QF621 Sample Report - Group 4

Portfolio Optimization Using Trend Following & Mean Reversion Strategies

Project Objective
Our objective is to compare and evaluate trend-following and mean-reversion strategies applied to a portfolio of stocks. The goal is to determine which approach provides superior risk-adjusted returns under various market conditions and how the two strategies can be applied to different sectors of the market.

Portfolio stock selection

Sector	Stocks
Consumer Staples & Discretionary	- **KO** – Coca-Cola Co. - **PEP** – PepsiCo, Inc. - **PG** – Procter & Gamble Co. - **MCD** – McDonald's Corporation - **NKE** – Nike, Inc. - **DIS** – The Walt Disney Company
Healthcare	- **PFE** – Pfizer Inc.
Industrials	- **GE** – General Electric Co.
Communication Services	- **T** – AT&T Inc. - **GOOGL** – Alphabet Inc.
Information Technology	- **INTC** – Intel Corporation - **AAPL** – Apple Inc. - **MSFT** – Microsoft Corporation - **NVDA** – NVIDIA Corporation - **AMD** – Advanced Micro Devices, Inc.
Financials	- **JPM** – JPMorgan Chase & Co. - **GS** – Goldman Sachs Group, Inc. - **BAC** – Bank of America Corporation - **WFC** – Wells Fargo & Company - **C** – Citigroup Inc.

We chose a set of 20 large-cap US stocks across various sectors. This gives us diversified exposure and allows us to test strategy performance across different market dynamics. These stocks are also liquid and widely traded, which makes them ideal for strategy implementation without worrying too much about slippage or trade execution costs. More importantly, the selection includes stocks that exhibit strong trending characteristics while others are more stable or mean-reverting. This makes our comparison between trend following and mean reversion strategies more robust and realistic, as we're applying them to a broad, representative sample of the market.

Trend Following for Portfolio Optimization

Introduction

Trend following is a systematic trading strategy designed to capture persistent directional movements in asset prices. Rather than attempting to predict future price behaviour, trend followers respond to existing trends by entering long positions in rising markets and exiting or shorting in falling markets. The strategy is built on the premise that assets which have performed well recently are likely to continue performing well in the near term, and vice versa. A key strength of trend following lies in its simplicity and objectivity. Decisions are governed by clearly defined rules based on technical indicators derived from historical price data, eliminating discretionary bias. By focusing on reacting to price behaviour rather than forecasting, trend following effectively exploits the momentum effect. This approach is particularly valuable in portfolio optimization, as it provides a disciplined framework for managing positions and controlling risk.

Technical Indicators

A wide range of technical indicators are used to identify and confirm trends. Among the most commonly employed are:

Technical Indicators	What is it?
Simple Moving Average (SMA)	The average of past prices over a specified period.
Exponential Moving Average (EMA)	Similar to SMA, but gives more weight to recent prices.
Moving Average Convergence Divergence (MACD)	Measures the difference between short- and long-term EMAs.
Relative Strength Index (RSI)	Identifies overbought or oversold conditions.
Breakout Systems	Signals generated when prices break above/below recent highs/lows

For our project, we focused on SMA crossover as the trend following strategy as it is easy to implement, transparent and interpretable and historically tested across many asset classes.

Standard 20/51 SMA Crossover Strategy

The 20-51 SMA crossover is a commonly used strategy that captures short-term momentum (20-day) against longer-term trends (51-day). A buy signal is triggered when the short SMA crosses above the long SMA, and a sell signal when it crosses below. This standard configuration balances responsiveness and reliability, making it a popular baseline before further optimization.

Backtesting Methodology

The 20-51 SMA crossover is a widely used strategy that compares short-term momentum with longer-term trends to generate trading signals. A buy occurs when the 20-day SMA crosses above the 51-day SMA, and a sell when it crosses below. This configuration balances responsiveness and reliability, serving as a common baseline before optimization.

To evaluate its performance, the strategy was backtested on daily adjusted closing prices from January 2015 to December 2024 for the stocks listed in the above section. This 10-year period spans multiple market regimes—including bull and bear markets—ensuring a robust assessment of the strategy's effectiveness across varying conditions. Portfolio-level returns were computed by averaging daily strategy returns across all stocks. A risk-free rate of 2.8%, derived from the average U.S. 10-year Treasury yield over the sample period, was used to calculate risk-adjusted metrics such as the Sharpe ratio. Additional evaluation metrics included annualized return, annualized volatility, and maximum drawdown.

This strategy was benchmarked against a buy-and-hold strategy for the same stocks listed across the same timeframe.

Results of 20/51 Strategy compared to Buy-and-hold Strategy

Performance Metrics	Buy and Hold (Equal Weights)	Standard 20/51
Annualized Return	14.39%	0.76%
Annualized Volatility	19.6%	15.71%
Sharpe Ratio	0.68	-0.1301
Maximum Drawdown	– 36.4%	– 32.49%

The 20/51 SMA strategy underperformed the buy-and-hold benchmark, with a much lower annualized return (0.76% vs. 14.39%) and a negative Sharpe ratio (-0.13 vs. 0.68). Although it slightly reduced volatility and drawdown, the strategy's poor risk-adjusted performance and significant losses highlight that it lacks robustness on its own and requires further refinement. This may be due to its rigid parameter choice, which fails to adapt to shifting market conditions, leading to delayed signals and poor responsiveness during trend reversals or sideways markets.

To further investigate this limitation, we conducted a sensitivity analysis using alternative SMA crossover configurations. These included the Golden Cross (50/200), a more conservative setup (50/100), and a volatile market configuration (20/100). Each strategy was applied to the same portfolio and time period to enable a direct comparison of performance, helping to evaluate whether other parameter choices offer improved robustness and effectiveness.

Sensitivity Analysis with Common Crossovers

We evaluated three widely used and well-regarded SMA crossover strategies: the Golden Cross (50/200), the conservative mid-range (50/100), and a more reactive setup for volatile markets (20/100). These configurations differ in their sensitivity to price movements—longer-term crossovers like the 50/200 are slower but more stable, while shorter-term combinations like 20/100 are quicker to react but prone to noise. Using the same methodology as the 20/51 strategy—including the same portfolio, time period, and performance metrics—we backtested each configuration to assess their relative robustness and effectiveness.

Evaluation of Sensitivity Analysis

Performance Metrics	Buy and Hold (Equal Weights)	Golden Cross 50/200	Standard Conservative 50/100	Volatile Market 20/100
Annualized Return	14.39%	6.57%	3.85%	1.79%
Annualized Volatility	19.6%	13.49%	12.72%	13.01%
Sharpe Ratio	0.68	0.2796	0.0827	-0.0779
Maximum Drawdown	– 36.4%	– 32.3%	– 33.69%	– 29.41%

Among the tested strategies, the 50/200 Golden Cross delivered the strongest performance after buy-and-hold, with an annualized return of 6.57% and a Sharpe ratio of 0.28. Its long-term configuration reduced noise and false signals, making it well-suited for capturing sustained trends. This robustness is especially effective for large-cap stocks, which tend to trend more steadily and exhibit lower short-term volatility—conditions where long-term crossovers excel.

The 50/100 strategy showed moderate results but lagged in risk-adjusted returns (Sharpe: 0.08), often reacting too slowly to trends. The 20/100 setup, while more reactive, suffered from whipsaws in volatile periods, leading to negative risk-adjusted performance.

Despite its higher volatility, the buy-and-hold approach outperformed all SMA crossover strategies in both total and risk-adjusted returns (Sharpe: 0.68). This reflects the long-term upward drift of large-cap equities, where passive exposure has historically outperformed rule-based strategies that rely on rigid signal thresholds and may miss recoveries after downturns.

However, the underperformance of fixed SMA strategies highlights the need for further refinement. Stocks exhibit heterogeneous behaviors—some trend persistently, while others are more mean-reverting or reactive to macro news. A one-size-fits-all SMA configuration fails to account for this diversity.

Bayesian Optimization of SMA Parameters

To systematically identify profitable SMA crossover configurations, we applied Bayesian Optimization, a probabilistic model-based method that is well-suited for optimizing non-convex and noisy objective functions often encountered in financial modeling. The algorithm models the reward surface using a Gaussian Process surrogate and iteratively selects new points in the parameter space by maximizing an acquisition function based on expected improvement.

Search Scope and Parameter Bounds

To constrain the optimizer to meaningful regions of the parameter space and reduce the risk of overfitting, we defined the following bounds:

Parameter	Range
Short SMA Window	20 to 50 days

Long SMA Window	50 to 200 days

These ranges are grounded in technical analysis conventions, where short SMAs (20–50 days) are used to capture near-term price action and long SMAs (100–200 days) represent broader market trends. This search domain allows flexibility while avoiding excessively short or long periods that can yield erratic or lagging signals.

Optimization Constraints

To further guide the optimization towards realistic and robust strategies, we embedded several constraints and penalties into the objective function:

1. **Structural Constraint**:
   A minimum gap of 30 days between the short and long SMA is enforced (Short +30 < Long). This constraint preserves a meaningful crossover signal and avoids choppy behavior. Given the relatively low volatility of large-cap equities in our universe, this buffer enhances signal reliability while remaining responsive to trend shifts.
2. **Volatility Penalty**:
   To suppress overly volatile strategies, the 20-day rolling standard deviation of the return series was penalized. This discourages models that produce erratic profit-and-loss profiles.
3. **Trade Count Penalty**:
   To reduce turnover, the number of trades executed in the training window was penalized. Lower turnover strategies are less susceptible to slippage, spread costs, and commissions.

