Quantification of Credit Exposures

QF622 Credit Risk Models

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Recap and Review

Key concepts of CCR

- CCR represents a combination of market risk (EAD) and credit risk (PD & LGD).
- Settlement risk vs. pre-settlement risk.
- Exposure is time-dependent and always conditional on counterparty default.
- Different types of market participants: major dealers, medium-sized players, financial end users, and clearing houses.
- Risk mitigants: netting, margining, central clearing, and hedging.

Risk mitigants of CCR

Netting

- Payment netting, currency netting, and portfolio compression.
- Close-out netting.

Collateral

- Variation margin, margin period of risk, and initial margin.
- Rehypothecation and segregation.
- CSA: two-way, one-way, thresholds, and minimum transfer amount.

Credit exposure

- Exposure is asymmetric – a party may lose but will not gain if the counterparty defaults.
- Current exposure vs. future exposure.
- Potential future exposure, expected positive exposure, and expected negative exposure.
- The loan equivalent amount as the time-weighted average of expected positive exposure.

Key Drivers of Credit Exposure

Future uncertainty (I)

- The most obvious driver of exposure is future uncertainty.
- This could be demonstrated by contracts with a single payoff at maturity, such as forward contracts or vanilla options.
- Intuitively, the exposure is a monotonically increasing function of time. The longer the time horizon, the higher the uncertainty.
- This is similar to the time value of an option.
- With the commonly made assumption, the exposure grows as a linear function of the square-root-of-time.

Future uncertainty (II)

Example

Assuming that the value of a contract follows

$$dV_t = \mu dt + \sigma dW_t,$$

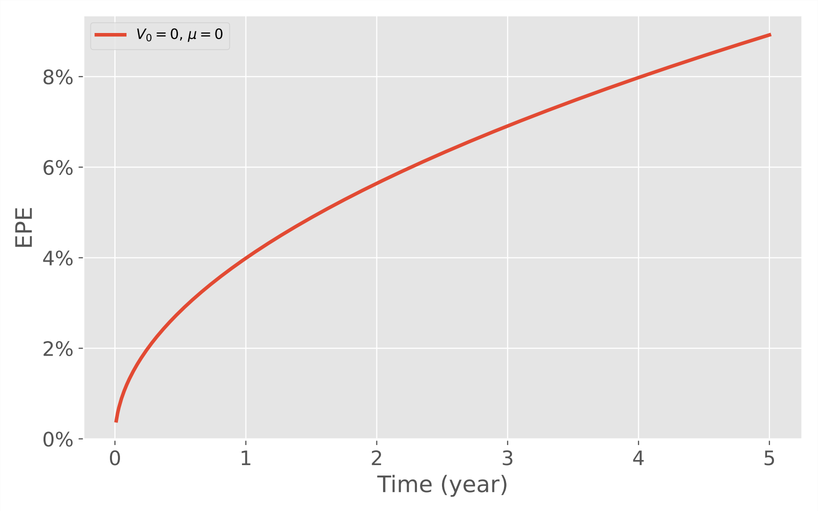
V_t is normally distributed with mean $V_0 + \mu t$ and standard deviation $\sigma\sqrt{t}$. As demonstrated previously,

$$E_t^+ = (V_0 + \mu t)\,\Phi\left(\frac{V_0 + \mu t}{\sigma\sqrt{t}}\right) + \sigma\sqrt{t}\phi\left(\frac{V_0 + \mu t}{\sigma\sqrt{t}}\right).$$

Specifically, when $V_0 = \mu = 0$, $E_t^+ = \sigma\sqrt{t}\phi\,(0)$.

This cannot be used in production to measure exposures but is good enough to get some intuition.

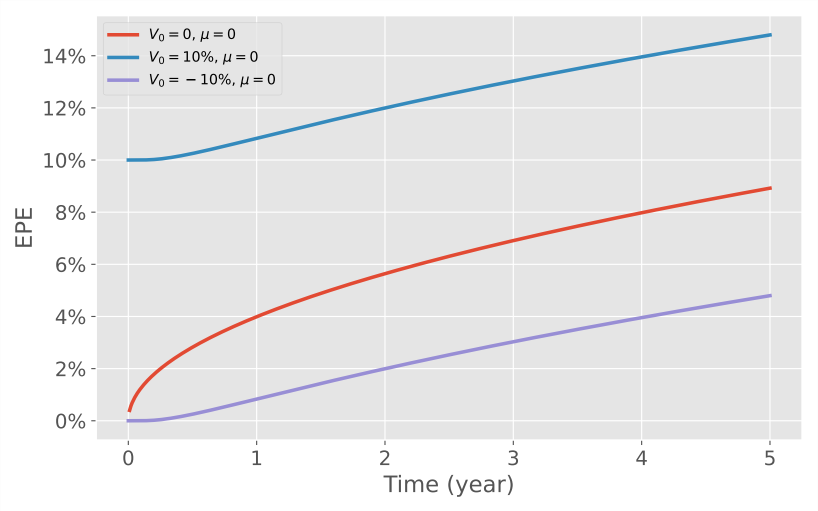
Future uncertainty (III)



