Bond Portfolio Immunization

The process of obtaining a portfolio with zero dollar duration and dollar convexity is called **portfolio immunization**, and can be done by taking positions in other bonds available in the market.

Let V be the value of a portfolio with dollar duration $D_{\$}(V)$ and dollar convexity $C_{\$}(V)$. Take positions of sizes B_1 and B_2, respectively, in two bonds with duration and convexity D_i and C_i for $i = 1, 2$.

The value of the new (hedged) portfolio is

$$\Pi = V + B_1 + B_2.$$

Note that $D_{\$}(B_1) = B_1 D_1$, $D_{\$}(B_2) = B_2 D_2$, $C_{\$}(B_1) = B_1 C_1$, $C_{\$}(B_2) = B_2 C_2$.

Choose B_1 and B_2 such that the dollar duration and dollar convexity of the portfolio Π are equal to 0, i.e. such that

$$\begin{cases} B_1 D_1 + B_2 D_2 = -D_{\$}(V) \\ B_1 C_1 + B_2 C_2 = -C_{\$}(V) \end{cases}$$

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Example Suppose we have just invested \$1 million in a bond with duration 3.2 and convexity 16, and \$2.5 million in a bond with duration 4 and convexity 24.

1. What are the dollar duration and the dollar convexity of your portfolio?
2. If the yield curves go up by 10 basis points, estimate the new value of the portfolio.
3. You can buy or sell two other bonds, one with duration 1.6 and convexity 12, and another one with duration 3.2 and convexity 20. What positions would you take in these bonds to immunize your portfolio, i.e. to obtain a portfolio with zero dollar duration and dollar convexity?

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Solution

1. We have

$$D_{\$}(V) = B_1 D_1 + B_2 D_2 = \$13,200,000$$
$$C_{\$}(V) = B_1 C_1 + B_2 C_2 = \$76,000,000$$

2. We have

$$\Delta V \approx -D_{\$}(V)\Delta r + \frac{C_{\$}(V)}{2} \cdot (\Delta r)^2$$
$$\approx -\$13,162$$

Hence $V' \approx V + \Delta V = \$3,500,000 - \$13,162 = \$3,486,838.$

3. Let B_3 and B_4 be the values of the positions taken in the two bonds. We have $\Pi = V + B_3 + B_4$, and

$$D_{\$}(\Pi) = \$13.2mil + D_3 B_3 + D_4 B_4$$
$$= \$13.2mil + 1.6B_3 + 3.2B_4$$
$$C_{\$}(\Pi) = \$76mil + 12B_3 + 20B_4.$$

Solving, we obtain $B_3 = \$3.25mil$ and $B_4 = -\$5.75mil.$

Bonds with Negative Yields

Why did investors buy bonds with negative yields?

1. **Regulatory requirements**

   ⇒ Central banks, pension funds, insurance companies, banks are required to hold government bonds to meet liquidity requirements and also to pledge as collateral when they borrow.

2. **Potential capital gain**

   ⇒ Further interest rate cuts will cause bond prices to appreciate.
   ⇒ If a negative yielding currency might appreciate, investors buy bonds to benefit from currency appreciation.
   ⇒ If deflation is likely, investors buy bonds to profit in "real" terms (buy more goods and services), even though there is nominal loss.

3. **Cash is not optimal**

   ⇒ If the government bond yields are negative, cash interest rates are likely to be negative too, so holding cash is not better.