Financial Data Science

Lecture 5
Portfolio Optimization with Machine Learning

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Introducing portfolio management

- Systematic management of a collection of financial instruments, such as stocks and bonds, to achieve pre-defined objectives on portfolio risk and return
- Active management vs. passive management
- Mathematical frameworks:
    - Modern Portfolio Theory (MPT)
    - Capital Asset Pricing Model (CAPM)
    - Multiple factor models

Modern portfolio theory

A mathematical framework that seeks

- the highest expected return given a certain level of risk, or
- the lowest risk given a certain level of expected return, in a portfolio.

Portfolio expected return

Portfolio risk/volatility

Diversification

Efficient frontier

Risk free rate

Optimization

Portfolio expected return

- Each asset return $i \in \{1, \dots, N\}$ is denoted by random variable R_i
- Recall that the return for asset i on day t is calculated as $R_i^t = \frac{S_i^t - S_i^{t-1}}{S_i^{t-1}}$
- Portfolio consists of a collection of random variables $\mathbf{R}^T = [R_1, \dots, R_N]$
- Expected return of individual asset is denoted as $\mu_i = E[R_i]$; this will be our estimate for future return of the asset
- Collecting a total of N assets gives $\boldsymbol{\mu}^T = [\mu_1, \dots, \mu_N]$
- Similarly, portfolio allocation takes the form of a set of weights $\mathbf{w}^T = [w_1, \dots, w_N]$
- Portfolio expected return is expressed as a weighted sum of individual asset returns: $E[R_P] = \mu_P = \sum_{i=1}^{N} w_i R_i = \mathbf{w}^T \mathbf{R}$

Portfolio risk/volatility

- Variance of individual asset: $\mathrm{Var}[R_i] = \sigma_i^2 = E[(R_i - \mu_i)^2]$
- Covariance between two assets: $\mathrm{Cov}[R_i, R_j] = \sigma_{ij} = E[(R_i - \mu_i)(R_j - \mu_j)]$
- Portfolio variance: $\mathrm{Var}[R_P] = \mathbf{w}^T \mathbf{Q} \mathbf{w} = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \,\mathrm{Cov}[R_i, R_j]$
- $\mathrm{Var}[R_P] = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq i} w_i w_j \sigma_{ij}$
- Decompose into a sum of individual asset variances (diagonal elements) and between-asset covariances (off-diagonal elements)
- $\mathrm{Var}[R_P] = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \,\sigma_i \sigma_j \rho_{ij}$ using correlation $\rho_{ij} = \frac{\mathrm{Cov}[R_i, R_j]}{\sigma_i \sigma_j}$
- Portfolio risk/volatility: $\sqrt{\mathrm{Var}[R_P]}$

Diversification

- Leads to reduced risk without sacrificing the expected return
- The only free lunch in finance
- When aggregating individual assets into a portfolio
  - $\mu_P = \sum_{i=1}^{N} w_i R_i$, no magic
  - $\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \, \sigma_{ij} < \left(\sum_{i=1}^{N} w_i \sigma_i\right)^2$, magic here! (why?)

In-class Quiz

- Q1-3

Group Discussion

- What is considered as a good portfolio?
- How to be a good portfolio manager?

Mathematical formulation of MPT

Objective:

$$\max_{\mathbf{w}} \boldsymbol{\mu}^T \mathbf{w}$$

subject to:

$$\mathbf{w}^T \mathbf{Q} \mathbf{w} \leq \sigma_0^2$$ User-defined target variance

$$\sum_{i=1}^{N} w_i = 1$$ Budget constraint

$$w_i \geq 0 \text{ for all } i = 1, \ldots, N$$ Nonnegativity constraint

Objective:

$$\min_{\mathbf{w}} \mathbf{w}^T \mathbf{Q} \mathbf{w}$$

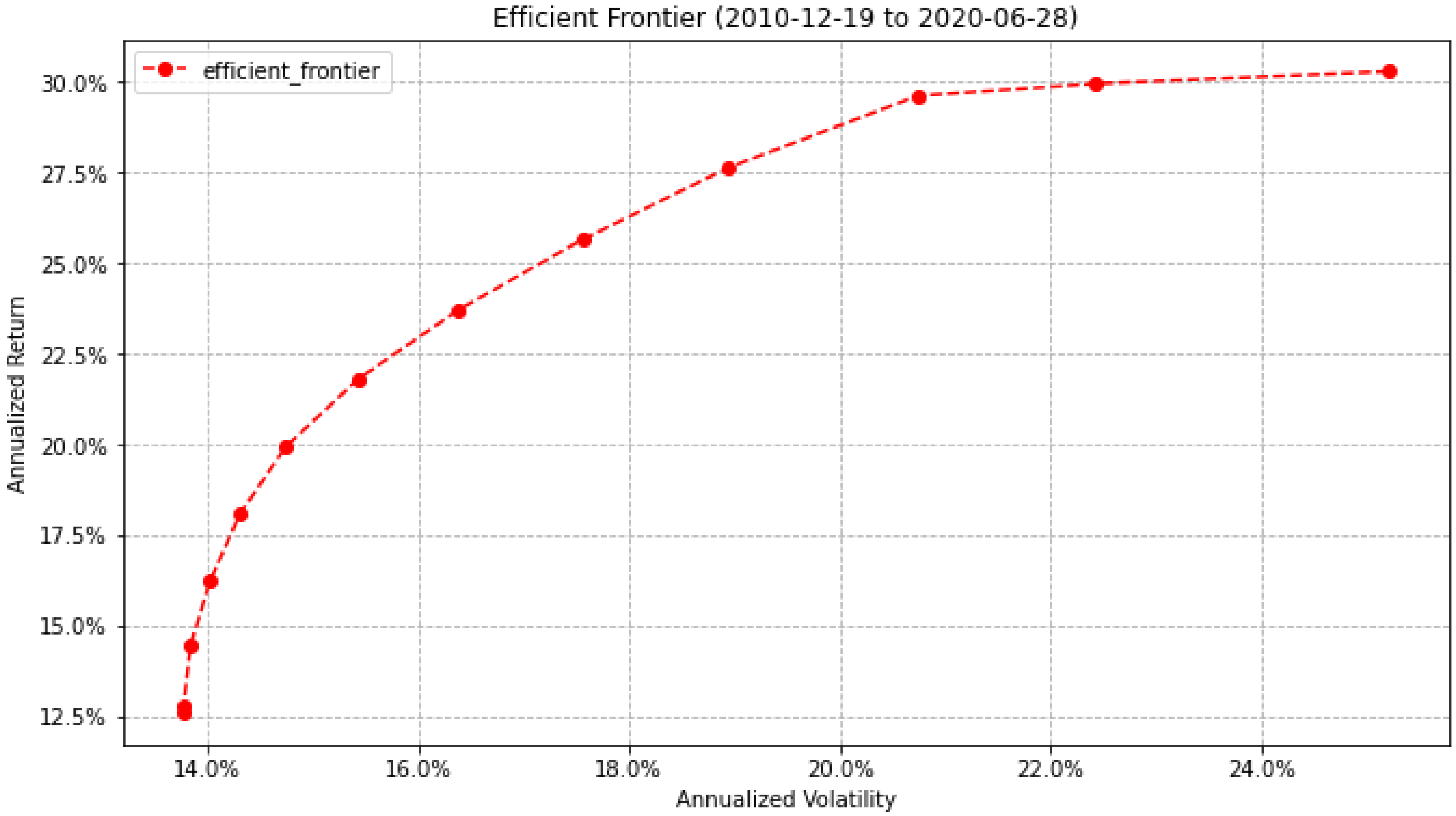
subject to:

$$\boldsymbol{\mu}^T \mathbf{w} \geq \mu_0$$ User-defined target return

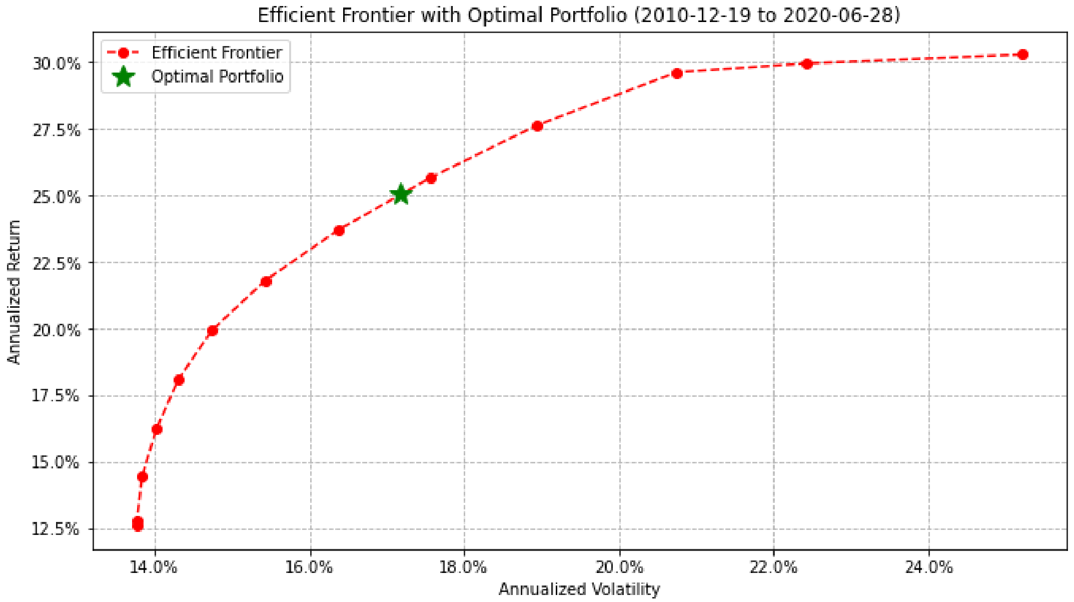
$$\sum_{i=1}^{N} w_i = 1$$ Budget constraint

$$w_i \geq 0 \text{ for all } i = 1, \ldots, N$$ Nonnegativity constraint

Efficient frontier



Maximum Sharpe Ratio Portfolio



Objective:

$$\max_{\mathbf{w}} \frac{\boldsymbol{\mu}^T \mathbf{w} - R_f}{\sqrt{\mathbf{w}^T \mathbf{Q} \mathbf{w}}}$$

subject to:

$$\sum_{i=1}^{N} w_i = 1$$

$$w_i \geq 0 \text{ for all } i = 1, \dots, N$$

R_f is the risk-free rate

In-class Quiz

- Q4-6

Capital Asset Pricing Model (CAPM)

- A linear regression model, also called single factor model, that connects the expected excess return of a portfolio to that of the market portfolio

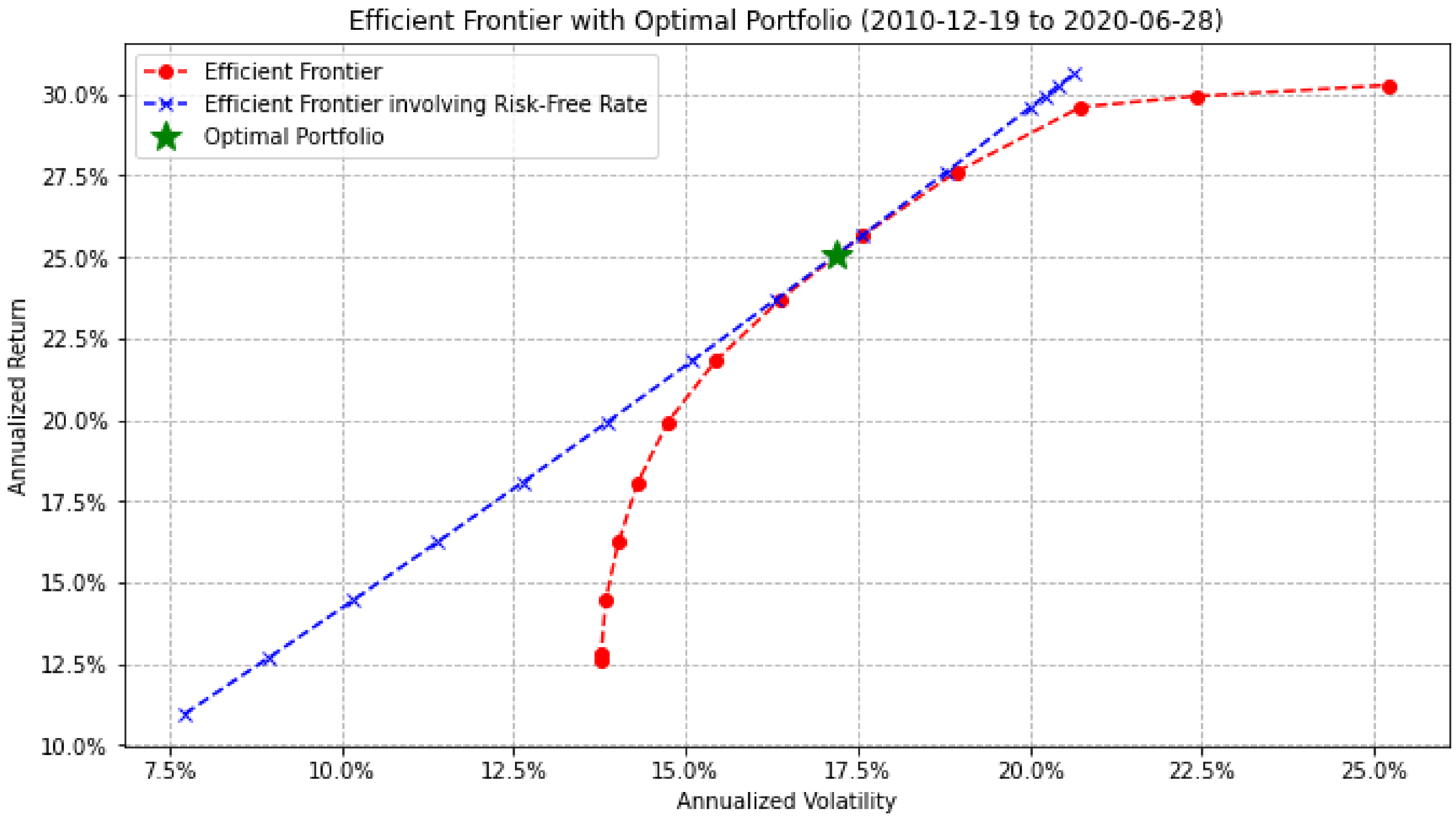
$$(R_{it} - R_{ft}) = \alpha_i + \beta_i(R_{Mt} - R_{ft}) + \epsilon_{it} \qquad E[R_i] = R_f + \hat{\beta}_i(E[R_M] - R_f)$$