Moneyness (I)

- The previous example assumes that the contract was entered into at par, i.e., with zero initial value.
- However, the exposure may have some initial moneyness: in-the-money (ITM) when it has positive value and vice versa for OTM.
- This is similar to the intrinsic value of an option.
- A portfolio that is ITM (OTM) for one party is OTM (ITM) for its counterparty.

Moneyness (II)

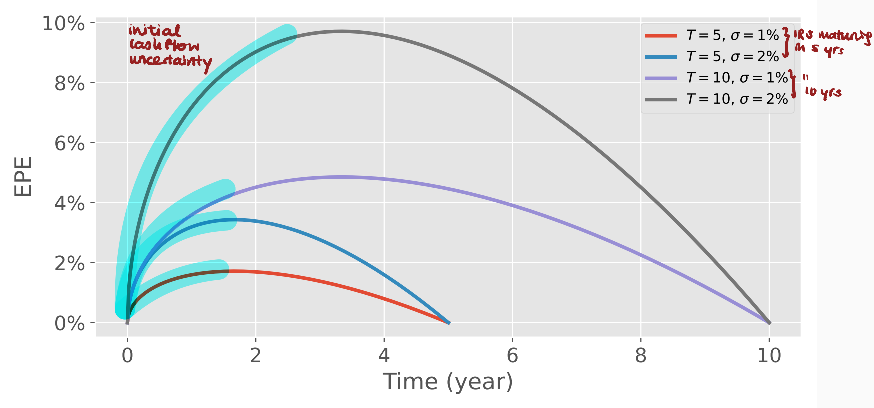


Cashflow uncertainty (I)

- Many derivative contracts have periodic cashflow, which offset the future uncertainty.
- The most common example is the exposure profile of an interest rate swap, which shows a peaked shape.
- The shape is the result of the interplay between future uncertainty and periodic cashflow.
- Roughly, the exposure is proportional to $(T - t)\sqrt{t}$, where T is the maturity of the contract.

Example

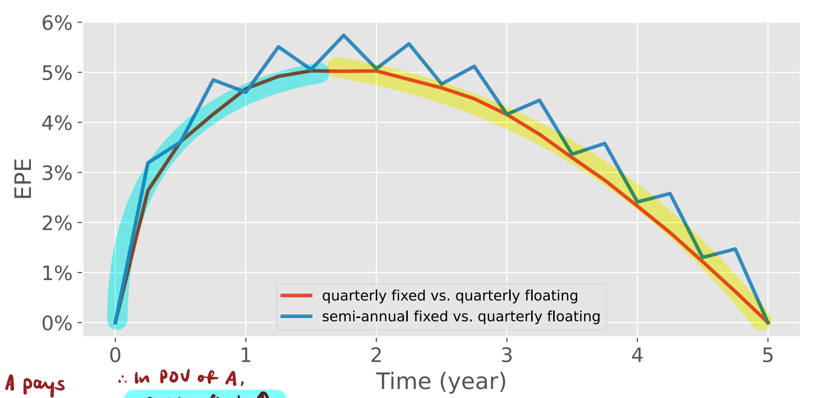
Assuming that V_t is normally distributed with zero mean and standard deviation $\sigma\sqrt{t}(T-t)$. We have $E_t^+ = \sigma\sqrt{t}(T-t)\phi(0)$.



Cashflow uncertainty (III)

Example – unmatched cashflow

When two legs have different frequencies, the trade may be "temporarily" ITM or OTM, which alters the exposure locally.



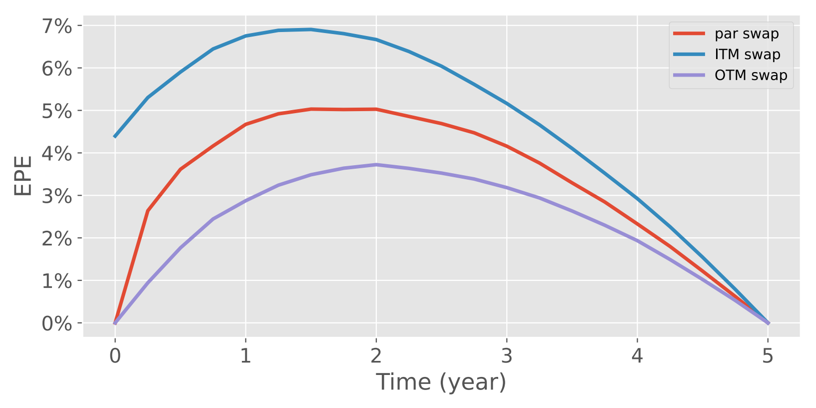
party A pays before party B, exposure of A ↑

∴ in POV of A,
∴ exposure first ↑
then exposure ↓
when party B's turn to pay

Cashflow uncertainty (IV)

Example – the effect of moneyness

Moneyness of the swap also shifts its exposure profile.



