SINGAPORE MANAGEMENT UNIVERSITY

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QF605 Fixed Income Securities

Project Report

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Part I (Bootstrapping Swap Curves)

Recall that interest rate swap (IRS) is a contract where two parties agree to exchange the floating leg in terms of interest rate payments based on LIBOR rates for the fixed leg. Overnight Index Swaps (OIS) pay a fixed swap rate vs. a floating leg whose rate is compounded according to the daily overnight rate. For this project, we adopt the $\frac{30}{360}$ convention. Here, we use the semi-annual fixed leg frequency and semi-annual floating leg frequency for IRS, alongside daily O/N leg frequency and annual fixed leg frequency for OIS. The datasets from the `IRS` and `OIS` tabs of `IR Data.xlsm`, with tenor measured in years, are shown below.

Tenor	0.5	1.0	2.0	3.0	4.0	5.0	7.0	10.0	15.0	20.0	30.0
Product	LIBOR	IRS	IRS	IRS	IRS	IRS	IRS	IRS	IRS	IRS	IRS
Rate	0.025	0.028	0.03	0.0315	0.0325	0.033	0.035	0.037	0.04	0.045	0.05

Tenor	0.5	1.0	2.0	3.0	4.0	5.0	7.0	10.0	15.0	20.0	30.0
Product	OIS	OIS	OIS	OIS	OIS	OIS	OIS	OIS	OIS	OIS	OIS
Rate	0.0025	0.003	0.00325	0.00335	0.0035	0.0036	0.004	0.0045	0.005	0.00525	0.0055

Let f_0 denote the daily compounded overnight rate for $[0, 6m]$. Using the $6m$ OIS, we can solve for f_0:

$$PV_{fix}^{6m\,\text{OIS}} = PV_{flt}^{6m\,\text{OIS}} \Rightarrow D_o(0,6m) \times \frac{6m}{1y} \times 0.25\% = D_o(0,6m) \times \left[\left(1+\frac{f_0}{360}\right)^{360\times\frac{6m}{1y}} - 1\right]$$

$$\therefore f_0 = 360 \times \left[(1+0.5\times 0.25\%)^{\frac{1}{180}} - 1\right].$$

Similarly, using the $1y$ OIS, we can solve for f_1, the daily compounded overnight rate for $[6m, 1y]$:

$$PV_{fix}^{1y\,\text{OIS}} = PV_{flt}^{1y\,\text{OIS}} \Rightarrow D_o(0,1y) \times 0.3\% = D_o(0,1y) \times \left[\left(1+\frac{f_0}{360}\right)^{180}\left(1+\frac{f_1}{360}\right)^{180} - 1\right]$$

$$\therefore f_1 = 360 \times \left[\frac{(1+0.3\%)^{\frac{1}{180}}}{1+\frac{f_0}{360}} - 1\right].$$

Using the $1y$ and $2y$ OIS, as well as the formula $D_o(t,T) = \frac{D_o(t_0,T)}{D_o(t_0,t)}$ with $t_0 < t < T$, we can solve for f_2, the daily compounded overnight rate for $[1y, 2y]$, using the equation $PV_{fix}^{2y\,\text{OIS}} = PV_{flt}^{2y\,\text{OIS}}$:

$$[D_o(0,1y) + D_o(0,2y)] \times 0.325\% = D_o(0,1y) \times 0.3\% + D_o(0,2y) \times \left[\left(1+\frac{f_2}{360}\right)^{360} - 1\right].$$

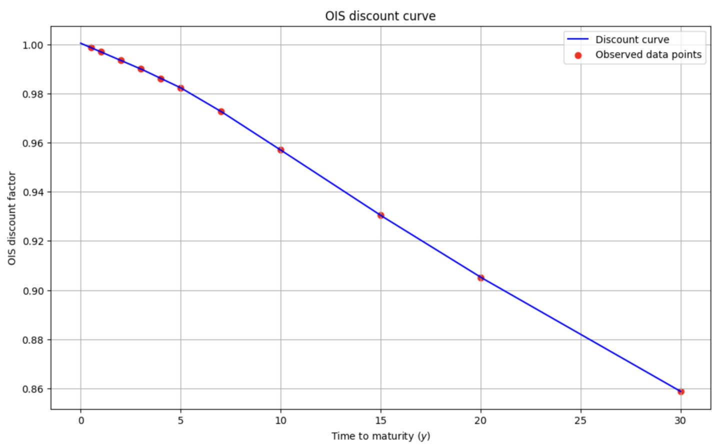
Similarly, we can solve for the approximate values of f_i, where $i \in \{3,4,5,7,10,15,20,30\}$ is the tenor measured in years. In general, we have $f_T \approx 360\times\left[(1+\text{OIS Rate})^{\frac{1}{360}} - 1\right]$ for all $T \geq 2$. We can then compute the OIS discount factors $D_o(0,T_i) = \frac{1}{1+f_iT_i}$, where $T_0 = 0.5$ and $T_i = i$ (for $i \in \{1,2,3,4,5,7,10,20,30\}$). Next, we use the forward LIBOR rate $L(0,6m)$ to obtain the first LIBOR discount factor $D(0,6m) = \frac{1}{1+0.5\times L(0,6m)}$. Assuming that the swap market is collateralized in cash and overnight interest is paid on collateral posted, we solve for $L(6m,1y)$ and hence $D(0,1y)$ using the equation $PV_{fix}^{1y\,\text{IRS}} = PV_{flt}^{1y\,\text{IRS}}$:

