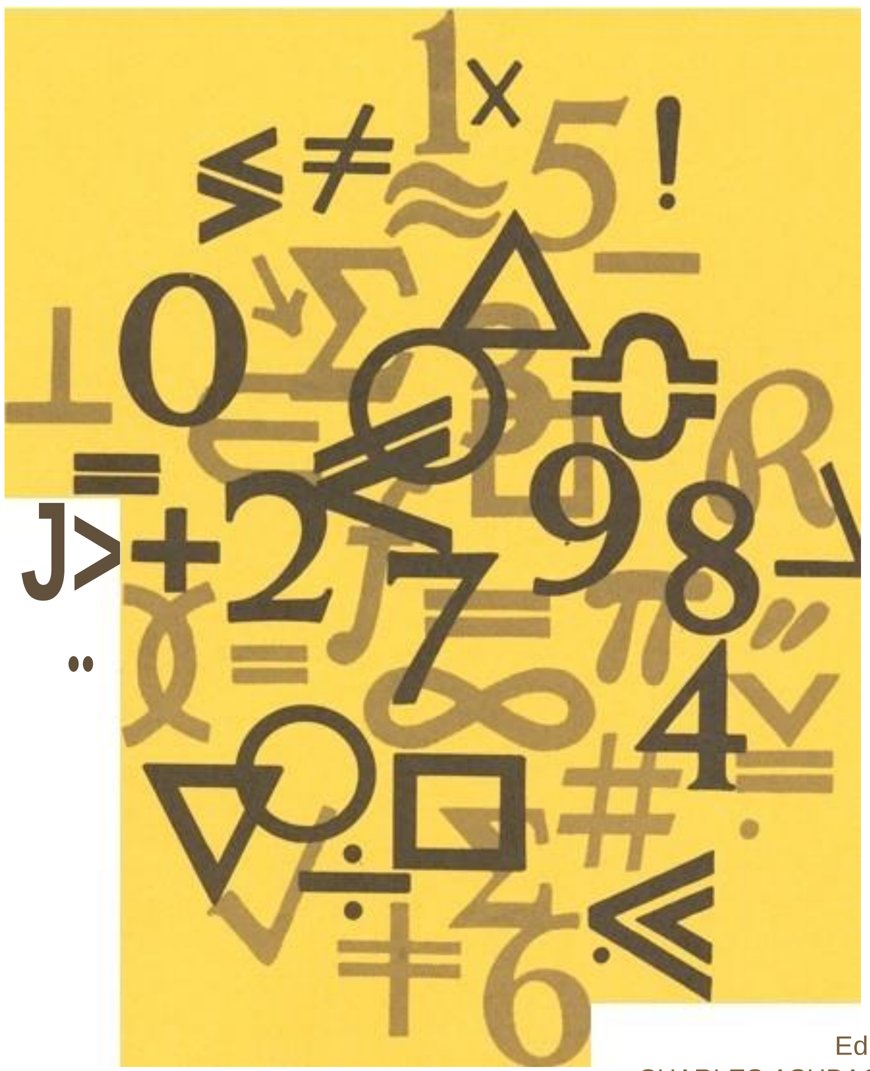
Volume 35, Number 1 - 2006

JOURNAL OF RECREATIONAL MATHEMATICS



Baywood Publishing Company, Inc.

ISSN0022-412X

Journal of RECREATIONAL MATHEMATICS

The *Journal of Recreational Mathematics* 1spublished by the Baywood Publishing Company. Inc.. 26 Austin Ave., P.O. Box 337. Amityville, NY 11701.

Indexed in *Academrc Search Premier; Book Review Index (BRIJ; H.W. Wilson; InternatHJnal Bibliography of Periodical Uterature; International Bibliography of Book Reviews;* and *The* Galo *Group.*

Subscription rate per volume (4 issues): 1ns11tu1tonat {includes postage) $258.00; Individual (includes postage) $67.00 (prepaidby personal check 0< credit card). Back listvolumesare availableat 10%abovecurrent price.Subscription is on a volumebasis onlyand mustbe pre-paid.Copies of single issues are not available. No part of this journal may be reproduced in any form withoutwrittenpermission from the publisher.

