2 T}\Phi(-x^* + 2\sigma_i\sqrt{T}) - L_i(0)K\Phi(-x^* + \sigma_i\sqrt{T})$, $-x^* = d_2$
Black-Scholes: $dS_t = rS_t\, dt + \sigma S_t\, dW_t \Rightarrow S_t = S_0 e^{\left(r-\frac{\sigma^2}{2}\right)t + \sigma W_t}$; Bachelier: $dS_t = \sigma\, dW_t$
Vasicek: $dr_t = \kappa(\theta - r_t)\, dt + \sigma\, dW_t$ $[X_t = e^{\kappa t}r_t] \Rightarrow r_t = r_0 e^{-\kappa t} + \theta(1-e^{-\kappa t}) + \sigma\int_0^t e^{\kappa(u-t)}\, dW_u$ (θ: long-run mean; if $r_t < \theta$, the drift term is positive, and dr_t is expected to drift upward, and vice versa. κ is a positive multiplier and controls the mean reversion speed)
Cash-or-nothing digital call: $V_0 = e^{-rT}\mathbb{E}[1_{S_T>K}]$; asset-or-nothing digital call: $V_0 = e^{-rT}\mathbb{E}[S_T 1_{S_T>K}]$
Ito's isometry: $\mathbb{E}\left[\left(\int_0^T X_t\, dW_t\right)^2\right] = \mathbb{E}\left[\int_0^T X_t^2\, dt\right]$; Black76: $dF_t = \sigma F_t\, dW_t$, $F_t = S_t e^{r(T-t)}$ (expansion pt)

Displaced-diffusion: $dF_t = \sigma[\beta F_t + (1-\beta)F_0]\, dW_t$, $\beta \in [0,1] \Rightarrow F_t = \frac{F_0}{\beta}e^{-\frac{(\beta\sigma)^2}{2}t + \beta\sigma W_t} - \frac{1-\beta}{\beta}F_0$

Model	**Option type**		
	Vanilla call/put	Digital cash-or-nothing call/put	Digital asset-or-nothing call/put
Black-Scholes $d_1 = \frac{\log\frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ $d_2 = d_1 - \sigma\sqrt{T}$	Call: $C_{BS,v} = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2)$ Put: $P_{BS,v} = Ke^{-rT}\Phi(-d_2) - S_0\Phi(-d_1)$	Call: $C_{BS,c} = e^{-rT}\Phi(d_2)$ Put: $P_{BS,c} = e^{-rT}\Phi(-d_2)$	Call: $C_{BS,a} = S_0\Phi(d_1)$ Put: $P_{BS,a} = S_0\Phi(-d_1)$
Bachelier $d = \frac{S_0 - K}{\sigma\sqrt{T}}$	$C_{Ba,v} = e^{-rT}\left[(S_0-K)\Phi(d) + \sigma\sqrt{T}\phi(d)\right]$ $P_{Ba,v} = e^{-rT}\left[(K-S_0)\Phi(-d) + \sigma\sqrt{T}\phi(-d)\right]$	$C_{Ba,c} = e^{-rT}\Phi(d)$ $P_{Ba,c} = e^{-rT}\Phi(-d)$	$C_{Ba,a} = S_0\Phi(d) + \sigma\sqrt{T}\phi(d)$ $P_{Ba,a} = S_0\Phi(-d) + \sigma\sqrt{T}\phi(-d)$
Black $d_1 = \frac{\log\frac{F_0}{K} + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$ $d_2 = d_1 - \sigma\sqrt{T}$	$C_{B,v} = e^{-rT}\left[F_0\Phi(d_1) - K\Phi(d_2)\right]$ $P_{B,v} = e^{-rT}\left[K\Phi(-d_2) - F_0\Phi(-d_1)\right]$	$C_{B,c} = e^{-rT}\Phi(d_2)$ $P_{B,c} = e^{-rT}\Phi(-d_2)$	$C_{B,a} = F_0e^{-rT}\Phi(d_1)$ $P_{B,a} = F_0e^{-rT}\Phi(-d_1)$
Displaced-diffusion $d_1 = \frac{\log\left(\frac{F_0}{F_0+\beta(K-F_0)}\right) + \frac{(\sigma\beta)^2}{2}T}{\sigma\beta\sqrt{T}}$ $d_2 = d_1 - \sigma\beta\sqrt{T}$	$C_{D,v} = C_{B,v}\left(\frac{F_0}{\beta}, K + \frac{1-\beta}{\beta}F_0, r, \beta\sigma, T\right)$ $= e^{-rT}\left[\frac{F_0}{\beta}\Phi(d_1) - \left(K + \frac{1-\beta}{\beta}F_0\right)\Phi(d_2)\right]$ $P_{D,v} = P_{B,v}\left(\frac{F_0}{\beta}, K + \frac{1-\beta}{\beta}F_0, r, \beta\sigma, T\right)$ $= e^{-rT}\left[\left(K + \frac{1-\beta}{\beta}F_0\right)\Phi(-d_2) - \frac{F_0}{\beta}\Phi(-d_1))\right]$	$C_{D,c} = e^{-rT}\Phi(d_2)$ $P_{D,c} = e^{-rT}\Phi(-d_2)$	$C_{D,a} = \frac{F_0}{\beta}\Phi(d_1)$ $P_{D,a} = \frac{F_0}{\beta}\Phi(-d_1)$