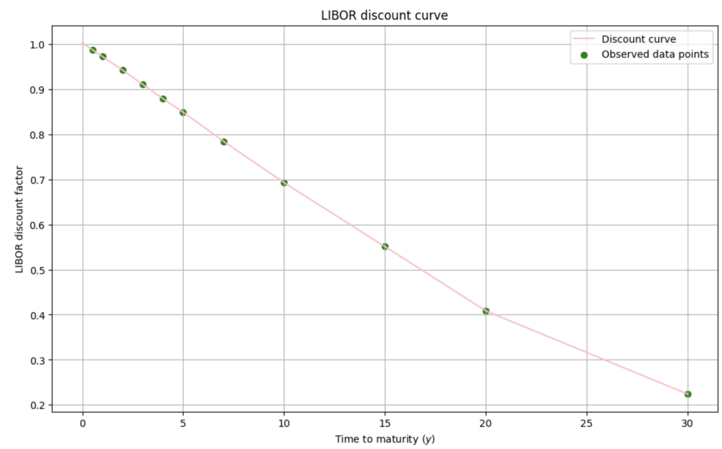
$$0.5 \times 0.025 \times [D_o(0,6m) + D_o(0,1y)] = 0.5 \times [(D_o(0,6m) \times L(0,6m)) + (D_o(0,1y) \times L(6m,1y))].$$

To obtain subsequent forward LIBOR and discount rates, we use the generalized relation:

$$PV_{fix}^{T\,\text{IRS}} = PV_{flt}^{T\,\text{IRS}} \Rightarrow K\sum_{i=1}^{n}\Delta_{i-1}D(0,T_i) = \sum_{i=1}^{n}D(0,T_i)\Delta_{i-1}L(T_{i-1},T_i),$$

which equates the fixed and floating legs at each tenor T. Using linear interpolation based on tenors and discount factors, we obtain the OIS and LIBOR discount curves:





From the plots, we see that the OIS discount curve reflects the market's expectation of future overnight rates. The decreasing trend in the discount factors indicates that the PV of future cash flows decreases as the tenor increases, which is consistent with the time value of money. The OIS curve is typically used for discounting collateralized cash flows in the swap market. Similarly, the LIBOR discount curve shows a decreasing trend. However, the LIBOR curve is generally steeper than the OIS curve, reflecting the credit risk and liquidity premium associated with LIBOR rates. The LIBOR curve is used for discounting uncollateralized cash flows. Using the formula for the par swap rate (see below), we obtain the following forward swap rates:

$$S(T_i, T_{i+m}) = \frac{\sum\limits_{n=i+1}^{i+m} D_o(0,T_n)\times 0.5 \times L(T_{n-0.5},T_n)}{0.5\times \sum\limits_{n=i+1}^{i+m} D_o(0,T_n)},$$

These rates provide insights into the market's expectations of future interest rates. For example, the $1y \times 5y$

	1Y	2Y	3Y	5Y	10Y
1Y	0.032007	0.033259	0.034011	0.035255	0.038427
5Y	0.039273	0.040074	0.040070	0.041091	0.043629
10Y	0.042179	0.043105	0.044086	0.046238	0.053433

forward swap rate reflects the market's expectation of the 1-year swap rate five years from now. The forward swap rates can be used to assess the shape of the yield curve and to identify potential arbitrage opportunities.

- $1y \times 1y, 1y \times 2y, 1y \times 3y, 1y \times 5y, 1y \times 10y$: These rates ($1y$ to maturity/expiry) increase with the tenor, reflecting the upward-sloping yield curve.
- $5y \times 1y, 5y \times 2y, 5y \times 3y, 5y \times 5y, 5y \times 10y$: These rates are generally higher than the $1y$ forward rates, indicating that the market expects higher interest rates in the future.

- $10y \times 1y, 10y \times 2y, 10y \times 3y, 10y \times 5y, 10y \times 10y$: These rates are higher than both the $1y$ and $5y$ forward rates, indicating a steepening of the yield curve over longer tenors.

The bootstrapped OIS and LIBOR discount curves, along with the computed forward swap rates, provide valuable insights into the market's expectations of future interest rates and the shape of the yield curve. These results are essential for pricing and risk management of fixed income securities and interest rate derivatives.

Part II (Swaption Calibration)

We need to calibrate IR swaptions as it ensures that the pricing model used to value them accurately reflects current market conditions, by adjusting the model parameters to match the observed market prices of similar instruments. This would produce more reliable valuations for complex derivative products like swaptions, especially when used for risk management and hedging strategies. For this project, we use the `Swaption` tab of `IR Data.xlsm` comprising lognormal implied volatilities for IR swaptions (100 bps = 1%). Note that the swaption is at-the-money (ATM) if its strike price, K = Forward price + Basis points (bps), is exactly the same as the current market price of the underlying asset, and the swaption has a zero basis point (meaning no additional premium or cost beyond the intrinsic value) associated with it.

