QF623: Portfolio management I
Bottom-up equity anomalies and risk premia I

Risk factor models

- A risk factor is a pattern that explains the cross-section (not the time variation) of asset returns and that will explain the cross-section of asset returns in the future.

- Factor models are used to
  - Decompose risk and return attribution (systematic vs. idiosyncratic)
  - Generate estimates of abnormal return (new alpha source)
  - Factorize covariance matrix of returns

General model specification

- Factor models decompose the returns of a security, $r_{i,t}$, into factor-specific and asset-specific returns and have the following general form:

$$r_{i,t} = \alpha_i + \beta_{i,1} f_{1,t} + \cdots + \beta_{i,K} f_{k,t} + \varepsilon_{i,t}$$

  where $\beta_{i,K}$ id the factor loading/ exposures/ betas of stock i on the k^{th} factor, $f_{k,t}$ is the k^{th} common factor and $\varepsilon_{i,t}$ is the asset specific factor (idiosyncratic return)

- The above specification can be rewritten in matrix format. Collecting data from assets $i = 1, \ldots, N$ across time $t = 1, \ldots, T$, we get:

$$\underset{(T \times N)}{\mathbf{R}} = \underset{(T \times 1)}{\mathbf{1}} \underset{(1 \times N)}{\boldsymbol{\alpha}^{\mathbf{T}}} + \underset{(T \times K)}{\mathbf{F}} \underset{(K \times N)}{\mathbf{B}^{\mathbf{T}}} + \underset{(T \times N)}{\mathbf{E}}$$

Types of factor models

- Macroeconomic factor model
  - Factors are observable economic and financial time series (i.e. $\mathbf{F}$ is known and $\mathbf{B}$ is estimated given observable $\mathbf{F}$)
  - Time-series regression
  - Sharpe's single factor model (CAPM)

- Fundamental factor model
  - Factor betas are created from observable asset characteristics (fundamentals) such as industry classification, market capitalization, style classification (e.g. value, growth), etc (i.e. $\mathbf{B}$ is known)
  - Factor returns or realizations, $\mathbf{F}$, are estimated for each time period t given $\mathbf{B}$.

- Statistical factor model
  - Factors are unobservable and extracted from asset returns
  - Traditional factor analysis and principal component analysis (PCA) are usually applied to extract factor realizations
  - Axioma risk models

Estimation of fundamental factor models

- Barra approach
  - Pioneered by Bar Rosenberg and discussed at length in Grinold and Kahn (2000), Conner et al (2010) and Carino et al (2010)
  - In this approach, the observable asset specific fundamentals (or some transformation of them, e.g. z-scoring) are treated as the factor betas, $\mathbf{B}$, which are time invariant
  - Econometric problem is then to estimate the factor returns (realizations), $\mathbf{F}$, at time t given $\mathbf{B}$. This is done by running T cross-sectional regressions, where T is the number of periods used to estimate the factor model

Estimation of fundamental factor models

- Fama-French (1992) approach
  - For a given observed asset specific characteristic, e.g. size, they determine factor realizations using a two step process.
  - First, they sort the cross-section of assets based on the values of the asset specific characteristic.
  - Second, they form a hedge portfolio which is long in the top quintile of the sorted assets and short in the bottom quintile of the sorted assets (Q5-Q1 factor mimicking portfolios). The observed return on this hedge portfolio at time *t* is the observed factor realization for the asset specific characteristic.
  - Process is reported for each asset specific characteristic
  - Given the observed factor realizations for $t = 1, \ldots, T$, the factor betas, $\mathbf{B}$, for each asset are estimated using N time series regressions.

From risk factor models to factor investing

- Risk factors were initially based on systematic and common risks. They now embed other dimensions, such as anomalies or trading strategies.

Example of a fundamental factor model (Bloomberg Japanese Equity Fundamental Model)

- If there is predictability in the performance of the risk factor, then it becomes factor investing!

Why factor investing?

- At the security level, there is a lot of idiosyncratic risk or alpha.

	Common Risk	Idiosyncratic Risk
GOOGLE	47%	53%
NETFLIX	24%	76%
MASTERCARD	50%	50%
NOKIA	32%	68%
TOTAL	89%	11%
AIRBUS	56%	44%

- Idiosyncratic risk decreases with the breath of the investment portfolios => Diversification

Portfolio	Common Risk	Idiosyncratic Risk
Renaissance Europe	69.2%	30.8%
Threadneedle Pan European SC	87.5%	12.5%
Franklin Mutual European	90.2%	9.8%
SG Actions Euro Value	91.7%	8.3%
Metropole Selection	91.8%	8.2%
Allianz Europe Equity Growth	92.0%	8.0%

Why factor investing?

- Fundamental law of active management (to be derived later)

$$\mathrm{E}(R_A) = IC\sqrt{N}\sigma_A$$

  where R_A is the active return, IC is the information coefficient (manager skill), N is the number of bets and σ_A is the active risk

- Factor investing is scalable.
  - Coverage
  - Capacity

Risk premium, risk premia and risk factors

- CAPM
  - There is one risk premium, which is captured by the market portfolio.

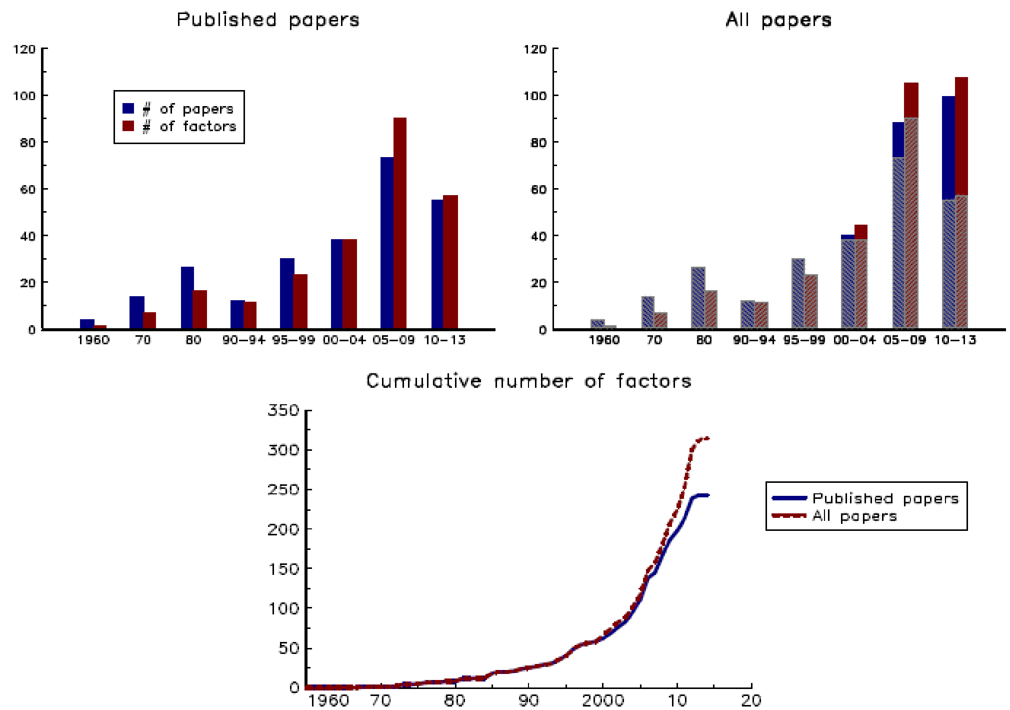
- Factor investing
  - Apart from the market risk premium, there are other risk premia which corresponds to rewarded risk factors.
  - If XMY (e.g. HML) is a risk factor, -XMY (e.g. LMH) is a risk factor
  - If XMY (e.g. HML) is a risk premia, -XMY (e.g. LMH) is not a risk premia
  - XMY is a risk premia $\Rightarrow$ XMY is a risk factor
  - XMY is a risk factor $\not\Rightarrow$ XMY is a risk premia

Quick facts about risk factors

- Risk factors are a powerful tool to understand the cross-section of (expected) returns
- Common risk factors explain more variance than idiosyncratic risks in diversified portfolios
- Risk premia (or persistence of risk factors) are time-varying and low-frequency mean-reverting, i.e. the cycle can span years (and hence the validity of factor timing)
- Risk factors are local, not global, i.e. risk factors are not homogenous
- Factor investing (or “smart beta”) is nothing new, and has been used by asset managers and hedge funds for a long time.