Visa/MasterCard Accepted

J. RECREATIONAL MATHEMATICS, Vol. 35(1) 44-51, 2006

SMYOZ: A NEW INDUCTION GAME

ZOLTAN HENNYEY

MIKLOS N. SZILAGYI

Department of Electrical and Computer Engineering
University of Arizona
Tucson, AZ 85721
e-mail: mns*171717@yahoo.com*

Everyone knows the Battleships game: one of the players has to find the ships of the other player on a graph paper by guessing. We propose an induction game based on this simple game but with strict rules to replace guessing with logic.

SMYOZ is played on a 9-by-9 size graph paper (i.e., 81 square cells) by two players. Let us assume that the player placing the ships on the grid is He and the player finding the ships is She. He places IO ships on the "sea": one a-ship consisting of four adjacent cells, two P-ships (three cells each), three y-ships (two cells each), and four 8-ships (one cell each). All ships are horizontally or vertically placed rectangles: no L-shaped or squared a- or P-ships are allowed.

There are three kinds of mesh points:

I. if a point belongs to a shore line, it is a *shore point;* all 36 points on the border of the graph are shore points;
2. if a point is on the border of a ship, it is a *ship point;*
3. if four water cells surround a point, it is a *water point.*

A *water point chain* is composed ofadjacent water points that can be connected by horizontal and vertical lines without reaching the border of any ship. Diagonal lines are not allowed.

The ships must be placed on the grid so that five basic rules are satisfied:

1. The ships cannot touch each other (i.e., a ship point belongs to one ship only).
2. When a ship is docked at a shore line, exactly two ship points must be simultaneously also shore points (i.e., ships cannot be docked at any corner and the α-, }-, and ψ-ships cannot be parallel to the shore line at which the ship is docked).
3. The geometric center of each ship is determined as the center of gravity of the corresponding rectangle. The geometric centers of ships of the same category (}, ψ, or o) cannot be on the same horizontal or vertical line.
4. If two or more ships are in line, they must be separated from each other by at least two cells. The placement of ships shown in Figure 1a is permissible because the two ships are not in line. In Figure 1b a forbidden placement is shown.
5. If two or more ships are parallel to each other and they are separated by one cell only, they are not allowed to be abreast to the extent of more than one cell. Therefore, the situation of Figure 1c is permissible but that of Figure 1d is not.

When all the ships are placed according to these rules, He proceeds further. In each row and column, if two or more ships occupy that row or column, a broken (or red) line must be drawn linking the inner sides of the two outermost ships in that row or column. If there are more than two ships in a given row or column, the broken line will pass through all ships that are in between the two outermost ones (see Figure 2).

If there are no ships in a row or column, a dotted (or green) line must be drawn throughout that row or column. In this case the line is called a *water line*.

If there is exactly one ship in a row or column, a dotted (or green) line must be drawn through those cells of the ship that belong to that particular row or column. This line runs in the direction of the given row or column but it does not extend beyond the ship (see Figure 2).

Now He declares the number of water lines (w), the number of docked ships (d), and the water point chains c_i ($i = 1, 2, \ldots, n$), where c_i is the number of water points in the i-th chain and n is the number of chains. Obviously,

$$\Lambda_{i=1}^{n} c_i = 2d + 4 \tag{1}$$

These data are listed above the graph (see Figure 2). The value of d can be calculated from Equation (1) if it is not given. These data can be found at the top of Figure 2.

