

QF605 Fixed-Income Securities

Assignment 3, Due Date: 12-Mar-2025

1. (a) Write down the LIBOR Market Model (LMM), and identify under what numeraire is the LIBOR process a martingale.

- (b) A contract pays

$$\Delta_i \times \sqrt{L_i(T)}$$

at $T = T_{i+1}$. Derive a valuation formula for this contract using LIBOR market model.

- (c) Consider a contract with the following payoff at time $T = T_{i+1}$:

$$\begin{cases} \$1 & \text{if } K_1 \leq L_i(T) \leq K_2 \\ 0 & \text{otherwise} \end{cases}$$

Derive a valuation formula for this contract using LIBOR market model.

2. Under the Swap Market Model (SMM), the forward swap rate follows the stochastic differential equation

$$dS_{n,N}(t) = \sigma_{n,N} S_{n,N}(t) dW^{n+1,N}.$$

- (a) What is the numeraire security associated with the risk-neutral measure $\mathbb{Q}^{n+1,N}$, under which $W^{n+1,N}$ is a standard Brownian motion?

- (b) A floating-leg-or-nothing digital option pays

$$P_{n+1,N}(T) S_{n,N}(T) \mathbb{1}_{S_{n,N}(T) > K}$$

on maturity T , where $P_{n+1,N}$ is the *present value of a basis point*. Derive a valuation formula for this contract.

- (c) A contract pays

$$S_{n,N}(T)$$

on maturity T . Briefly explain why we cannot value this simple contract directly using the Swap Market Model without applying convexity correction.