Factor zoo

Figure: Harvey *et al.* (2014)



"Now we have a zoo of new factors" (Cochrane, 2011)

The alpha puzzle or evolution
Cochrane, 2011

- Chaos

$$\mathbb{E}[R_i] - R_f = \boxed{\alpha_i}$$

- Sharpe (1964)

$$\mathbb{E}[R_i] - R_f = \beta_i^m (\mathbb{E}[R_m] - R_f)$$

- Chaos again

$$\mathbb{E}[R_i] - R_f = \boxed{\alpha_i} + \beta_i^m (\mathbb{E}[R_m] - R_f)$$

- Fama and French (1992)

$$\mathbb{E}[R_i] - R_f = \beta_i^m (\mathbb{E}[R_m] - R_f) + \beta_i^{smb}\mathbb{E}[R_{smb}] + \beta_i^{hml}\mathbb{E}[R_{hml}]$$

The alpha puzzle or evolution
Cochrane, 2011

- Chaos again

$$\mathbb{E}[R_i] - R_f = \boxed{\alpha_i} + \beta_i^m(\mathbb{E}[R_m] - R_f) + \beta_i^{smb}\mathbb{E}[R_{smb}] + \beta_i^{hml}\mathbb{E}[R_{hml}]$$

- Carhart (1997)

$$\mathbb{E}[R_i] - R_f = \beta_i^m(\mathbb{E}[R_m] - R_f) + \beta_i^{smb}\mathbb{E}[R_{smb}] + \beta_i^{hml}\mathbb{E}[R_{hml}] + \beta_i^{wml}\mathbb{E}[R_{wml}]$$

- Chaos again

$$\begin{aligned}\mathbb{E}[R_i] - R_f \quad = \quad & \boxed{\alpha_i} + \beta_i^m(\mathbb{E}[R_m] - R_f) + \beta_i^{smb}\mathbb{E}[R_{smb}] + \\ & \beta_i^{hml}\mathbb{E}[R_{hml}] + \beta_i^{wml}\mathbb{E}[R_{wml}]\end{aligned}$$

- Etc.

Asset pricing model keeps getting updated!

A note on equity backtesting

- Use of historical constituents important
  - **Survivorship bias** is the tendency for mutual funds with poor performance to be dropped by mutual fund companies, generally because of poor results or low asset accumulation. This phenomenon, which is widespread in the fund industry, results in an overestimation of the past returns of mutual funds
- Price adjustments
  - Corporate actions such as stock splits, etc
- Price return vs. total return
- Implementation lag

Fama-French (FF) risk factors

- Fama-French three factor model

$$\mathrm{E}(R_i) - R_f = \beta_i^m\left(\mathrm{E}(R_m) - R_f\right) + \beta_i^{smb}\mathrm{E}(R_{smb}) + \beta_i^{hml}E(R_{hml})$$

where R_{smb} is the return of small stocks minus large stocks, and R_{hml} is the return of stocks with high book-to-market ratios minus the return of stocks with low book-to-market ratios

- The factors are defined as follows:

$$SMB_t = \frac{1}{3}(R_t(SV) + R_t(SN) + R_t(SG)) - \frac{1}{3}(R_t(BV) + R_t(BN) + R_t(BG))$$

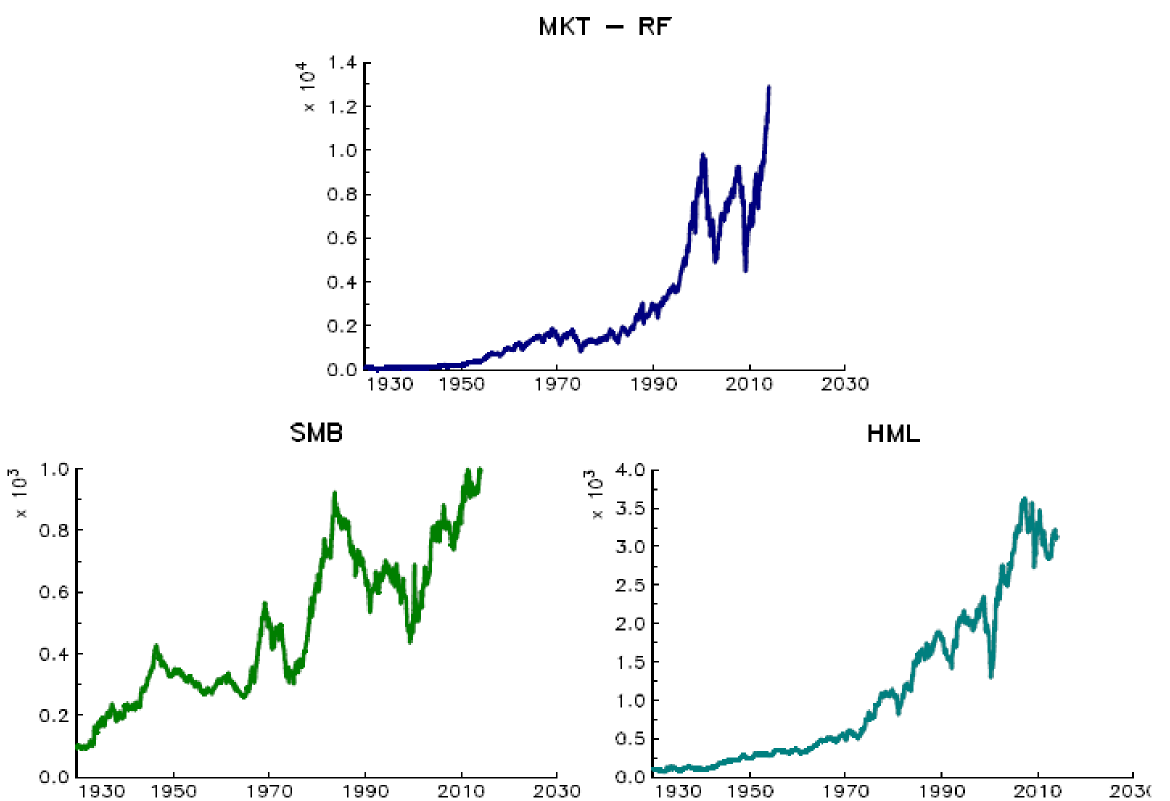
$$HML_t = \frac{1}{2}(R_t(SV) + R_t(BV)) - \frac{1}{2}(R_t(SG) + R_t(BG))$$

with the following portfolio definitions:

	Value	Neutral	Growth
Small	SV	SN	SG
Big	BV	BN	BG

FF US risk factor performance

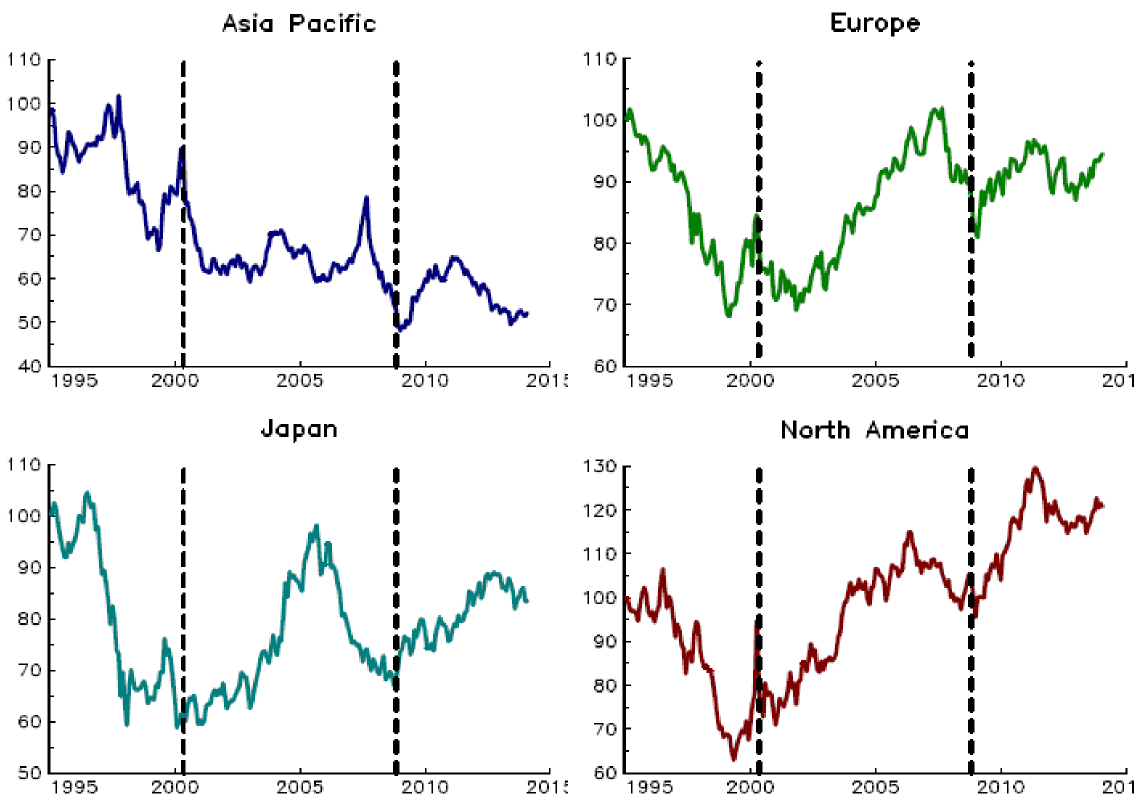
Figure: Fama-French US risk factors (1930 - 2013)



Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing; Data is from Kenneth French's website; US data is from CRSP, non-US data is from a combination of MSCI and Bloomberg.

