QF634 APPLIED QUANTITATIVE RESEARCH METHODS LECTURE 8

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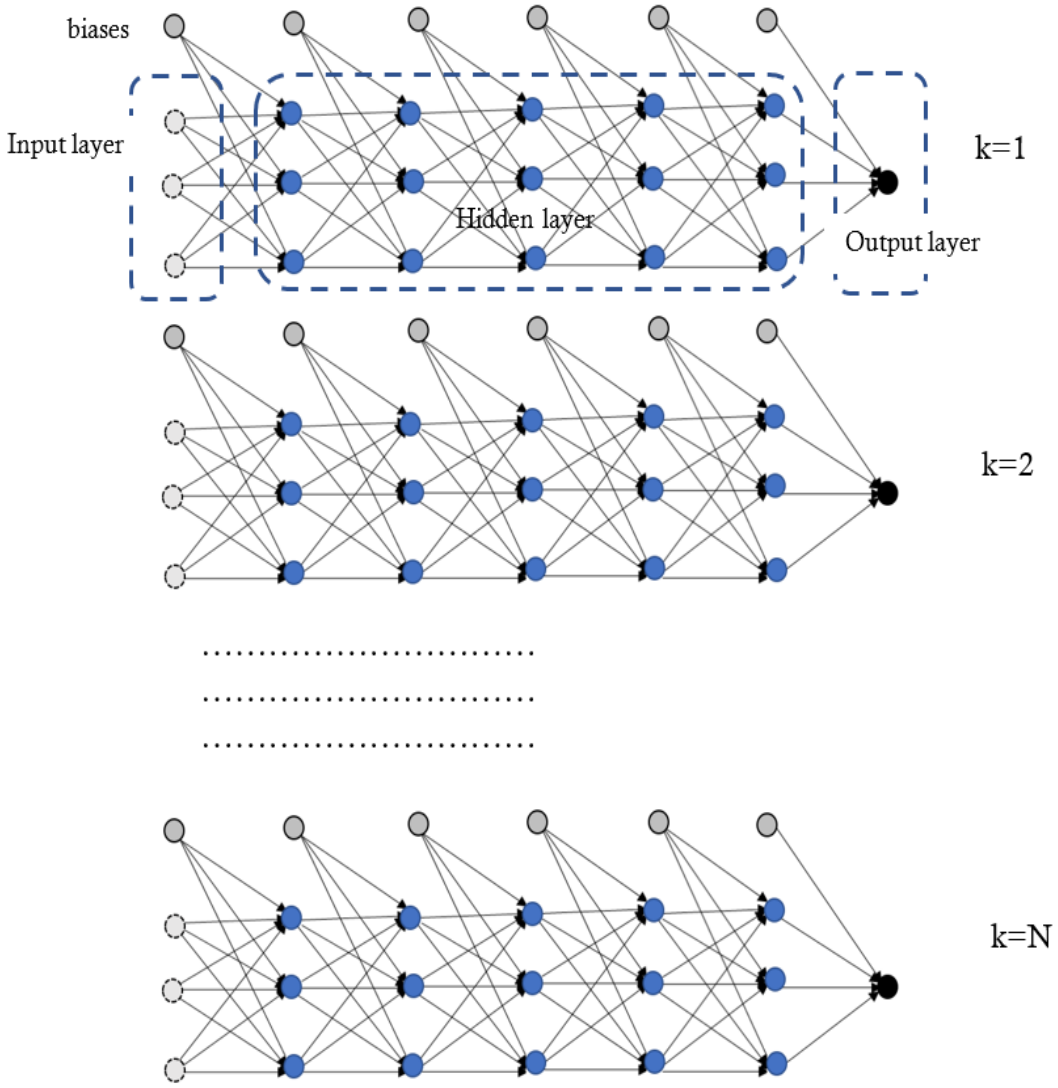
ARTIFICIAL INTELLIGENCE

Artificial Neural Network II: Deep Learning NN

In Chapter 7, we see that when cases k are independent of one another, i.e., their inputs and output do not affect other cases, then the common (across k) input weights and biases, and weights and biases in hidden layer(s) are revised via backpropagation to minimize the error/loss function L(.) that combines all cases $\sum_{k=1}^{N} \frac{1}{N} L(Y_k, \hat{Z}_k)$. $\hat{Z}_k$is the corresponding predicted output.

These computations in a single iteration can be done in parallel for each case k since the ANN is the same for each k.

In Figure 8.1, we show the parallel multiprocessing using N identical multilayer perceptrons (MLP), one to forward-propagate each case/subject or sample point k in the training data set of size N or N sample points.



Recurrent Neural Network (RNN)

Suppose now the input features for each case, instead of given altogether at the initial input layer, are given one at a time in a sequence. In a common setup to this new way of input, the number of hidden layers is now equal to the number of sequential input features so that at each additional hidden layer, there is a fresh sequential input feature – see dotted arrow in Figure 8.2. We consider the simple case where each sequential input feature is a scalar number. More general cases include an input feature which is a vector of numbers.

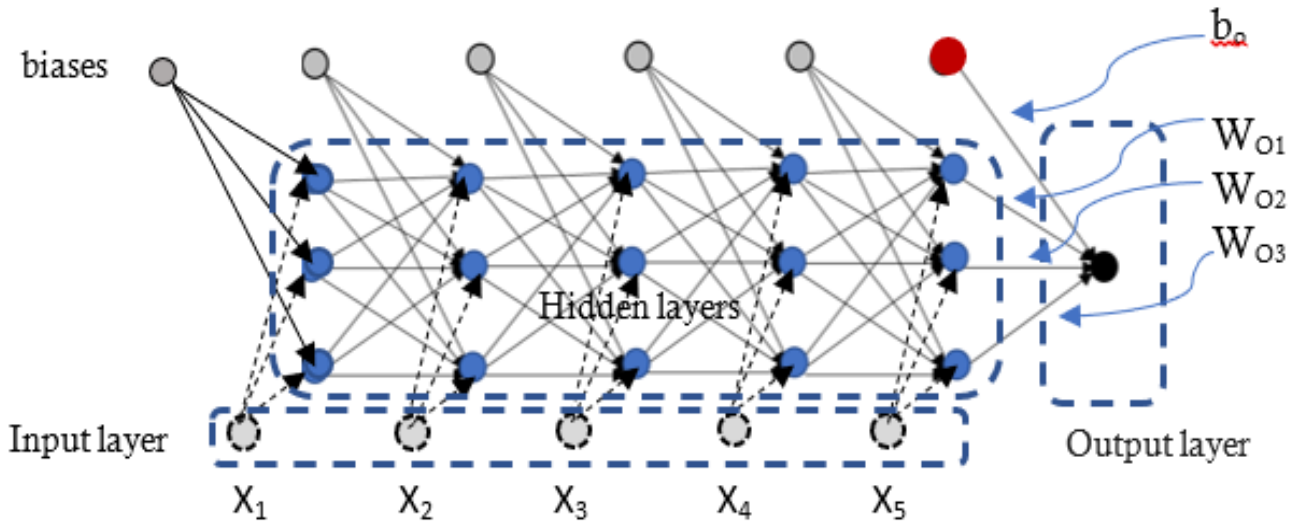
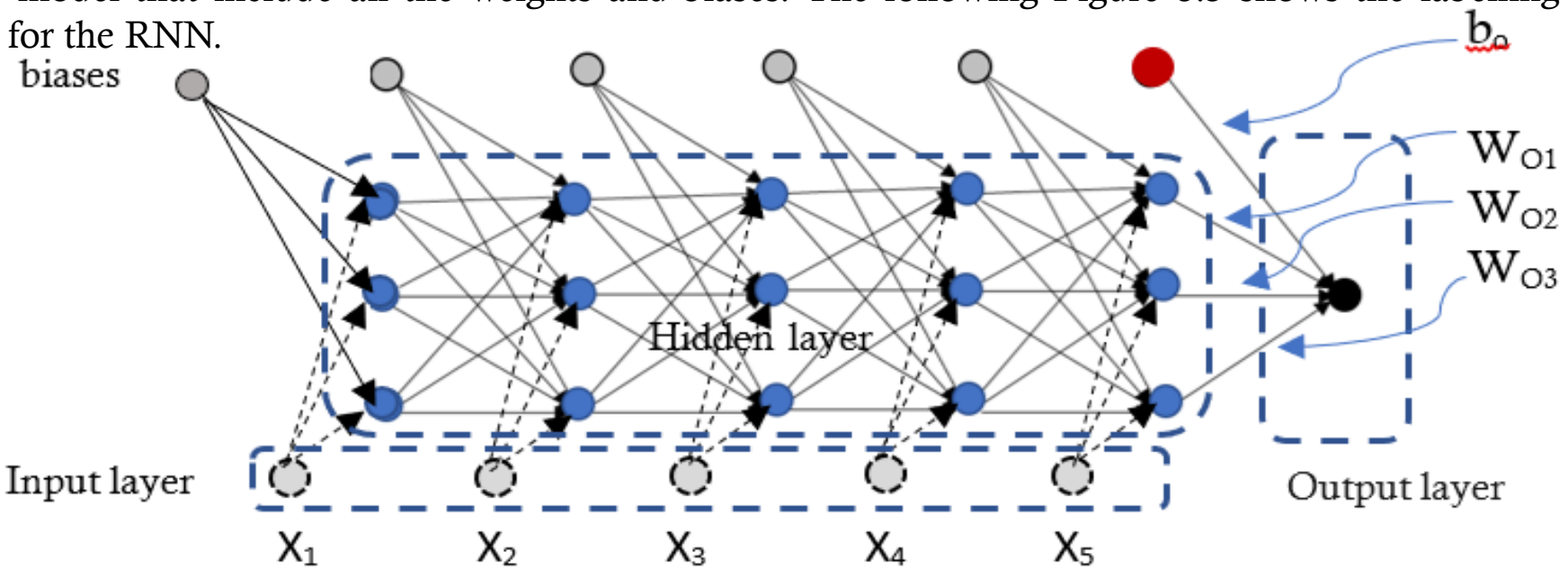


Figure 8.2

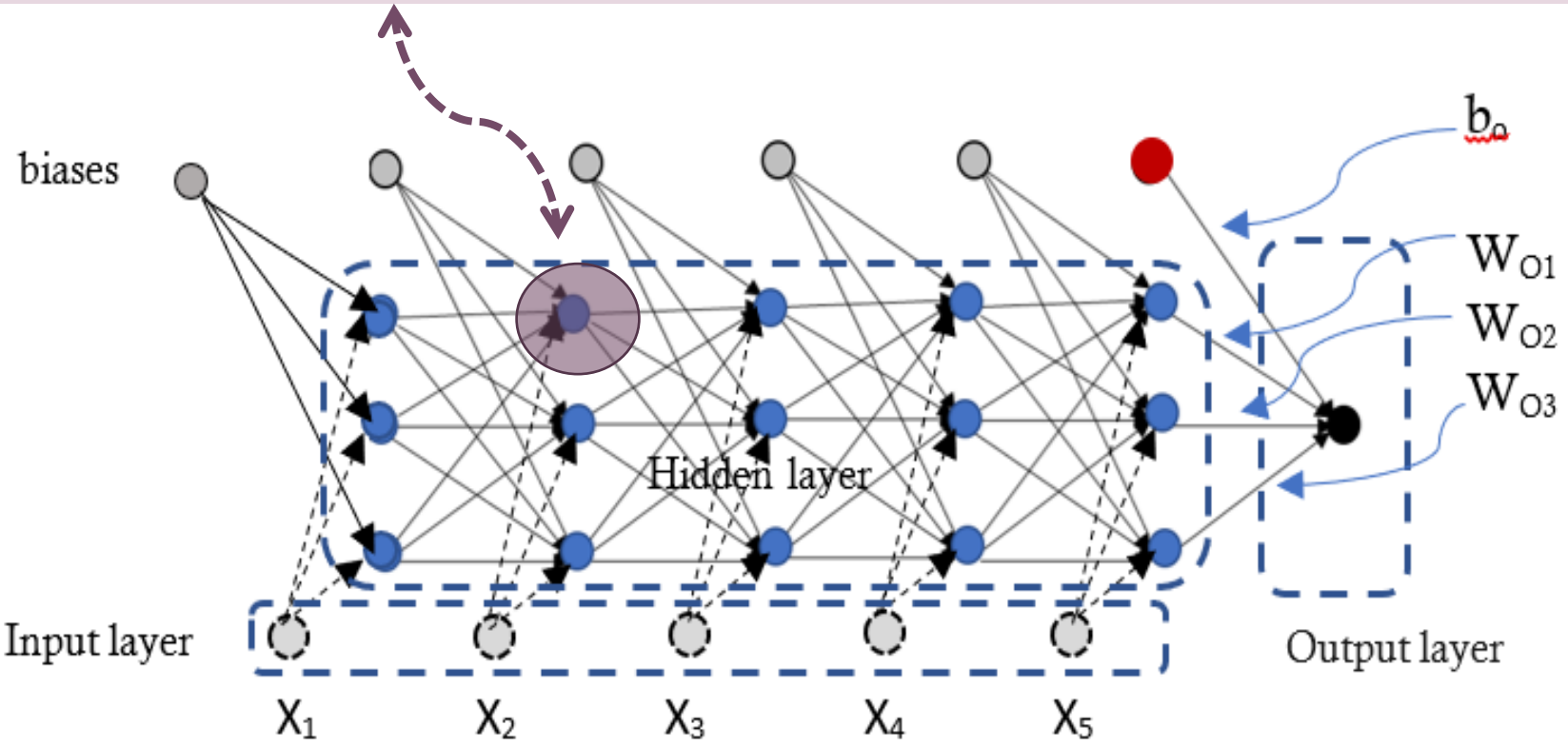
This new architecture is common in time series prediction such as predicting the next day stock price (label or target) using lagged prices in the past 5 days. These lagged prices form the features of this label. Each of these lagged prices becomes one feature in this one training case. A different window of price with lagged 5 prices forms another case, and so on.

- Figure 8.2 shows an architecture of 3 neurons per hidden layer whereby one training case is passed through the NN. The inputs for this case consist of 5 sequential features X_1, X_2, X_3, X_4, and X_5. There is one output for the case in the output layer of one neuron. As in ANN or MLP, many cases in a batch or mini batch are required to pass through this RNN before the loss function, comprising sum of loss in each case, is to be reduced via revising the parameters in the model that include all the weights and biases. The following Figure 8.3 shows the labelling convention we adopt for the RNN.



- In RNN, the weights and biases at each neuron level do not change from one hidden layer to another. This is unlike that in the ANN. There are 3 input weights (W_{P1}, W_{P2}, and W_{P3}), 3×3 weights W_{pq} from prior layer of neurons, and 3 biases. In addition, there are the weights W_{o1}, W_{o2}, W_{o3} and b_0 representing weights and bias from last hidden layer to the output neuron.

For example, the parameters to the first (uppermost) neuron in the 2nd layer are (1) W_{P1}, the weight on exogenous input X_2 from the input layer I, (2) the bias b_1, (3) W_{11}, the weight on forward pass from previous hidden layer first neuron, (4) W_{21}, the weight on forward pass from previous hidden layer second neuron, (5) W_{31}, the weight on forward pass from previous hidden layer third neuron.



In general, when the hidden layer has m neurons, then the total number of parameters to be fitted is (m+2)(m+1) – 1.

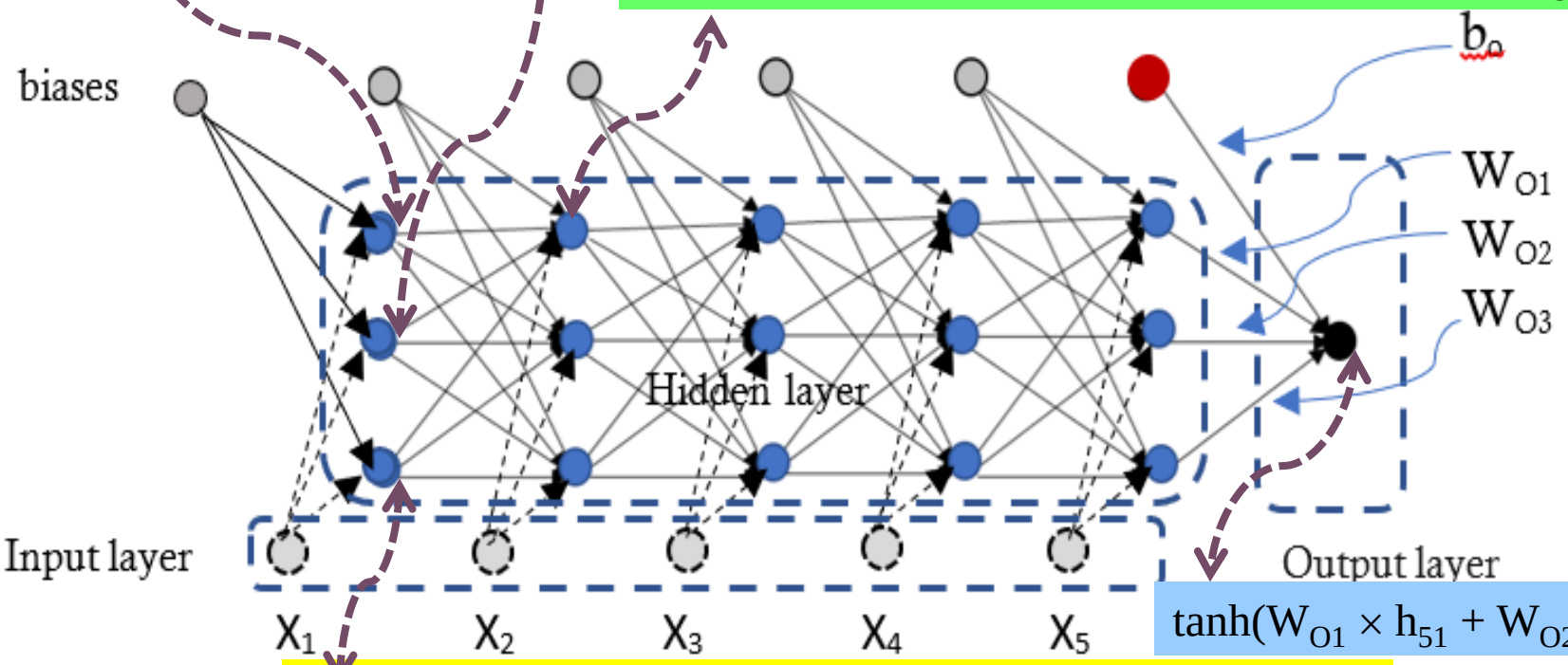
Output from the first neuron in the first hidden layer is

$h_{11} \equiv \tanh(W_{P1} \times X_1 + b_1 + W_{11}\, h_{01} + W_{21}\, h_{02} + W_{31}\, h_{03})$

Outputs (hidden state) h_{tj} at sequence step t and j^{th} position neuron

$h_{12} \equiv \tanh(W_{P2} \times X_1 + b_2 + W_{12}\, h_{01} + W_{22}\, h_{02} + W_{32}\, h_{03})$

$h_{21} \equiv \tanh(W_{P1} \times X_2 + b_1 + W_{11}\, h_{11} + W_{21}\, h_{12} + W_{31}\, h_{13})$



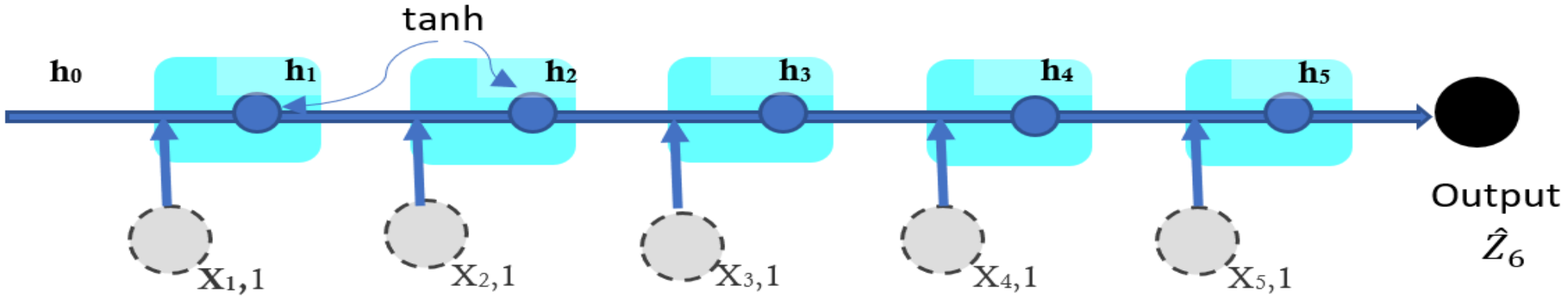
$\tanh(W_{O1} \times h_{51} + W_{O2}\, h_{52} + W_{O3}\, h_{53} + b_0)$

$h_{13} \equiv \tanh(W_{P3} \times X_1 + b_3 + W_{13}\, h_{01} + W_{23}\, h_{02} + W_{33}\, h_{03})$

A concise representation of Recurrent Neural Network

Each box represents a hidden layer of neurons

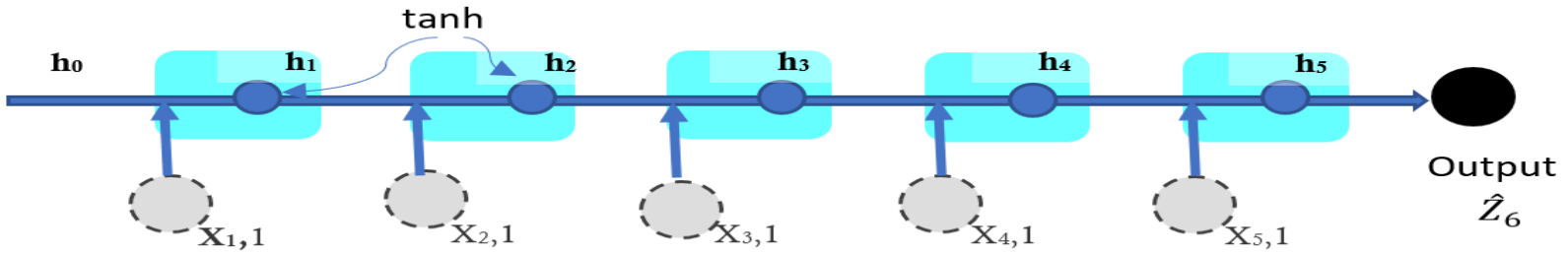
The RNN may be represented in a concise manner as follows. Each box represents a hidden layer of neurons. It is noted that the hidden states (at each level of neuron) change with each step in the sequence. Each hidden state at sequence step t, h_{tj}, contains information of lagged exogenous input features X_{t-1}, X_{t-2}, X_{t-3}, and so on. This type of architecture is important if indeed current input features are not independent from lagged input features that have useful information in predicting the output. This type of neural network that allows for recurrence of influences of past features is thus called Recurrent Neural Network (RNN).



The additional "1" in the input box reflects the "input" to multiply with the bias coefficient.

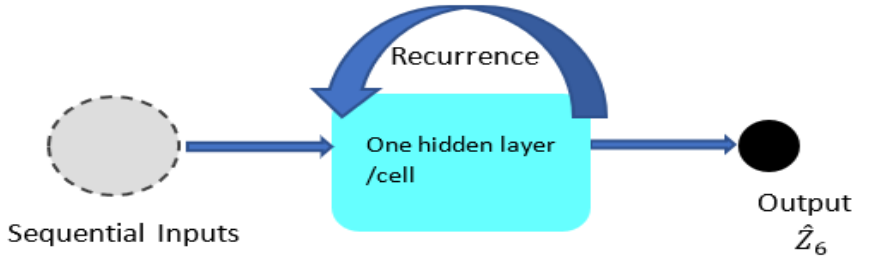
A concise representation of Recurrent Neural Network

Each box represents a hidden layer of neurons



There is concatenation of the inputs (X_t , 1) and $\mathbf{h_{t-1}} \equiv (h_{t-1,1}, h_{t-1,2}, h_{t-1,3})$ at their junction, and this is passed through the hidden layer where the dark circle represents the activation function on the dot product of the concatenated inputs and parameters. The output of that hidden layer is then $\mathbf{h_t}$ which enters into the computations at the next hidden layer. Each of the hidden layer may be viewed as a recurrent cell since the computations are recurrent on the updated hidden states as the sequence progresses until the output.

Above is sometimes called the unfolded NN of
the diagram on the right where recurrence is explicitly indicated.
In the recurrence representation on the right:
Each of the hidden layer may be viewed as a recurrent cell since the computations are recurrent on the updated hidden states as the sequence progresses until the output.



Recurrent Neural Network

Revision/Updating of Parameters

- In general, a training data set could have a time series or a sequence of T data points. The T sample points are divided into cases or packs each with R, e.g., 5, number of recurrent cells/hidden layers. The number of cases or packs is T/R. A certain number of cases/packs are grouped into batches with batch size S. So, there are T/(RS) number of mini batches (each size S).
- Each case in a mini batch yields one set of partial derivatives of the loss function with respect to the (m+2)(m+1) – 1 number of parameters. While the sequence within each case or pack clearly considers dependence and recurrence, the different cases or packs are treated as independent. Thus, S number of cases in each mini batch yield S sets of partial derivatives of the loss function. The aggregate loss function is the sum of losses of the S number of cases in each batch.
- Thus, over each mini batch, the forward propagation and then backward propagation in updating the parameters based on the sum of S partial derivatives is considered one iteration. The training sample cases can be run in parallel for each iteration. Over E number of epochs and T/(RS) number of mini batches, there will be E × T/(RS) number of iterations to update/revise the parameters.

Backward Propagation in RNN

m=3 neurons per layer, R=5 inputs per case or instance

Consider the loss function of one case

$$\frac{\partial L(Y_{t+6},\hat{Y}_{t+6})}{\partial W_{P1}} = \frac{\partial L(Y_{t+6},\hat{Y}_{t+6})}{\partial \hat{Y}_{t+6}} \times \frac{\partial \hat{Y}_{t+6}}{\partial W_{P1}} = \frac{\partial L(Y_{t+6},\hat{Y}_{t+6})}{\partial \hat{Y}_{t+6}} \times \left(1 - \hat{Y}_{t+6}{}^2\right) \times \frac{\partial (W_{O1} \times h_{51} + W_{O2} \times h_{52} + W_{O3} \times h_{53} + b_o)}{\partial W_{P1}}$$

where

$$\frac{\partial h_{51}}{\partial W_{P1}} = \left(1 - h_{51}{}^2\right) \frac{\partial (W_{P1}\, X_5 \,+\, W_{11}\, h_{41} \,+\, W_{21}\, h_{42} \,+\, W_{31}\, h_{43} \,+\, b_1)}{\partial W_{P1}} = \left(1 - h_{51}{}^2\right) \left(X_5 + W_{11} \frac{\partial h_{41}}{\partial W_{P1}} + W_{21} \frac{\partial h_{42}}{\partial W_{P1}} + W_{31} \frac{\partial h_{43}}{\partial W_{P1}} \right)$$

$$\frac{\partial h_{52}}{\partial W_{P1}} = \left(1 - h_{52}{}^2\right) \frac{\partial (W_{P2}\, X_5 \,+\, W_{12}\, h_{41} \,+\, W_{22}\, h_{42} \,+\, W_{32}\, h_{43} \,+\, b_2)}{\partial W_{P1}} = \left(1 - h_{52}{}^2\right) \left(0 + W_{12} \frac{\partial h_{41}}{\partial W_{P1}} + W_{22} \frac{\partial h_{42}}{\partial W_{P1}} + W_{32} \frac{\partial h_{43}}{\partial W_{P1}} \right)$$

$$\frac{\partial h_{53}}{\partial W_{P1}} = \left(1 - h_{53}{}^2\right) \frac{\partial (W_{P3}\, X_5 \,+\, W_{13}\, h_{41} \,+\, W_{23}\, h_{42} \,+\, W_{33}\, h_{43} \,+\, b_3)}{\partial W_{P1}} = \left(1 - h_{53}{}^2\right) \left(0 + W_{13} \frac{\partial h_{41}}{\partial W_{P1}} + W_{23} \frac{\partial h_{42}}{\partial W_{P1}} + W_{33} \frac{\partial h_{43}}{\partial W_{P1}} \right)$$

$$\frac{\partial h_{41}}{\partial W_{P1}} = \text{.....}$$

and so on. This clearly involves summation of partial derivative terms across each time-step or each sequential recurrent unit or hidden layer, e.g., $\frac{\partial h_{5j}}{\partial W_{P1}}$, $\frac{\partial h_{4j}}{\partial W_{P1}}$, $\frac{\partial h_{3j}}{\partial W_{P1}}$, $\frac{\partial h_{2j}}{\partial W_{P1}}$, $\frac{\partial h_{1j}}{\partial W_{P1}}$ for every j, involving terms of inputs X_1, …., X_5 and previous parameter values $W_{ij}^{[t]}$, and so on. This is unlike the feedforward NN case where a partial derivative would involve summation only across paths originating from the parameter edge and not across all edges in each layer.

Backward Propagation in RNN

- The above partial derivatives are for one case. They are summed across all cases in one mini batch size S.
- If loss across all cases in the mini batch of size S is $L = 1/S\ \Sigma^{S}\ L\left(Y_{t+R}, \hat{Y}_{t+R}\right)$, then $\partial L/\partial W_{ij}^{[t]} = 1/S\ \Sigma^{S}\ \partial L\left(Y_{t+R}, \hat{Y}_{t+R}\right) / \partial W_{ij}^{[t]}$ at the t^{th} iteration.
- After the updated partial derivatives are found, the usual adjustments of the parameter weights are done, viz.

$$W_{ij}^{[t+1]} = W_{ij}^{[t]} - \alpha(t+1)\frac{\partial L}{\partial W_{ij}^{[t]}}$$

Advantages and Disadvantages of RNN

- While the advantage of RNN over MLP is the ability to recognize the input of lagged (historical) information in affecting current output, and the ability to take in long sequence of inputs (since the number of parameters is not blown up as the parameters are constant at each recurrent unit), the computational time can be longer as the back propagation involves more computations of the complicated partial derivatives.
- RNN also has two drawbacks: (1) the partial derivatives in the RNN backpropagation may in some situations progressively "collapse to zero" or else they may explode exponentially ("vanishing" or else "exploding" gradient with deeper layers); (2) the design does not allow it to consider future or forward input for training since the time-sequence uses information or input from the past or back till the current and is meant to predict the future or forward data. This limitation is real for problems such as predicting a picture by neighboring pixels or a word sequence as in a sentence. However, in general, limitation (2) is appropriate for financial time series where known future state cannot be used to predict an earlier state as this may introduce serious issues of spuriously high accuracies.

Remedies

Problem (1) may be remedied by pruning the model such as reducing the number of recurrent cells to make the model less complex. It may also be remedied by a variant RNN called the Long Short-term Memory NN (LSTM) or use additional gates to control/reduce loss of gradient. Problem (2) may be remedied by using features at current time but are other market-based forecasts of the future state, e.g., a forward contract price on a future spot price on the same underlying commodity.

Besides the example in Figure 8.2 of many inputs to one output (many-to-one structure), there are also other structures such as one-to-many, e.g., for music generation or predicting a sequence of notes from one input, where at each step the activation becomes an explicit output that can be trained with actual output, and many-to-many as in text translation. We can also use the structure in Figure 8.2 for many-to-many if there is training on an output at the end of each hidden layer in a case.

Worked Example I -- Data

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

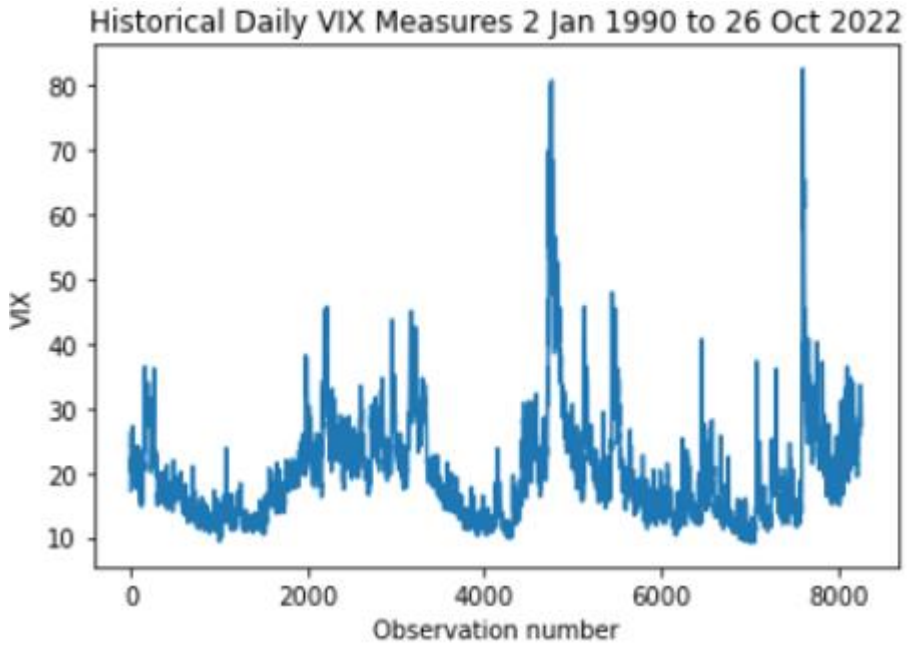
- The CBOE Volatility Index (VIX) is a real-time index derived from the prices of SPX (S&P 500) index options with near-term (approximately 1 month) expiration dates. It is market expectation of the SPX risk-neutral volatility over the short term. VIX has been called a 'fear gauge' and is often seen as a measure of negative market sentiment. Data is downloaded from Yahoo Finance.
- The following exercise uses daily VIX data from 2 Jan 1990 to 26 Oct 2022 (8270 time-sequenced sample points) to perform a RNN prediction of next day VIX based on the observed VIX of the past 10 days (approximately 2 trading weeks), the inputs. The total data set is split into 80% training data (6616 observations) and 20% test data (1654 observations) – see code line [5], [6].
- The financial time series has the property that under reasonable market efficiency, past information and not future information should be part of an investor's consideration for future forecast. Future observations could not have been perfectly known beforehand.

Snapshot of the data is shown as follows.

	Date	Open	High	Low	Close	Adj Close	Volume
0	2/1/1990	17.240000	17.240000	17.240000	17.240000	17.240000	0
1	3/1/1990	18.190001	18.190001	18.190001	18.190001	18.190001	0
2	4/1/1990	19.219999	19.219999	19.219999	19.219999	19.219999	0
3	5/1/1990	20.110001	20.110001	20.110001	20.110001	20.110001	0
4	8/1/1990	20.260000	20.260000	20.260000	20.260000	20.260000	0
...	...	...	...	...	...	...	...
8265	20/10/2022	31.299999	31.320000	29.760000	29.980000	29.980000	0
8266	21/10/2022	30.209999	30.440001	29.240000	29.690001	29.690001	0
8267	24/10/2022	30.650000	30.950001	29.780001	29.850000	29.850000	0
8268	25/10/2022	29.799999	30.000000	28.219999	28.459999	28.459999	0
8269	26/10/2022	28.440001	28.520000	27.270000	27.280001	27.280001	0

[8270 rows x 7 columns]

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook



Transforming the Data

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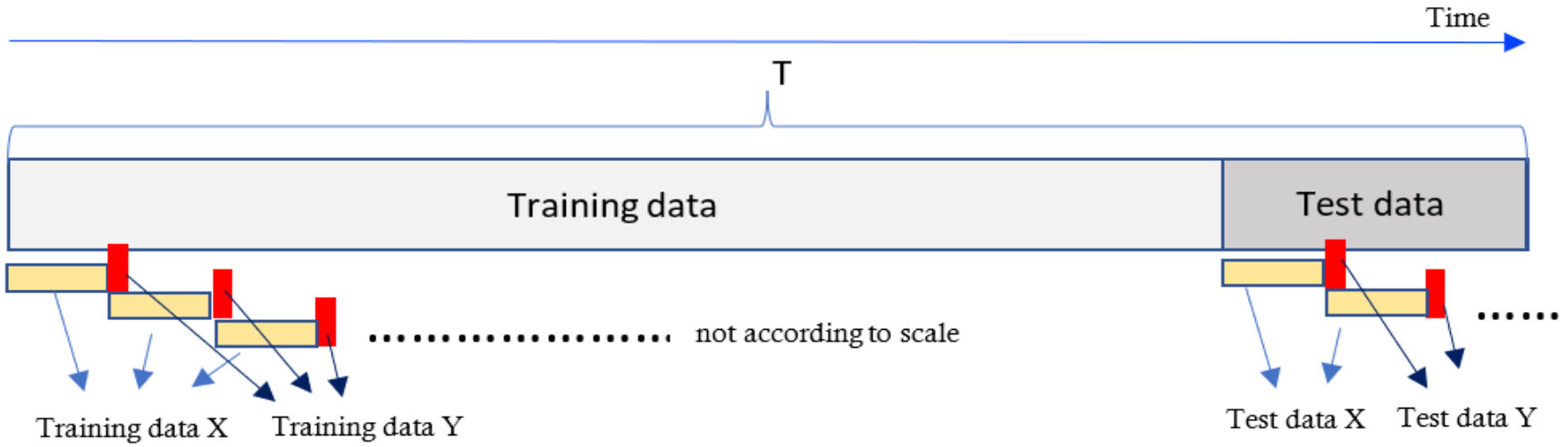
```
In [9]: #Performing Feature Scaling
        #from sklearn.preprocessing import StandardScaler
        from sklearn.preprocessing import MinMaxScaler
        scaler = MinMaxScaler(feature_range=(0, 1))  ### putting all positive is suitable as vola are all pos nos.
        train_data = scaler.fit_transform(train_data.values.reshape(-1, 1)) ### (-1,1) reshapes it to a 2D array;
        ### without .values - it may not work; .values .values convert to np array, the axes labels will be removed.
        test_data = scaler.fit_transform(test_data.values.reshape(-1, 1)) ### (-1,1) reshapes it to a 2D array
```

Arranging Data for Training and Testing



Arranging Data for Training and Testing

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

In [13]:
```
### Preparing the input X and target Y
def get_XY(dat, time_steps):
    ### Indices of target array
    C_ind = np.arange(time_steps, len(dat), time_steps)
        ### example np.arange(start=1, stop=10, step=3) gives array([1, 4, 7]), ends up to/before stop
    C = dat[C_ind]
        ### example: ray=np.arange(2, stop=10, step=3); print(ray) --- gives [2 5 8]
        ### c=np.array([1,3,6,8,9,10,12,15,18,20]); c[ray] --- gives array([ 6, 10, 18])
        ### with elements from the 2nd, 5th, 8th positions of c. c's 1st position starts at '0'

    ### Prepare X
    rows_x = len(C)                              ### here len(C) is 661
    X = dat[range(time_steps*rows_x)]            ### range(L) is 0,1,2,..., L-1. L is 10 x 661 = 6610. X is array 1,2,...,6610
    X = np.reshape(X, (rows_x, time_steps, 1))   ### X rshaped as (661,10,1)
    return X, C

time_steps = 10  ### hence C_ind = array ([10, 20, 30, 40, ....,661]), 661 number of 10 steps
                 ### C = array(10th position, 20th position of dat, etc.)
                 ### -- approx two weeks (10 trading days) interval for one prediction point of VIX
trainX, trainY = get_XY(train_data, time_steps)
testX, testY = get_XY(test_data, time_steps)
```

We use a sequence of 10 steps (approximately two weeks of 10 trading days) to form one pack of 10 recurrent units. These 10 daily VIX prices of training data X (inputs) are then followed by the following 11^{th} day of VIX price as the training data for output Y. The training data set is divided into T/10 number of the packs. The test data are similarly arranged into packs of 10 inputs followed by 1 output. The cases or packs are non-overlapping.

RNN Structure

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

In [18]:
```
def create_RNN(hidden_units, dense_units, input_shape, activation):
    model = Sequential()
    model.add(SimpleRNN(hidden_units, input_shape=input_shape, activation=activation[0]))
       ### See SimpleRNN apps in https://www.tensorflow.org/api_docs/python/tf/keras/layers/SimpleRNN
    model.add(Dense(units=dense_units, activation=activation[1]))
       ### Using model = Sequential() allows defining -- no. of inputs,#neurons in hidden,#neurons in output layer
    model.compile(loss='mean_squared_error', optimizer='adam')
       ### .compile in Sequential carries loss and optimizer options
    return model
```

In [19]:
```
# Create model and train
model1 = create_RNN(hidden_units=8, dense_units=1, input_shape=(time_steps,1), activation=['tanh', 'tanh'])
   ### calls function create_RNN, fills in the arguments that were sub-defined in last codeline via .add that defines operations
   ###   at the input layer and at the hidden layer and at dense/output layer
### hidden_units = 8 means that there are 8 neurons in each hidden layer
### input_shapes = (time-steps,1) with time_steps=10 means that each time-step in a pack of 10 is an input
###   (the output for that time-step is ignored) -- only the end of pack time variable is used in trainY
```

Fitting and Prediction

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

Model1.fit is then called (via the Sequential app) to execute the training fit (minimizing loss based on number of iterations specified in number of epochs 30. Batch size in one batch is 661 (T/10)). # parameters is (m+2)(m+1) – 1 = 10 × 9 – 1 = 89.

```
model1.fit(trainX, trainY, epochs=30, batch_size=1, verbose=2)
   ### time series of trainX is reshaped as (661,10,1), trainY is (661,)
   ### time series of testX is (165, 10, 1)), testY is (165,)
```

This is followed by making predictions on trainY based on trainX (using the optimized .fit parameters), and then making predictions on testY based on testX (using the optimized .fit parameters).

```
### make predictions
train_predict = model1.predict(trainX) ### Using the fitted model with trainX, trainY
test_predict = model1.predict(testX)   ### Using the same fitted model with trainX, trainY

### above, more appropriately test_predict is run after train_predict error is minimized or loss is minimized after the
### training set trainX, trainY are used in .fit -- and then the hyperparameters are tuned, before applying in a separate
### model1.fit to the testX, testY data. Above assumes the model1.fit on trainX, trainY is already the optimal one
```

Minimized Loss

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

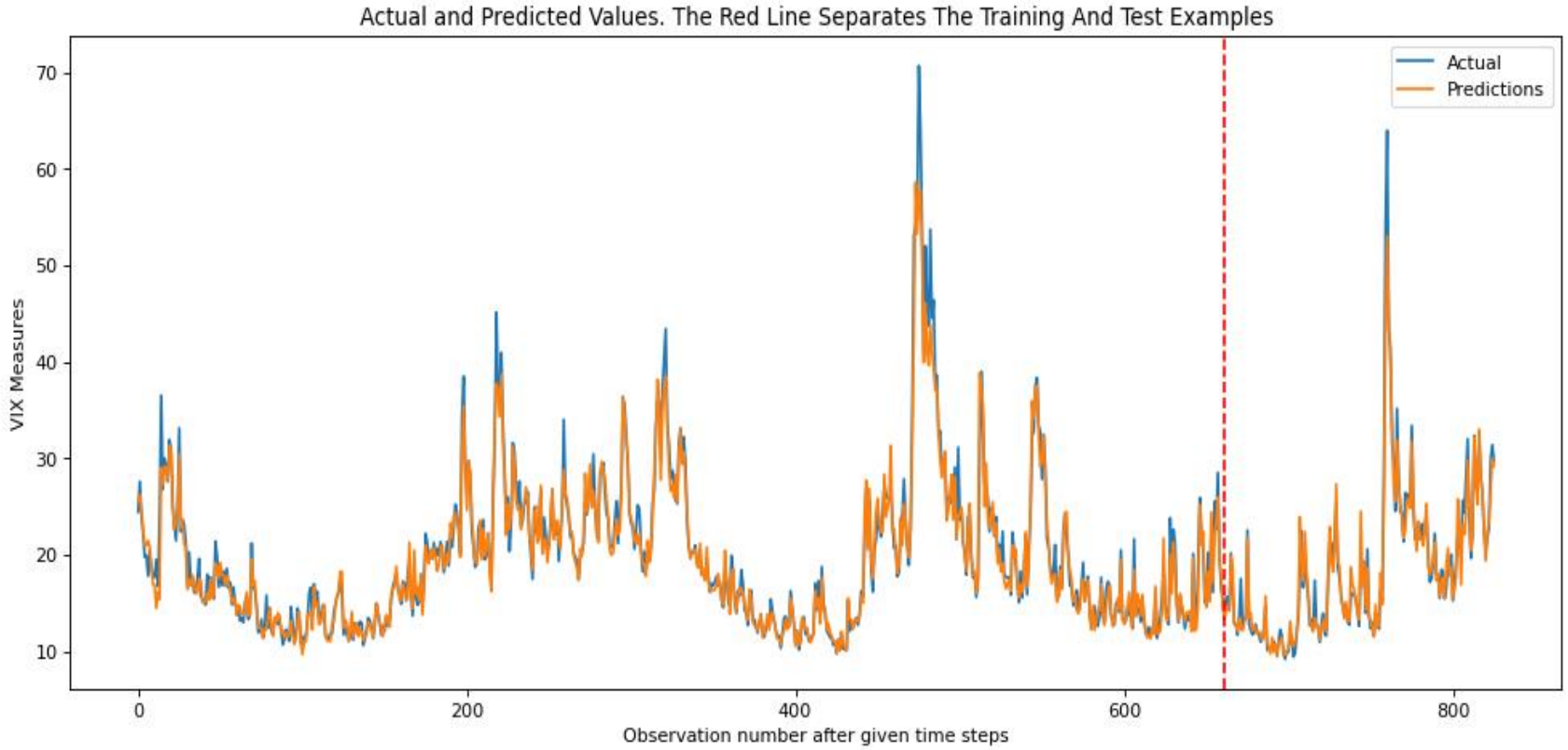
After iterating over 661 cases (packs of 10) each epoch, and then repeatedly over 30 epochs, the optimized .fit parameters produce the following root-mean-square-errors (RMSE) in the prediction of training data Y (trainY) and test data Y (testY) respectively:

```
In [21]: def print_error(trainY, testY, train_predict, test_predict):
             ### Error of predictions
             train_rmse = math.sqrt(mean_squared_error(trainY, train_predict))
             test_rmse = math.sqrt(mean_squared_error(testY, test_predict))
             ### Print RMSE
             print('Train RMSE: %.3f RMSE' % (train_rmse))
             print('Test RMSE: %.3f RMSE' % (test_rmse))

         print_error(trainY, testY, train_predict, test_predict)

         Train RMSE: 0.023 RMSE
         Test RMSE: 0.024 RMSE
```

Prediction versus Actual

Please upload Chapter8-1.ipynb and follow the computing steps in Jupyter Notebook

The RMSEs are about 2.3% for the training and 2.4% for the testing. The plot of the rescaled (inverse of scaling in [9] actual versus predicted outputs in both the training set and the test set are shown below.

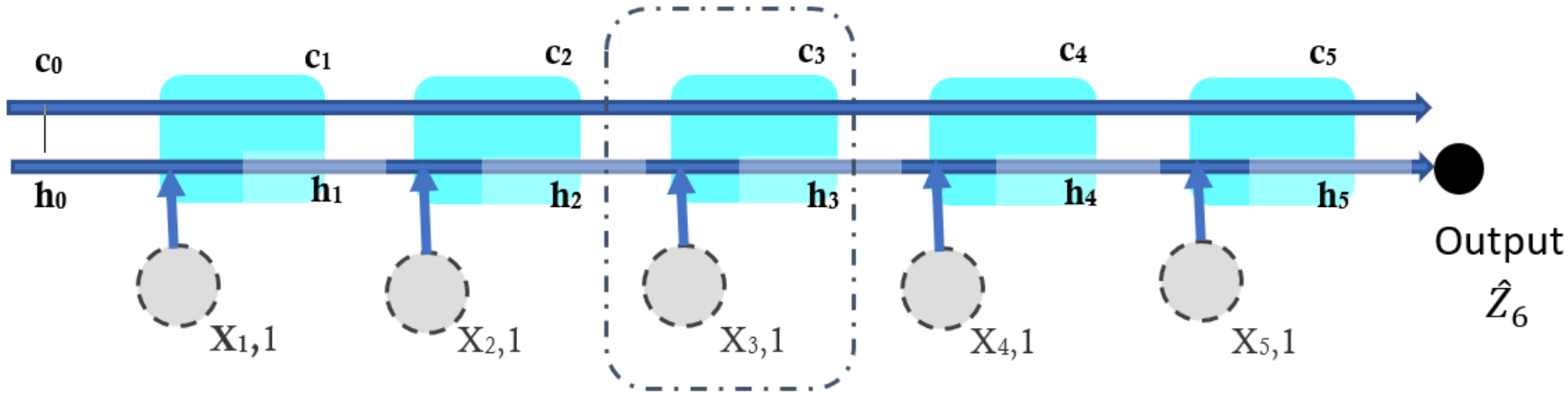


This level of accuracy is not significantly improved in the RNN for this study even if we perform overlapping data using all 6606 number of cases for training and increasing the number of epochs to 100.

Variants of RNN

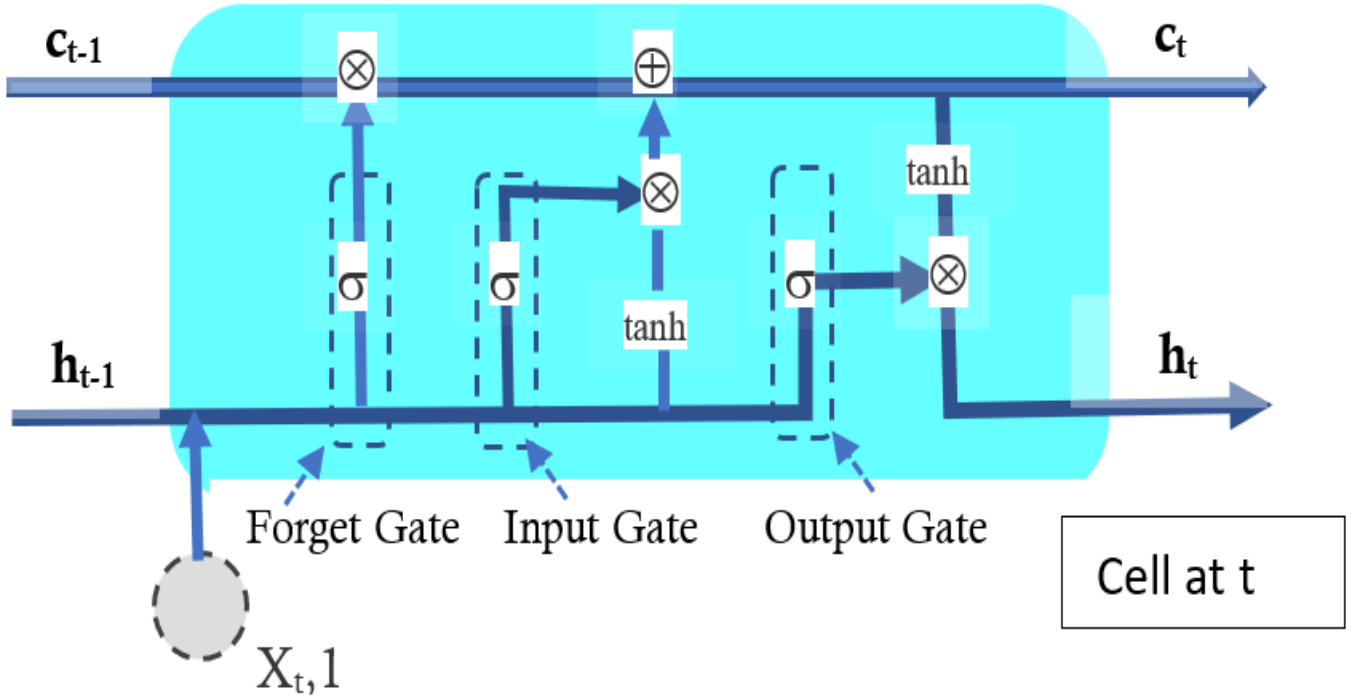
Mitigating lack of long term memory with disappearing gradients

In the LSTM architecture, the recurrent cell in Figure 8.4 is redesigned as follows in Figure 8.7 There is an additional state (also not explicitly observed) called the 'cell state'. Like the hidden states $\mathbf{h}_t$ in a traditional RNN as in Figure 8.3 that stores 'short-term' memory of effects of past inputs, the cell state $\mathbf{c}_t$ stores 'long-term' memory of effects of past inputs. As in Figure 8.3 each "box" with each fresh input represents one hidden layer in the RNN. Each "box" can contain multiple neurons as width.



Variants of RNN

Long Short Term Memory NN



The concatenated inputs (X_t,1) and the previous hidden states (outputs of last hidden layer) h_{t-1} are passed through (1) the Forget Gate before they flow through to update the last cell state c_{t-1} , (2) the Input Gate before they flow through to make the final update on the cell state that is preliminarily updated by (1), (3) the Output Gate before they update the hidden state $\mathbf{h_{t-1}}$ to $\mathbf{h_t}$.

Long Short Term Memory NN

- Suppose we create 50 neurons in the hidden layer or the cell. In the Forget Gate (1), concatenated inputs (X_t,1, $\mathbf{h_{t-1}}$) after weighting is transformed by activation σ -function, viz.

$$\sigma\,(W_{FX}\,X_t + W_{Fh}\,\mathbf{h_{t-1}} + b_F\,)$$

- In (2), input gate is multiplied by cell update candidate

$$\sigma\,(W_{IX}\,X_t + W_{Ih}\,\mathbf{h_{t-1}} + b_I\,) \otimes \tanh\,(W_{CX}\,X_t + W_{Ch}\,\mathbf{h_{t-1}} + b_C\,)$$

This Hadamard product is used to update cell state

$$\mathbf{c_t} = \sigma\,(W_{FX}\,X_t + W_{Fh}\,\mathbf{h_{t-1}} + b_F\,) \otimes \mathbf{c_{t-1}} + \sigma\,(W_{IX}\,X_t + W_{Ih}\,\mathbf{h_{t-1}} + b_I\,) \otimes \tanh\,(W_{CX}\,X_t + W_{Ch}\,\mathbf{h_{t-1}} + b_C\,)$$

- In Output Gate (3), $\mathbf{h_t} = \sigma\,(W_{OX}\,X_t + W_{Oh}\,\mathbf{h_{t-1}} + b_O\,) \otimes \tanh\,(\mathbf{c_t})$

This yields the next hidden state h_t that flows into the next LSTM layer.

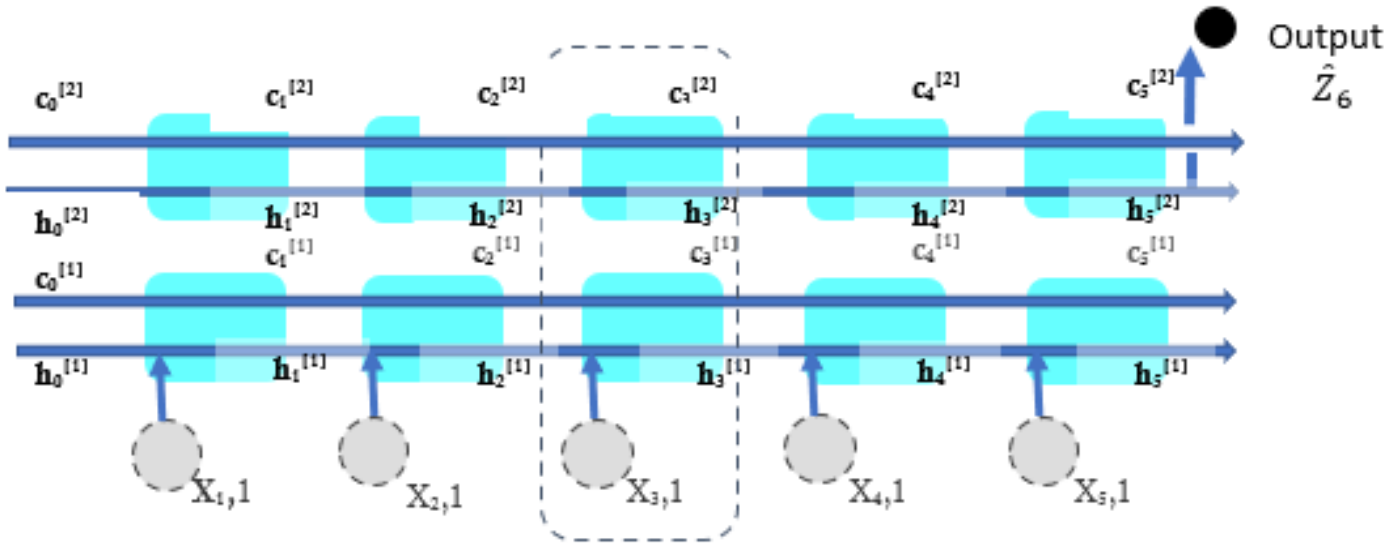
- In a many-to-one structure, the **final output** could be a scalar predicted number provided by the output layer comprising 51 weights (dimension of output h_t and the bias), viz. $f(W_{Yh}\,h_t + b_Y)$ where W_{Yh} and b_Y denote final output weights ("Y" being the output state) from a RNN/LSTM and f(.) is the final activation function.

Variants of RNN

- If in this LSTM NN with one hidden layer in each input point, there are 50 neurons (Keras App names it as number of units in the hidden layer/cell), then there is a total of $4 \times [(50 + 1) \times 50 + 50]$ parameters in the first hidden layer. In general, for N neurons or units in the first hidden layer of LSTM, and J number of inputs/features, there are $4 \times [(N + J) \times N + N]$ parameters to be optimized, where the (N+J) denotes the number of previous hidden state and feature inputs. This is multiplied into N, the number of neurons in the hidden layer.
- As said, the number of hidden states is typically structured to be the same as the number of neurons in the layer. The remaining N in the [ ] denotes the N biases, one for each neuron in the hidden layer. Thus, for the 50 neurons (units) in one layer of LSTM, there are $4 \times [(50 + 1) \times 50 + 50] = 10{,}400$ parameters. All adjacent layers of 50 neurons in same LSTM share the same $4 \times [(50 + 1) \times 50 + 50] = 10{,}400$ parameters.

Stacked Long Short Term Memory NN

However, single LSTM (even with a large number of neurons per layer or width) and many epochs may not be able to produce good training results. A deeper version is the stacked LSTM model with more hidden layers (that are stacked up at an input time-sequence). The LSTM with two stacked hidden layers at each time-sequence is shown below.



Stacked Long Short Term Memory NN

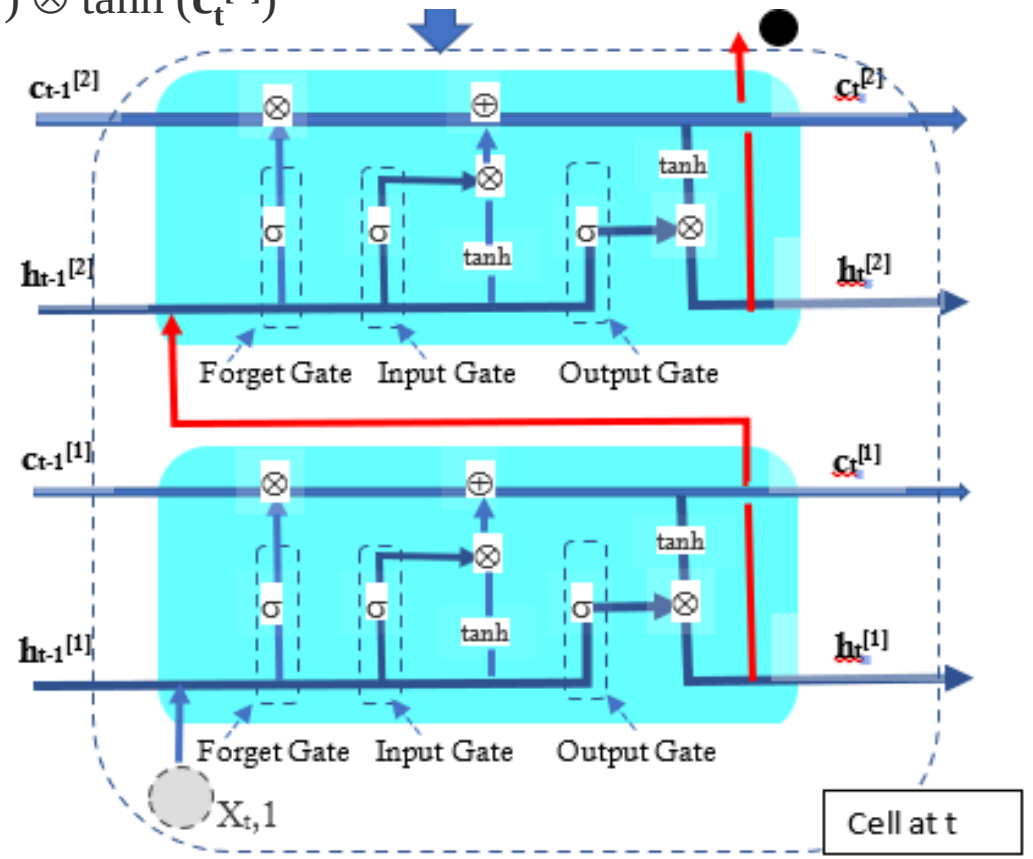
Superscripts to hidden states h_t and cell states c_t denote the first hidden layer 1 at t and the second (stacked) layer at t. The key idea is that the hidden state output $h_t^{[1]}$ at t in level 1 passes up to the second level layer as input together with the hidden state input $h_{t-1}^{[2]}$ at level 2. Note $h_{t-1}^{[2]}$ (level 2 hidden state at t-1) needs not be the same as $h_{t-1}^{[1]}$, the level 1 hidden state at t-1. The last output layer, if any, at t uses the level 2 hidden state $h_t^{[2]}$, i.e. $f(W_{Yh}\, h_t^{[2]} + b_Y)$ where $\mathbf{h_t^{[2]}} = \sigma\,(W_{OX}^{[2]}\, \mathbf{h_t^{[1]}} + W_{oh}^{[2]}\, \mathbf{h_{t-1}^{[2]}} + b_O^{[2]}) \otimes \tanh\,(\mathbf{c_t^{[2]}})$

The Output Gate from first level cell provides 50 hidden states at level 1, viz.

$$h_t^{[1]} = \sigma\,(W_{OX}^{[1]}\, X_t + W_{Oh}^{[1]}\, h_{t-1}^{[1]} + b_O^{[1]}) \otimes \tanh\,(c_t^{[1]})$$

Forget Gate output fraction vector
$\sigma(W_{FX}^{[2]}\, h_t^{[1]} + W_{Fh}^{[2]}\, h_{t-1}^{[2]} + b_F^{[2]})$

The same operations as in level 1 occur at the Input Gate, the candidate for cell update, and the Output Gate. Hence in total there are $4 \times [(N + N) \times N + N]$ parameters at the second level LSTM cell/stacked layer.



Worked Example II -- Data

Please upload Chapter8-2.ipynb and follow the computing steps in Jupyter Notebook

In this second worked example, we show how a LSTM RNN can be used to predict Google stock prices. Historical data are collected from Yahoo Finance consisting of daily (distribution and split) adjusted closing Google stock prices from 3 Jan 2012 till 29 Dec 2016, with a total of 1257 sample points.

```
In [2]: ### import training set
        dataset=pd.read_csv('GOOG.csv')
        dataset.head()
```

Out[2]:

	Date	Open	High	Low	Close	Adj Close	Volume
0	3/1/2012	16.262545	16.641375	16.248346	16.573130	16.573130	147611217
1	4/1/2012	16.563665	16.693678	16.453827	16.644611	16.644611	114989399
2	5/1/2012	16.491436	16.537264	16.344486	16.413727	16.413727	131808205
3	6/1/2012	16.417213	16.438385	16.184088	16.189817	16.189817	108119746
4	9/1/2012	16.102144	16.114599	15.472754	15.503389	15.503389	233776981

Please upload Chapter8-2.ipynb and follow the computing steps in Jupyter Notebook

The data structure is created to feed the inputs to the NN. T = 1100 for the training data set. The first 60 data points are used as a case in raining data X (X_train) while the next data point (point 61) is used as the corresponding label Y or Y_train. The concatenated X_train is reshaped into 1040 rows (overlapping cases) each with 60 columns (second dimension size) of timed inputs each.

In [7]:
```
### creating data structure with 60 time-steps and 1 output
X_train=[]
y_train=[]
for i in range(60,1100):
    X_train.append(training_set_scaled[i-60:i, 0])
    y_train.append(training_set_scaled[i, 0])
X_train, y_train = np.array(X_train), np.array(y_train)
print(X_train.shape, y_train.shape)
X_train=np.reshape(X_train, (X_train.shape[0], X_train.shape[1],1))
### this step converts X_train to 3D from (1040,60) to (1040,60,1) for input to the keras app
```

```
(1040, 60) (1040,)
```

Stacked LSTM NN is used with four stacked LSTM cells at each timed input for each case of 60 inputs followed by prediction of the output at the end of 60 time-sequenced inputs. There are 1040 cases as we use overlapping cases here. Each stacked LSTM cell contains 50 neurons. The output layer specified one neuron, so there is only one predicted scalar output.

Stacked LSTM NN is used with four stacked LSTM cells at each timed input for each case of 60 inputs followed by prediction of the output at the end of 60 time-sequenced inputs. There are 1040 cases as we use overlapping cases here. Each stacked LSTM cell contains 50 neurons. The output layer specified one neuron, so there is only one predicted scalar output. Notice that activation functions throughout are built into the Keras LSTM App as discussed in the last section. The d**ropout** rate is a regularization tool applied to reduce overfitting. In general, it is used in deep learning NN with many neurons and layers. If the dropout rate is set to say 0.20 at a particular layer (including input layer), then at that layer, 20% of the neurons will be randomly selected to "disappear" in that iteration of forward and backward propagation through that layer. In that iteration, there is no forward pass from the temporarily dropped out neurons, and there is also no updates or revisions in weights connected to those neurons during the backward propagation. In the next iteration, other neurons will be dropped out and revisions of weights continue as before. When dropout rate is added to the training, it is sometimes suggested that the width of the layer be scaled up by 1/(1 – dropout rate) so that the sum of all effective neurons in each iteration remains the same. Dropout can prevent overfitting and allow better prediction in generalized data.

Stacked LSTM

[10]:
```
### Initializing RNN
model = Sequential()
```

[11]:
```
### Add first LSTM layer and add Dropout Regularization

model.add(LSTM(units=50,return_sequences=True,input_shape=(X_train.shape[1],1)))

### Sequential reads input as 3D. Add return_sequences=True for all LSTM layers except
###  the last one. Setting this flag to True lets Keras know that LSTM output should
###  be 3D. So, next LSTM layer can work further on the data. If this flag is false, then
###  LSTM only returns last output (2D). Such output does not feed to another LSTM layer.

model.add(Dropout(0.2))
### This is a hyperparameter that prevents overfit by dropping 20% of nodal values
```

[12]:
```
### Add second LSTM layer and Dropout
model.add(LSTM(units=50,return_sequences=True))
model.add(Dropout(0.2))
```

[13]:
```
### Add third LSTM layer and Dropout
model.add(LSTM(units=50,return_sequences=True))
model.add(Dropout(0.2))
```

[14]:
```
### Add fourth LSTM layer and Dropout
model.add(LSTM(units=50))
### note: last LSTM layer does not carry argument 'return_sequences=True'
model.add(Dropout(0.2))
```

[15]:
```
### Add output layer
model.add(Dense(units=1))  ### not capital "U"nit
```

Stacked LSTM

```
[16]: ### Compiling the RNN
      model.compile(optimizer='adam',loss='mean_squared_error')

[17]: ### Run the training set with the LSTM (specialized RNN here)
      model.fit(X_train,y_train,epochs=100,batch_size=10)
```

In codeline [16], we employ optimizer ‘Adam’ for the optimal adjustment toward loss function minimum in the back propagation, and mean squared error MSE as the loss function to be minimized.

Code line [17] performs the fitting or optimization of loss function based on the built stacked LSTM model. The batch size = 10, which means that 10 cases are grouped together into one batch for parameter update each time, so there are 1040/10 = 104 batches. Updating through the 104 batches is one epoch. Repeating the updates involve more epochs – we use 100 epochs.

Fitting and Prediction

Next we perform the prediction (getting the LSTM output layer output) by feeding in the feature data in X_train. See code line [19]. As the output 'predict_train' is multiple array format, we use code line [21] to retrieve the column containing the 1040 predicted outputs with each case.

[19]:
```
predict_train=model.predict(X_train)
print(predict_train.shape)
### this output is (1040,60,1), we want only the first no. in each row of 1040
### this 3D structure makes it more difficult to interpret comparison of prediction
###  in training set vs output in trg set
```

```
33/33 ━━━━━━━━━━━━━━━━━━━━ 2s 43ms/step
(1040, 1)
```

[20]:
```
print(predict_train)
```

```
[[0.08865437]
 [0.08858573]
 [0.08465725]
 ...
 [0.86496216]
 [0.86246455]
 [0.87109554]]
```

[21]:
```
predict_train=predict_train[:, 0]
### select first column or first day of 60 days of each day prediction from 1 to 1040
```

[22]:
```
print(predict_train)
```

```
[0.08865437 0.08858573 0.08465725 ... 0.86496216 0.86246455 0.87109554]
```

Fitting and Prediction

By this step, the model is trained using the training set data. We now structure the test set data to perform prediction using the built/trained stacked LSTM model. Inputs to the model from the test set are in X_test and the prediction is compared with actual y_test using the MSE loss function.

[28]:
```
from sklearn.metrics import mean_squared_error
import math
def print_error(trainY, testY, train_predict, test_predict):
    ### Error of predictions
    train_rmse = math.sqrt(mean_squared_error(trainY, train_predict))
    test_rmse = math.sqrt(mean_squared_error(testY, test_predict))
    ### Print RMSE
    print('Train RMSE: %.3f RMSE' % (train_rmse))
    print('Test RMSE: %.3f RMSE' % (test_rmse))

print_error(y_train, y_test, predict_train, predicted_stock_price)
```

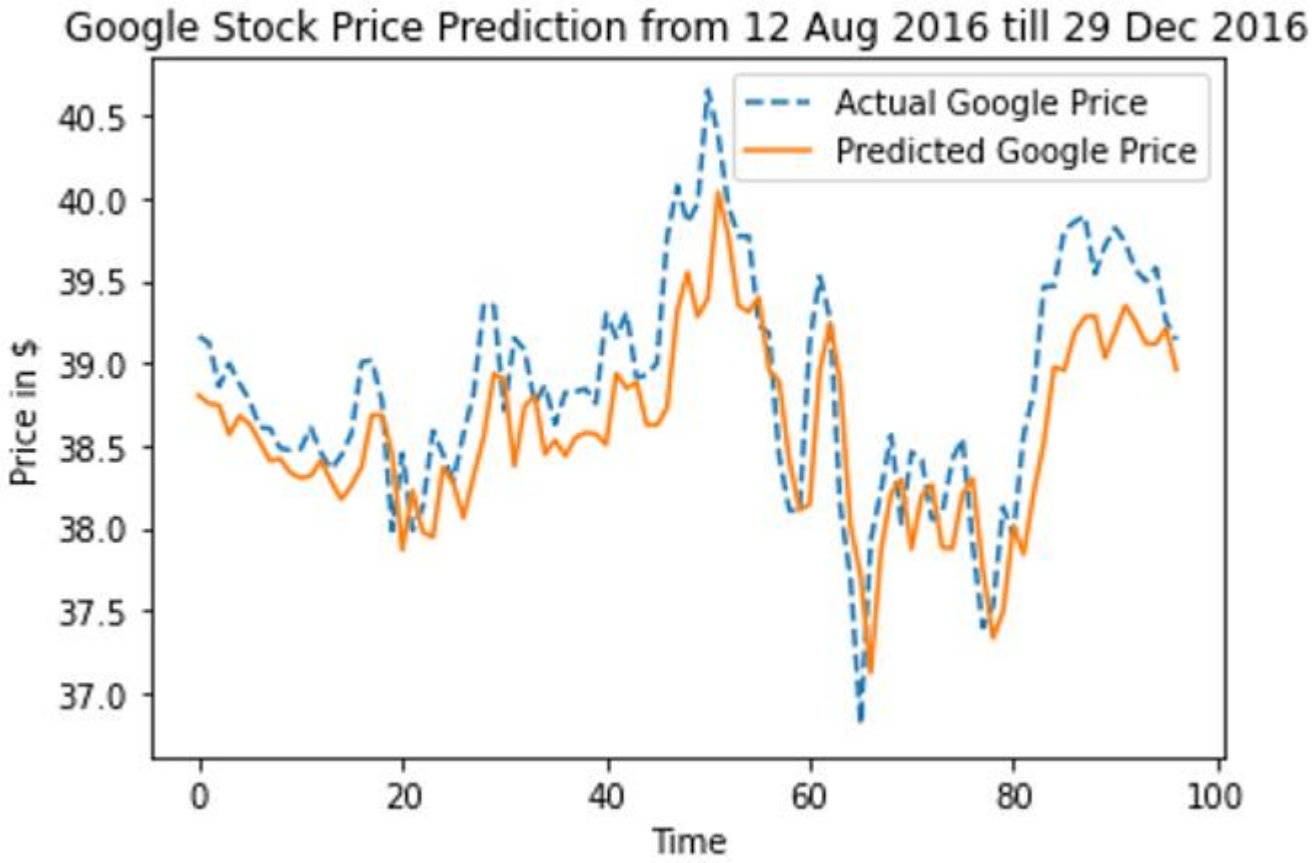
```
Train RMSE: 0.021 RMSE
Test RMSE: 0.022 RMSE
```

The RMSE for the training set fitting and then the test set prediction are about 2.1 % and 2.2% respectively. Recall that a different run will produce slightly different RMSEs due to the random seeds in the NN.

Overlapping Cases

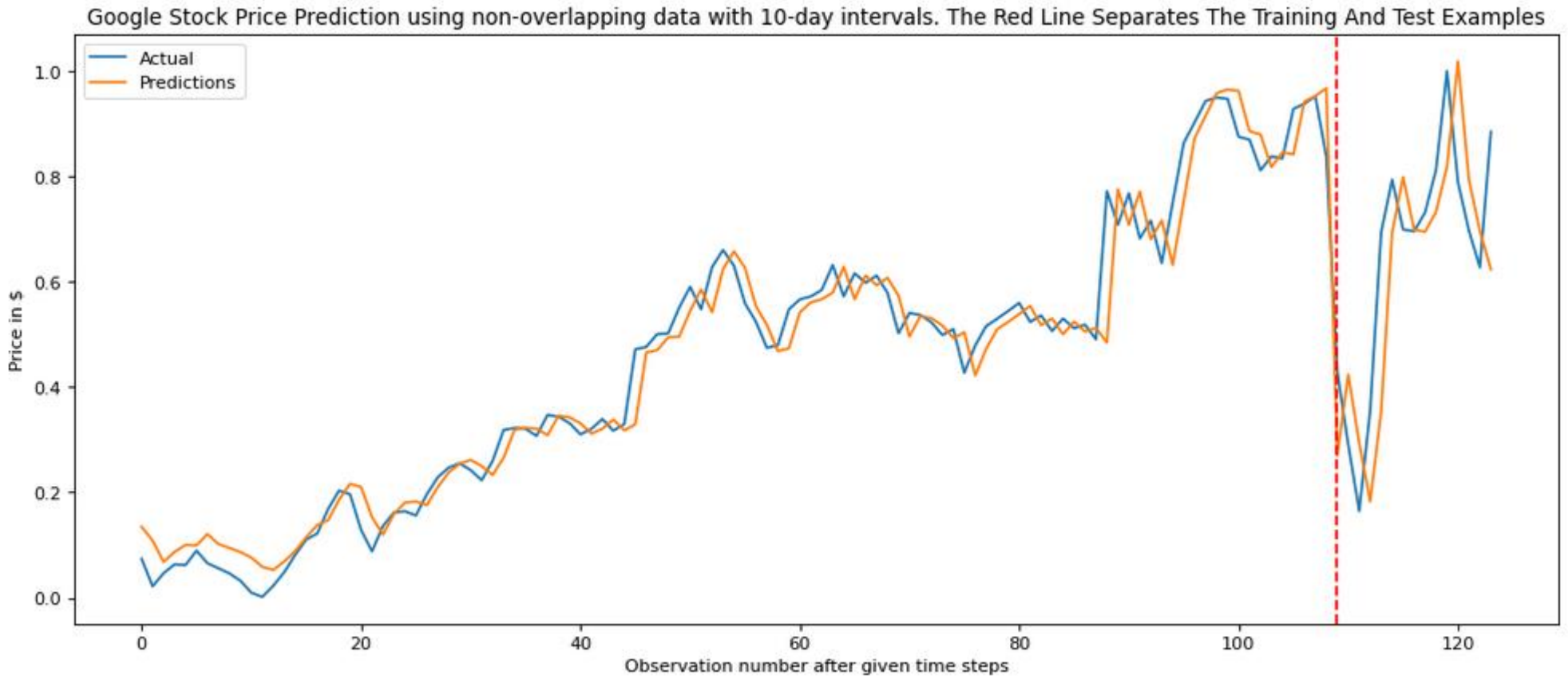
Please upload Chapter8-2.ipynb and follow the computing steps in Jupyter Notebook

The time series of the actual Google price from 12 Aug 2016 till 29 Dec 2016 is compared with the predicted price based on the LSTM model above – this is shown in the following matplotlib.pyplot graph. It is seen that the predicted price tended to follow/lag the actual price.



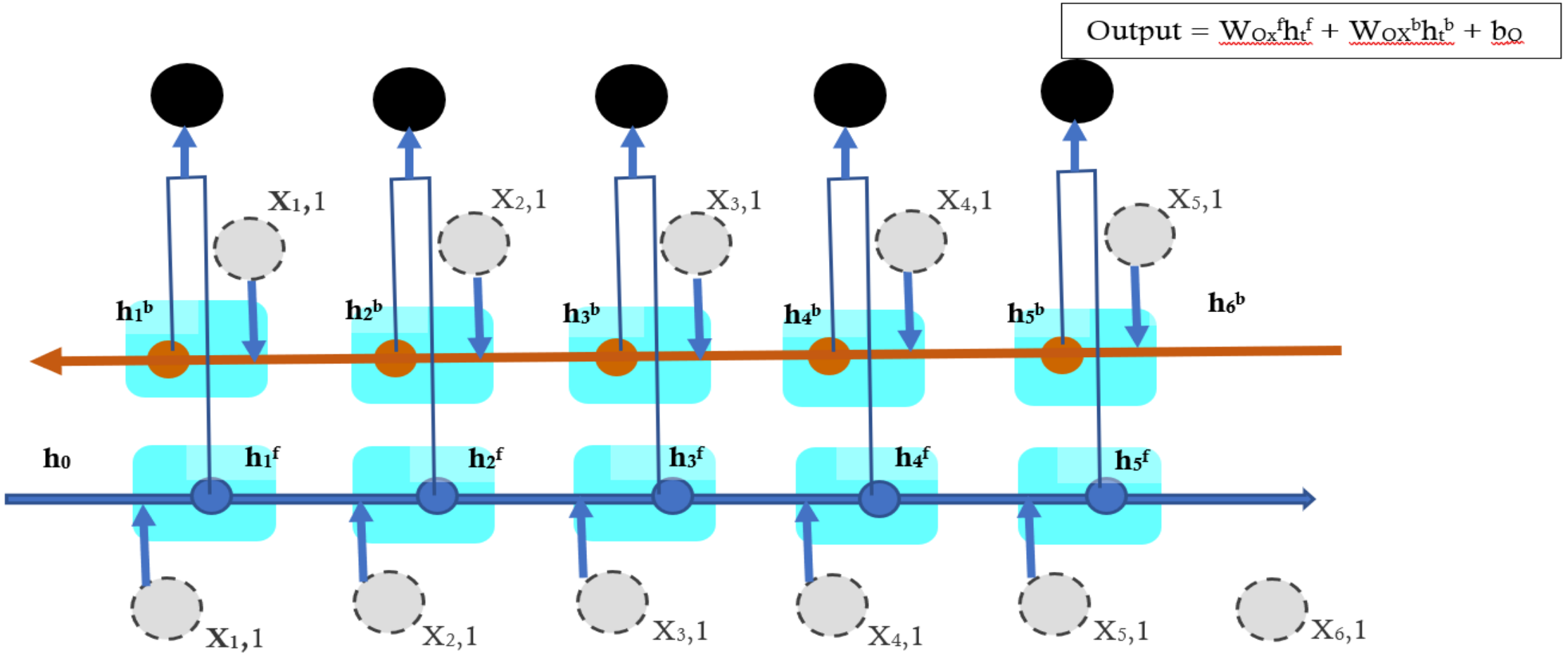
Non-overlapping Cases

Please upload Chapter8-3.ipynb and follow the computing steps in Jupyter Notebook



Bidirectional RNN/LSTM

A BRNN is a combination of two RNNs (also LSTMs) - one RNN (LSTM) moving forward from the start of the data sequence, and the other moving backward from the end of the data sequence.



Gated Recurrent Unit (GRU)

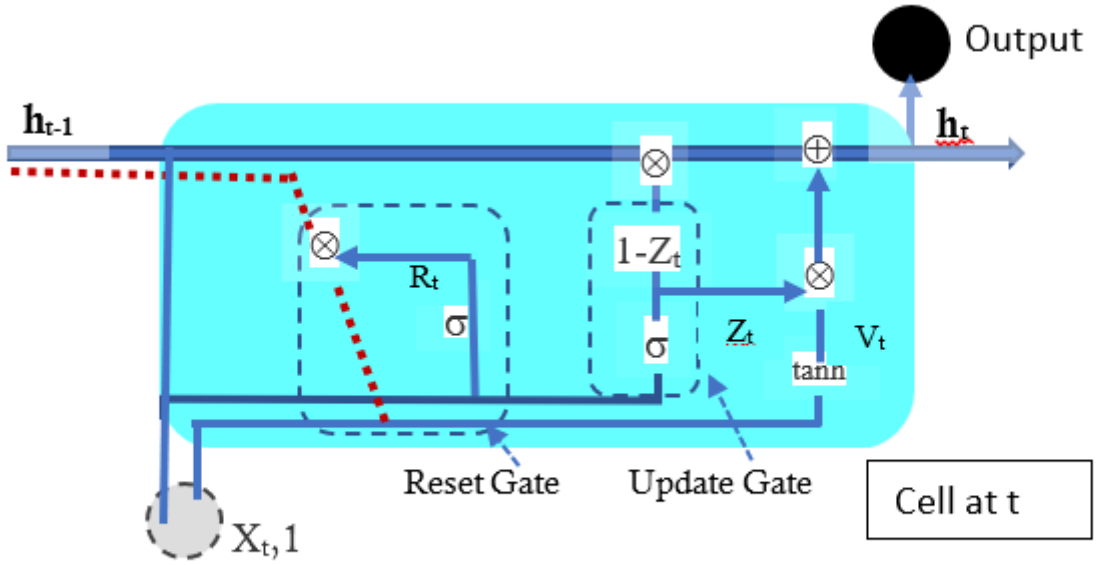


Figure 8.10

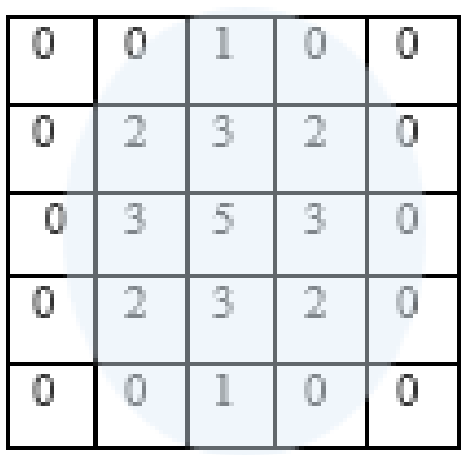
In the Reset Gate, the output R_t is given by $R_t = \sigma (W_{RX} X_t + W_R \mathbf{h_{t-1}} + b_R)$. This R_t is multiplied with $\mathbf{h_{t-1}}$ in a Hadamard product (see dotted line) and then activated using tanh function to create a candidate activation vector, $\mathbf{V_t}$. R_t is between 0 and 1 and could be close to zero, implying small V_t , where $V_t = \tanh (W_{VX} X_t + W_V (R_t \otimes \mathbf{h_{t-1}}) + b_V)$.

The Reset Gate decides how much of the past information in $\mathbf{h_{t-1}}$ is to be neglected or if the hidden state is important or not.

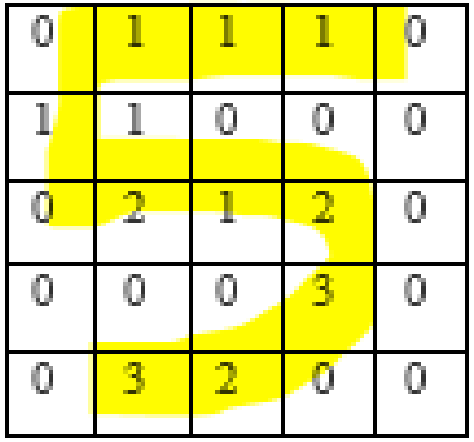
In the Update Gate, the output Z_t is given by $Z_t = \sigma (W_{UX} X_t + W_Z \mathbf{h_{t-1}} + b_Z)$. The output hidden state $\mathbf{h_t} = Z_t \otimes V_t + (1 - Z_t) \otimes \mathbf{h_{t-1}}$. The Update Gate determines a weighted average of V_t and past $\mathbf{h_{t-1}}$.

Convolutional Neural Network (CNN)

Another type of deep-learning neural network is convolutional neural network (CNN) that is frequently applied in image identification and classification. The CNN approach can also be applied to time series data for prediction and classification. In image recognition or classification, the input is typically a 2-dimensional matrix of pixel values. A pixel value could be the representation of shade of a pixel (smallest unit or building block of a digital image) at a location. A rugby ball could perhaps be represented as follows on the left and the figure 5 could be represented by 5 x 5 matrix on the right.