Combination of profiles (I)

- Some products could be viewed as a combination of other products.
- In the case of cross-currency swaps, it is a combination of two interest rate swaps (one for each currency) and an FX forward (for the final exchange of notional).
- In such cases, not only are the volatility of each individual risk factors important, the correlations among them drives the exposure too.

Combination of profiles II

Example

Assuming that the values of three components are

$$V_d^{IRS}(t) \sim N\left(0, \sigma_d\sqrt{t}(T-t)\right)$$

$$V_f^{IRS}(t) \sim N\left(0, \sigma_f\sqrt{t}(T-t)\right)$$

$$V^{FX}(t) \sim N\left(0, \sigma_X\sqrt{t}\right).$$

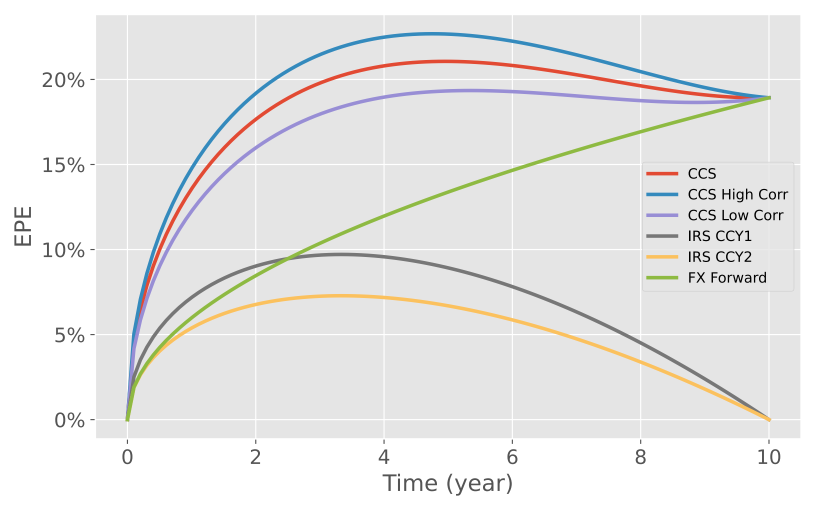
The sum of the three random variates:

$$V^{CCS}(t) \sim N\left(0, \sqrt{\mathbb{V}}\right)$$

$$\mathbb{V} = \left(\sigma_d^2 + \sigma_f^2 + 2\rho_{d,f}\sigma_d\sigma_f\right) t(T-t)^2 + \sigma_X^2 t + 2\sigma_X t(T-t)(\rho_{d,X}\sigma_d + \rho_{f,X}\sigma_f)$$

Combination of profiles III

Example



Optionality and physical settlement

- Options could be settled in cash upon exercise or physically into a position in the underlying.
- When settled in cash, the exposure is extinguished immediately. However, when settled physically, the underlying position may continue to generate exposures.
- A swaption being physically settle leads to a position in the underlying interest swap.
- As an example, a 5Y-into-5Y swaption have non-zero exposure for 10 years if it is physically settled but for 5 years if it is cash settled.

Aggregation and netting (I)

Aggregated exposure cannot be greater than the sum of of individual exposures as

$$\max\left(\sum V_i, 0\right) \leq \sum \max(V_i, 0).$$

	Trade 1	Trade 2	Portfolio	Trade 1	Trade 2	Portfolio
Scenario 1	20	30	50	20	-5	15
Scenario 2	-10	-5	-15	-10	30	20
Scenario 3	20	45	65	20	45	65
Scenario 4	30	0	30	30	-5	25
Scenario 5	5	-5	0	5	0	5
EFV	13	13	26	13	13	26
EPE	15	15	29	15	15	26
ENE	-2	-2	-3	-2	-2	0

Aggregation and netting (II)

Example

Assuming that we have a portfolio of N trades and $V_i \sim N(0, \sigma_i)$.