		Strike (Forward + basis point)										
Expiry	**Tenor**	***-200bps***	***-150bps***	***-100bps***	***-50bps***	***-25bps***	***ATM***	***+25bps***	***+50bps***	***+100bps***	***+150bps***	***+200bps***
1Y	1Y	91.57	62.03	44.13	31.224	26.182	22.5	20.96	21.4	24.34	27.488	30.297
1Y	2Y	83.27	61.24	46.57	35.807	31.712	28.72	27.12	26.84	28.51	31.025	33.523
1Y	3Y	73.92	56.87	44.77	35.745	32.317	29.78	28.29	27.8	28.77	30.725	32.833
1Y	5Y	55.19	44.64	36.51	30.242	27.851	26.07	24.98	24.56	25.12	26.536	28.165
1Y	10Y	41.18	35.04	30.207	26.619	25.351	24.47	23.98	23.82	24.25	25.204	26.355
5Y	1Y	67.8	49.09	38.4	31.485	29.06	27.26	26.04	25.32	24.94	25.32	25.98
5Y	2Y	57.88	46.41	39.033	33.653	31.531	29.83	28.56	27.65	26.71	26.54	26.76
5Y	3Y	53.43	44.44	38.18	33.437	31.536	29.98	28.76	27.82	26.67	26.2	26.15
5Y	5Y	41.99	36.524	32.326	29.005	27.677	26.6	25.73	25.02	24.06	23.57	23.4
5Y	10Y	34.417	30.948	28.148	25.954	25.136	24.51	23.99	23.56	22.91	22.49	22.25
10Y	1Y	55.16	44.32	37.368	32.259	30.21	28.54	27.31	26.45	25.61	25.52	25.78
10Y	2Y	51.17	42.9	37.078	32.622	30.8	29.28	28.09	27.2	26.12	25.72	25.71
10Y	3Y	48.22	41.43	36.4	32.439	30.796	29.4	28.27	27.38	26.18	25.58	25.37
10Y	5Y	40.55	35.891	32.181	29.144	27.857	26.74	25.8	25.02	23.87	23.17	22.8
10Y	10Y	33.601	30.509	27.978	25.926	25.086	24.37	23.76	23.24	22.44	21.9	21.56

Through calibration of the displaced-diffusion model to the swaption market data, the swaption price becomes:

$$V_{n,N}(0) = P_{n+1,N}(0)\,\text{Black}\left(\frac{S_{n,N}(0)}{\beta}, K + \frac{1-\beta}{\beta}S_{n,N}(0), \sigma\beta, T\right),$$

$$\text{where Black}(S_{n,N}(0), K, \sigma, T) = \begin{cases} S_{n,N}(0)\Phi(d_1) - K\Phi(d_2) \text{ (payer)} \\ K\Phi(-d_2) - S_{n,N}(0)\Phi(-d_1) \text{ (receiver)}, \end{cases}$$

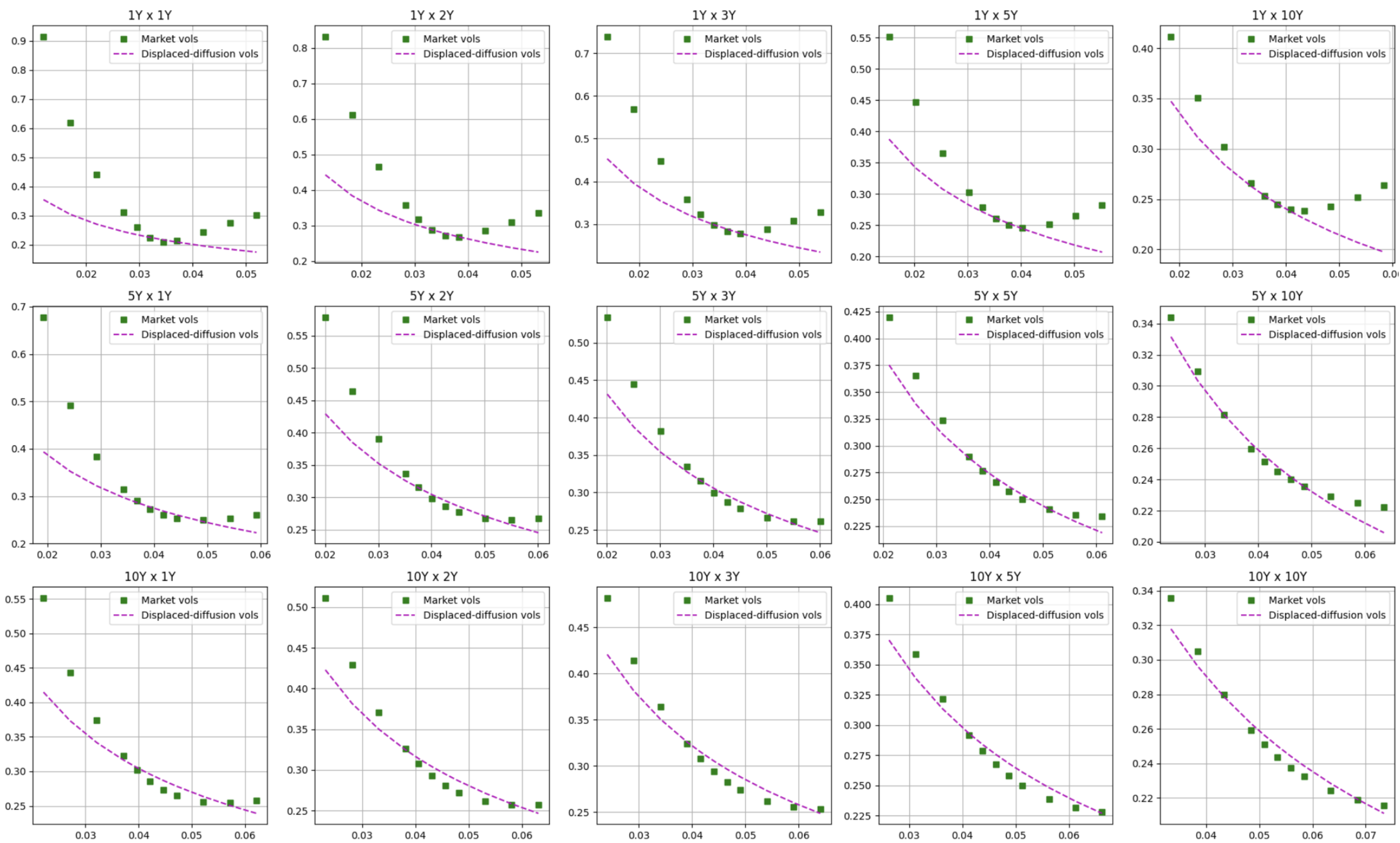
$$d_1 = \frac{\log\frac{S_{n,N}(0)}{K} + \frac{1}{2}\sigma_{n,N}^2 T}{\sigma_{n,N}\sqrt{T}}, \quad d_2 = d_1 - \sigma_{n,N}\sqrt{T}.$$

