QF620 Additional Examples
Session 9: Replication of Payoffs with Discontinuities

1 Examples

1. An asset-or-nothing digital call option pays (on maturity T)

$$V_T = S_T \mathbb{1}_{S_T \geq K^*}$$

Suppose $K^* > F$, starting with

$$\int_F^\infty h(K) \frac{\partial^2 C(K)}{\partial K^2} dK,$$

use static-replication approach to value the contract.

2 Suggested Solutions

1. We have

$$\begin{aligned}\int_F^\infty h(K)\frac{\partial^2 C(K)}{\partial K^2}dK &= \int_{K^*}^\infty K\frac{\partial^2 C(K)}{\partial K^2}dK \\ &= \left[K\frac{\partial C(K)}{\partial K}\right]_{K^*}^\infty - \int_{K^*}^\infty 1\cdot\frac{\partial C(K)}{\partial K}dK \\ &= -K^*\frac{\partial C(K^*)}{\partial K} - [C(K)]_{K^*}^\infty + \int_{K^*}^\infty 0\cdot C(K)dK \\ &= -K^*\frac{\partial C(K^*)}{\partial K} + C(K^*) \quad \triangleleft\end{aligned}$$

This expression carries important insights about replication — it is a statement that an asset-or-nothing digital call option can be replicated with K^* amount of cash-or-nothing digital call option and a vanilla call, as shown in the figure below: