QF632-2025-W3

Number of participants: 40

1. **Suppose X has PDF $f(x) = 3x^2, \quad 0 \leq x \leq 1$ and zero elsewhere. What is $P(X = 0.5)$?**

8 correct answers out of 31 respondents

	$f(0.5) = 3 \cdot (0.5)^2$	48%	15 votes
	$\int_{0.5}^{0.5} 3x^2 \, dx$	19%	6 votes
✓	0	26%	8 votes
	$P(0 \leq X \leq 0.5)$	6%	2 votes

2. Let X be a continuous random variable with CDF F and PDF f. Which of the following statements must hold (wherever the derivatives exist)?

17 correct answers out of 31 respondents

	$F(x) = \int_x^{\infty} f(t)\,dt$	26%	8 votes
✓	$f(x) = \frac{d}{dx}F(x)$	55%	17 votes
	$F(x) = f(x)$	3%	1 vote
	$\int_{-\infty}^{x} f(t)\,dt = 1 - F(x)$	16%	5 votes

3. Let $Y = aX + b$, where a, b are constants and $X \sim \mathcal{N}(\mu, \sigma^2)$. What is $\mathrm{Var}(Y)$?

24 correct answers
out of 31 respondents

✓	$a^2\sigma^2$	77%	24 votes
	$a\sigma^2$	3%	1 vote
	σ^2	19%	6 votes
	$b^2\sigma^2$	0%	0 votes

4. Which of the following distance measures does OLS minimize?

29 correct answers out of 32 respondents

✓	Sum of squared vertical distances $(y_i - \hat{y}_i)^2$	91%	29 votes
	Sum of squared perpendicular distances to the hyperplane	6%	2 votes
	Maximum absolute deviation from the hyperplane	0%	0 votes
	Sum of absolute vertical distances $\lvert y_i - \hat{y}_i \rvert$	3%	1 vote

5. In the fitted model $\hat{y} = \beta_0 + \sum_{j=1}^{p} \beta_j x_j$, the coefficient β_k is interpreted as:

26 correct answers out of 27 respondents

	The change in $\hat{y}$ when \textit{all} predictors increase by one unit.	0%	0 votes
	The fraction of total variance in y explained by x_k alone.	4%	1 vote
✓	The change in $\hat{y}$ when x_k increases by one unit, holding all other x_j fixed.	96%	26 votes
	The correlation between x_k and the residuals.	0%	0 votes

6. **Consider the following causal diagram: $Z \longrightarrow X \longrightarrow Y, \quad Z \longrightarrow Y$. Which of the following is \textbf{true} about the marginal correlation $\mathrm{Corr}(X, Y)$ vs.\ the partial correlation $\mathrm{Corr}(X, Y \mid Z)$?**

24 correct answers out of 32 respondents

	They must always agree in sign.	13%	4 votes
✓	They can disagree in sign if Z is a confounder.	75%	24 votes
	Marginal correlation is zero if and only if the partial correlation is zero.	6%	2 votes
	Partial correlation can only be larger in magnitude than the marginal correlation. \end{itemize}	6%	2 votes

7. **Consider the model $mpg_i = \beta_0 + \beta_1 qsec_i + \beta_2 am_i + \beta_3 (qsec_i \times am_i) + \varepsilon_i$, where $am_i = 0$ for automatic and $am_i = 1$ for manual. Which of the following is the slope of $qsec$ for manual cars?**

26 correct answers
out of 29 respondents

	β_1	0%	0 votes
	β_2	0%	0 votes
✓	$\beta_1 + \beta_3$	90%	26 votes
	β_3	10%	3 votes

8. At what value of $qsec$ do the two fitted lines ($\beta_0 + \beta_1 q$ and $\beta_0 + \beta_2 + (\beta_1 + \beta_3)q$) intersect?

19 correct answers
out of 25 respondents

✓	$q = -\frac{\beta_2}{\beta_3}$	76%	19 votes
	$q = \frac{\beta_2}{\beta_1}$	12%	3 votes
	$q = -\frac{\beta_1}{\beta_2}$	4%	1 vote
	$q = \frac{\beta_3}{\beta_2}$	8%	2 votes

9. **You fit the models** $(\mathrm{A}) \quad y = \beta_0 + \beta_1 x + \varepsilon$ **and** $(\mathrm{B}) \quad y = \beta_0 + \beta_1 x + \beta_2 x^2 + \varepsilon$**. Which formal test assesses whether the quadratic term adds significant explanatory power?**

17 correct answers out of 28 respondents

Test $H_0 : \beta_1 = 0$ under model (B)	4%	1 vote
✓ Test $H_0 : \beta_2 = 0$ in model (B)	61%	17 votes
Compare R^2 of (A) and (B) without an F-test	25%	7 votes
Test $H_0 : \beta_0 = \beta_1 = \beta_2 = 0$	11%	3 votes

10. **The least-squares objective $S(\beta) = (y - X\beta)^\top (y - X\beta)$ has Hessian $2X^\top X$. Which property of $X^\top X$ ensures $S(\beta)$ is convex?**

19 correct answers
out of 24 respondents

	$X^\top X$ is diagonal.	17%	4 votes
✓	$X^\top X$ is positive semidefinite.	79%	19 votes
	$X^\top X$ has full row rank.	4%	1 vote
	$\det(X^\top X) = 0$.	0%	0 votes

11. **Consider the ridge loss $L_{\text{ridge}} = \sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij}\right)^2 + \lambda\sum_{j=1}^{p}\beta_j^2$. As $\lambda \to \infty$, the ridge estimates $\hat{\beta}_j^{\text{ridge}}$ for $j = 1, \ldots, p$ tend to:**

13 correct answers out of 25 respondents

	the ordinary-least-squares estimates.	20%	5 votes
✓	0 (while β_0 remains the OLS intercept).	52%	13 votes
	infinity in magnitude.	8%	2 votes
	random values with mean zero.	20%	5 votes

12. Why is it essential to standardize (center and scale) predictors before fitting a lasso model?

25 correct answers out of 28 respondents

	The lasso penalty $\sum_j \lvert\beta_j\rvert$ is invariant to predictor scaling.	11%	3 votes
✓	Without standardization, predictors with larger scale incur disproportionately larger penalties.	89%	25 votes
	Standardization guarantees a unique solution.	0%	0 votes
	Only binary predictors must be standardized for lasso.	0%	0 votes

13. In the high-dimensional regime where $p > n$, a Lasso solution can have at most how many nonzero coefficients?

17 correct answers
out of 27 respondents

✓	n	63%	17 votes
	p	19%	5 votes
	$\sqrt{n}$	19%	5 votes
	$\log(p)$	0%	0 votes