The final objective function used for optimization was:

Score = Sharpe Ratio − 0.5 × Volatility − 0.001 × Number of Trades

This formulation balances return and risk while embedding practical trading constraints, improving the model's real-world viability.

Rolling Walk-Forward Backtesting

To evaluate the out-of-sample robustness of the optimized SMA strategy, we conducted a rolling walk-forward backtest. This technique simulates a real-world scenario where a trader continuously re-optimizes and redeploys models over time using only historical data.

Backtest Structure

Each iteration of the backtest uses the following data partition:

Window Type	Duration
Training Window	3 years (~756 days)
Testing Window	1 year (~252 days)

This 3+1 design was selected for the following reasons:

1. **Adequate Data Coverage**:
   The 3-year training window ensures the optimizer has sufficient historical context,

especially since the long SMA window can span up to 200 days. This ensures accurate performance estimation and parameter tuning.

2. **Robustness Across Market Regimes**:
A 3-year horizon captures varying macroeconomic and volatility conditions, enhancing the generalizability of the learned parameters.
3. **Practical Re-Optimization Frequency**:
Annual retraining aligns with industry practices, striking a balance between adapting to new trends and avoiding excessive model churn.
4. **Test Horizon Matches SMA Scale**:
The 1-year test window provides enough time for long SMA signals to play out and mitigates bias from delayed signal activation.

Implementation Logic

The walk-forward loop proceeds as follows:

- **Step 1: Train-Test Split**
Define the first 3-year training and 1-year test windows from the start of the price history for each stock.
- **Step 2: Bayesian Optimization**
Optimize SMA parameters on the training window using the constrained objective function defined above.
- **Step 3: Out-of-Sample Evaluation**
Apply the optimized parameters to the 1-year test set. Track strategy performance, trade frequency, and Sharpe Ratio.
- **Step 4: Logging Results**
Store the optimized parameters and performance metrics for each period and stock. Return series are accumulated for final portfolio construction.
Step 5: Roll Forward
Shift the entire window forward by one year and repeat the above steps until the dataset is exhausted.

This rolling process is conducted independently for each stock, allowing for dynamic calibration that adapts to individual price behavior and evolving market structure. It also supports cross-sectional portfolio construction using up-to-date parameterizations.

Similarly, returns using these optimised parameters were tracked, however since rolling backtesting was used, the bayesian optimised strategy was calculated starting from 2018. A persistent method was used, with a warm-up period, to allow for stocks to start in 2018 with an existing long or short position based on the 2018 optimal crossover parameters. These positions would rollover into the following year, with new optimal parameters, and positions would then only be flipped when a new trade signal is generated based on the new parameters for that year.

Returns were then aggregated for the period 2018-2024. To address our overarching objective of comparing trend-following and mean-reversion strategies in a portfolio, we explored using Mean-Variance Optimization (MVO) to evaluate the robustness of this strategy with different allocated weights for each stock in the portfolio.

Portfolio Construction using Mean-Variance Optimization (MVO)

MVO allows us to evaluate whether trend-following signals—when optimally weighted—can deliver superior risk-adjusted returns across different market regimes and sectors, providing a robust benchmark for comparison against mean-reversion-based approaches.

Objective and Constraints

The optimization was conducted under realistic trading constraints:

Constraint Type	Specification
Net exposure	Sum of weights = 1
Short exposure limitation	Net short exposure must not exceed net short
Individual asset bounds	Weight ∈ [−0.2, 0.2]

This bounded exposure ensures that the portfolio maintains reasonable diversification and avoids excessive concentration in any single asset or directional bet. Short positions are allowed but limited, which reflects typical risk control in professional long-short portfolios.

Rolling Train-Test Design

To simulate practical deployment, we employed a 1-year rolling window approach, mirroring the time structure of the SMA backtest:

Window Type	Duration
Training Window	1 year (~252 days)
Testing Window	1 year (~252 days)

Each MVO iteration uses the previous year of SMA strategy returns to estimate expected returns and covariances, and then applies the optimized weights to the subsequent year of out-of-sample SMA returns.

Implementation Steps

1. **Preprocessing and Data Validation**
   The input dataset consists of the SMA strategy returns from the persistent crossover strategy. Prior to optimization, stocks with insufficient return history in the training window were excluded. A minimum of 5 valid assets was required to proceed.
2. **Parameter Estimation**
   For each training window:
   - **Expected returns** were calculated as the mean of daily returns, annualized by multiplying by 252.

    - **Covariance matrix** was similarly annualized.
3. **Optimization Procedure**
   The optimization was performed using the scipy.optimize.minimize function with Sequential Least Squares Programming (SLSQP). Custom constraints were implemented to enforce:

    - Sum of weights = 1
    - Net long-short balance
    - Individual weight caps and floors
4. **Out-of-Sample Testing**
   The resulting optimal weights were applied to the test window. Daily portfolio returns were computed as the dot product of weights and test-period SMA returns. These were then aggregated into a cumulative return series.
5. **Rolling Forward**
   The window advanced by one year, and the process was repeated until the full data range was covered.

Performance Summary

The table below summarizes key performance metrics across four strategies:

Performance Metrics	Buy and Hold (2018-2024)	Golden Cross 50/200 (2018-2024)	Bayesian Optimised Parameters	MVO Portfolio
Annualized Return	12.65%	5.18%	0.64%	2.73%
Annualized Volatility	21.55%	15.23%	12.73%	21.59%
Sharpe Ratio	0.54	0.1565	-0.17	-0.00324
Maximum Drawdown	– 36.4%	– 32.3%	– 22.75%	– 43.42%

Performance of Bayesian Optimized SMA Strategy

Despite the appeal of adaptive parameter tuning, the Bayesian-optimized SMA strategy underperformed across most metrics. It delivered a low annualized return of 0.64% and a negative Sharpe ratio (−0.17), suggesting worse risk-adjusted performance than cash. However, it showed improved downside protection, with a lower maximum drawdown of −22.75%, likely due to the dynamic adjustment of parameters.

Several factors may explain this underperformance. First, the strategy may have overfit to the training data despite rolling backtests, resulting in poor out-of-sample generalization. Second, SMAs are lagging indicators; the enforced 30-day gap between short and long windows likely reduced noise but introduced delays in capturing new trends. Finally, the use of low-volatility large-cap stocks may have dampened the effectiveness of trend-following signals in the absence of strong directional moves.

Performance of MVO Portfolio

The MVO portfolio, built on top of the Bayesian-optimized SMA returns, also underperformed. It recorded a modest annualized return of 2.73% and a near-zero Sharpe ratio (−0.00324), along with the highest maximum drawdown of −43.42%, reflecting weak downside resilience, however, likely attributed to the leverage aspect of short-selling as part of the constraints included.

This poor performance is largely due to noisy input signals—since the base SMA strategy was already weak, MVO amplified noise rather than enhancing returns. Additionally, the covariance matrix, estimated from short rolling windows, was likely unstable, resulting in erratic and unreliable weight allocations. Lastly, even with constraints, the optimizer may have tilted towards high-volatility assets, introducing leverage-like behavior that increased drawdown risk.

Summary of Trend Following

This study highlights the limitations of applying complex models to weak or unstable signals. Both the Bayesian-Optimized SMA and the MVO portfolio underperformed simpler benchmarks like the Equal-Weighted Buy & Hold and the Golden Cross. While the Bayesian strategy offered improved downside protection, it suffered from low returns and negative Sharpe ratios. The MVO overlay, intended to enhance risk-adjusted performance, instead amplified noise due to unstable covariance estimates and low-quality inputs.

To improve robustness and performance, several refinements are proposed:

- **Regime Detection:** Dynamic parameter adjustments using regime-switching models (e.g., Markov Switching, volatility filters) can help align strategies with prevailing market conditions (Baek et al., 2020).
- **Volatility-Scaled Signals:** Scaling position sizes by rolling volatility or ATR may improve drawdown control and Sharpe ratios.
- **Hybrid Strategies:** Blending trend-following with mean-reversion indicators (e.g., z-scores, Bollinger Bands) can enhance adaptability.
- **Better Covariance Estimation:** Techniques like shrinkage estimators (Ledoit & Wolf, 2004) or factor models can reduce instability in MVO.
- **Robust Portfolio Objectives:** Alternative optimization goals such as Minimum Variance or CVaR (Rockafellar & Uryasev, 2000) may improve out-of-sample reliability.

  Execution Cost Modeling: Incorporating slippage and transaction costs is essential for evaluating real-world performance, especially for high-turnover strategies.

Mean Reversion for Portfolio Optimisation

Mean reversion refers to the statistical tendency for extreme observations to move closer to the average over time. If an observed value is significantly above or below the mean, it is likely that subsequent observations will be closer to the average due to a combination of persistent and transient effects (Stigler, 1997).

In finance, mean reversion assumes that an asset's price tends to converge to its average historical price. Researchers and financial analysts have been applying numerous models to capture and exploit the mean reversion phenomenon, particularly in trading strategies and risk management. We will explore 2 trading strategies— **Extended Bollinger Bands and Relative Strength Index strategy,** and **Extended Pairs Trading strategy.**

Extended Bollinger Bands and Relative Strength Index

1. **Introduction**

Extended Bollinger Bands and Relative Strength Index (or Extended BB & RSI for short) is a mean reversion strategy that utilises two technical indicators, Bollinger Bands and Relative Strength Index, to identify buy and sell signals when price deviates significantly from its average and momentum appeared exhausted.

This dual-indicator approach aims to reduce false signals and improve timing accuracy by incorporating both price dispersion and momentum exhaustion.

2. Technical Indicators

Bollinger Bands
- **Definition:** A volatility-based envelope drawn around a moving average.
- **Purpose:** To identify relative high or low price levels by tracking standard deviations from a mean.
- **Common use:** Traders use the bands to detect overbought and oversold conditions or breakout signals.

Bollinger Bands are computed as follows:

- Middle Band
$$MB_t = SMA_t(P_t)$$
- Upper and Lower Bounds
$$BB_{\text{upper/lower}} = SMA_t(P_t) \pm k\sigma$$

Throughout this section, we set k=2, corresponding to two standard deviations from the mean—a standard convention in Bollinger Band analysis that captures significant price deviations under the assumption of normality.

Relative Strength Index
- **Definition:** A momentum oscillator that measures the speed and change of price movements.
- **Purpose:** To identify overbought or oversold market conditions.
- **Common use:** RSI < 30 typically signals oversold conditions (potential buying opportunity), while RSI > 70 indicates overbought conditions (potential sell signal).

The RSI is calculated using the standard formula based on average gains and losses over a designated lookback period.

$$RSI = 100 - \left(\frac{100}{1 + \frac{\text{average gain}}{\text{average loss}}}\right)$$

3. Strategy Implementation
3.1. Single Asset

To initiate the implementation of our Extended BB & RSI trading strategy, we first begin by a simple single asset implementation of the strategy to establish some foundation frameworks.

Setting
- **Asset:** Apple Inc. (AAPL)
- **Period:** 2015-01-01 to 2024-12-31
- **Indicators:** Bollinger Bands (20 days); Relative Strength Index (14 days)

Strategy Mechanics
- **Position Management:** Positions are maintained until a new signal is triggered
- **Transaction Cost:** Assume to be 0.1% per trade
- **Returns:** Returns are net of costs and are compounded over time

Signal

- **Buy:** RSI < 30 and Price < Lower Bollinger Bands
- **Sell:** RSI > 70 and Price < Upper Bollinger Bands
- **Execution:** All signals are applied with 1-day lag to reflect practical trading constraint

These three components—Setting, Strategy Mechanics, and Signal Generation—constitute the core framework for our subsequent implementations. While the underlying logic of the strategy remains consistent across extensions, modifications are made primarily to the **Setting**, and incorporation of additional elements such as portfolio construction, parameter optimisation, and sector-specific characteristics are included for strategy extensions.

Result

Our strategy shows limited success in terms of performance metrics; it delivers a modest 0.61% annualised return, with a high annualised volatility of 28.59%, and a sharpe ratio of 0.13.

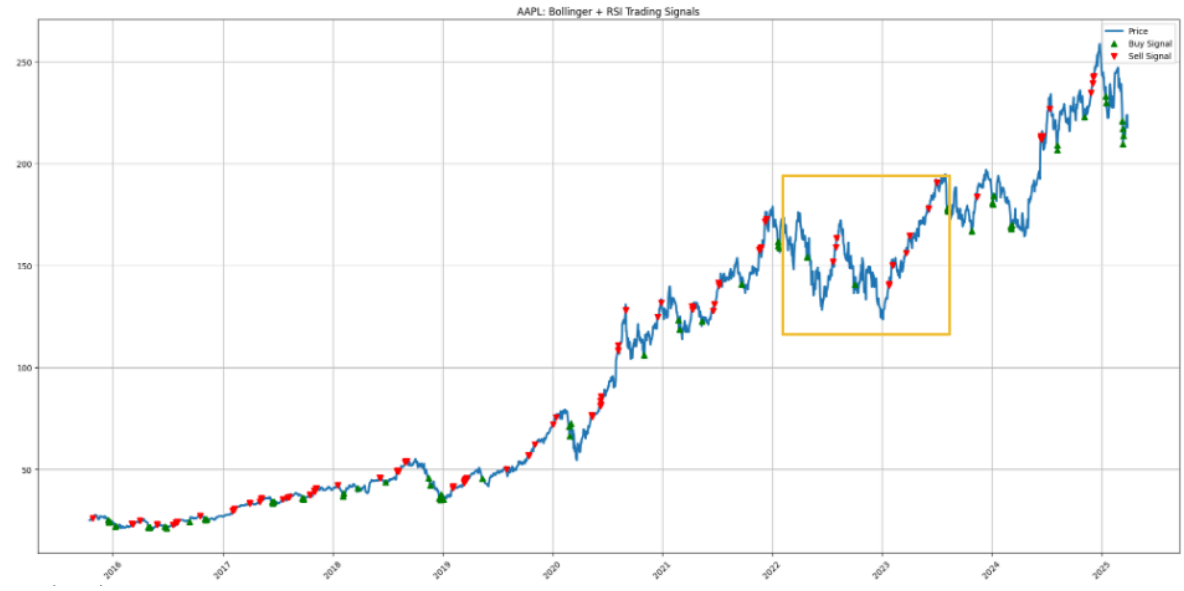
Performance Metrics	Value
Annualized Return	0.61%
Annualized Volatility	28.59%
Sharpe Ratio	0.13
Maximum Drawdown	-65.79%

The strategy, while preventing wealth erosion, achieves modest success in its performance metrics — oversees marginal gains and suffers from high volatility and significant drawdowns. This indicates that the strategy exhibits poor risk-adjusted returns.

Limitation

One observed limitation of this strategy is its excessive conservatism, which results in missed trading opportunities. For example, as illustrated in the annotated chart, the strategy failed to initiate trades when there were trading opportunities to be made. The strategy failed to execute the fundamental **"buy-low-and-sell-high"** that any good trading strategy should.

Moreover, while the strategy correctly identified and responded to the 2020 market crash for AAPL, such success is not consistently observed across time or assets. Its reliance on historical mean reversion patterns renders it ill-suited for predicting black swan events, where price movements deviate sharply and unpredictably from previous behavior.



3.2. Static Portfolio Optimization

We continue our implementation of the extended strategy to static portfolio optimization by applying it to a static multi-asset portfolio, in which we further narrow our stocks of choice into 3 main sectors — Essential, Tech, and Financial. The motivation for this categorisation is to identify sector-specific optimal parameters and investigate how structural differences across industries may influence the efficacy of our trading strategy.

By segmenting our investment pool into distinct sectors, we aim to assess whether tailored configurations of Bollinger Bands and RSI can yield superior performance relative to a uniform parameter set, and to determine how these differences affect overall portfolio construction and optimisation.

Portfolio Optimisation
We apply mean-variance optimisation to allocate portfolio weights, which is achieved by using the **scipy.optimize** library. The portfolio is constrained to long-only positions, such that $w_i \geq 1\%$ for all asset weights. This ensures diversification and prevents concentrated exposure to individual stocks while aligning with practical investment constraints.

Parameter Optimization
We perform a grid search over combinations of Bollinger Band and RSI periods to identify optimal parameters for our trading strategy. The objective is to maximise the Sharpe ratio within each sector while only accounting for transaction costs as the risk-free interest rate is ignored due our long-only trading positions.

The following parameter grids are evaluated:
- `bollinger_periods = [10, 15, 20, 40]`
- `rsi_periods = [5, 9, 14, 31]`

Each sector's parameter set is selected based on in-sample performance using these discrete search intervals.

Result
Our strategy shows great performances with all of the selected portfolios outperforming the buy-and-hold strategy. After the portfolio optimisation, the Tech and Financial sectors favour

a higher RSI period of RSI(9), suggesting that short-term price fluctuations in these sectors are often noisy and require a slightly longer window to effectively filter momentum signals. In contrast, the Essential sector exhibits a preference for a lower RSI period at RSI(5), indicating that price reversals in this sector are typically short-lived and more reactive to rapid shifts in momentum.

Intuitively, Tech and Financial sectors are usually noisy due to the rapid sentiment changes caused by many involving factors such as macro news, earnings shocks, and market competitions, which might trigger false alarm. One recent example is how NVDA oversaw large fluctuations caused by the introduction of Deepseek. Meanwhile for the Essential sector, prices are less noisy but when they move, they tend to be quick and meaningful. For example, due to inflation, KO saw its price drop in October 2024 due to disappointing earnings and warning on demand slowing due to high inflation.

On the other hand, the Technology sector exhibits the shortest optimal Bollinger Band period at BB(10), followed by Financials at BB(15), while the Essential sector selects the longest window at BB(40). This implies that tech stocks tend to mean-revert more quickly and benefit from tighter band sensitivity, whereas essential sector stocks revert more slowly and require a broader volatility filter to capture reliable trading signals.

The intuition here is clear: volatile sectors tend to mean-revert more quickly due to frequent overreactions to news, sentiment shifts, and speculative trading, which create sharp but temporary deviations from the mean. In contrast, less volatile sectors exhibit slower mean reversion as price movements are more stable and typically driven by fundamentals, requiring broader filters to distinguish meaningful reversals from normal fluctuations.