FF global risk factor performance
Size (SMB)

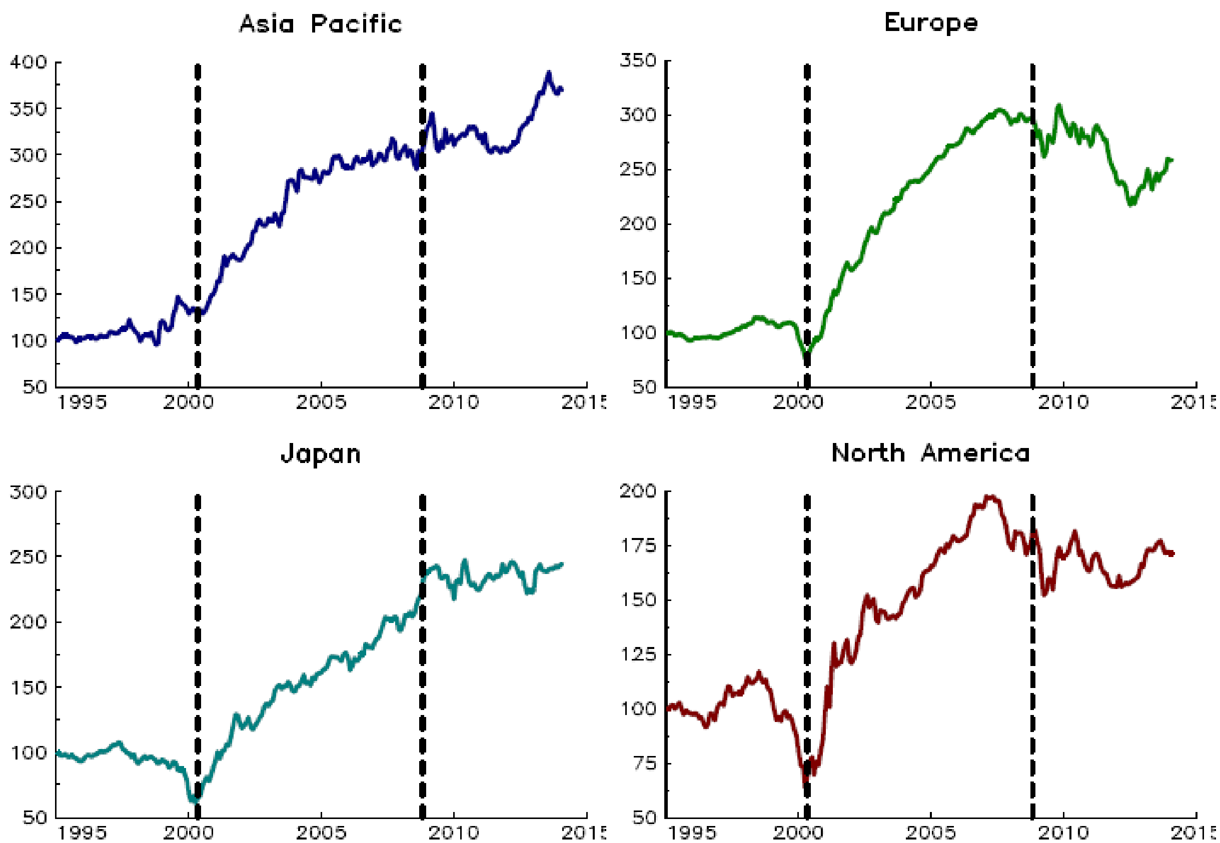
Figure: Fama-French SMB factor (1995 – 2013)



Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing; Data is from Kenneth French's website; US data is from CRSP, non-US data is from a combination of MSCI and Bloomberg.

FF global risk factor performance
Value (HML)

Figure: Fama-French HML factor (1995 – 2013)

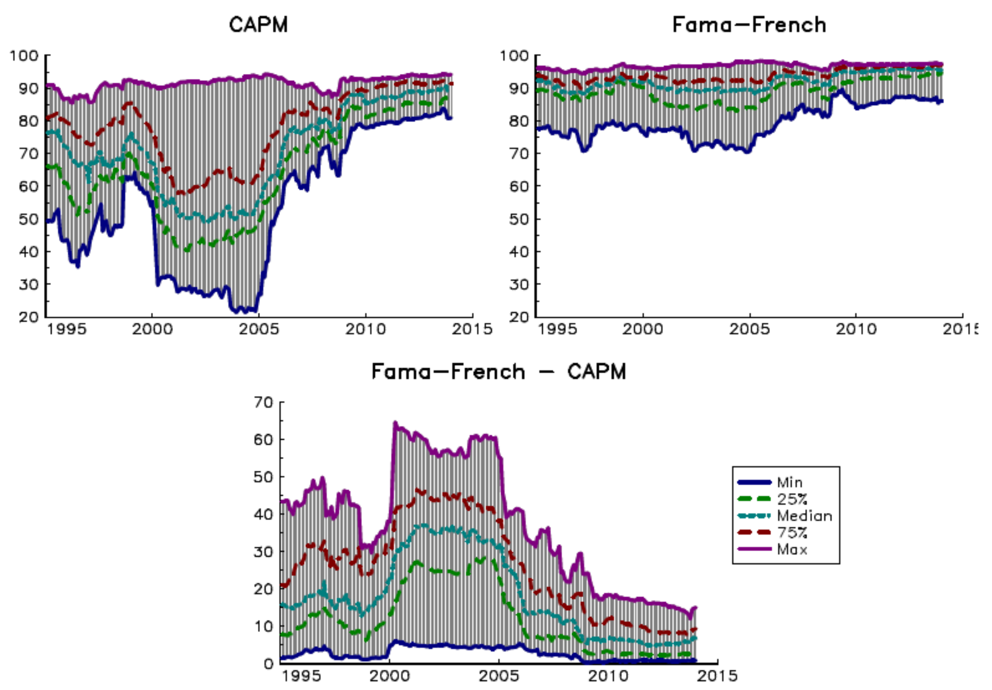


Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing; Data is from Kenneth French's website; US data is from CRSP, non-US data is from a combination of MSCI and Bloomberg.

Cross-section of US stock returns
Incremental explanatory power of FF model

- Performed regressions on 100 value-weighted portfolios based on independent sorts into 10 size and book-to-market groups

Figure: R^2 coefficient (in %) – US



Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

Cross-section of global stock returns
Incremental explanatory power of FF model

Table 7: Average of ΔR^2 (in %) for different periods

Model	Asia Pacific	Europe	Japan	North America	US
			1995 / 2013		
SMB	7.1	7.8	13.3	11.8	11.0
HML	2.7	3.4	4.0	6.4	8.0
FF	9.9	11.2	16.9	17.5	19.8
			2004 / 2013		
SMB	4.9	4.5	12.2	5.0	5.3
HML	1.5	1.1	2.9	1.8	2.4
FF	6.4	5.6	14.7	6.7	7.6
			2009 / 2013		
SMB	5.1	4.3	9.4	4.5	4.7
HML	1.7	1.4	3.0	1.5	2.1
FF	6.7	5.7	12.3	5.8	6.8

Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

Market beta has been strong recently

Table: Average of $R^2_{\mathrm{FF}} - R^2_{\mathrm{CAPM}}$ (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	12.1	13.7	10.0	17.9	18.0
1996	11.7	14.4	9.8	22.5	23.0
1997	12.7	17.6	11.1	22.4	20.2
1998	13.0	19.1	14.0	21.1	18.4
1999	12.8	19.9	15.2	19.2	19.2
2000	13.1	27.2	20.4	29.5	31.6
2001	13.0	26.4	21.1	30.3	36.1
2002	12.3	23.4	20.9	28.6	35.0
2003	13.3	20.3	19.4	27.3	34.4
2004	13.5	17.5	19.3	27.1	33.2
2005	11.5	11.6	13.9	17.7	23.7
2006	11.3	8.8	14.2	13.0	15.7
2007	12.5	7.5	15.4	11.3	13.6
2008	9.6	6.3	15.8	10.0	11.4
2009	6.1	5.0	15.5	7.1	7.8
2010	5.9	5.7	15.0	6.8	7.9
2011	5.4	5.1	14.1	5.9	6.9
2012	4.8	4.9	13.7	5.3	6.3
2013	5.3	5.1	12.1	5.3	6.3

Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

The size effect in the HML risk factor

- SHML is the HML factor for small stocks

- BHML is the HML factor for big stocks

$$
\begin{aligned}
HML_t &= \frac{1}{2}(R_t(SV) + R_t(BV)) - \frac{1}{2}(R_t(SG) + R_t(BG)) \\
&= \frac{1}{2}(R_t(SV) - R_t(SG)) + \frac{1}{2}(R_t(BV) - R_t(BG)) \\
&= \frac{1}{2}SHML_t + \frac{1}{2}BHML_t
\end{aligned}
$$

- The HML factor may be biased towards a size factor because:
  - SHML factor contributes more than the BHML factor
  - BHML factor is itself biased by a size effect

The size effect in the HML risk factor

Figure: Fama-French SHML, BHML and HML factors (1995 – 2013

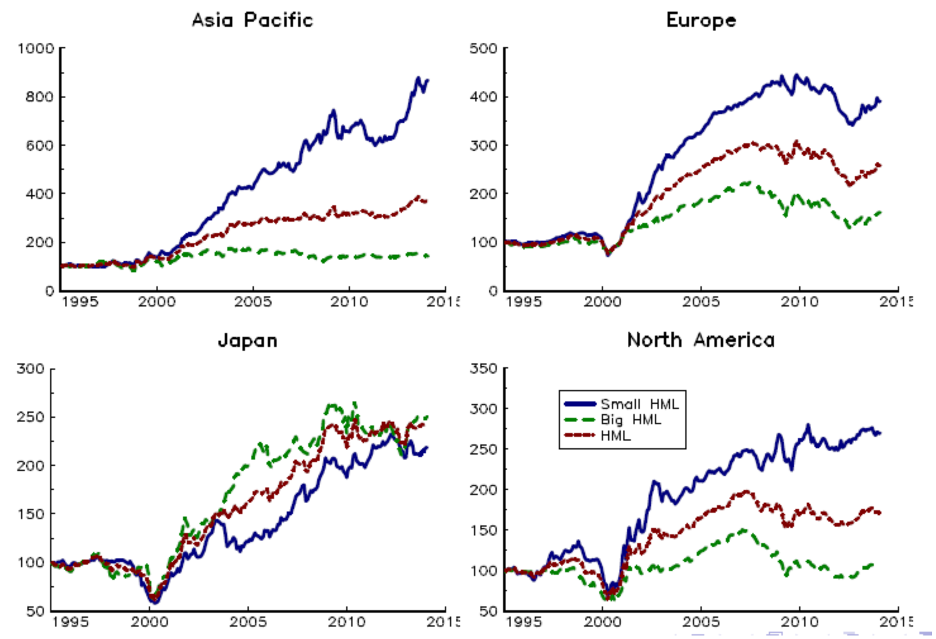


Table: Performance of the SHML, BHML and HML factors (1995 – 2013)

Statistic	Factor	Asia Pacific	Europe	Japan	North America	US
$\mu(x)$	SHML	12.0	7.4	4.2	5.4	5.1
	BHML	1.8	2.6	5.0	0.2	−0.6
	HML	7.1	5.2	4.8	2.9	2.4
$\sigma(x)$	SHML	11.7	10.0	11.0	15.2	13.4
	BHML	15.2	11.0	13.3	11.2	11.9
	HML	11.5	9.0	10.3	12.1	11.5
$\mathrm{SR}(x \mid r)$	SHML	1.03	0.74	0.38	0.35	0.38
	BHML	0.12	0.24	0.38	0.02	−0.05
	HML	0.61	0.57	0.47	0.24	0.20

Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

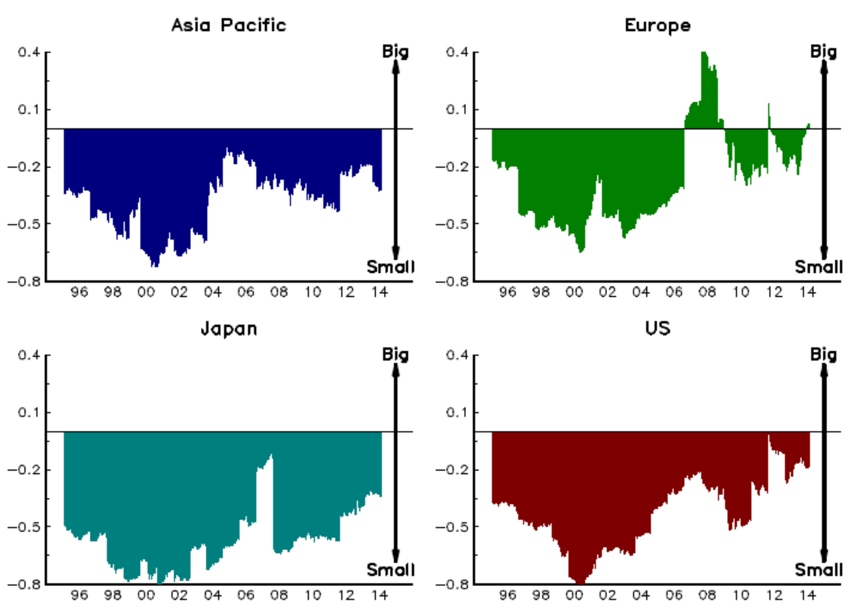
The size bias of the BHML factor

- Define the size ratio between big value and big growth portfolios as:

$$\mathrm{I}\{\overline{ME}_t(BG) \geq \overline{ME}_t(BV)\}.\left(\frac{\overline{ME}_t(BV)}{\overline{ME}_t(BG)} - 1\right) - \mathrm{I}(\overline{ME}_t(BG) < \overline{ME}_t(BV)).\left(\frac{\overline{ME}_t(BG)}{\overline{ME}_t(BV)} - 1\right)$$

where $\overline{ME}_t(BV)$ and $\overline{ME}_t(BG)$ are the average market values of the stocks in the big value and big growth portfolios

Figure: Size ratio between the big value and the big growth portfolios



Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

Implementation considerations
FF vs. fund-based risk factors

- Cazalet (2014) build SMB and HML factors for the period 2000-2014 from the Morningstar database which contains monthly returns for different mutual funds invested globally.
- Differentiate styles to obtain small Value (SV), small blend (SB), small growth (SG), large value (LV), large blend (LB) and large growth (LG)
- Mutual fund (MF) based SMB and HML are calculated as per the definition provided by Fama-French, substituted with the appropriate mutual fund styles

Table 13: Correlation between FF and MF risk factors (1999 – 2013)

Factor	Europe	Japan	US
SMB	79.8	86.0	93.9
HML	55.5	54.3	84.8

Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

FF versus fund-based risk factors
US and Europe

Figure: Comparison between FF and MF HML risk factors (US, 1999-2014))

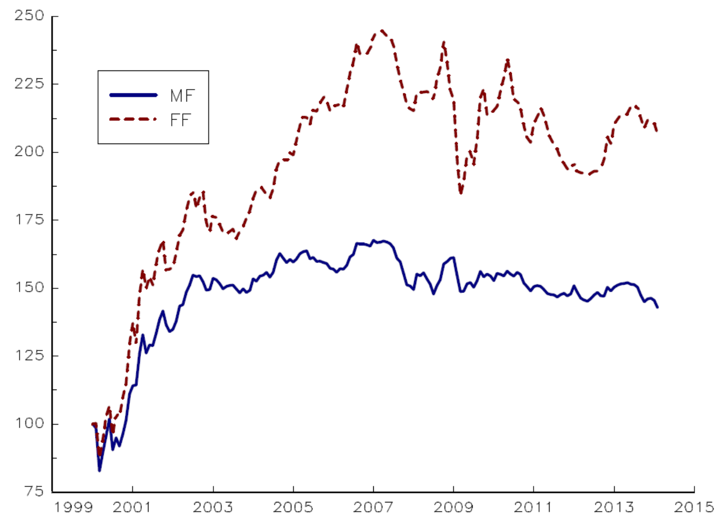
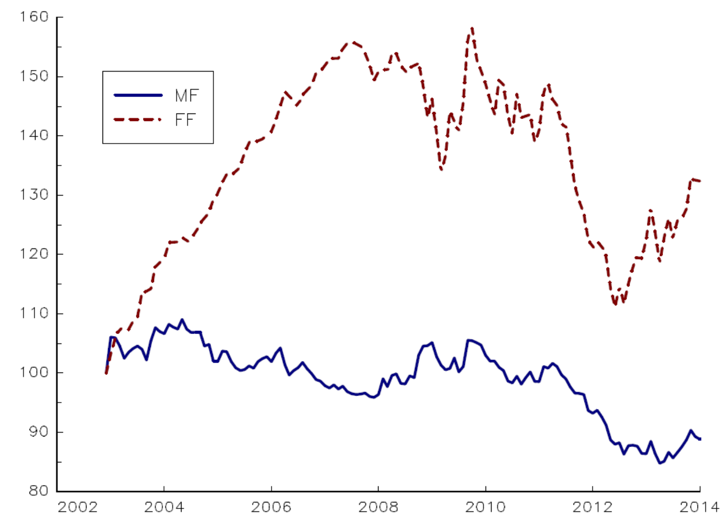