The calibrated σ and β parameters obtained are tabulated below.

Sigma (Expiry\Tenor)	1Y	2Y	3Y	5Y	10Y
1Y	0.2250	0.2872	0.2978	0.2607	0.2447
5Y	0.2726	0.2983	0.2998	0.2660	0.2451
10Y	0.2854	0.2928	0.2940	0.2674	0.2437

Beta (Expiry\Tenor)	1Y	2Y	3Y	5Y	10Y
1Y	4.831147e-07	3.578737e-08	1.910510e-07	6.681484e-07	2.586166e-07
5Y	2.162272e-07	2.101644e-06	2.361982e-08	3.387585e-06	5.017631e-02
10Y	3.216240e-07	1.000058e-07	1.992773e-06	2.447620e-05	4.459102e-03

Here, the calibrated σ values represent the implied volatilities of the swaptions. These values are crucial for pricing swaptions, as they directly influence the option's premium. Such values typically increase with the tenor, reflecting higher uncertainty and risk over longer periods. On the other hand, the β values represent the displacement factor, which adjusts the forward rate to account for the possibility of negative interest rates. In this calibration, β values are very small, close to zero. This indicates that the model assumes almost no displacement of the forward rate, meaning that the forward rate behaves similarly to a standard lognormal process. A β value close to zero suggests that the market does not expect significant shifts in the forward rate, and the model is effectively behaving like a standard Black-Scholes model for swaptions. The associated calibration results are then plotted as follows:



These results show that while the displaced-diffusion model can considerably capture the market's implied volatilities, i.e., the volatility skew, especially at longer expiries and tenors, the small β values imply that the model is not significantly adjusting for displacement, which may limit its ability to accurately price swaptions in environments where interest rates are highly volatile or negative. For the SABR model, we consider the case where $\beta = 0.9$ with zero risk-free rate, i.e., $r = 0$. By calibrating this model, the swaption price becomes:

$$V_{n,N}(0) = P_{n+1,N}(0)\ \text{BlackScholes}(S_{n,N}(0), K, 0, \sigma_{\text{SABR}}, T),$$

$$\text{where BlackScholes}(S_{n,N}(0), K, 0, \sigma_{\text{SABR}}, T) = \begin{cases} S_{n,N}(0)\Phi(d_1) - K\Phi(d_2) & \text{(payer)} \\ K\Phi(-d_2) - S_{n,N}(0)\Phi(-d_1) & \text{(receiver)}, \end{cases}$$

$$d_1 = \frac{\log \frac{S_{n,N}(0)}{K} + \frac{1}{2}\sigma_{\text{SABR}}^2 T}{\sigma_{\text{SABR}}\sqrt{T}},\ \ d_2 = d_1 - \sigma_{\text{SABR}}\sqrt{T}.$$