Martingale pricing theorem: $\frac{V_0}{B_0} = \mathbb{E}^*\left[\frac{V_T}{B_T}\right]$, $dB_t = rB_t\,dt$; static replication (Breeden-Litzenberger):
$V_0 = e^{-rT}h(F) + \int_0^F h''(K)P(K)\,dK + \int_F^\infty h''(K)C(K)\,dK$ [$h(K)$: payoff function]
Change of numéraire: $dW_t^* = dW_t + \kappa\,dt$; multi-currency case: $\mathbb{E}^{i+1,D}[L_i^F(T)] = \mathbb{E}^{i+1,F}\left[L_i^F(T)\frac{d\mathbb{Q}^{i+1,D}}{d\mathbb{Q}^{i+1,F}}\right]$

Domestic numeraires N^D and XM^F (foreign: $\frac{1}{X}$ times): $d\mathbb{Q}^{XM,D} = \frac{d\mathbb{Q}^{N,D}}{d\mathbb{Q}^{M,F}} = \frac{\frac{N_T^D}{N_0^D}}{\frac{X_T M_T^F}{X_0 M_0^F}} = \frac{\frac{\frac{1}{X_T}N_T^D}{\frac{1}{X_0}N_0^D}}{\frac{M_T^F}{M_0^F}} = d\mathbb{Q}^{\frac{N}{X},F}$

$\frac{d\mathbb{Q}^{i+1,D}}{d\mathbb{Q}^{i+1,F}} = \frac{\frac{D_{i+1}^D(T_{i+1})}{D_{i+1}^D(0)}}{\frac{X_T D_{i+1}^F(T_{i+1})}{X_0 D_{i+1}^F(0)}} = \frac{\frac{D_{i+1}^D(T_{i+1})}{X_T D_{i+1}^F(T_{i+1})}}{\frac{D_{i+1}^D(0)}{X_0 D_{i+1}^F(0)}} = \frac{\frac{1}{F_{T_{i+1}}}}{\frac{1}{F_{T_0}}}$; convexity corr: $\frac{d\mathbb{Q}^i}{d\mathbb{Q}^{i+1}} = \frac{\frac{D_i(T_i)}{D_i(0)}}{\frac{D_{i+1}(T_{i+1})}{D_{i+1}(0)}}$; $\mathbb{E}^i[L_i(T_i)] = \frac{L_i(0)+\Delta_i L_i(0)^2 e^{\sigma^2 T_i}}{1+\Delta_i L_i(0)}$
Cholesky decomposition: correlation of two BMs $A = LL^T$ (A: matrix w/ diag entries being 1; L: LT)
$\mathbb{E}^{i+1,F}\left[e^{\sigma_i W^{i+1}}e^{\sigma_X W^F}\right] = \mathbb{E}^{i+1,F}\left[e^{\sigma_i Z_1}e^{\sigma_X\left(\rho Z_1+\sqrt{1-\rho^2}Z_2\right)}\right] = e^{\frac{\sigma_i^2}{2}T}e^{\frac{\sigma_X^2}{2}T}e^{\sigma_i\sigma_X\rho T}$