The value of the portfolio is therefore normally distributed with zero mean and standard deviation of

$$\sqrt{\sum_{i=1}^{N} \sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} \rho_{i,j}\sigma_i\sigma_j} = \bar{\sigma}\sqrt{N + N(N-1)\bar{\rho}},$$

by further assuming $\rho_{i,j} = \bar{\rho}$ and $\sigma_i = \bar{\sigma}$.

portfolio EPE

decreases with decreasing $\bar{\rho}$ or increasing N

$$\frac{E_p^+}{\sum_i E_i^+} = \frac{\bar{\sigma}\sqrt{N + N(N-1)\bar{\rho}}\,\phi(0)}{N\bar{\sigma}\phi(0)} = \sqrt{\frac{1 + (N-1)\bar{\rho}}{N}}$$

sum of individual EPEs

Collateralisation

- If the collateral position is known, it could be subtracted from the value of the portfolio in calculating the exposure.
- However, margin is not a a perfect risk mitigant due to various operational considerations.
- Thresholds and minimum transfer amount may lead to imperfect (over- or under-) collateralisation.
- There is always a delay in receiving margin and there could be margin disputes.
- The value of the margin may also change.

A Simulation-Based Framework

Introduction

- Monte Carlo simulation is the most generic approach to deal with the high dimensionality and complexities in modelling counterparty credit exposures.
- It is considered the state-of-the-art approach for years.
- Monte Carlo simulation is time consuming and trade-off must be made in modelling choices.
- Other simpler approaches have been in use too.

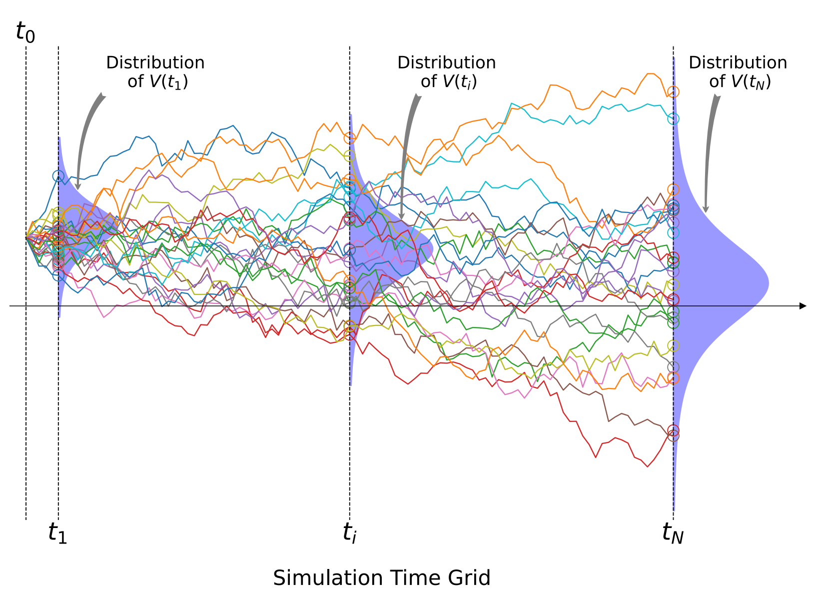
Parametric Approaches

- Parametric approaches are not model-based. They are parametrised simply and calibrated conservatively.
- They are simple to implement and not risk-sensitive.
- Examples include the current exposure method (CEM) and the new standardised approach for counterparty credit risk (SA-CCR) in the Basel framework.
- Both approximate the future exposure as the current exposure plus an add-on component representing the uncertainty.
- At trade level, the approach may not be able account for all specifics, e.g., moneyness, cashflow, maturity etc.
- At portfolio level, the approach usually accounts for netting and margin in a crude way.

Semi-analytical Approaches

- Semi-analytical approaches are more sophisticated but are usually limited in their applicability.
- They are based on simple distributional assumptions on the underlying risk factors.
- Based on the assumptions, exposure metrics are derived semi-analytically.
- Path-dependent aspects, including margining, are hard to incorporate.

A simulation-based approach



Key components

Scenario Generation

Future market scenarios are simulated for a fixed set of dates using evolution models of the risk factors.

Instrument Valuation

For each simulation date and for each realisation of the underlying market risk factors, instrument valuation is performed for each trade in the counterparty portfolio.

Portfolio Aggregation

For each simulation date and for each realisation of the underlying market risk factors, counterparty-level exposure is obtained by applying netting rules.

Some numerical aspects

Time grid

- Granular enough to capture local features of the exposure but not too much.
- Usually not evenly spaced with denser initial points.

Number of paths

- High enough to achieve desired level of convergence but not too high.
- Usually a lower number of paths is used when compared the typical configuration used for pricing.

Some numerical aspects – a demanding task

Example

Let's say we have a trading book with a number of trades transacted with various counterparties.

