qf621 prep review qns 2a

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0.0.1 Question 5

Answer: A

The daily max drawdown is generally higher than the monthly max drawdown, as it captures the intra-day volatility which is not reflected in monthly closing prices. - Daily max drawdown: This measures the largest peak-to-trough decline in price within a single trading day, capturing all the intra-day fluctuations. - Monthly max drawdown: This measures the largest peak-to-trough decline in price over an entire month, only considering the opening and closing prices of each day, not the intra-day volatility.

Other options are incorrect: - B: This is the opposite of what is true. Since monthly drawdowns only consider closing prices, they will generally be smaller than the daily drawdowns which capture all price movements throughout the day. - C: Daily and monthly drawdowns are not identical because they measure different timeframes and include different levels of detail. Daily drawdowns encompass all price fluctuations within a day, while monthly drawdowns only consider opening and closing prices. - D: While the calculations for both are similar, they can definitely be compared as they measure the same concept (peak-to-trough decline) just over different timeframes. The key takeaway is that daily drawdowns will almost always be higher than monthly drawdowns for the same asset due to the inclusion of intra-day volatility.

0.0.2 Question 11

Answer: False

While a limit order book provides a key indication of market liquidity, it's not a complete representation. The book shows visible orders, but it doesn't capture all forms of liquidity, including hidden liquidity or "dark pools" where large orders might be executed without being displayed on the public order book.

What the Limit Order Book shows: - It displays all buy and sell orders at various price levels for a specific asset, says Investopedia. - It indicates the best bid (highest price buyers are willing to pay) and best ask (lowest price sellers are willing to accept) at any given moment. - It shows the depth of the market, which reflects the quantity of outstanding buy and sell orders at each price level.

Why it's not a complete picture: - Hidden liquidity: Some large orders, especially those from institutional investors, might be placed in "dark pools" or submitted as hidden orders (like iceberg orders) that don't appear in the public order book says Investopedia. - Liquidity providers: Limit order books show liquidity provided by those willing to post buy and sell orders, but it doesn't capture all types of liquidity. - Market depth and volume: While the order book shows market depth and volume, it doesn't necessarily mean a high-volume stock also has good market depth. Imbalances in orders can still lead to high volatility, says Investopedia. - Trade execution: The order book shows the potential for trades, but it doesn't guarantee that all orders will be filled, especially for larger orders in less liquid securities says Investopedia.

0.0.3 Question 16

```python
# (a) Correlation for x2 = a * x1 + b, a > 0
# Correlation is +1 if a > 0, -1 if a < 0 (perfect linear relationship)
a = 2
corr_x1_x2 = 1 if a > 0 else -1

# (b) Portfolio risk and return for two assets
mu1 = 0.08
mu2 = 0.12
sigma1 = 0.15
sigma2 = 0.2
rho = 0.5
w1 = 0.6
w2 = 0.4

# Portfolio return
portfolio_return = w1 * mu1 + w2 * mu2

# Portfolio variance
portfolio_variance = (
    w1**2 * sigma1**2 +
    w2**2 * sigma2**2 +
    2 * w1 * w2 * sigma1 * sigma2 * rho
)
portfolio_std = np.sqrt(portfolio_variance)

# (c) d assets, all mean mu, std sigma, correlation 0
d = 10
mu = 0.1
sigma = 0.2

# All-in-one-asset portfolio
return_one = mu
risk_one = sigma

# Equal-weighted portfolio
w_eq = 1/d
return_eq = mu
# Variance: sum_i w_i^2 * sigma^2 = d*(1/d^2)*sigma^2 = sigma^2/d
risk_eq = sigma / np.sqrt(d)
```

Short calculation process - (a) Correlation is +1 for positive a in a linear model. - (b) Portfolio return = w1 * mu1 + w2 * mu2; variance = w1^2 * sigma1^2 + w2^2 * sigma2^2 + 2 * w1 * w2 * sigma1 * sigma2 * rho - (c) One-asset: risk = sigma, return = mu; Equal-weighted: risk = sigma/sqrt(d), return = mu. - Diversification reduces risk by a factor of sqrt(d) when assets are uncorrelated.

Final answers - (a) Correlation between x1 and x2: 1 - (b) Portfolio return: 0.0960, portfolio risk (std): 0.1473 - (c) One-asset portfolio: return = 0.1, risk = 0.2; equal-weighted portfolio: return = 0.1, risk = 0.0632

0.0.4 Question 17

(a) Return & risk profiles

- **Long position**
  - **Definition**: You buy (own) an asset today hoping its price will rise.
  - **Return**: $R_{\text{long}} = \frac{P_{\text{sell}} - P_{\text{buy}} + D}{P_{\text{buy}}}$, where D is any dividends or coupons received.
  - **Payoff diagram**: As price P goes up, your profit rises linearly; downside is limited to -100% (if the asset goes to zero).
  - **Risk**: Downside capped at total loss of your investment; upside is theoretically unlimited.
- **Short position**
  - **Definition**: You borrow shares and sell them today, hoping to buy them back cheaper later to return to the lender.
  - **Return**: $R_{\text{short}} = \frac{P_{\text{sell}} - P_{\text{buy}} - C}{P_{\text{sell}}}$, where C is borrowing costs (and you forfeit any dividends).
  - **Payoff diagram**: Profit is capped at +100% of proceeds (if price goes to zero); losses are unbounded if the price shoots up.
  - **Risk**: Unlimited upward risk, limited upside reward; often requires margin and can incur margin calls in a rally.

```
# (b) Simple average (arithmetic mean) of returns
r1 = 0.20
r2 = -0.10
arithmetic_avg = (r1 + r2) / 2

# (b) Geometric average return (CAGR)
geometric_avg = ((1 + r1) * (1 + r2))**0.5 - 1

# (c) Dollar-weighted average (money-weighted return, IRR approximation)
# Year 1: $1M, Year 2: $10M
# Weighted average = (r1 * 1 + r2 * 10) / (1 + 10)
```