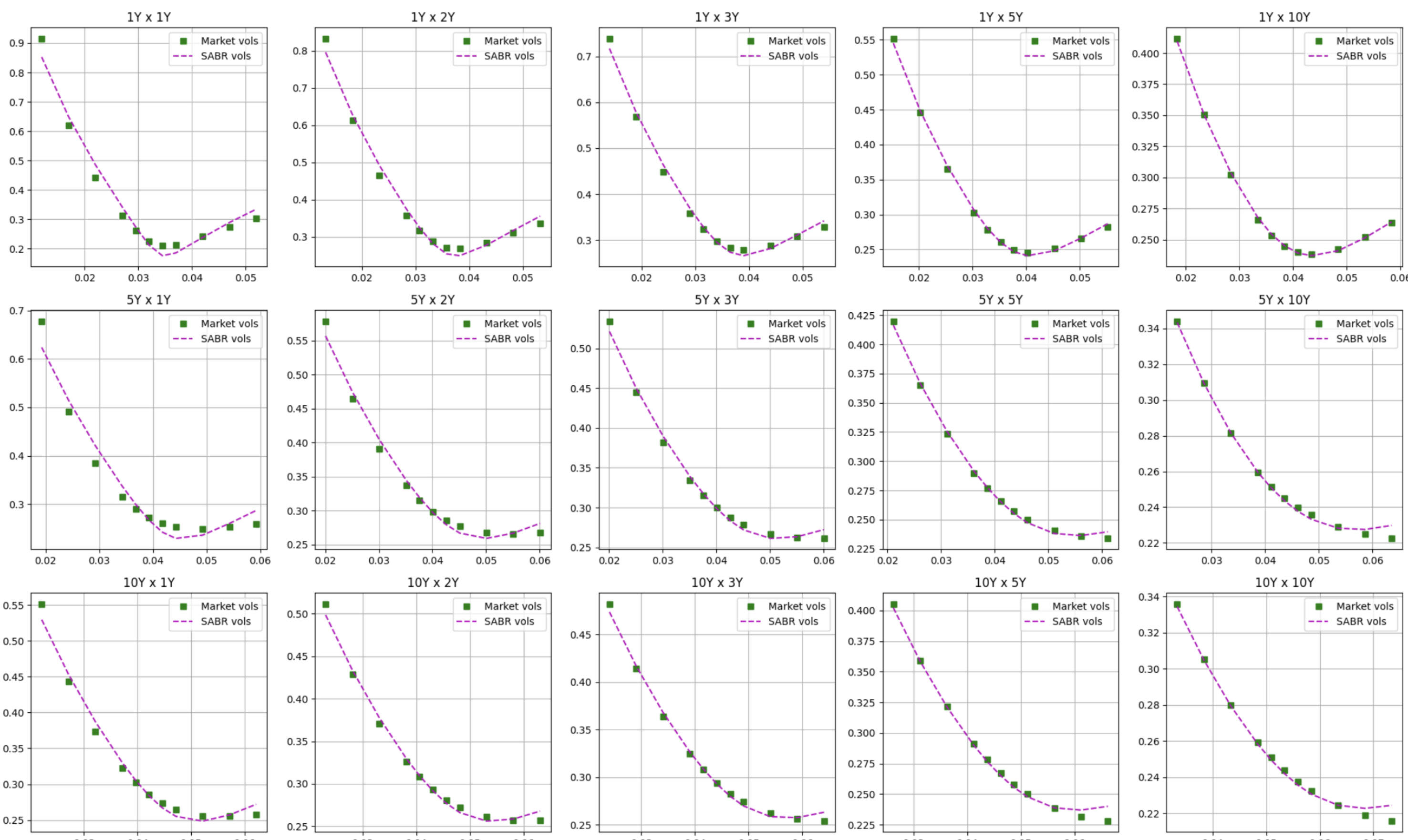
The calibrated α, ν and ρ parameters obtained are tabulated below.

Alpha (Expiry\Tenor)	1Y	2Y	3Y	5Y	10Y
1Y	0.139067	0.184652	0.196852	0.178052	0.171083
5Y	0.166263	0.198497	0.207231	0.188238	0.171191
10Y	0.176061	0.191402	0.200019	0.181849	0.164772

Nu (Expiry\Tenor)	1Y	2Y	3Y	5Y	10Y
1Y	2.049536	1.677369	1.438123	1.064876	0.777804
5Y	1.337576	1.058899	0.933328	0.680006	0.530820
10Y	1.000675	0.914422	0.853172	0.704893	0.600363

Rho (Expiry\Tenor)	1Y	2Y	3Y	5Y	10Y
1Y	-0.633241	-0.525115	-0.482846	-0.414420	-0.264656
5Y	-0.583927	-0.542490	-0.536731	-0.493564	-0.375759
10Y	-0.540240	-0.532954	-0.531122	-0.490882	-0.420789

Here, the α values represent the initial volatility of the forward rate. These values are typically higher for shorter tenors, reflecting higher uncertainty in the near term. Next, the ρ values represent the correlation between the forward rate and its volatility. Negative ρ values indicate an inverse relationship, which is common in markets where interest rates are expected to decrease as volatility increases. Finally, the ν values represent the volatility of volatility, capturing the extent to which the volatility itself is volatile. Higher ν values indicate a more pronounced volatility smile or skew. The associated calibration results are then plotted:



When determining the optimality of a pricing model, a standard market practice would be to check how close can it fit observable market prices (Neo & Tee, 2019). These results demonstrate that the SABR model can accurately capture the market's implied volatilities, i.e., both the volatility skew and smile, compared to the displaced-diffusion model whose calibration results are poorer. The model's flexibility in adjusting α, ρ and ν makes it a powerful tool for pricing swaptions, especially in markets with complex volatility structures.