- For end-of-day P&L, we need to price each trade once.
- For end-of-day market risk VaR, we need to price each trade 250 times, assuming a one-year look-back period.
- For end-of-day CCR / CVA calculations, we need price each trade 500,000 times, assuming a time grid of 100 points and 5,000 paths.

Overview for risk factor modelling

1. Coverage vs. accuracy: the number of risk factors could be very substantial. Portfolio characteristics drive the choice of models.
2. Cross-asset dependency: stochastic interest rate for all other asset classes, with a much higher complexity.
3. Realistic vs. parsimonious: yield curves may twist, volatility may be stochastic. But should the model capture them? What's the cost and benefit?
4. Generic vs. bespoke: many derivatives are priced with dedicated valuation models. Exposure measurement must be done consistently. What's the implication?
5. For the CCR purpose, a simpler model is generally preferred.

Risk factor modelling – key considerations (I)

Compelxity of portfolio

A simple portfolio does not warrant an overly complex model. Most counterparty portfolios are dominated by linear products, such as interest rate swaps and FX forwards, etc.

Collateralisation

For margined portfolios, the treatment of margin period of risk is the key whereas simpler models could be used for risk factor modelling.

Wrong-way risk

If wrong-way risk is to be captured in the model, the basic setup needs to be simple enough for the model to be viable.

Risk factor modelling – key considerations (II)

Dimensionality

When the number of risk factors is high, a simpler model should be preferred. Complex marginal distribution modelling makes it harder to capture the co-dependence. The more factors to capture, the more unobservable correlations to mark.

Computational cost

The more complex the model is, the the higher the requirement on hardware and the longer it takes to generate exposure profiles.

Risk-neutral vs. Real-world measure (I)

Risk-neutral measure (aka the Q-measure)

- Routinely employed in valuing and risk managing derivative contracts through non-arbitrage pricing theory.
- Model parameters are typically calibrated to the market price of derivatives to the extent possible. Non-observable model parameters (e.g., correlations) may be estimated from historical data.
- For example, the famous Black-Scholes model is based on risk-neutrality and its key parameter, the volatility, is implied from market prices of options.
- The Q-measure is inherently more forward-looking, incorporating market expectation and risk premia.

Risk-neutral vs. historical measure (II)

Real-world measure (aka the P-measure)

- Used widely in risk management application with models calibrated to historical data, with the underlying assumption that history is a good indicator of the future.
- For example, a market risk VaR model, whether based on historical simulation or Monte Carlo simulation, is typically calibrated to historical data with a certain look-back period.
- P-measure models may be slow in responding to rapid changes in market condition. And when they do respond timely, its results may be procyclical.
- Calibration to a stressed historical period (as currently required for both market risk IMA and counterparty credit risk IMM) may produce more countercyclical results.

Risk-neutral vs. historical measure (III)

For CVA calculation

Both the market practices and regulatory requirements have converged to the use of Q-measure, where the drift and volatility are estimated from market prices. CVA is ultimately a pricing (and hedging) task, which demands the Q-measure to be used.

For CCR calculation

P-measure is widely used with volatility calibrated from historical data. However, drifts are often calibrated to market forward prices. This ensure the repricing of simple products in the CCR calculation.

Same models but different measures

It is possible to have one set of models but two set of parameters corresponding to the Q-measure and P-measure.

Interest rate

Interest rate is the most important risk factor for exposure quantification, not only for its product coverage but also for the fact that it impacts models for other asset class through stochastic interest rates.

Factors to consider

- Correlation / de-correlation among rates of different tenors.
- Intra-currency basis among different curves.
- Implementation / numerical complexity.
- Flexibility to calibrate to swaption prices (tenors, maturities, and strikes).
- Availability of analytical calibration.

The Hull-White one-factor model (HW1F)

The Hull-White one-factor model is the simplest model that may provide a reasonable compromise. It models the short rate as a Gaussian process with mean reversion. Both the mean reversion rate and the volatility could be time-dependent.

- It cannot capture the de-correlation across tenors.
- Intra-currency basis is assumed constant.
- The model is analytically tractable and easy to implement.
- Mean reversion rate is usually marked with the term structure of volatility calibrated to ATM co-terminal swaptions.
- Calibration to swaption prices are analytical and stable.
- Easier to account for in models for other asset classes.

Foreign exchange

The FX rate is typically modelled with a lognormal process with time-dependent volatility and stochastic interest rate.

Things to note

- Defining a base currency and treatment of cross pairs with the triangle rule.
- Treatment of pegged currencies, which are of low-volatility. Diffusion-based models cannot capture the depeg risk.
- Two HW1F models with a lognormal FX may be calibrated efficiently to ATM options for both IR and FX.
- Options on cross pairs and the marking of correlations.