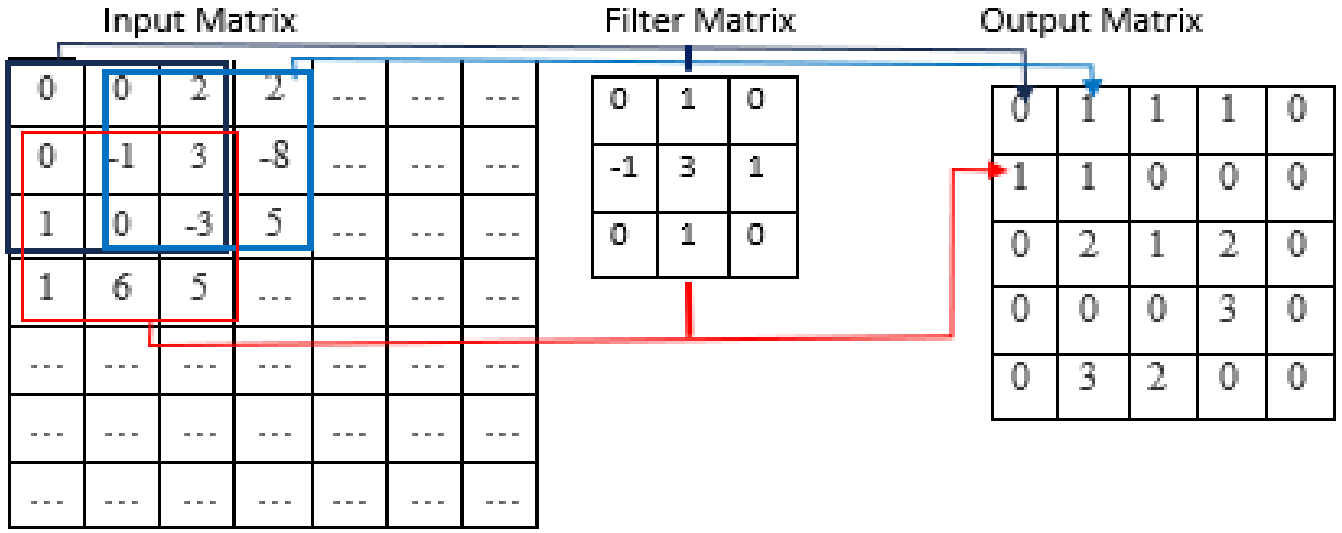
0	0	1	0	0
0	2	3	2	0
0	3	5	3	0
0	2	3	2	0
0	0	1	0	0



0	1	1	1	0
1	1	0	0	0
0	2	1	2	0
0	0	0	3	0
0	3	2	0	0

Convolutional Neural Network (CNN)

A colored picture could be represented by 3 sets of matrices, each corresponding to a color (RGB). In the color context, the 3 sets of filters can also be termed 3 channels. An input picture with pixel values in a huge, e.g., 10,000 × 10,000 matrix, could be reduced to a 5 × 5 matrix using several operations such as convolution filters, activation, and pooling. As an example, a 7 × 7 input matrix on the left when convolved with a 3 × 3 filter based on element by element multiplication and then summing all elements in the matrix produces 0 and is entered in the output matrix. This operation is a convolution. The output matrix is also called a feature map. Often, an activation function such as ReLU is put on this summing operation to ensure the output is a positive number.



Convolutional Neural Network (CNN)

Each filter in a convolution layer typically produces one output feature map. The outputs or feature maps could be passed through several alternating convolution and pooling layers before they are then flattened into a single dimension array of feature inputs into a dense (fully connected) neural network (layer(s) of neurons). The flattening concatenates all the final feature maps. So, if each final feature map is of dimension $k \times k$, then m feature maps mean a flattening to an input feature array of $k^2 \times m$. In a single time series prediction, the original image could be a one-dimensional array of numbers such as stock prices. The filter could be also a $k \times 1$ vector that shifts down the array of numbers to produce the feature map. This is shown in Figure 8.12.

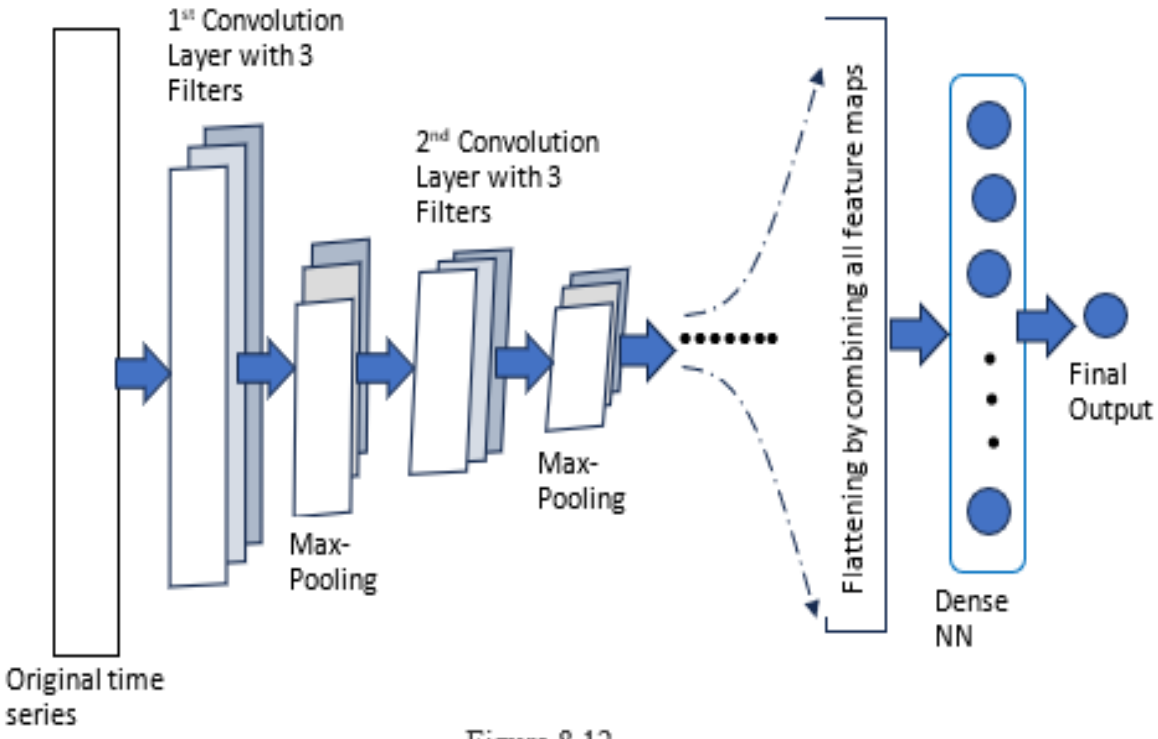


Figure 8.12

Convolutional Neural Network (CNN)

Using the same data set of Google prices in Worked Example II (demonstration file Chapter8-2.ipynb), the CNN is applied to predict the following day stock price. The algorithm uses 12 1-D filters with zero padding, kernel size of 3, and activation function ReLU. MaxPooling is done with a pool size equal to 2. 1,000 epochs and batch size of 10. The code lines are shown as follows.

[39]:
```
# Define model architecture
model = Sequential()
model.add(Conv1D(filters=12,kernel_size=3,activation='relu',input_shape=(60,1),padding='valid'))
model.add(MaxPooling1D(pool_size=2))
model.add(Flatten())
model.add(Dense(100,activation='relu'))
model.add(Dense(1))
model.compile(optimizer='adam',loss='mse')
```

[40]:
```
# fit CNN model
history = model.fit(X_train,y_train,epochs=1000,batch_size=10,verbose=1)
print(history)
```

Prediction is shown in code lines [41], [42], and the RMSE results are reported in code line [44].

[42]:
```
CNN_y_train = model.predict(X_train, verbose=0)
print(CNN_y_train)
```

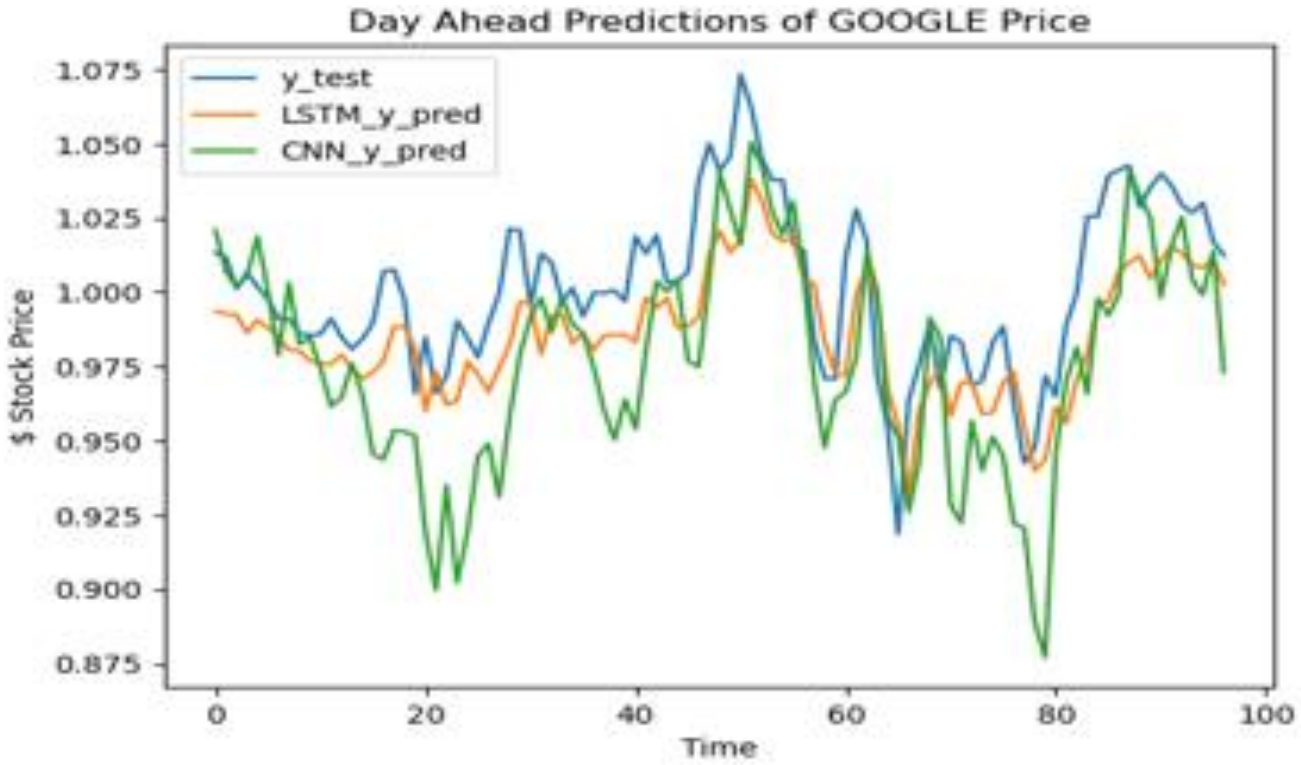
[43]:
```
# Testing the model
CNN_y_pred = model.predict(X_test, verbose=0)
print(CNN_y_pred)
```

```
print_error(y_train, y_test, CNN_y_train, CNN_y_pred)
```

```
Train RMSE: 0.011 RMSE
Test RMSE: 0.036 RMSE
```

Convolutional Neural Network (CNN)

The CNN training error of RMSE 1.1% is smaller than that of LSTM 2.1%. However, for the testing data set, CNN RMSE of 3.6% is larger than the 2.2% of LSTM. The predicted CNN prices in the test set are compared with the LSTM predicted price. The prediction results shown in the following graph are comparable.



CAVEATS

- For stock price prediction for trading purposes, there are two major caveats. One is transactions costs. Any apparent trading profits can be decimated or vanquished by transaction fees/costs, impact costs/widening bid-ask spread if more want to buy (ask goes up) or if more want to sell (bid goes down), liquidity risk – when it is not possible to trade as frequently as in the training/testing using past data, and slippages in market order – getting higher purchase price or lower selling price for your order (effect same as in execution risk when latency - time delay between order and actual trade – is not as fast as competitors.
- Two is the technical aspect of the prediction by minimizing mean-square errors. The results for Google indicate that the NNs are able to provide forward predictions with a small MSE. However, a closer look may reveal that most of the time, the algorithm is trend chasing. Hence the predicted curve appears to be a forward shift of the past price curve. Trading profit may be possible if the trend persists and that there is no major price reversals. Otherwise, a buy on an upward trend (or a sell on a downward trend) in the face of a major market fall (rise) would face huge losses (unless one has the capital to hold on and suffer marked-to-market losses) that can easily wipe out all the previous trend-chasing gains.

CAVEATS

- More meaningful ML models for trading could include training based not just on past prices, but using market microstructure features in high frequency trading (HFT) such as queue order level 2 quotes (market depth) or even level 3 (who are the buyers/sellers in the queue), the momentum such as average price movements in the last few intervals, bid-ask spread, volume, a market-sentiment measure, firm size, price-earnings ratios, price-to-book ratios, profit news, competitor prices, industry price, market movements, interest rates, and so on. And importantly, the model is effective only when back-testing on a trading strategy, e.g., buy low and sell high or sell high and buy low, can yield a significant profit.

In-Class Practice Exercise (not graded):

Chapter8-4.ipynb

Use the VIX price data for prediction – VIX.csv

Construct a stacked LSTM NN with 50 neurons in each stacked layer, and 4 layers, plus an output layer. Use batch size = 10 and # epochs = 100. Here time-steps = 60. This is an input in the input_shape=(X_train.shape[1],1)) of Keras LSTM App.

Use Adam optimizer, and mean-squared-error loss.

Use Overlapping training cases.

Find the training and the test prediction RMSEs.

End of Class