Finally, we obtain approximate swaption prices (payer $2y \times 10y$ and receiver $8y \times 10y$) with $1\% \leq K \leq 8\%$ (with step size 1%), using cubic spline interpolation and the following formulae, with step size 0.5:

$$\text{Payer } 2y \times 10y \text{ par swap rate} = \frac{\sum_{i=2.5}^{12} D_o(0,t_i) \times L(t_{i-0.5}, t_i)}{0.5 \sum_{i=2.5}^{12} D_o(0,t_i)}$$

$$\text{Payer } 8y \times 10y \text{ par swap rate} = \frac{\sum_{i=8.5}^{18} D_o(0,t_i) \times L(t_{i-0.5}, t_i)}{0.5 \sum_{i=8.5}^{18} D_o(0,t_i)}$$

Calibrated swaption\Strike price	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
Pay 2y x 10y under displaced-diffusion	0.288851	0.196827	0.115971	0.055779	0.020693	0.005643	0.00109	0.000145
Pay 2y x 10y under SABR	0.291743	0.202217	0.119619	0.053665	0.020036	0.009649	0.006029	0.004354
Rec 8y x 10y under displaced-diffusion	0.02475	0.041342	0.065483	0.098652	0.141833	0.195308	0.258599	0.33057
Rec 8y x 10y under SABR	0.049404	0.079499	0.102818	0.12464	0.154958	0.212199	0.294126	0.385348

These results show that the premiums for payer swaptions increase with the strike price, reflecting the higher cost of entering into a swap with a higher fixed rate. The displaced-diffusion model produces prices that are consistent with a standard lognormal model. The SABR model, on the other hand, captures the volatility smile more accurately, leading to slightly different premiums for out-of-the-money options. On the other hand, the premiums for receiver swaptions decrease with the strike price, reflecting the lower benefit of entering into a swap with a lower fixed rate. Again, the SABR model provides more accurate pricing for out-of-the-money options due to its ability to capture the volatility smile. The calibrated models provide reliable tools for pricing and risk management of swaptions. The ability to accurately price swaptions is crucial for hedging interest rate risk and for the valuation of complex derivative products. The forward swap rates and swaption prices can be used to identify potential arbitrage opportunities. For example, discrepancies between the model prices and market prices may indicate mispricing, which can be exploited for arbitrage.

Part III (Convexity Correction)

A constant maturity swap (CMS) is a type of interest rate swap that has one floating leg (CMS leg) that resets periodically to a fixed maturity. The PV is the sum of the discounted values of the CMS rates, multiplied by the day count fraction. Each CMS rate is calculated using the static-replication approach, i.e., a CMS contract paying the swap rate $S_{n,N}(T)$ at time $T = T_n$ can be expressed as:

$$\frac{V_0}{D(0,T)} = \mathbb{E}^T\left[\frac{V_T}{D(T,T)}\right] \Rightarrow V_0 = D(0,T)\mathbb{E}^T[S_{n,N}(T)].$$

By choosing the forward swap rate $F = S_{n,N}(0)$ as our expansion point, we can express the CMS rate as:

$$\begin{aligned} V_0 &= D(0,T)g(F) + h'(F)[V^{pay}(F) - V^{rec}(F)] + \int_0^F h''(K)V^{rec}(K)\,dK + \int_F^\infty h''(K)V^{pay}(K)\,dK \\ &= D(0,T)g(F) + \int_0^F h''(K)V^{rec}(K)\,dK + \int_F^\infty h''(K)V^{pay}(K)\,dK. \end{aligned}$$

In other words, we can write:

$$\underbrace{\mathbb{E}^T[S_{n,N}(T)]}_{\text{CMS rate}} = g(F) + \frac{1}{D(0,T)}\left[\int_0^F h''(K)V^{rec}(K)\,dK + \int_F^\infty h''(K)V^{pay}(K)\,dK\right].$$

Here, the IRR-settled option pricer (V^{pay} or V^{rec}) is given by:

$$V(K) = D(0,T)\cdot \mathrm{IRR}(S_{n,N}(0))\cdot \mathrm{Black}(S_{n,N}(0), K, \sigma_{\mathrm{SABR}}, T)$$

so the discount factor $D(0,T)$ can be cancelled away. Since the payoff function is simply $g(K) = K$, we have:

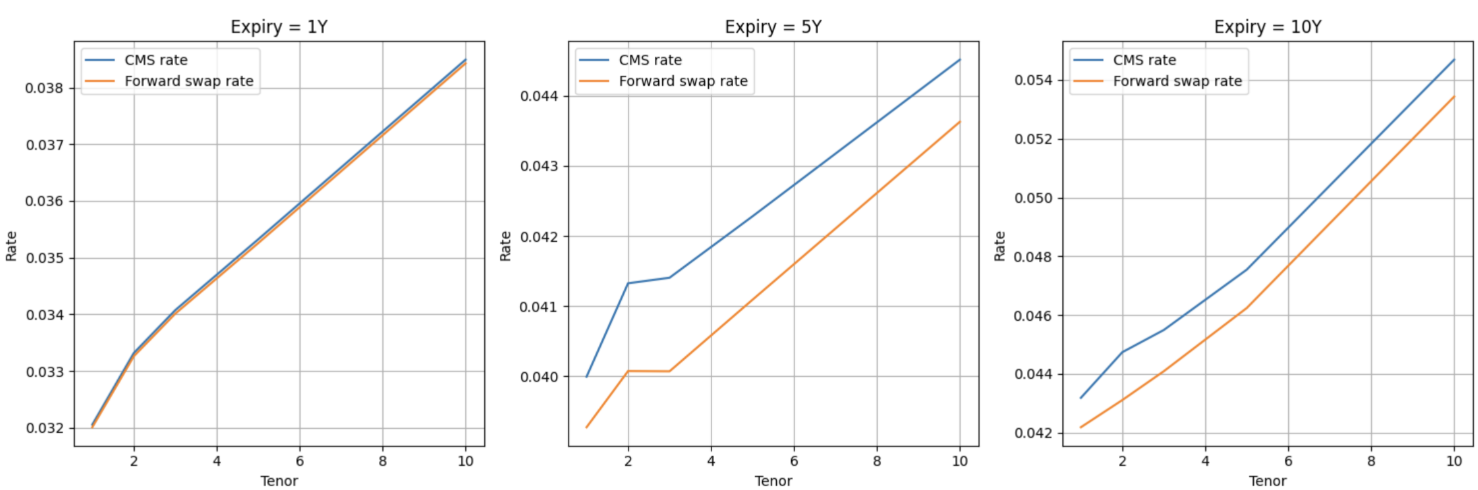
$$h'(K) = \frac{\mathrm{IRR}(K)g'(K) - g(K)\mathrm{IRR}'(K)}{\mathrm{IRR}(K)^2},\quad h''(K) = \frac{-\mathrm{IRR}''(K)\cdot K - 2\cdot \mathrm{IRR}'(K)}{\mathrm{IRR}(K)^2} + \frac{2\cdot \mathrm{IRR}'(K)^2\cdot K}{\mathrm{IRR}(K)^3}.$$