Other asset classes

Equity

Equity spot is typically modelled with a lognormal process. A factor model may be used to avoid simulating a large number equity underlyings and the marking of a huge correlation matrix.

Inflation

Inflation has an FX analogy, where the inflation index (FX forward) is driven by the nominal-real (domestic-foreign) rate differential. Therefore, inflation could be modelled in the FX framework.

Commodities

Key considerations include price mean reversion, seasonality, spot vs. forward.

Cross-asset correlation

Example

We have a counterparty portfolio with vanilla interest rate and FX transactions, involving 10 currencies and hence 9 FX pairs. The number of correlation parameters is $\frac{19 \times 18}{2} = 171$. This number grows quadratically with the number of currencies.

The materiality consideration

- When a corporate client trades in the a single asset class, the intra-asset-class correlations are important.
- For a typical portfolio, inter-currency correlations are more important than intra-currency correlations (say across tenors).
- Materiality could be assessed through sensitivity analyses.

Cross-asset correlation – estimation

Correlation estimation

- It is generally not possible to calibrate correlations to market prices, barring some specific cases such as quantos or implied volatility on cross FX pairs.
- They are usually estimated from historical data, even when the Q-measure is adopted. The same is routinely practiced for derivative valuation.
- To avoid instability, a large dataset may be used while the re-estimation frequency is lower than that of other model parameters.

Overview of trade revaluation

Trade revaluation for exposure modelling is far more complex than running the same pricers for official valuations many times.

1. The sheer volume of vanilla trades means that their pricing functions must be optimised. This may require the pricers to be standardised leaving out some finer details for certain trades.
2. The Monte Carlo simulation needs to have memory on the trade lifecycle. For example, whether a barrier option is already knocked out at certain time point must be known for each path.
3. The time grid may not include all the important dates for all contracts in everyone counterparty portfolio.
4. Some more complex trades, priced with numerical methods (e.g, PDE or Monte Carlo) for official valuation, are too numerically expensive to be treated in the same way.

Treatment of complex products

Approximation

This tends to be product specific and ad hoc. For example, pricing Bermudan swaptions as European swaptions. The latter could be priced analytically while the former must be treated numerically.

Grids

A grid of values are generated using the official pricer, from which future MtM is interpolated given each simulated scenario. It works reasonably well when the number of risk factors is relatively low.

Regression-based pricing

This is the technique used to price American options with Monte Carlo simulation, where future simulated value of risk factors are used to approximate exposures.

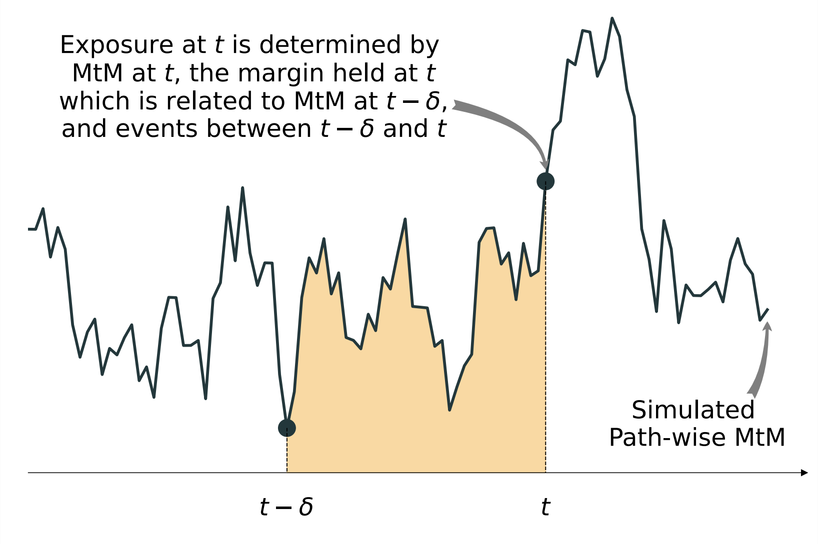
Correction for approximations

- It is generally accepted that the revaluations in exposure quantification do not fully represent the value of transactions.
- Some deviation from official valuation is not only expected but also unavoidable in many cases.
- A value of interest is the valuation difference at time zero.
- This difference, $\Delta V(t_0)$, may be used to adjust the simulated MtMs, e.g., a fixed shift or a scaling of $\frac{\Delta V(t_0)(T-t)}{T}$.
- This needs to be done at the trade level for each path.