Forward FX process: $d\frac{1}{F_t} = \sigma_X\frac{1}{F_t}dW_t^F \Rightarrow \frac{1}{F_{T_{i+1}}} = \frac{1}{F_0}e^{-\frac{\sigma_X^2}{2}T+\sigma_X W^F}$
Short-rate r_t (spot/zero): $D(t,T) = e^{-R(t,T)(T-t)}$; $R(t,T) = \frac{1}{T-t}(-A(t,T)+r_t B(t,T))$ (affine in r_t)
Discount factor $D(0,T) = \mathbb{E}\left[e^{-\int_0^T r_t\,dt}\right]$
Ho-Lee model: $dr_t = \theta(t)\,dt + \sigma\,dW_t^*$; fitting requires $\theta(T) = -\frac{\partial^2}{\partial T^2}\log D(0,T) + \sigma^2 T$
Ho-Lee binomial tree: $D(0,3) = \mathbb{E}^*[D(0,1)D(1,2)D(2,3)] = D(0,1)\mathbb{E}^*[D(1,2)\mathbb{E}_1^*[D(2,3)]]$

$$\begin{tikzcd}[column sep=small]
 & & \begin{array}{c} r{=}7\%{+}\theta_0{+}\theta_1 \\ D(2,3){=}e^{-(7\%+\theta_0+\theta_1)\cdot 1} \end{array} \\
 & \begin{array}{c} r{=}6\%{+}\theta_0 \\ D(1,3){=}\ldots \end{array} \arrow[ur] \arrow[dr] & \\
\begin{array}{c} r{=}5\% \\ D(0,3){=}\ldots \end{array} \arrow[ur] \arrow[dr] & & \begin{array}{c} r{=}5\%{+}\theta_0{+}\theta_1 \\ D(2,3){=}e^{-(5\%+\theta_0+\theta_1)\cdot 1} \end{array} \\
 & \begin{array}{c} r{=}4\%{+}\theta_0 \\ D(1,3){=}\ldots \end{array} \arrow[ur] \arrow[dr] & \\
 & & \begin{array}{c} r{=}3\%{+}\theta_0{+}\theta_1 \\ D(2,3){=}e^{-(3\%+\theta_0+\theta_1)\cdot 1} \end{array}
\end{tikzcd}$$

Hull-White model: $dr_t = \kappa(\theta(t)-r_t)\,dt + \sigma\,dW_t^*$
Rule of thumb: 1. If we think that the 1m spot rate will remain unchanged a month later, we should **short** (o.w. long) the 1×2 FRA if $F(1m,2m) > L_{1m}$ (o.w. $F(1m,2m) < L_{1m}$), i.e., we can borrow (o.w. lend) at L_{1m} to deposit (o.w. borrow) at $F(1m,2m)$ if we were right.
2. If we see that $FX_T = FX_0 = \$1.42$, then enter into the FX forward to lock-in to this forward exchange rate. We long 1 unit of USD bond by shorting some SGD bond to generate a cash amount of \$1.36888 in SGD ($\because 0.964\,[D_{USD}(0,T)]\times\$1.42 = \$1.36888$). When the USD bond matures, we convert the \$1 USD back to SGD to get \$1.42 SGD. The short SGD bond position now becomes: $\$1.36888\times\frac{1}{0.98\,[D_{SGD}(0,T)]} = \$1.3968$. The difference is the arbitrage.