Limitation

While we see great performance metrics, there are inevitable shortcomings to our strategy. One main problem is that our optimised portfolio allocates a disproportionately large weight to the Tech sector, as reflected in its parameter preference for BB=10 and RSI=9. This introduces a sector concentration risk, making the portfolio more susceptible to drawdowns driven by tech-specific market events or regime shifts.

Moreover, this outcome raises concerns about potential overfitting to historical performance within the tech sector, highlighting the importance of incorporating out-of-sample validation or regularisation techniques in future portfolio construction efforts.

3.3. Dynamic Portfolio Optimization

We conclude our static portfolio implementation at this point. In subsequent extensions, we build upon the existing framework and introduce time-varying dynamics through rolling window re-optimisation and frequency-adjusted rebalancing. These extensions include

Rebalancing Frequencies:

$$\{\text{Monthly, Quarterly, Annually}\}$$

Rolling Window Training:

- For each rebalancing frequency, we use a rolling window of the past 756 trading days (approximately 3 years) as the training period.
- At each rebalancing point, we re-optimise both the strategy parameters and the portfolio weights using only historical information up to that point.

Rebalancing Procedure:

- Portfolio weights are estimated using return data from the most recent 252 trading days (approximately 1 year) preceding each rebalancing point.

- The optimised weights are then applied to compute portfolio returns for the next holding period (i.e., the next month, quarter, or year, depending on the frequency selected).

This dynamic extension enables the strategy to adapt to evolving market conditions by continually updating its parameters and asset allocations based on recent performance and volatility characteristics.

Evaluation

Among the dynamic configurations, the monthly rebalancing frequency yields the most favourable performance, followed by annual rebalancing with modest results. The quarterly frequency performs the worst across all metrics.

Performance Metrics	ME	QE	Y
Annualized Return	3.35%	-1.19%	0.26%
Annualized Volatility	21.23%	17.18%	17.76%
Sharpe Ratio	0.13	-0.15	-0.05
Maximum Drawdown	-50.71%	-52.86%	-44.20%

The superior performance of monthly rebalancing can be attributed to its ability to adapt more rapidly to recent price trends, capturing short-to-medium term mean-reverting behaviour more effectively. However, this comes at the cost of higher transaction frequency, which may erode returns if transaction costs are not adequately controlled.

Despite fewer trades, annual rebalancing outperforms quarterly, likely due to a more consistent alignment with long-term mean reversion, while quarterly rebalancing appears to strike an ineffective balance between responsiveness and stability.

Limitation

The dynamic framework, while more adaptive, is prone to overfitting when re-optimisation is performed too frequently on limited historical data. Additionally, increased rebalancing frequency introduces greater transaction costs and turnover, which can significantly reduce net returns, especially under volatile market conditions.

The framework also assumes stationarity in the underlying return distributions during the training windows, which may not hold in real-world financial markets, thereby reducing robustness. Further research could explore hybrid models that incorporate regime-switching or macroeconomic filters to improve signal reliability.

Conclusion

This section examined the effectiveness of mean reversion strategies in portfolio optimisation using Extended Bollinger Bands and Relative Strength Index across single assets, sector-based portfolios, and dynamic rebalancing. The strategy's rule-based design proved intuitive and adaptable, with sector-level customisation yielding notable improvements—particularly in tech and financials, where tighter bands and smoother momentum filters performed well. Essential sectors responded better to broader volatility thresholds. When optimally tuned, the strategy outperformed a Buy & Hold benchmark in selected contexts.

Dynamic rebalancing further improved responsiveness, with monthly rebalancing delivering the best results. However, this came at the cost of higher transaction frequency, potential overfitting, and signal lag. The strategy's assumption of mean reversion limits its effectiveness in strongly trending or extreme market environments. Future research could address these limitations by incorporating regime-switching signals that alternate between mean-reversion and trend-following modes based on market volatility.

Suggested Further Extension

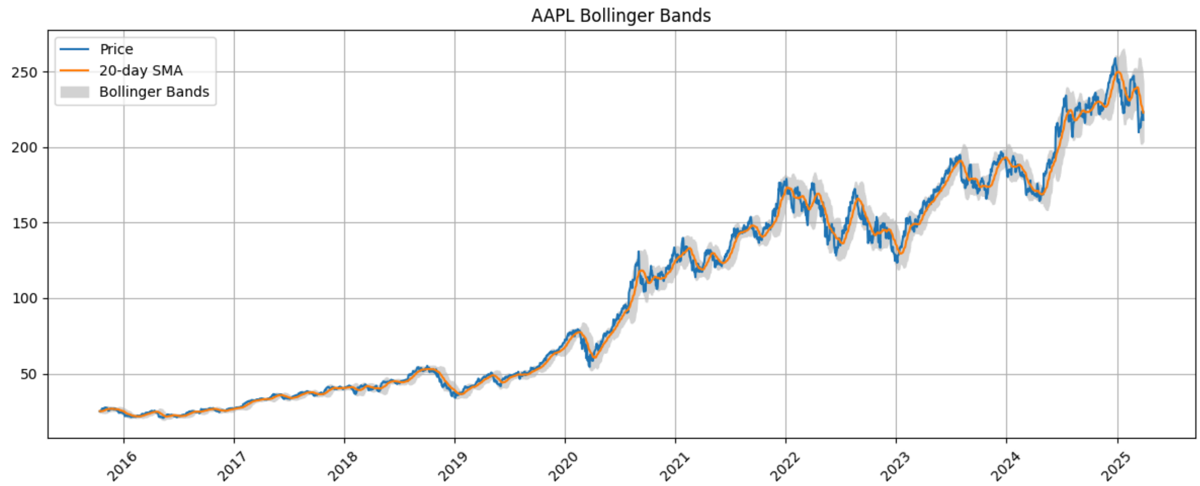
One promising direction for further development is to enhance the Extended BB & RSI framework with a regime-based signal that incorporates elements of trend-following. Specifically, we propose augmenting the strategy with a signal derived from the relationship between short- and long-term moving averages, as well as observed market volatility.

This extension would enable the strategy to switch between mean reversion and trend-following modes based on observed market volatility regimes. When volatility is low, the strategy favours mean-reverting signals (e.g., BB & RSI), while in high-volatility environments, it may incorporate moving average crossovers or momentum indicators to align with trending behaviour. Such an approach could improve robustness across market cycles and enhance risk-adjusted returns.

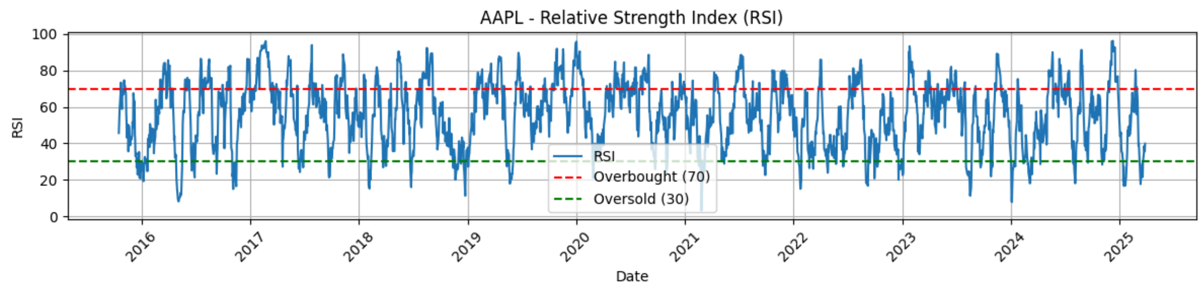
Appendix

Single Asset

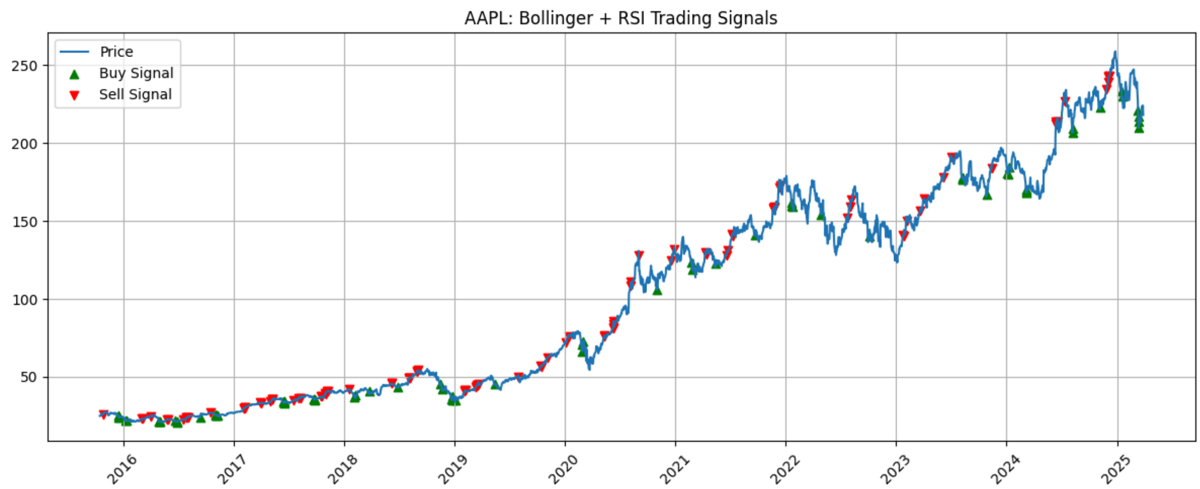
AAPL - BB(20)