Having prepared the data by computing LIBOR forward rates and associated forward swap rates, we can use cubic spline interpolation and the calibrated SABR model to value the following CMS products:

- PV of a leg receiving CMS10y semi-annually over the next 5 years ≈ 0.20282,
- PV of a leg receiving CMS2y quarterly over the next 10 years ≈ 0.38488.

We then compare the following forward swap rates with the CMS rates:

	1Y×1Y	1Y×2Y	1Y×3Y	1Y×5Y	1Y×10Y	5Y×1Y	5Y×2Y	5Y×3Y	5Y×5Y	5Y×10Y	10Y×1Y	10Y×2Y	10Y×3Y	10Y×5Y	10Y×10Y
CMS rate	0.032053	0.033316	0.034072	0.035322	0.038492	0.039994	0.041326	0.041405	0.042280	0.044515	0.043179	0.044734	0.045490	0.047544	0.054690
Forward swap rate	0.032007	0.033259	0.034011	0.035255	0.038427	0.039273	0.040074	0.040070	0.041091	0.043629	0.042179	0.043105	0.044086	0.046238	0.053433
Diff	0.000046	0.000057	0.000062	0.000067	0.000065	0.000721	0.001252	0.001334	0.001189	0.000886	0.001000	0.001629	0.001404	0.001306	0.001257



From the results above, we observe the following:

- Short maturities (e.g., 1y): The convexity correction is minimal. For instance, the differences for $1y\times 1y$, $1y\times 2y$, and $1y\times 3y$ are close to 0.000046 to 0.000062. This is because, for shorter maturities, the interest rate volatility and the time horizon over which convexity can accumulate are limited.
- Longer maturities (e.g., 10y): The convexity correction increases as the maturity grows. For example, for $10y\times 10y$, the difference is 0.001257, which is significantly larger. This reflects the cumulative effect of convexity over time and the higher uncertainty in long-term interest rate movements.

- For a given maturity, convexity correction tends to increase with tenor as longer tenors imply more extended cash flow payments and larger potential deviations between the CMS and forward swap rates:
  1. At $5y$ maturity, $5y \times 1y$ (0.000721) has a smaller correction than $5y \times 3y$ (0.001334).
  2. At $10y$ maturity, $10y \times 1y$ (0.001000) is smaller than $10y \times 10y$ (0.001257).
- The convexity correction is smallest for shorter maturities and shorter tenors due to limited interest rate volatility and time horizon. Moreover, convexity correction grows non-linearly with tenor and maturity due to the compounding effects of volatility and the integration terms in the CMS valuation formula.
- The SABR model accounts for skew and smile in volatility, amplifying the convexity correction for longer maturities and tenors. The provided formulas for CMS rates incorporate integral terms dependent on strike price, volatility (via the SABR model), and IRR. These integrals increase with both maturity and tenor, leading to larger convexity corrections.
- The plots provided for $1y$, $5y$, and $10y$ expiries show a clear divergence between CMS rates and forward swap rates as tenor increases, reinforcing the mathematical findings. This divergence is most pronounced for $10y$ expiry, reflecting the combined effect of long maturity and tenor.

In essence, convexity correction increases with both maturity and tenor due to higher volatility, longer time horizons, and the non-linear effects captured by the SABR model and IRR terms. Short maturities and tenors exhibit minimal corrections, while longer durations experience significant corrections, driven by cumulative volatility and the forward-looking nature of CMS rates.

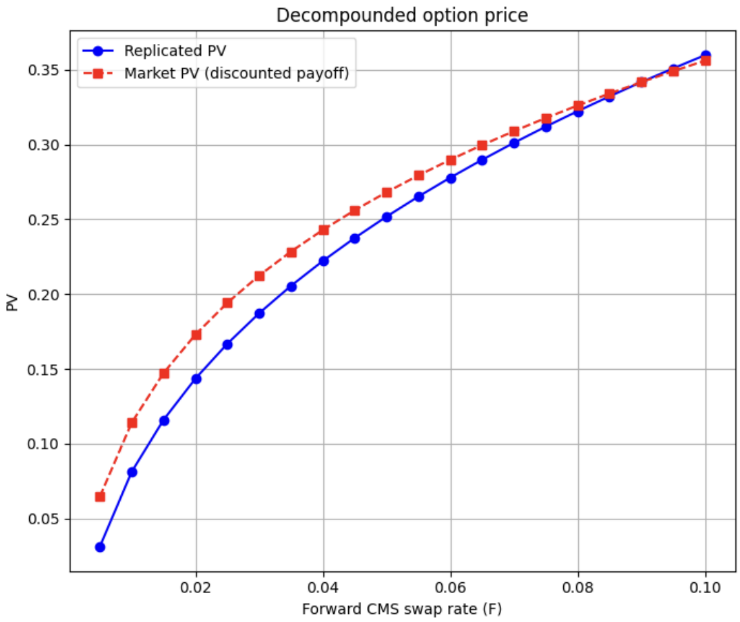
Part IV (Decompounded Options)