Collateral modelling – MPoR

- Uncollateralised exposures are long-term in nature, spanning across the entire life of the transaction.
- It is transformed into risk over a short period with the exchange of variation margins.
- This period is from the last successful margin call in advance of the eventual default to the time when the loss becomes known.
- Many different things could happen during the MPoR:
  - margin disputes,
  - delay in recognising the default,
  - missing trade flows – a more serious breach of contract,
  - legal procedures when a default is deemed to have happened,
  - hedging, replacement and unwinding.

Understanding the MPoR



Modelling the MPoR

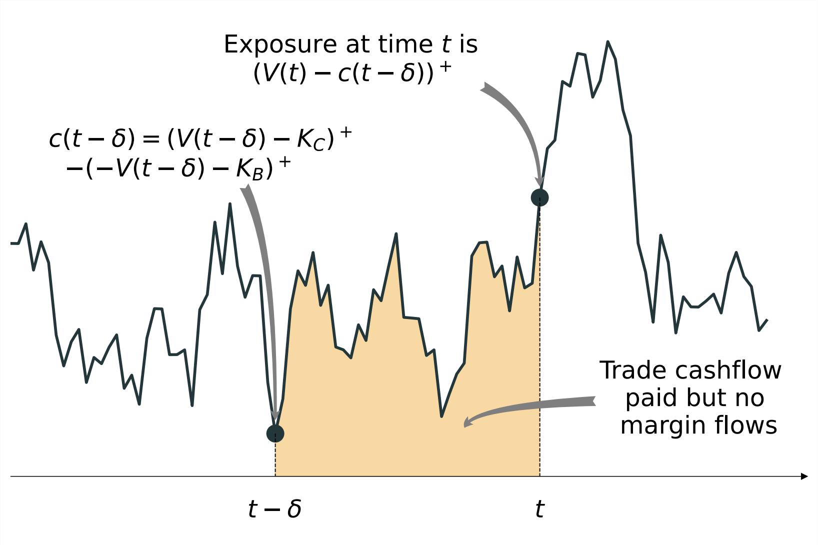
Common assumptions on the MPoR

- MPoR = 5d for centrally cleared portfolios.
- MPoR = 10d for bi-lateral uncleared portfolios as a baseline, doubled if the margin is illiquid or the netting set is large.
- Trade cash flows continued to be paid but not margin flows by both parties (the so called classical+ approach).

Rethinking MPoR

The 2018 quants of the year paper by Leif Andersen, Michael Pykhtin and Alexander Sokol titled Rethinking Margin of Period of Risk provides a detailed account on various events during the MPoR. Based on the timeline, various model setups were discussed.

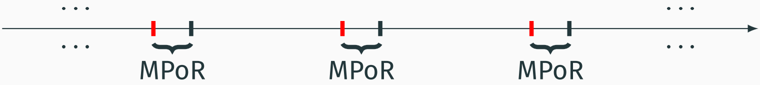
The classical+ approach (I)



The classical+ approach (II)

Time grid with look-back points

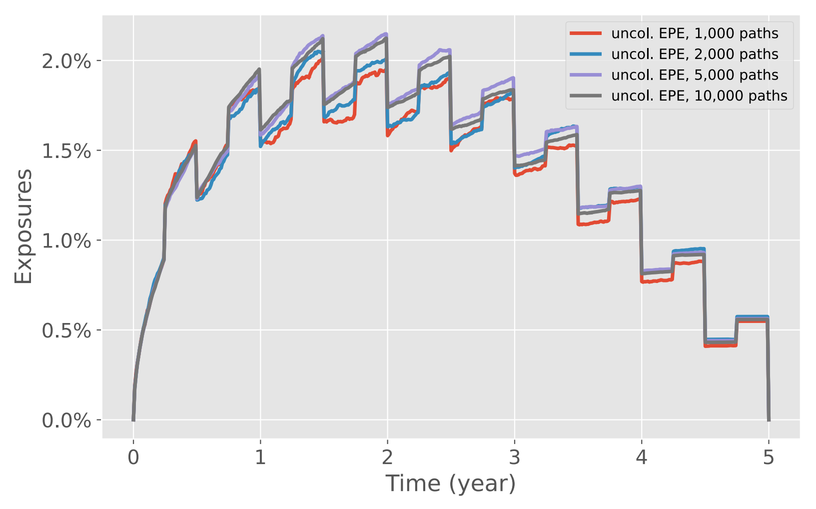
- Each time point in the time grid requires a look-back point.
- Exposure profiles are generated on **original time points** but not on the **look-back points**.
- This cannot account for minimum transfer amount, of which the impact is path-dependent.