AAPL - RSI(14)



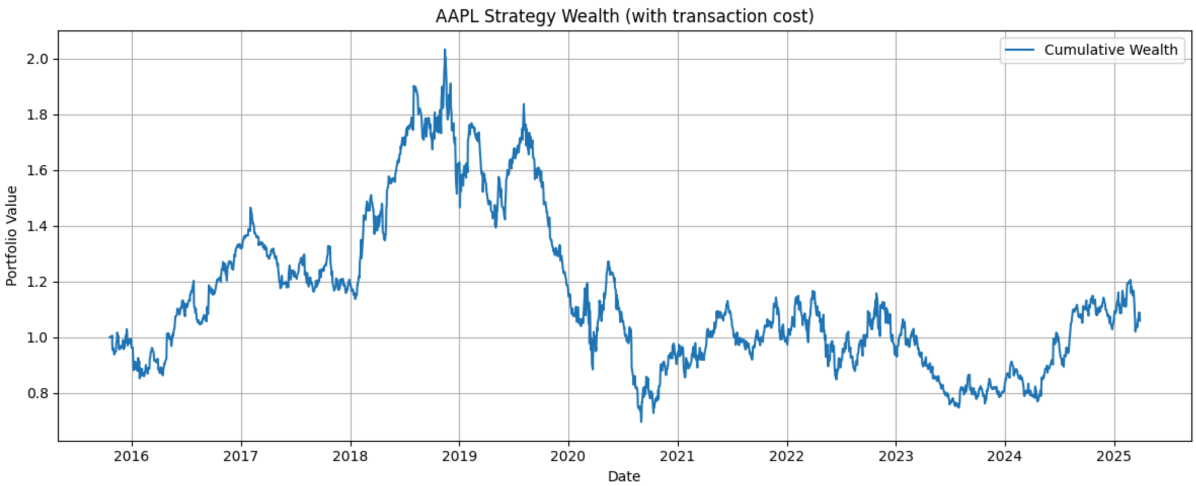
AAPL - BB(2) + RSI(14)



AAPL- Performance Metrics

Performance Metrics	Value
Annualized Return	0.61%
Annualized Volatility	28.59%
Sharpe Ratio	0.13
Maximum Drawdown	-65.79%

AAPL - Wealth Index



Static Portfolio Optimization

Overall Performance Metrics

Performance Metrics	Tech	Fin	Ess	Port
Annualized Return	37.80%	17.54%	11.73%	32.03%
Annualized Volatility	31.64%	26.77%	16.80%	27.83%
Sharpe Ratio	1.08	0.63	0.58	1.04
Maximum Drawdown	-33.15%	-48.78%	-27.74%	-31.90%

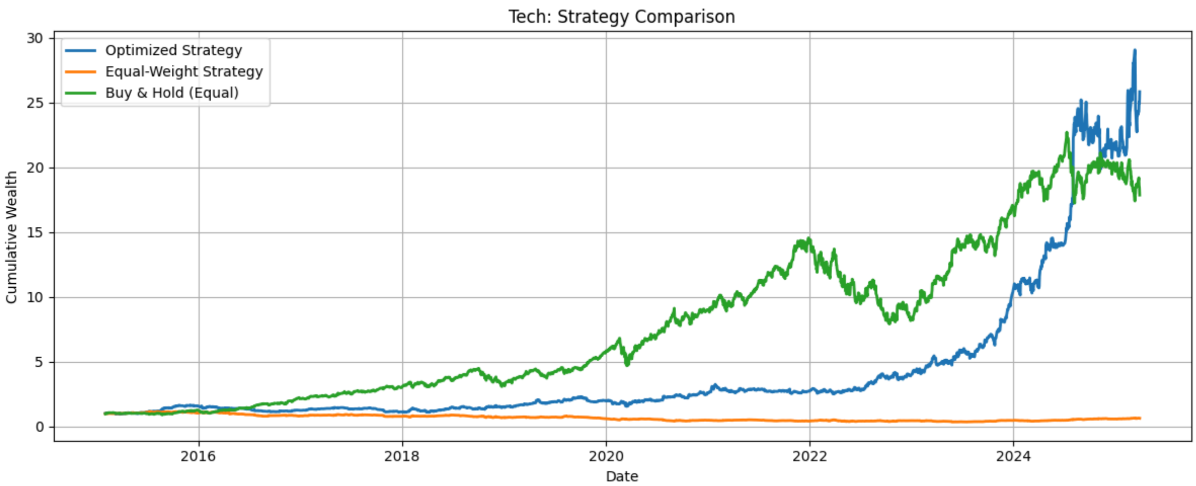
Tech Portfolio

BB = 10

RSI = 9

Ticker	Weight
AAPL	0.01
MSFT	0.01
NVDA	0.01
AMD	0.106
GOOGL	0.01
INTC	0.854

Tech Portfolio - Wealth Index



Financial Portfolio

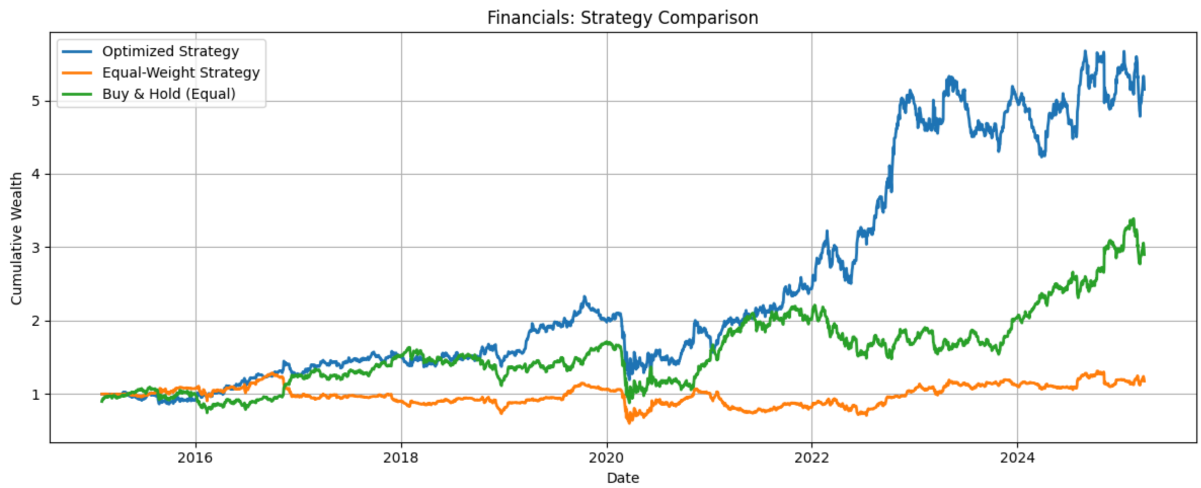
BB = 15

RSI = 9

Ticker	Weight
JPM	0.96
GS	0.01

BAC	0.01
WFC	0.01
C	0.01

Financial Portfolio - Wealth Index



Essential Portfolio

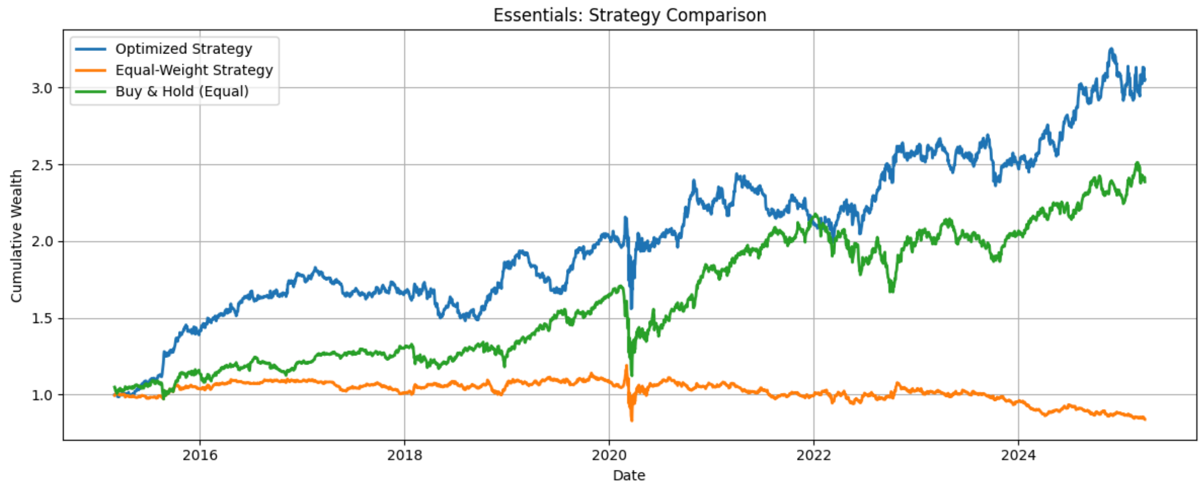
BB = 40

RSI = 5

Ticker	Weight
PEP	0.838
MCD	0.01
NKE	0.01
DIS	0.01
PG	0.01
KO	0.092
PFE	0.01
GE	0.01

