Master of Science in Quantitative Finance

COURSE CODE: QF620
COURSE TITLE: Stochastic Modelling in Finance

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PRE-REQUISITE/CO-REQUISITE/MUTUALLY EXCLUSIVE COURSE(S)
None

COURSE AREA
MSc in Quantitative Finance Core

GRADING BASIS
Graded

COURSE UNIT
1 CU

FIRST OFFERING TERM
Academic Year: AY2024-25
Academic Term: August

COURSE DESCRIPTION

Since the seminal work of Fisher Black, Myron Scholes and Robert Merton in the early seventies, an elegant theory has been developed for efficient derivatives securities valuation. The concepts of arbitrage, martingales and numerical methods are the three pillars on which the theory and implementation of the valuation of derivative securities rests. Today, advanced financial modelling techniques, developed by collaboration between academics and practitioners, have fundamentally transformed the financial market landscape. The derivatives business alone is a multi-trillion-dollar market. The objective of this course is to provide students a first course in financial calculus and the modelling of financial derivatives.

Although the main emphasis of this course is on models based on stochastic processes, we start with binomial trees to illustrate 3 key concepts: hedging, no-arbitrage, and risk-neutrality. This lays down the foundation for more advanced topics, including martingales, stochastic integrals, stochastic differential equations, Ito's formula, change of measure etc. The insight that created this subject and led to the development of the field of financial derivatives is the pricing and hedging of vanilla European options. There are two main approaches – 1) stochastic calculus with partial differential equation and 2) probability theory with martingale pricing formula. Both will be covered in this course.

The same framework has subsequently been applied to the pricing and hedging of other more exotic financial products. Regardless of the approach taken, the theory is based on the assumption on no-arbitrage, and hence strong emphasis is placed on the importance of hedging and dynamic replication.

This course is an interesting mix of finance and mathematics. Students will see that simple financial concepts like the no-arbitrage principle, coupled with careful mathematical reasoning, can lead to a sophisticated formulation of pricing and hedging framework. Students are encouraged to form a rigorous understanding of financial modelling, valuation techniques and hedging.

LEARNING OBJECTIVES

At the end of the course, participants will be able to understand the following concepts:

- Random walk and Brownian motion, scaling and reflection principle
- Use binomial tree model to illustrate the concepts of hedging, risk neutrality and no-arbitrage
- Solve several important forms of stochastic differential equations
- Ito's formula, Girsanov Theorem, Martingale Representation Theorem, Feynman-Kac representation
- Equivalent Martingale Measure, Radon-Nikodym derivative and Change of Numeraire Theorem
- Black-Scholes Model, its assumptions, and possible extensions
- Risk-neutral option valuation framework and the hedging of risk
- Static Replication of European Payoffs, Dynamic Replication
- European options, exotic options, and path-dependent options

ASSESSMENT METHODS

The various key assessment components are as follows:

Class Participation	10%
Project	20%
Assignments	20%
Exam	50%

ACADEMIC INTEGRITY

Academic integrity is of utmost importance for all work presented and submitted for grading. As such, work presented must be one's own with appropriate citation from sources used, including the Internet.

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INSTRUCTIONAL METHODS AND EXPECTATIONS

Seminars
Weekly seminars will cover the course materials and go through relevant examples.

Assignments
Question packs will be provided for students to practice on concepts taught in the class.

Projects
Students will complete a project developing a basic valuation library.

Examination
This will take the form of a (closed-book) exam for a duration of **three hour**.

CLASS TIMINGS

The course is taught in ten 3-hour sessions over Term 1. There will be a 15-minute break after 1.5-hour in each class.

RECOMMENDED TEXT AND READINGS

A course pack will be offered for this course. The following textbooks are recommended for additional readings:

1. Steven Shreve (2004) "Stochastic Calculus for Finance I & II"
2. Martin Baxter and Andrew Rennie (1996) "Financial Calculus: An Introduction to Derivative Pricing" *17th edition*.
3. Mark Joshi (2008) "The Concepts and Practice of Mathematical Finance" *2nd edition*.
4. Timothy Falcon Crack (2017) "Basic Black-Scholes: Option Pricing and Trading" *4th edition*.

WEEKLY LESSON PLAN

Week	Topics
1	**Probability Theory, Random Walk, and Lattice Model**
2	**Discrete Time Model and the Risk-Neutral Measure**
3	**Continuous Time Model, Brownian Motion, and Martingale**
4	**Stochastic Integrals and Ito's Formula**
5	**Stochastic Differential Equations**
6	**Equivalent Martingale Measures and Girsanov Theorem**
7	**Martingale Valuation Framework, Stochastic Volatility Models**
8	**Risk-Neutral Density and the Breeden-Litzenberger Formula**
9	**Static Replication of European Payoffs, Carr and Madan Formula**
10	**Greeks and Dynamic Hedging**