Final answers - (b) Arithmetic average return: 5.00%; geometric average return: 3.92% - (c) Dollar-weighted average return: -7.27%

0.0.5 Question 18

(a) Definition: Simple returns are calculated as (Ending Value − Starting Value) / Starting Value, expressed as a percentage. They represent the percentage increase or decrease in an asset's price over a specific period.

Interpretation: - A positive simple return indicates that the asset's price has increased over the period, and the investor has made a profit. - A negative simple return indicates that the asset's price has decreased over the period, and the investor has incurred a loss. - A zero simple return indicates that the asset's price has remained unchanged over the period.

(b) The lower bound for simple returns is $-\infty$. This means that the percentage decrease in an asset's value can theoretically be infinitely large. In practice, the lower bound is limited by the underlying asset and market conditions, but theoretically, it can be as low as $-\infty$.

0.0.6 Question 19

Let a portfolio hold weights $w = (w_1, \dots, w_N)^\top$ in N risky assets with covariance matrix Σ.

(a) Portfolio risk (variance)

$$\sigma_p^2 \;=\; \mathrm{Var}(w^\top r) \;=\; w^\top \Sigma\, w,$$

where r is the vector of asset returns.

(b) Why diversification reduces risk

- The weighted sum of individual variances alone would be $\sum_i w_i^2 \sigma_i^2$.
- But the full variance includes covariance terms:

$$\sigma_p^2 \;=\; \sum_i w_i^2 \sigma_i^2 \;+\; 2 \sum_{i<j} w_i\, w_j\, \sigma_{ij}.$$

- If assets are not perfectly positively correlated ($\sigma_{ij} < \sigma_i\, \sigma_j$), then the covariance terms reduce total variance below $\sum_i w_i^2 \sigma_i^2$.
- In the extreme of zero correlation, $\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 < \sum_i w_i \sigma_i^2$ as long as no single $w_i = 1$.
- Hence combining assets "smooths out" individual swings, yielding a portfolio whose risk is strictly less than the naive weighted sum of standalone risks.

0.0.7 Question 20

(a) Definition & mechanism The exponentially smoothed value s_t of a time series x_t is given by:

$$s_t \;=\; \alpha\, x_t \;+\; (1-\alpha)\, s_{t-1}, \quad 0 < \alpha \le 1,$$

with some initialization $s_0 = x_0$ (or the sample mean).

- **How it works**: At each step you take a fraction α of the new observation x_t plus a fraction $(1-\alpha)$ of last period's smoothed value.
- Recent observations receive exponentially more weight; older data "fade" at rate $(1-\alpha)$.

(b) Weights sum to one Unfold the recursion once:

$$s_t = \alpha\, x_t + (1-\alpha)\big[\alpha\, x_{t-1} + (1-\alpha)\, s_{t-2}\big] = \alpha\, x_t + \alpha(1-\alpha)\, x_{t-1} + (1-\alpha)^2 s_{t-2},$$

and so on. After k steps:

$$s_t = \sum_{i=0}^{k} \alpha\,(1-\alpha)^{\,i}\, x_{t-i} \;+\; (1-\alpha)^{\,k+1} s_{t-(k+1)}.$$

As $k \to \infty$, the residual term $(1-\alpha)^{k+1} s_{t-k-1} \to 0$, and the weights on the raw x-values are

$$w_i = \alpha\,(1-\alpha)^{\,i}, \quad i = 0, 1, 2, \ldots$$

Their sum is a geometric series:

$$\sum_{i=0}^{\infty} w_i = \sum_{i=0}^{\infty} \alpha\,(1-\alpha)^i = \alpha\,\frac{1}{1-(1-\alpha)} = 1.$$

Thus the exponential smoother is a proper weighted average.

0.0.8 Question 21

```
# Given data
mu1 = 0.07
mu2 = 0.10
var1 = 0.09
var2 = 0.16
sigma1 = var1**0.5
sigma2 = var2**0.5
rho = 0.5
cov12 = rho * sigma1 * sigma2

# Minimum variance portfolio weights (w1 for Asset 1, w2 = 1 - w1)
# Portfolio variance = w1^2 * var1 + w2^2 * var2 + 2 * w1 * w2 * cov12
# Take derivative on both sides and set LHS = 0 yields: 0 = w1 * var1 - (1 - w1) * var2 + (1 -
# This yields: w1 = (var2 - cov12) / (var1 + var2 - 2*cov12)
w1 = (var2 - cov12) / (var1 + var2 - 2 * cov12)
w2 = 1 - w1
```

Short calculation process - sigma1 = sqrt(0.09) = 0.3 - sigma2 = sqrt(0.16) = 0.4 - cov12 = 0.5 * 0.3 * 0.4 = 0.06 - w1 = (0.16 - 0.06) / (0.09 + 0.16 - 2*0.06) = 0.10 / 0.13 0.7692 - w2 = 1 - 0.7692 = 0.2308