T	0.01

Essential Portfolio - Wealth Index



Optimised Portfolio

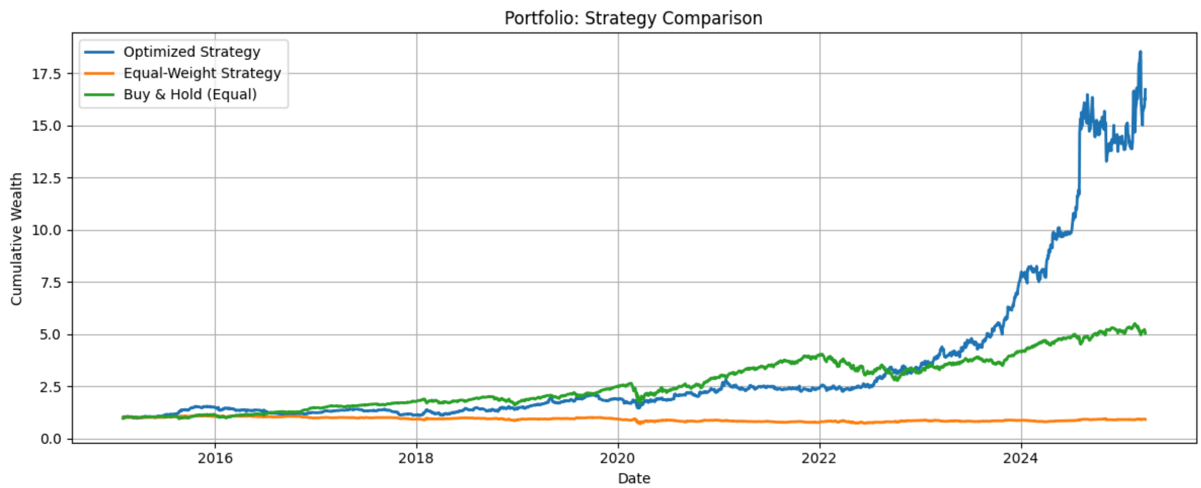
BB = 10

RSI = 9

Ticker	Weight
AAPL	0.01
MSFT	0.01
NVDA	0.01
AMD	0.084
GOOGL	0.01
INTC	0.736
JPM	0.01
GS	0.01
BAC	0.01

WFC	0.01
C	0.01
PEP	0.01
MCD	0.01
NKE	0.01
DIS	0.01
PG	0.01
KO	0.01
PFE	0.01
GE	0.01
T	0.01

Optimised Portfolio - Wealth Index

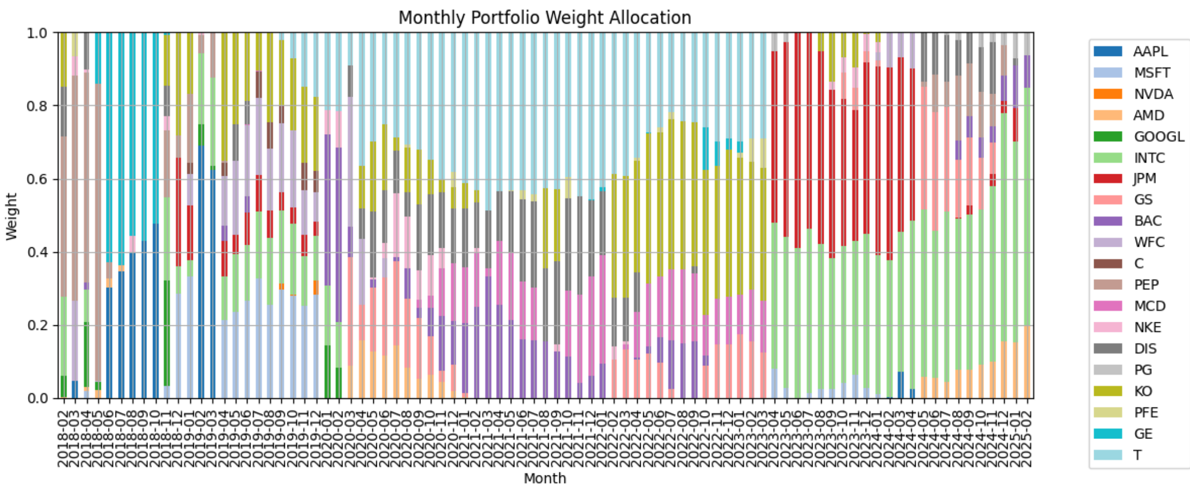


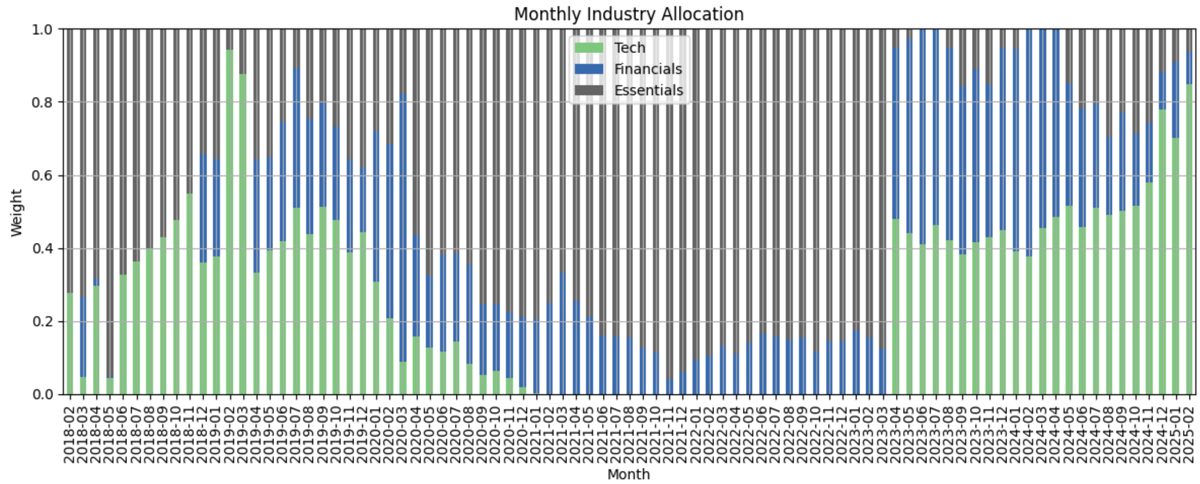
Dynamic Portfolio Optimization

Overall Performance Metrics

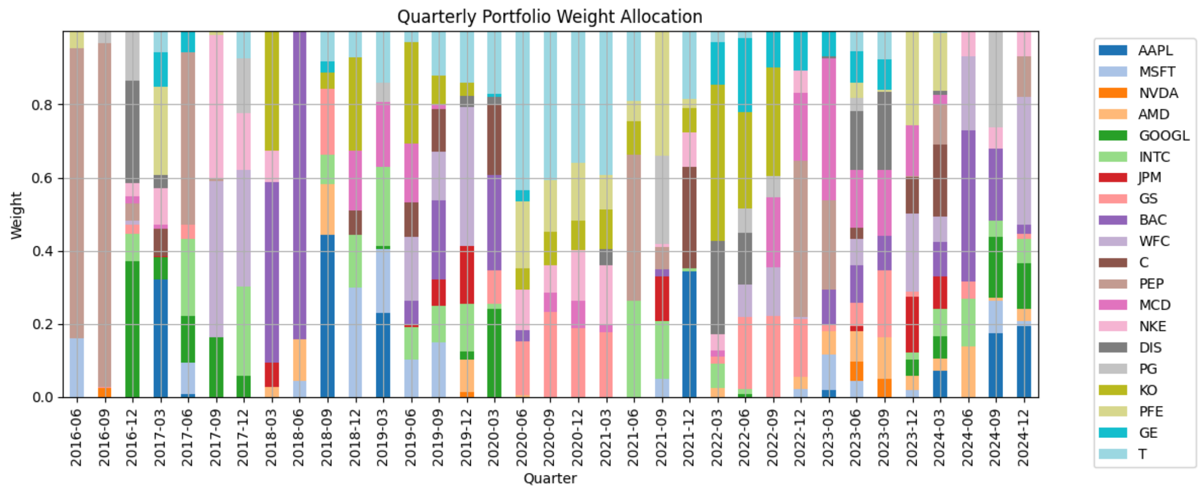
Performance Metrics	ME	QE	Y
Annualized Return	3.35%	-1.19%	0.26%
Annualized Volatility	21.23%	17.18%	17.76%
Sharpe Ratio	0.13	-0.15	-0.05
Maximum Drawdown	-50.71%	-52.86%	-44.20%