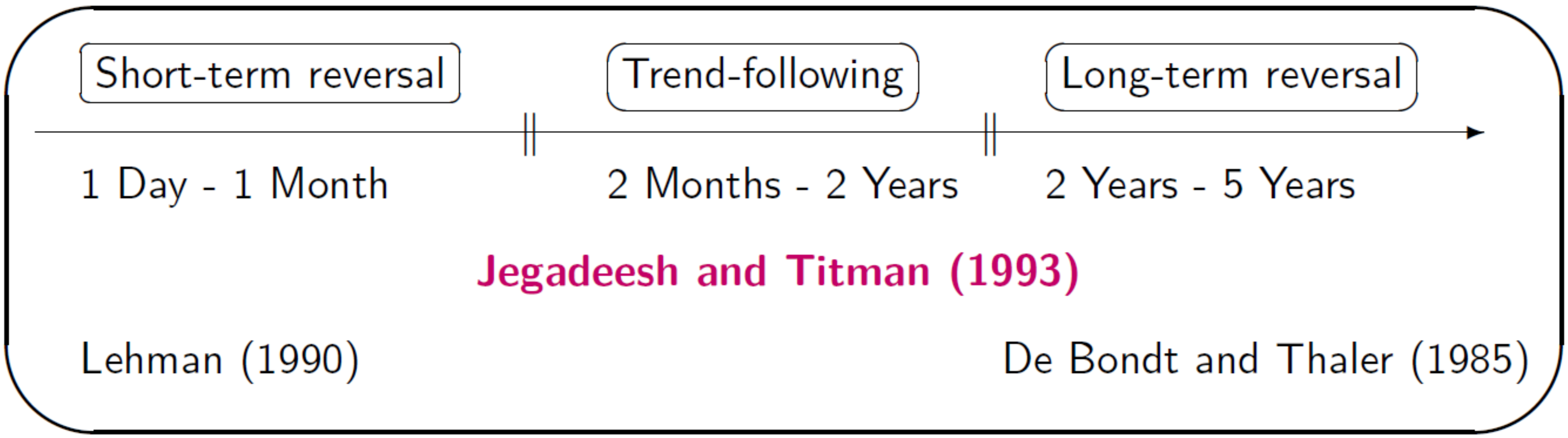
Figure: Comparison between FF and MF HML risk factors (Europe, 1999-2014))



Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

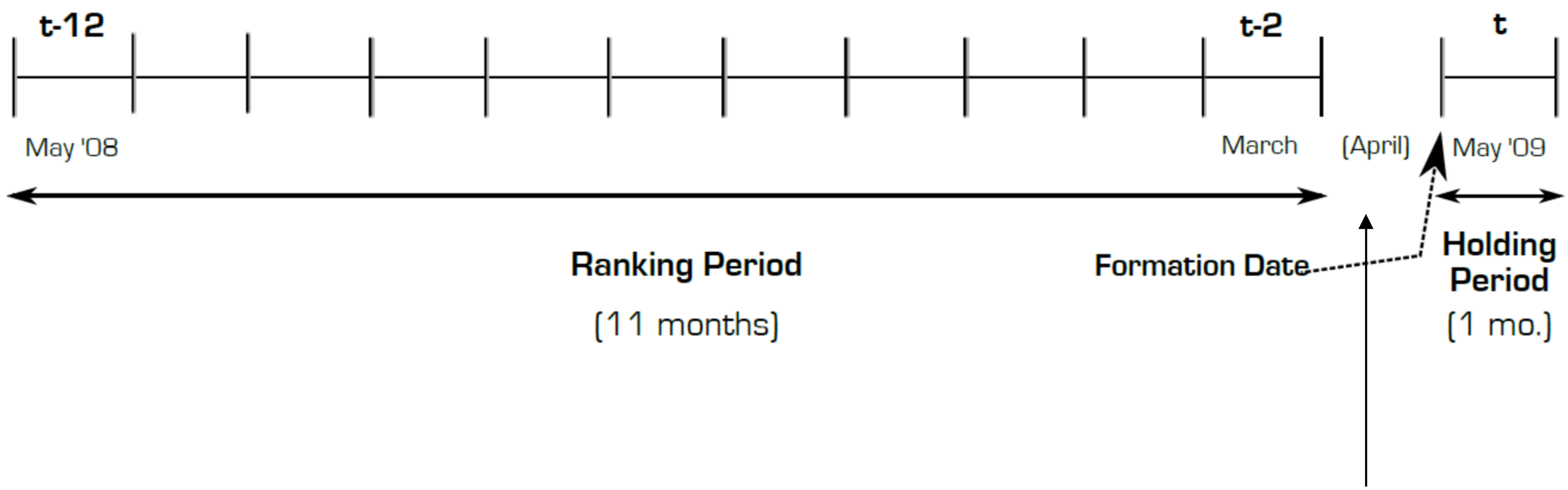
- Why the difference?
  - Liquidity issues
  - Shorting constraints
  - Risk budget

Momentum



Short-term reversal
Trend-following
Long-term reversal
1 Day - 1 Month
2 Months - 2 Years
2 Years - 5 Years
Jegadeesh and Titman (1993)
Lehman (1990)
De Bondt and Thaler (1985)

Momentum portfolio formation



To exclude short-term reversal effect

FF-Carhart four-factor model

- Fama-French-Carhart four factor model

$$\mathrm{E}(R_i) - R_f = \beta_i^m\left(\mathrm{E}(R_m) - R_f\right)$$
$$+\beta_i^{smb}\mathrm{E}(R_{smb}) + \beta_i^{hml}E(R_{hml}) + \beta_i^{wml}E(R_{wml})$$

  where R_{smb} is the return of small stocks minus large stocks, R_{hml} is the return difference between high and low book-to-market ratio stocks and R_{wml} is the return difference of winner and loser stocks of the past 2-12 months.

- The factors are defined as follows:

$$WML_t = \frac{1}{2}(R_t(SW) + R_t(BW)) - \frac{1}{2}(R_t(SL) + R_t(BL))$$

  with the following portfolio definitions:

	Loser	Average	Winner
Small	SL	SA	SW
Big	BL	BA	BW

Performance of the WML factor

Table: Performance of the WML factor

Statistic	Period	Asia Pacific	Europe	Japan	North America	US
$\mu(x)$	#1	3.8	19.9	8.7	22.6	17.6
	#2	11.9	11.5	−0.3	1.4	3.7
	#3	5.6	2.9	−2.0	−4.5	−9.3
$\sigma(x)$	#1	24.7	12.8	22.1	18.7	14.5
	#2	12.7	15.9	14.1	20.0	20.1
	#3	15.2	17.3	14.0	15.3	19.9
$\mathrm{SR}(x \mid r)$	#1	0.15	1.56	0.40	1.21	1.22
	#2	0.93	0.72	−0.02	0.07	0.19
	#3	0.37	0.17	−0.14	−0.30	−0.47

