

Early Exercise – A Game

Question Consider the following game of die: I will roll a single die up to 3 times. You can stop me immediately after the first, second or third roll. I will pay you the same number of dollars as there are dots on the up-turned face on the roll you stop me. What is your strategy and what is the fair value of this game?

Ans $\frac{14}{3} \approx 4.67$.

$$\begin{aligned}
 \mathbb{E}[X] &= \mathbb{E} \left[\underbrace{\mathbb{E} \left[\underbrace{\mathbb{E}[X \mid \text{play game 3}]}_{(a)} \mid \text{play game 2} \right]}_{(b)} \right] \\
 &\quad \underbrace{\hspace{10em}}_{(c)}
 \end{aligned}$$

(a) 3.5 $E[X] = (1 + 2 + 3 + 4 + 5 + 6)/6 = 3.5$

(b) $\frac{1}{2} \times 3.5 + \frac{1}{2} \times \frac{4+5+6}{3} = 4.25$

(c) $\frac{4}{6} \times 4.25 + \frac{2}{6} \times \frac{5+6}{2} = 4.67$

Strategy:

G1

G2

G3

continue if

1, 2, 3, 4

1, 2, 3

exercise if

5, 6

4, 5, 6

1, 2, 3, 4, 5, 6