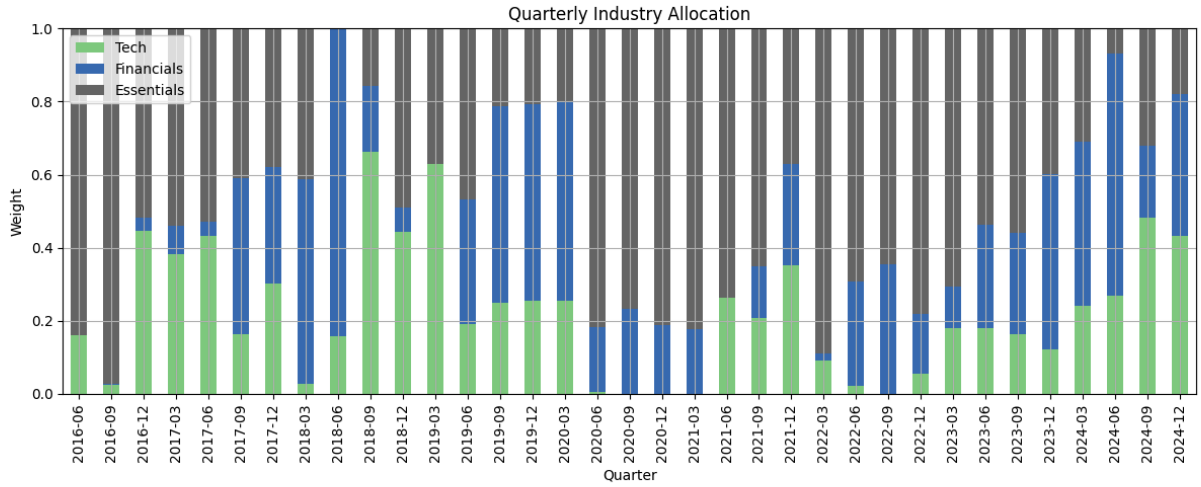
Monthly Rebalancing



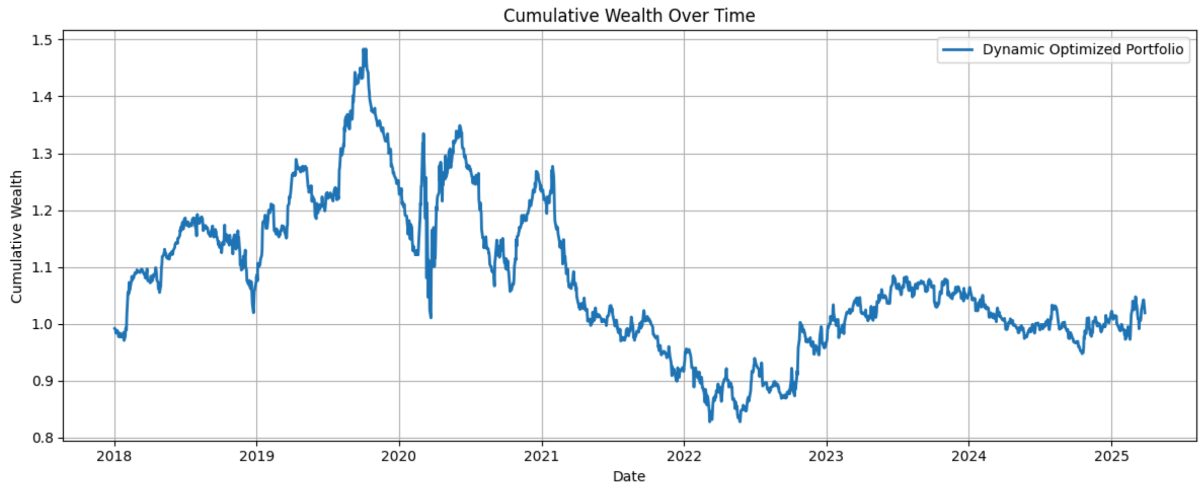


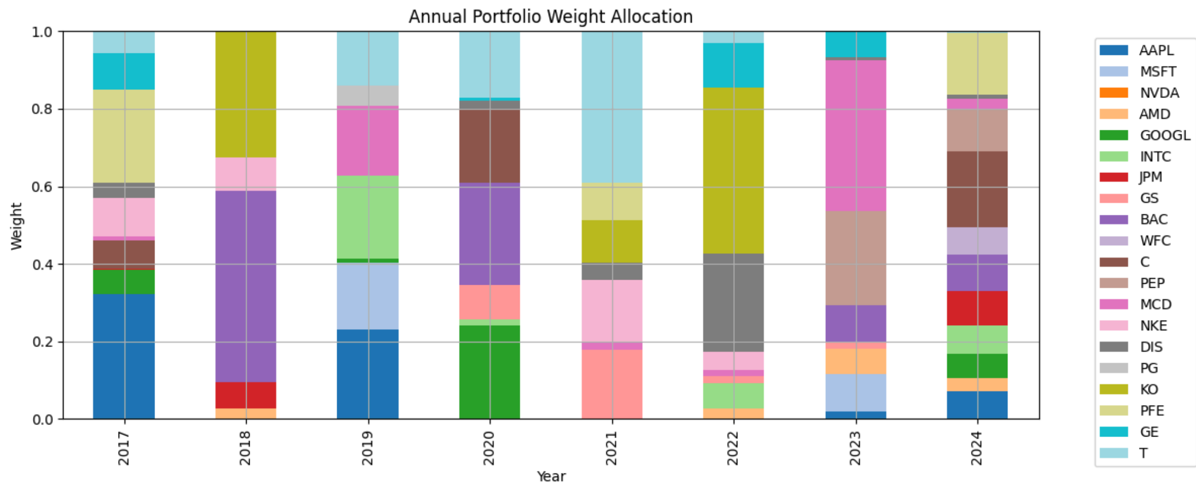
Quarterly Rebalancing

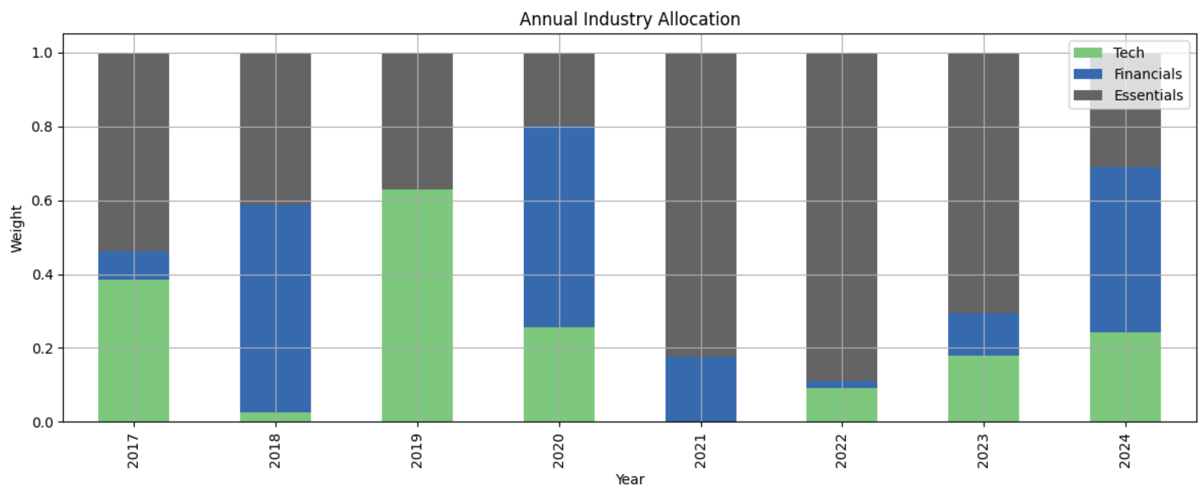




Annually Rebalancing







1. Introduction

This report outlines a systematic quantitative trading strategy that integrates cointegration-based pair selection with Mean-Variance Optimization (MVO) for capital allocation. Designed to capture mean-reverting behaviors between asset pairs, the strategy operates within a rolling window framework that enables annual model recalibration using the most recent historical data. This rolling mechanism ensures the strategy remains adaptive to evolving market conditions while allowing for robust out-of-sample performance evaluation. The core elements of the strategy involve identifying statistically cointegrated asset pairs through the Johansen test, confirming the stationarity of the spread using the Augmented Dickey-Fuller (ADF) test, and allocating capital among selected spreads using MVO to maximize the portfolio's Sharpe ratio. The effectiveness of the strategy is evaluated over multiple test periods using key financial performance metrics.

2. Problem Definition

Pairs trading is a classic market-neutral strategy that seeks to profit from the temporary divergence in the prices of historically related assets. By taking a long position in the undervalued asset and a simultaneous short position in the overvalued counterpart, the strategy anticipates a reversion of the spread to its historical mean. The success of such a strategy is contingent on three sequential processes: identifying pairs that exhibit long-run cointegration, verifying the stationarity of their spreads to ensure mean-reverting behavior, and efficiently allocating capital using a quantitative optimization framework. This project aims to construct a dynamic portfolio of cointegrated spreads, rebalanced annually using the most recent three years of data to maintain accuracy and responsiveness to market dynamics.

3. Data and Preprocessing

The dataset includes daily adjusted closing prices from January 2015 to December 2023 for ten U.S. equities, obtained via the `yfinance` API. Corporate actions are accounted for to ensure data consistency. A rolling window structure is applied: each training window spans 756 trading days (~3 years), followed by a 252-day testing window (~1 year). Within each training window, cointegrated pairs are identified, spread stationarity is verified, spread returns are calculated, and MVO determines the optimal portfolio weights. These weights are then tested out-of-sample in the following year.

4. Methodology

4.1 Cointegration Testing

Cointegration analysis is employed to identify pairs of assets that share a long-run equilibrium relationship. While individual stock prices are typically non-stationary and exhibit random walk behavior, a linear combination of two such series can sometimes be stationary. This stationary linear combination, or spread, forms the basis of the pairs trading signal. The Johansen test is used to examine the presence and number of cointegrating relationships among asset pairs within the training window. As a complementary method, the Engle-Granger two-step procedure is also applied. In the first step, a regression is performed between the two series, and in the second step, the residuals from the regression are tested for stationarity using the ADF test. Only pairs with statistically significant cointegration relationships are considered for further analysis.