#1 January 1995 – March 2000

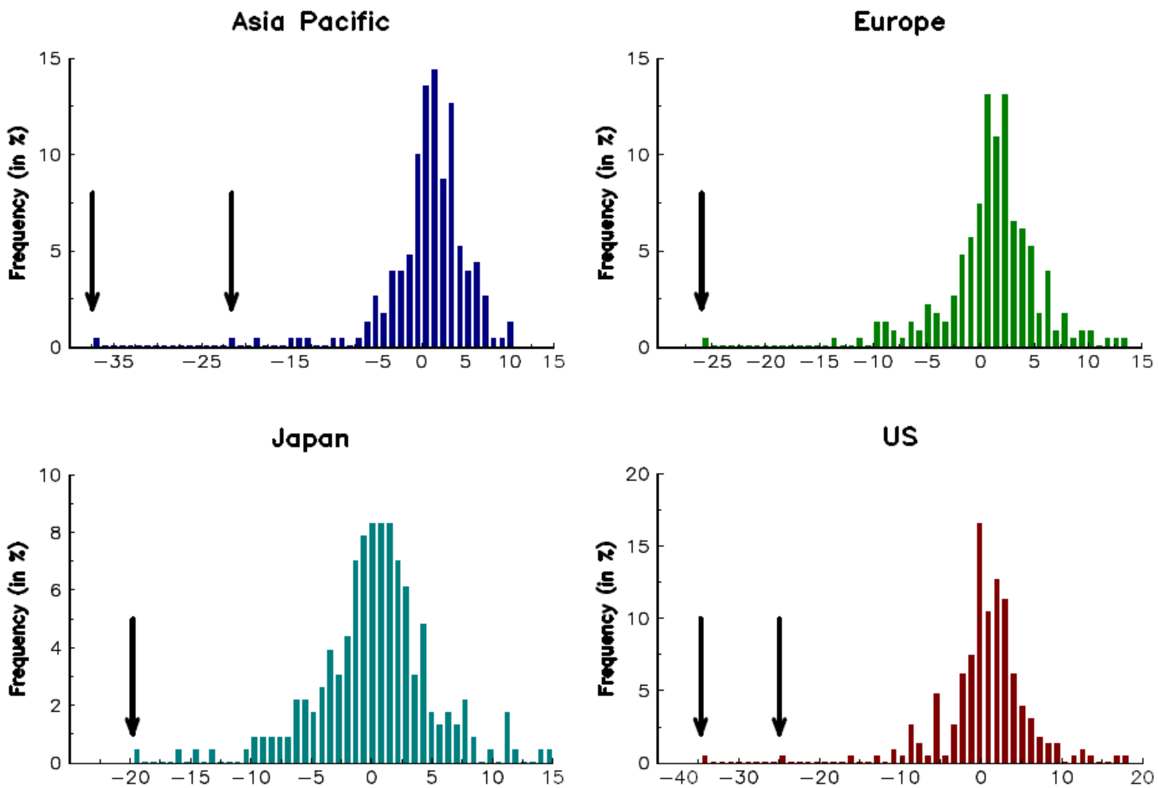
#2 April 2000 – March 2009

#3 April 2009 – December 2013

Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

Momentum crashes
Daniel and Moskowitz, 2013

Figure: Distribution of WML monthly returns



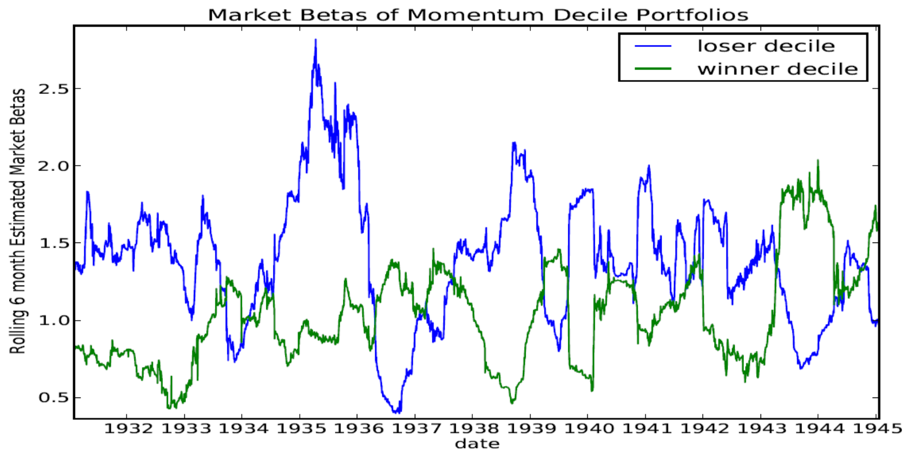
Momentum crashes
Daniel and Moskowitz, 2013

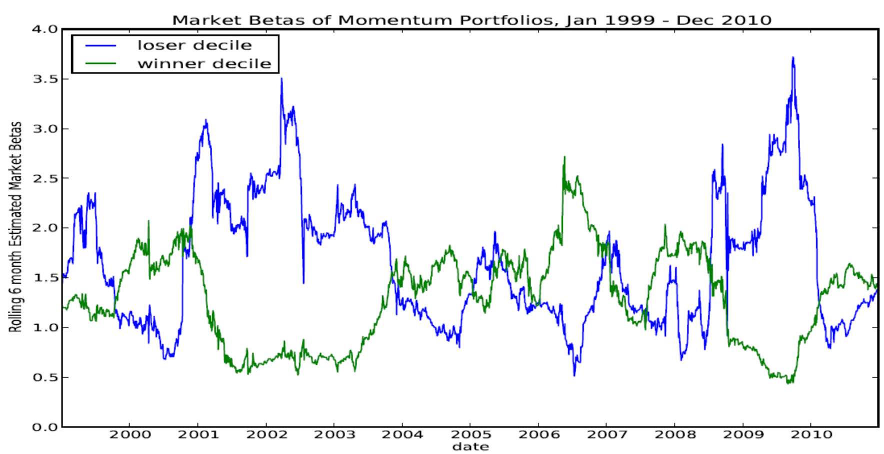
- Tail risk of momentum happens when a quick crash market is followed by a strong rebound
- Momentum or market beta?

Rank	Month	WML_t	Mkt-2y	Mkt_t
1	1932-08	-0.7896	-0.6767	0.3660
2	1932-07	-0.6011	-0.7487	0.3375
3	2009-04	-0.4599	-0.4136	0.1106
4	1939-09	-0.4394	-0.2140	0.1596
5	1933-04	-0.4233	-0.5904	0.3837
6	2001-01	-0.4218	0.1139	0.0395
7	2009-03	-0.3962	-0.4539	0.0877
8	1938-06	-0.3314	-0.2744	0.2361
9	1931-06	-0.3009	-0.4775	0.1380
10	1933-05	-0.2839	-0.3714	0.2119
11	2009-08	-0.2484	-0.2719	0.0319

Source: Daniel and Moskowitz (2011) – Momentum crashes; Table presents the 11 worst monthly returns to the WML US portfolio over the Jan 1927-December 2012 time period. Also tabulated are MKT-2Y, the 2-year market returns leading up to the portfolio formation date, and MKT_t, the market return in the same month

Momentum
Alpha or market beta?

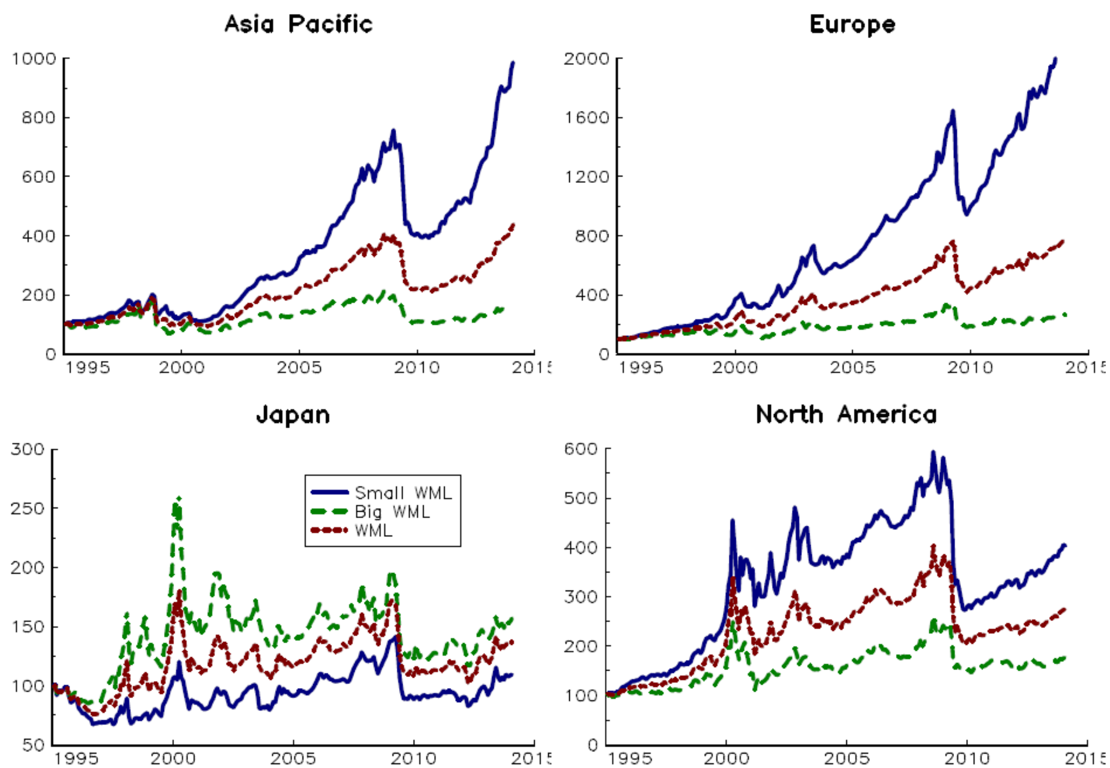




Source: Daniel and Moskowitz (2011) – Momentum crashes

The size effect in the WML factor

Figure: The SWML, BWML and WML factors (1995 – 2013)



Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

The size effect in the WML factor

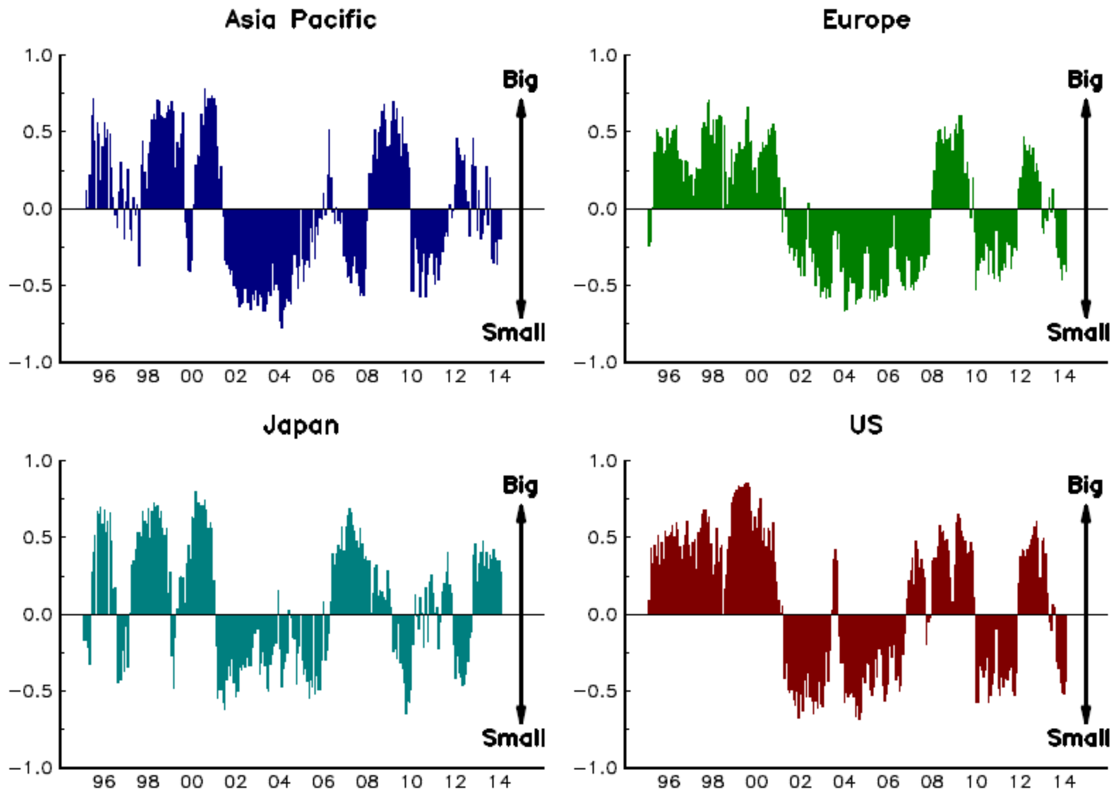
Table: Performance of the SWML, BWML and WML factors (1995 – 2013)

Statistic	Factor	Asia Pacific	Europe	Japan	North America	US
$\mu(x)$	SWML	12.7	17.7	0.4	7.4	4.9
	BWML	2.9	5.3	2.4	3.0	2.5
	WML	8.0	11.5	1.7	5.3	3.8
$\sigma(x)$	SWML	16.2	14.1	15.3	19.0	19.4
	BWML	20.9	18.3	20.4	19.8	19.7
	WML	17.3	15.5	16.6	18.7	18.8
$\mathrm{SR}(x \mid r)$	SWML	0.78	1.25	0.03	0.39	0.25
	BWML	0.14	0.29	0.12	0.15	0.13
	WML	0.46	0.74	0.10	0.28	0.20

Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

The size neutrality of the BWML factor

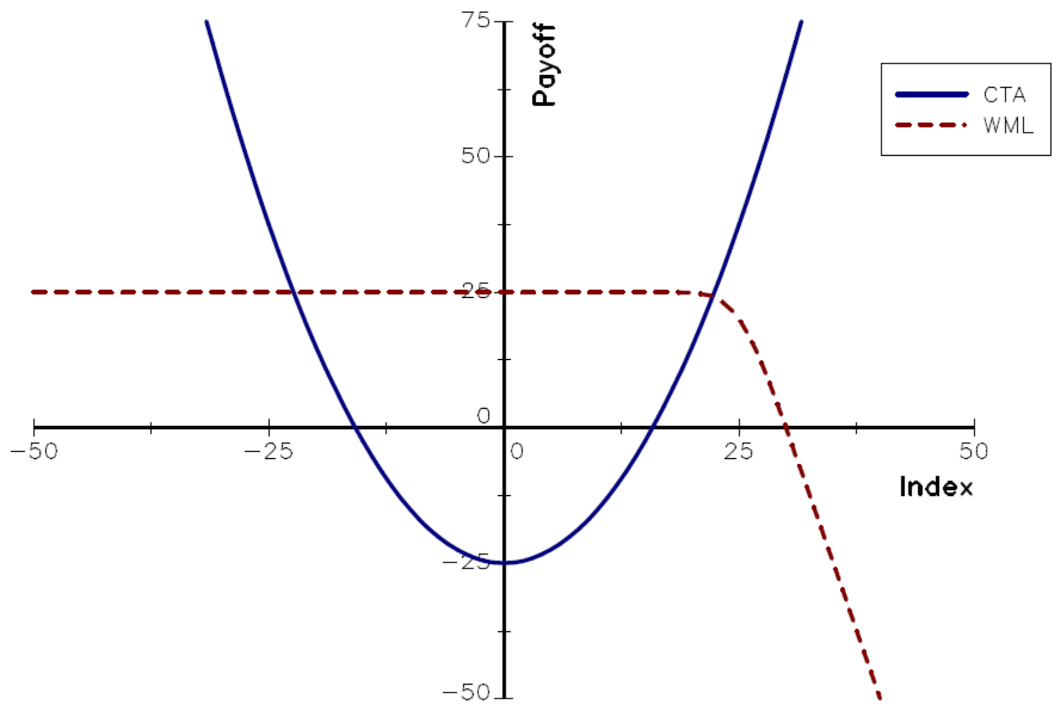
Figure 13: The size neutrality of the BWML factor



Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

Momentum in different forms
Cross-sectional vs. time-series

Figure: Payoff of CTA and conditional payoff of WML



Source: Cazalet and Roncalli. T. (2014) – Facts and fantasies about factor investing

Volatility

- Three anomalies in this category
  - Low volatility anomaly
  - Idiosyncratic volatility anomaly
  - Low beta anomaly

- The three anomalies are strongly related

Low volatility anomaly (VOL)

- Let x_1 and x_2 be two diversified portfolios. The expected return is an increasing function of the volatility of the portfolio (CAPM assumption):

$$\sigma(x_1) > \sigma(x_2) \Longrightarrow \mu(x_1) > \mu(x_2)$$

- However, empirical evidence suggests otherwise (Haugen and Baker, 1991; Clarke et al, 2006; Blitz and van Villet, 2007). The authors were able to build a "minimum variance" portfolio which performed better than the higher volatility market-cap portfolio.