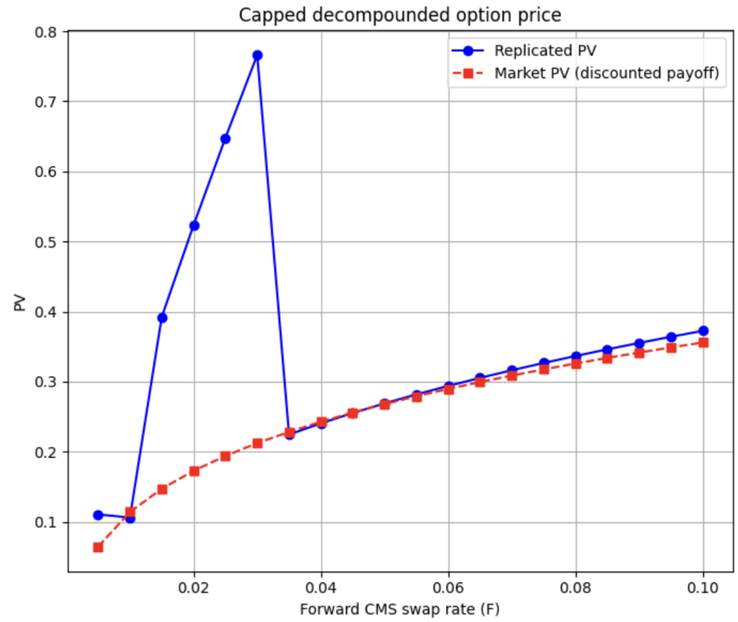
We can use the static replication formulae provided in Part III to value the PV of the following decompounded options with payoffs taken at time $T = 5y$:

1. $(\text{CMS } 10y)^{1/4} - 0.04^{1/2}$ with PV ≈ 0.23339,
2. $\left((\text{CMS } 10y)^{1/4} - 0.04^{1/2}\right)^+$ with PV ≈ 0.25145.

The steps involved in calculating these values are as follows:

1. Input parameters (α, ρ, ν, and $\beta = 0.9$) were extracted from the calibrated tables of SABR model parameters. The discount factor for $T = 5$ years was obtained from the discount factor table. The forward swap rate for a 10-year swap starting in 5 years was obtained from the par swap rate table.
2. For the first option, the payoff is $(\text{CMS}\,10y)^{1/4} - 0.04^{1/2}$. This payoff was replicated using a combination of put and call swaptions. The integrals for the put and call payoffs were computed numerically using the quadrature (`quad`) function, and the results were summed to obtain the PV of the payoff. For the second option, the payoff is $\left((\text{CMS}\,10y)^{1/4} - 0.04^{1/2}\right)^+$, which is the positive part of the first payoff. This payoff was replicated using a call swaption with a strike price $L = 0.04^{1/2}$. The numerical integral for the call payoff was computed, and the result was combined with the value of a payer swaption to obtain the PV of the payoff.
3. Finally, the payoffs corresponding to $0.005 \leq F \leq 0.1$ are multiplied by the discount factor to obtain the corresponding market PVs. The comparison is illustrated in the two graphs below:





These results provide insights into the valuation of decompounded options and the effectiveness of static replication in pricing complex derivatives. In particular, the first option's PV reflects the expected discounted payoff of the option, considering the current market conditions and the SABR model's calibrated parameters. On the other hand, the second option's PV is higher than the first option as the payoff is truncated at zero, meaning the option only pays out when the difference is positive. This truncation reduces the downside risk, leading to a higher PV. Moreover, the use of static replication allows for the accurate pricing of complex payoffs by breaking them down into simpler components (e.g., put and call swaptions). The results demonstrate that this technique is effective in capturing the value of decompounded options, even when the payoffs involve non-linear functions of the underlying swap rate.

The integration of the SABR model into the replication process ensures that the volatility smile and other market dynamics are accurately reflected in the option prices. Speaking of market implications, the higher PV of the second option (with the truncated payoff) indicates that the market assigns a premium to options that limit downside risk. This is consistent with the general preference for options that provide protection against adverse movements in the underlying rate. The results also highlight the importance of accurate model calibration, as the SABR model parameters (α, ρ, ν, and β) play a crucial role in determining the option prices. The calibrated parameters ensure that the model reflects current market conditions, leading to reliable valuations. The ability to accurately price decompounded options using static replication is valuable for risk management and hedging strategies. By replicating the payoffs with simpler instruments, financial institutions can hedge their exposure to complex derivatives more effectively. The results also provide insights into the cost of hedging such options, as the PVs represent the fair price of the options in the market.

References

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