The Johansen test assesses the rank rr of the cointegration matrix in the vector error correction model:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t$$

Where:

- X_t is a vector of non-stationary time series
- $\Pi = \alpha\beta'$ where β contains the cointegrating vectors

The trace statistic is used to test for cointegration:

$$\text{Trace Statistic} = -T \sum_{i=r+1}^{n} \ln(1 - \lambda_i)$$

Where:

- T is the sample size
- λ_i are the eigenvalues of the Johansen matrix

If the trace statistic exceeds the 5% critical value, cointegration is said to exist.

Engle-Granger Cointegration Test

The Engle-Granger (EG) test is a two-step procedure:

1. Estimate the regression:

$$Y_t = \alpha + \beta X_t + \varepsilon_t$$

2. Apply the ADF test to the residuals εtεt. If the residuals are stationary, the series are cointegrated.

4.2 Augmented Dickey-Fuller (ADF) Test

Once a cointegrated pair is identified, the resulting spread is subjected to the ADF test to ensure that it is stationary. The ADF test evaluates the null hypothesis that the spread has a unit root, implying non-stationarity. A p-value below the 0.05 threshold indicates statistical evidence of stationarity, confirming the mean-reverting nature of the spread. Only spreads that pass this stationarity criterion are included in the final portfolio.

The ADF test is used to verify the stationarity of the spread:

$$\Delta Z_t = \alpha + \rho Z_{t-1} + \sum_{i=1}^{p} \phi_i \Delta Z_{t-i} + \varepsilon_t$$

Where:

- Z_t is the spread
- The null hypothesis $H_0 : \rho = 0$ implies non-stationarity
- If the test statistic is less than the critical value at 5%, we reject H_0

Only spreads that pass this test are considered suitable for trading.

4.3 Spread Return Calculation

For each stationary pair, the spread return is computed using the percentage change in spread values over time. This calculation provides the historical return series necessary for optimization. The returns are computed using the hedge ratio estimated from the cointegration regression, ensuring that the constructed spread accurately reflects the long-run equilibrium relationship between the two assets.

For each stationary pair, the spread return is calculated using percentage change:

$$r_t^{\text{spread}} = \frac{Z_t - Z_{t-1}}{Z_{t-1}} = \frac{\Delta Z_t}{Z_{t-1}}$$

4.4 Mean-Variance Optimization

With a set of eligible spread returns obtained from the training period, Mean-Variance Optimization is applied to determine the portfolio weights that maximize the Sharpe ratio. The MVO framework considers both the expected returns and the covariance structure of the spread returns, allowing for a risk-adjusted allocation of capital. This approach systematically emphasizes spreads with higher return-to-risk profiles while managing the overall portfolio volatility.

Using spread returns from the training window, we apply MVO to determine the portfolio weights ww that maximize the Sharpe ratio:

$$\max_{\mathbf{w}} \frac{\mathbf{w}^T \mu}{\sqrt{\mathbf{w}^T \Sigma \mathbf{w}}}$$

subject to:

$$\sum w_i = 1, \quad 0 \leq w_i \leq 1$$

where μ is the vector of annualized expected returns, and Σ is the annualized covariance matrix of spread returns.

4.5 Portfolio Testing and Return Aggregation

In each test period, the optimized portfolio derived from the training window is deployed to compute spread returns based on the fixed hedge ratios and MVO weights. These test period returns are recorded and aggregated across multiple rolling windows to evaluate the strategy's overall performance. This approach simulates the real-world process of portfolio rebalancing and enables the assessment of strategy robustness over different market conditions.

During the test period, spread returns are computed using the estimated hedge ratio from linear regression:

$$Z_t^{\text{test}} = P_t^{(1)} - \beta P_t^{(2)}$$

Where:

- $P_t^{(1)}$ and $P_t^{(2)}$ are asset prices in the pair
- β is the hedge ratio from regression

Portfolio return for each day is computed using:

$$r_t^{\text{portfolio}} = \sum_{i=1}^{n} w_i \cdot r_t^{\text{spread}_i}$$

These returns are aggregated across all rolling test periods.

5. Results and Performance

5.1 Cointegrated Pairs

The strategy successfully identified robust cointegrated pairs such as MSFT–NVDA and AAPL–GOOGL across multiple training windows. These pairs consistently passed both the

Johansen and ADF tests, confirming the stability of their spread dynamics. In contrast, certain pairs like PFE–INTC were flagged by the Johansen test but failed the ADF test, indicating a lack of spread stationarity and were thus excluded. The Engle-Granger test confirmed approximately 65 percent of the Johansen-selected pairs, underscoring its role as a useful supplementary filter for increasing the reliability of pair selection.

5.2 Capital Allocation

Once the eligible pairs were selected, capital was allocated using MVO. In one illustrative instance, the optimization algorithm assigned significant weights to pairs with superior historical Sharpe ratios, such as MSFT–NVDA and AAPL–GOOGL. This allocation pattern reflects the optimizer's preference for spreads that offered a favorable balance between return and volatility during the training window.

In one instance, the optimal MVO weights resulted in:

```
Cointegrating vector for pair ('PFE', 'T'): [0.00405099 0.57021303]

=== Capital Allocation for Pair ('PFE', 'T') ===
Allocate $705.42 to PFE
Allocate $99294.58 to T
Cointegrating vector for pair ('GE', 'T'): [0.00907409 0.59328648]

=== Capital Allocation for Pair ('GE', 'T') ===
Allocate $1506.42 to GE
Allocate $98493.58 to T
Cointegrating vector for pair ('GE', 'JPM'): [ 0.02664515 -0.00551185]

=== Capital Allocation for Pair ('GE', 'JPM') ===
Allocate $82859.57 to GE
Allocate $17140.43 to JPM
Cointegrating vector for pair ('T', 'AAPL'): [0.56253436 0.00210617]

=== Capital Allocation for Pair ('T', 'AAPL') ===
Allocate $99626.99 to T
Allocate $373.01 to AAPL
Cointegrating vector for pair ('T', 'MSFT'): [0.56719233 0.00059615]
```

5.3 Performance Metrics

The aggregated out-of-sample performance revealed certain limitations. The annualized return was -0.5421, suggesting that the strategy lost value over the test periods. The portfolio also exhibited high volatility, with an annualized standard deviation of 0.6541. The resulting Sharpe ratio of -0.8287 indicates poor risk-adjusted performance. Furthermore, the maximum drawdown reached -0.9875, reflecting a significant capital decline during the worst-performing period. While the strategy demonstrated methodological soundness, the empirical results highlighted challenges in practical implementation, particularly under real-world constraints.

Let R_t be the daily portfolio return.

Annualized Return:

$$\mu_{\text{annual}} = \mathbb{E}[R_t] \times 252$$

Annualized Volatility:

$$\sigma_{\text{annual}} = \text{StdDev}(R_t) \times \sqrt{252}$$

Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{\mu_{\text{annual}}}{\sigma_{\text{annual}}}$$

Maximum Drawdown:

$$\text{Drawdown}_t = \frac{W_t - \max_{s \leq t} W_s}{\max_{s \leq t} W_s}$$

Where W_t is the cumulative wealth index over time.

Summary:

- Annualized Return: -0.5421
- Annualized Volatility: 0.6541
- Sharpe Ratio: -0.8287
- Maximum Drawdown: -0.9875

6. Conclusion

This project demonstrates a structured, quantitatively driven approach to market-neutral investing through the integration of cointegration analysis, stationarity verification, and Mean-Variance Optimization. By implementing the strategy within a rolling framework, the model remains adaptive to changes in market structure while facilitating robust out-of-sample testing. Despite the theoretical rigor and structured methodology, the realized portfolio performance was disappointing. High volatility and deep drawdowns offset the benefits of statistical precision, emphasizing the need for further enhancements to improve robustness and applicability.

7. Future Improvements

Several enhancements are proposed to improve the strategy's performance and real-world viability. First, the inclusion of risk management mechanisms, such as dynamic stop-loss rules and volatility-adjusted position sizing, could significantly reduce drawdown exposure. Second, expanding the analysis beyond two-asset spreads by leveraging the Johansen test in conjunction with Vector Error Correction Models (VECM) may unlock the benefits of multi-asset cointegrated portfolios, enhancing diversification and reducing idiosyncratic risk. Third, integrating machine learning techniques could improve asset selection. For example, clustering methods can help group structurally similar stocks, while predictive models may assess the persistence of cointegration relationships more accurately. Lastly, incorporating transaction cost modeling, including slippage and liquidity constraints, is essential for simulating realistic portfolio behavior and aligning backtest results with actual market execution.

This outlines a systematic quantitative trading strategy that integrates cointegration-based pair selection with Mean-Variance Optimization (MVO) for capital allocation. Designed to capture mean-reverting behaviors between asset pairs, the strategy operates within a rolling window framework that enables annual model recalibration using the most recent historical data. This rolling mechanism ensures the strategy remains adaptive to evolving market conditions while allowing for robust out-of-sample performance evaluation. The core elements of the strategy involve identifying statistically cointegrated asset pairs through the Johansen test, confirming the stationarity of the spread using the Augmented Dickey-Fuller (ADF) test, and allocating capital among selected spreads using MVO to maximize the portfolio's Sharpe ratio. The effectiveness of the strategy is evaluated over multiple test periods using key financial performance metrics

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