- α: measures the risk-adjusted performance of the portfolio manager; a positive α means the portfolio manager performs better than the market
- β: measures the sensitivity of asset return to the overall market return, or more specifically, the systematic risk premium $E[R_M] - R_f$ of the market factor; $\beta = \frac{Cov[R_p, R_M]}{Var[R_M]}$ which can be derived using first-order condition
- ϵ: idiosyncratic signal not explained by the market

Adding Risk-free Asset

- Risk-free asset: guaranteed return R_f with no risk, such as Treasury bills
- Nonnegativity constraint for risky portfolio (market portfolio) R_{M} but allows negative weight for a risk-free asset R_f
- A new efficient frontier is drawn, taking the shape of a straight line called the Capital Allocation Line (CAL)
- A linear risk-return trade-off:
  - $\mu_P = w\mu_M + (1-w)R_f$
  - $\sigma_P^2 = w^2\sigma_M^2$
  - Combining, we have $\mu_P = R_f + \frac{\mu_M - R_f}{\sigma_M}\sigma_P$
- The line that is tangent to the original efficient frontier is called the Capital Market Line (CML), which is obtained by maximizing the slope term $\frac{\mu_M - R_f}{\sigma_M}$
- Market equilibrium is reached at any point of CML when investors hold a combination of market portfolio and risk-free asset

The new efficient frontier



In-class Quiz

- Q7-9

Forecasting Asset Expected Returns

- Assets returns need to be forecasted to perform portfolio optimization
- Model Choices:
  - Linear: Ridge, Lasso (controls overfitting via regularization)
  - Tree-based: Random Forest, XGBoost (captures nonlinearities and interactions)
  - Deep Learning: LSTM/Transformer (models temporal and sequence dependencies)
- Feature Engineering: Price-momentum, volatility/volume indicators, firm fundamentals, macroeconomic variables

Forecasting Asset Covariance Matrix

Simple Estimators:

- Rolling-window: sample covariance over the last W observations
- EWMA: exponential weighting with decay factor λ for responsiveness

Multivariate GARCH:

- BEKK-GARCH: full-parameterization of dynamics
- DCC-GARCH: separate volatility GARCH plus dynamic correlation process

Factor & Shrinkage Methods:

- Statistical factors (e.g. PCA, Dynamic PCA) to reduce dimensionality
- Ledoit–Wolf shrinkage toward a structured target for improved estimation

Sparse/Regularized Estimators:

- Graphical Lasso: impose sparsity in the precision matrix
- Banding/Thresholding for high-dimensional covariance control

Coding session

Group Homework

- Choose at least three assets and time frame of interest
- Conduct the two-stage exercise in portfolio optimization following a monthly rebalancing schedule
  forecast + optimization
- Stage 1: build at least two forecasting models for asset returns and (optionally) covariance matrix
- Stage 2: form at least two strategies
- Report both forecasting and optimization performance