

## QF622 Credit Risk Models – Assignment

Due on Monday, 16 June 2025

40 Marks in Total

1. The value of a company's equity is \$5m and its volatility is 50%. It needs to pay its debt of \$10m in 3 years. The risk-free rate is 1%. Price the company's debt using the Merton's model. Solve for its default probability and recovery rate. [2 marks]
2. The number of corporate default follows a Poisson distribution with an average number of default being 2 every year.
  - (a) Calculate the probability of more than 3 defaults in a year. [1 mark]
  - (b) Calculate the probability of no default within two years. [1 mark]
3. A one-year zero-coupon bond with a face value of \$100 is trading at \$95. Assume the default could only happen at the one-year point with a recovery rate of 40%, and the one-year continuously compounded risk-free rate to be 2%, determine the one-year probability of default. [1 mark]
4. A ten-year zero-coupon corporate bond with a face value of \$100 has a continuously compounded yield of 5%. A ten-year zero-coupon government bond with a face value of \$100 has a continuously compounded yield of 2%.
  - (a) Price both bonds. [1 mark]
  - (b) Calculate the continuously compounded yield spread for the corporate bond. [1 mark]
  - (c) Calculate the change in price (for a face value of \$100) for a 1bp change in interest rate, i.e., the IRPVo1, for both bonds from the bondholder's point of view. [1 mark]
  - (d) Determine how the government bond could be used to hedge the interest rate risk for the corporate bond. [1 mark]

5. A 5-year corporate bond, which pays annual coupons of 5%, has an asset swap spread of 400 bps (annual payments for the floating leg). Assuming that continuously compounded risk-free rate is 2%, price the bond. [2 marks]
6. Suppose the continuously compounded risk-free rate is flat at 3% and the hazard rate is flat at 5%.
- (a) Calculate the unconditional default probability in the first year from now. [1 mark]
  - (b) Calculate the unconditional default probability in the second year from now. [1 mark]
  - (c) Calculate the default probability in the second year conditional on survival in the first year from now. [1 mark]
  - (d) Price the zero recovery risky zero coupon bond with \$1 face value and 5-year maturity. [1 mark]
  - (e) Price a fixed payment of \$1 at default with 5-year maturity. [1 mark]
  - (f) Price a random payment, which is uniformly distributed between \$0 and \$1 independent of interest rate or hazard rate, with 5-year maturity. [1 mark]
7. Suppose the annual default probability of a reference entity is 5% conditional on no earlier default. Estimate its 5-year CDS spread paid annually, assuming that the continuously compounded risk-free rate is 1%, recovery rate is 40%, and default could only occur exactly at the 1-year, 2-year, ..., 5-year anniversaries. [3 marks]
8. Suppose that in a one-factor Gaussian copula model the 5-year default probability of each of the 5 credit names is 5% and the pairwise correlation is 0.25. Calculate, for the common factor value of  $-2$ ,  $-1$ ,  $0$ ,  $1$ , and  $2$ :
- (a) The default probability conditional on the common factor value. [1 mark]
  - (b) The probability of at least one default conditional on the common factor value. [1 mark]
9. Explain why a long forward contract subject to counterparty credit risk is a combination of a short position in a put and a long position in a call subject to counterparty credit risk. [2 marks]

10. A bank has two OTC transactions, trade A and trade B with a counterparty. The mark-to-market of trade A as a function of time is  $V_A(t) = \mu_A + \sigma_A W_A(t)$  while the mark-to-market of trade B as a function of time is  $V_B(t) = \mu_B + \sigma_B W_B(t)$ . The two Brownian motions are correlated such that  $\mathbb{E}[W_A(t)W_B(t)] = \rho t$ .
- Derive the bank's trade-level expected positive exposure profile of trade A and trade B, respectively. [2 marks]
  - Derive the bank's portfolio-level expected positive exposure profile if there is no netting agreement between the bank and its counterparty. [2 marks]
  - Derive the bank's portfolio-level expected positive exposure profile if there is a netting agreement (with zero thresholds and no minimum transfer amount) between the bank and its counterparty. [3 marks]
11. A bank (B) has a portfolio of OTC transactions with a counterparty (C) covered by a two-way CSA. The CSA stipulated that the threshold for B is \$20m, the threshold for C is \$10m, and the minimum transfer amount is \$5m. Let  $t_1$  and  $t_5$  be five consecutive days on which variant margins are exchanged. Fill in the blanks (a)-(i) in the table below. [3 marks]

Time	Portfolio MtM (for B)	Credit Support Balance	Collateral Called (positive) or Posted (negative)
$t_1$	\$13m	\$0m	(a)
$t_2$	-\$26m	(b)	(c)
$t_3$	\$16m	(d)	(e)
$t_4$	-\$27m	(f)	(g)
$t_5$	\$11m	(h)	(i)

12. Identify transactions with wrong-way risk and right-way risk from the below. Explain for each transaction.
- An investor buys a call option on the stock of a major international bank from another major international bank. [1 mark]
  - A bank trades a forward contract on USD/CCY with a sovereign fund, where CCY is the domestic currency of the sovereign fund's home country. With the forward contract, the bank is committed to sell a certain amount of CCY at the fixed price with USD. [1 mark]
  - A corporate funds its business with floating rate loans. The corporate swaps its floating rate liability into fixed rates with a Bank through an interest rate swap, i.e, the bank pays floating rate and receives fixed rate. (Hint: Do interest rates rise or fall during recession?) [1 mark]

13. The chart below compares the potential future exposure (PFE) profiles of a long 5Y CDS contract (long protection where the recovery rate is 40%) at two different confidence levels, namely 95th percentile and 97.5th percentile. Explain the difference between the two profiles. [3 marks]