- Implementation: Minimum variance constrained portfolio, ranked-based portfolio methodology

Idiosyncratic volatility anomaly (IVOL)

- Ang et al (2006) defined IVOL as the volatility of the idiosyncratic risk $\epsilon_i(t)$ corresponding to the residual of the Fama-French regression

$$R_i(t) = \alpha_i + \beta_i^m R_m(t) + \beta_i^{smb} R_{smb}(t) + \beta_i^{hml} R_{hml}(t) + \epsilon_i(t)$$

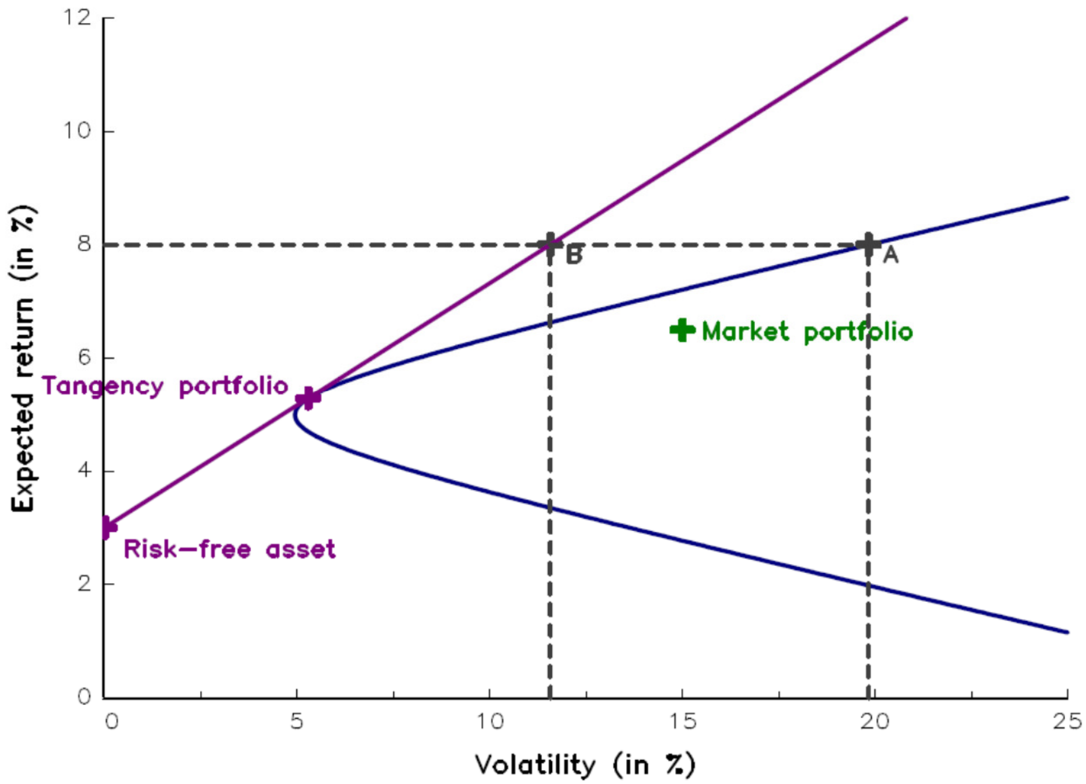
- By sorting stocks by exposure to IVOL, Ang et al (2006) observed that the return difference between the first quintile portfolio (long low IVOL) and the last quintile portfolio (short high IVOL) was over 1% in the US.

- Ang et al (2009) extended the study to G7 countries and found the low volatility effect to be significant.

- However, Bali and Cakini (2008) reported that the results are sensitive to the definition of idiosyncratic volatility, the weighting scheme and stock universe.

Low beta anomaly

- What is the impact of borrowing constraints on the market portfolio?



Low beta anomaly

- Frazzini and Pedersen (2014) show that if investors face some borrowing constraints, the relationship between the risk premium and the beta of asset i becomes

$$\mathrm{E}(R_i) - R_f = \alpha_i + \underbrace{\beta_i^m \left(\mathrm{E}(R_m) - R_f\right)}_{\pi_i \,=\, \text{implied risk premium}}$$

where $\alpha_i = \psi(1 - \beta_i^m)$ is a decreasing function of β_i^m

- Linked to the empirical evidence of Black et al (1972) which found that the slope of the security market line is lower than the theoretical slope given by the CAPM

- Black et al (1972) explains the puzzle of the lower security market line by the borrowing constraints of many investors. If the investor cannot leverage and targets an expected return higher than the expected return of the tangency portfolio, then high beta assets are preferred.

- Leverage constraints drive up the prices of high beta assets, and hence lower the expected return of high beta assets.

Low beta anomaly

Example

We consider four assets where $\mu_1 = 5\%$, $\mu_2 = 6\%$, $\mu_3 = 8\%$, $\mu_4 = 6\%$, $\sigma_1 = 15\%$, $\sigma_2 = 20\%$, $\sigma_3 = 25\%$ and $\sigma_4 = 20\%$. The correlation matrix C is equal to:

$$C = \begin{pmatrix} 1.00 & & & \\ 0.10 & 1.00 & & \\ 0.20 & 0.60 & 1.00 & \\ 0.40 & 0.50 & 0.50 & 1.00 \end{pmatrix}$$

The risk-free rate is set to 2%.

Table: Tangency portfolio $x^\star$ without any constraints

Asset	$x_i^\star$	$\beta_i(x^\star)$	$\pi_i(x^\star)$
1	47.50%	0.74	3.00%
2	19.83%	0.98	4.00%
3	27.37%	1.47	6.00%
4	5.30%	0.98	4.00%

$$\mu(x^*) = 6.07\%$$
$$\sigma(x^*) = 13.77\%$$

Low beta anomaly

- Let us suppose that the market include two investors. The first investor cannot leverage his risky portfolio, whereas the second investor must hold 50% of his wealth in cash. We obtain:

Asset	$x_{m,i}$	α_i	$\beta_i(x_m)$	$\pi_i(x_m)$	$\alpha_i + \pi_i(x_m)$
1	42.21%	0.32%	0.62	2.68%	3.00%
2	15.70%	0.07%	0.91	3.93%	4.00%
3	36.31%	−0.41%	1.49	6.41%	6.00%
4	5.78%	0.07%	0.91	3.93%	4.00%

Implied risk premium too high

$$\mu(x_m) = 6.3\%$$
$$\sigma(x_m) = 14.66\%$$

Table: Betting-against-beta (BAB) portfolios

Portfolio	#1	#2	#3	#4
$\tilde{x}_1$	1	0	1	5
$\tilde{x}_2$	0	1	1	0
$\tilde{x}_3$	−1	0	−3	−5
$\tilde{x}_4$	0	−1	1	0
$\mathbb{E}[R(\tilde{x})]$	0.79%	0.00%	1.51%	3.94%
$\sigma(R(\tilde{x}))$	26.45%	21.93%	46.59%	132.24%

Low beta anomaly (BAB)

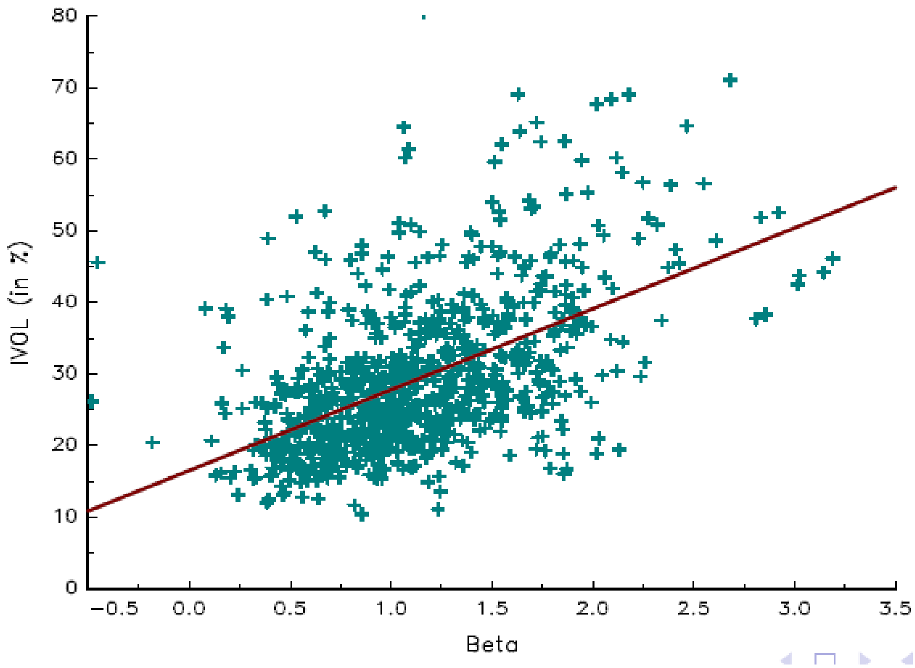
Table: Performance of the BAB factor (1995-2013)

Asset class	$\mu(x)$	$\sigma(x)$	$\mathrm{SR}(x \mid r)$
USD Equities	9.04%	14.96%	0.60
JPY Equities	2.65%	13.12%	0.20
DEM Equities	6.38%	17.98%	0.36
FRF Equities	−3.03%	26.26%	−0.12
GBP Equities	5.31%	14.41%	0.37
International Equities	7.73%	8.20%	0.94
US Treasury Bonds	1.73%	2.95%	0.59
US Corporate Bonds	5.43%	10.81%	0.50
Currencies	1.12%	8.64%	0.13
Commodities	−4.78%	17.76%	−0.27
All assets	5.36%	4.34%	1.24

Links between VOL, IVOL and BAB

$$\sigma_i^2 = (\beta_i^m)^2 \sigma_m^2 + \tilde{\sigma}_i^2$$

Figure: Relation between β_i^m and IVOL_i (Fama-French)



Links between VOL, IVOL and BAB

Figure: Difference between the low beta and low volatility anomalies