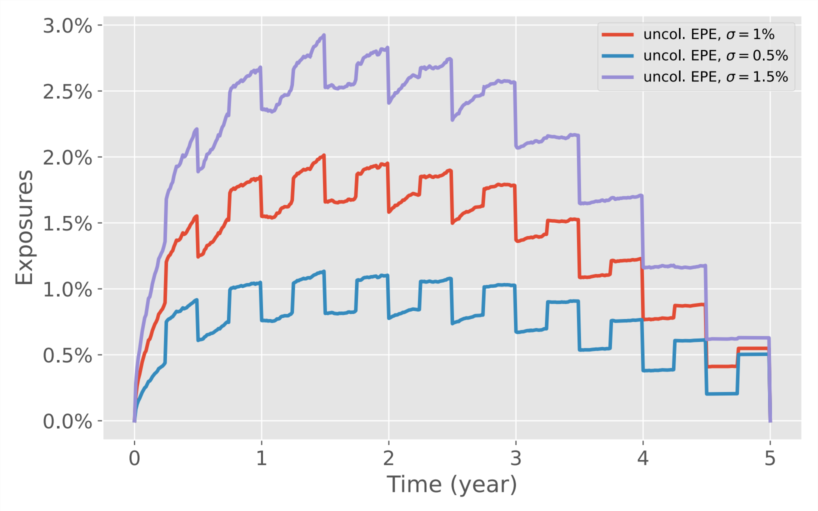
Examples with IRS

- All results generated on a single IRS using a toy model, where interest rate is flat and normally distributed.
- Base case market data: 2% interest rate and 1% interest rate volatility.
- Base case IRS feature: 5Y maturity, 2% fixed coupon paid semi-annually v.s. floating leg paid quarterly.
- Base case model configuration: 1,000 Monte Carlo paths – good enough to observe the exposure profile.

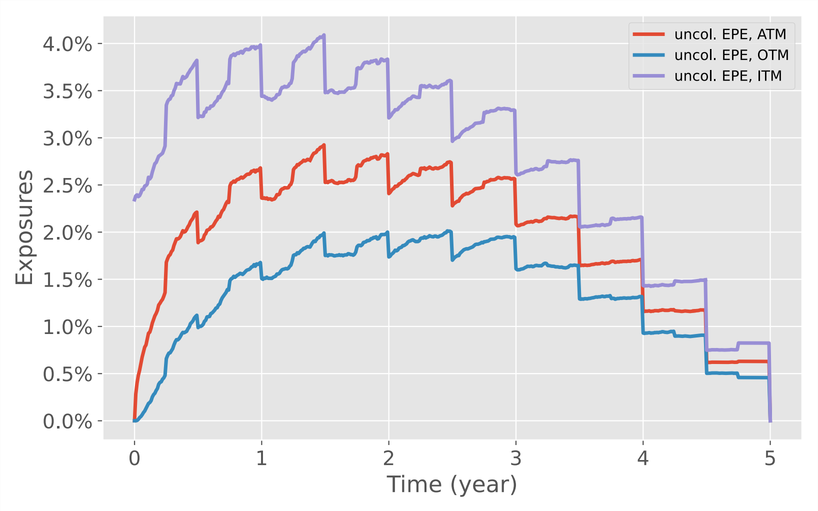
Examples with IRS – convergence



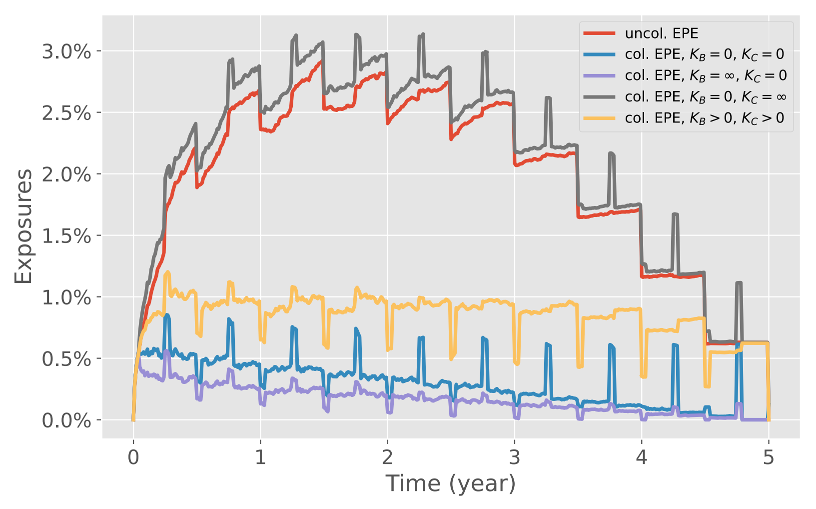
Examples with IRS – volatility



Examples with IRS – moneyness



Examples with IRS – collateral



Further Reading

1. John Gregory. *Counterparty Credit Risk and Credit Value Adjustment*. Wiley, 2010.

   Chapter 9.