He now declares the positions of the }- and ψ-ships with respect to the position of the α-ship. If a ship is parallel to the α-ship, it is not marked. If it is perpendicular, it is marked with an asterisk (see the left-hand side of Figure 2). He also declares the number of lines inside each ship by 2-digit numbers. The first

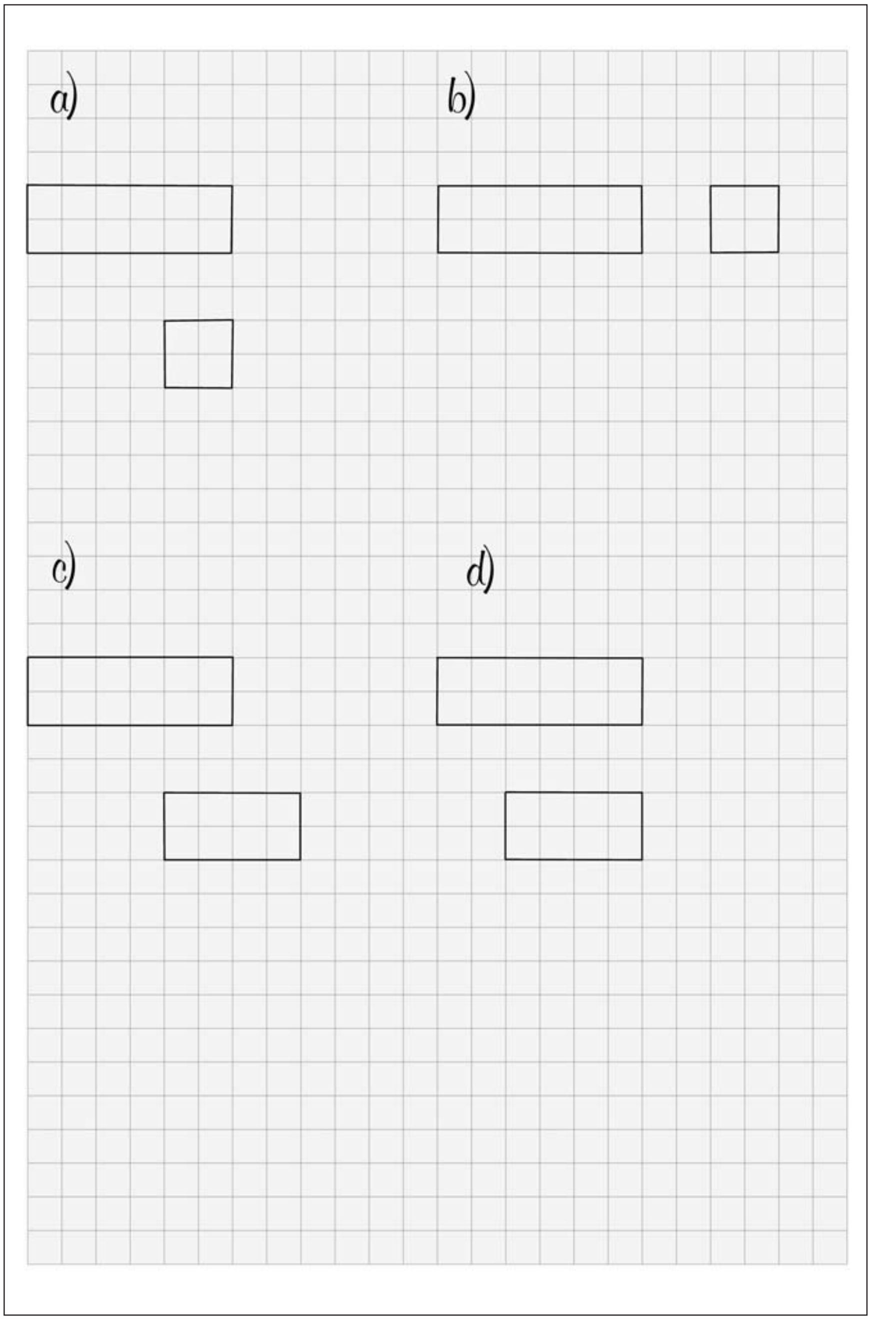
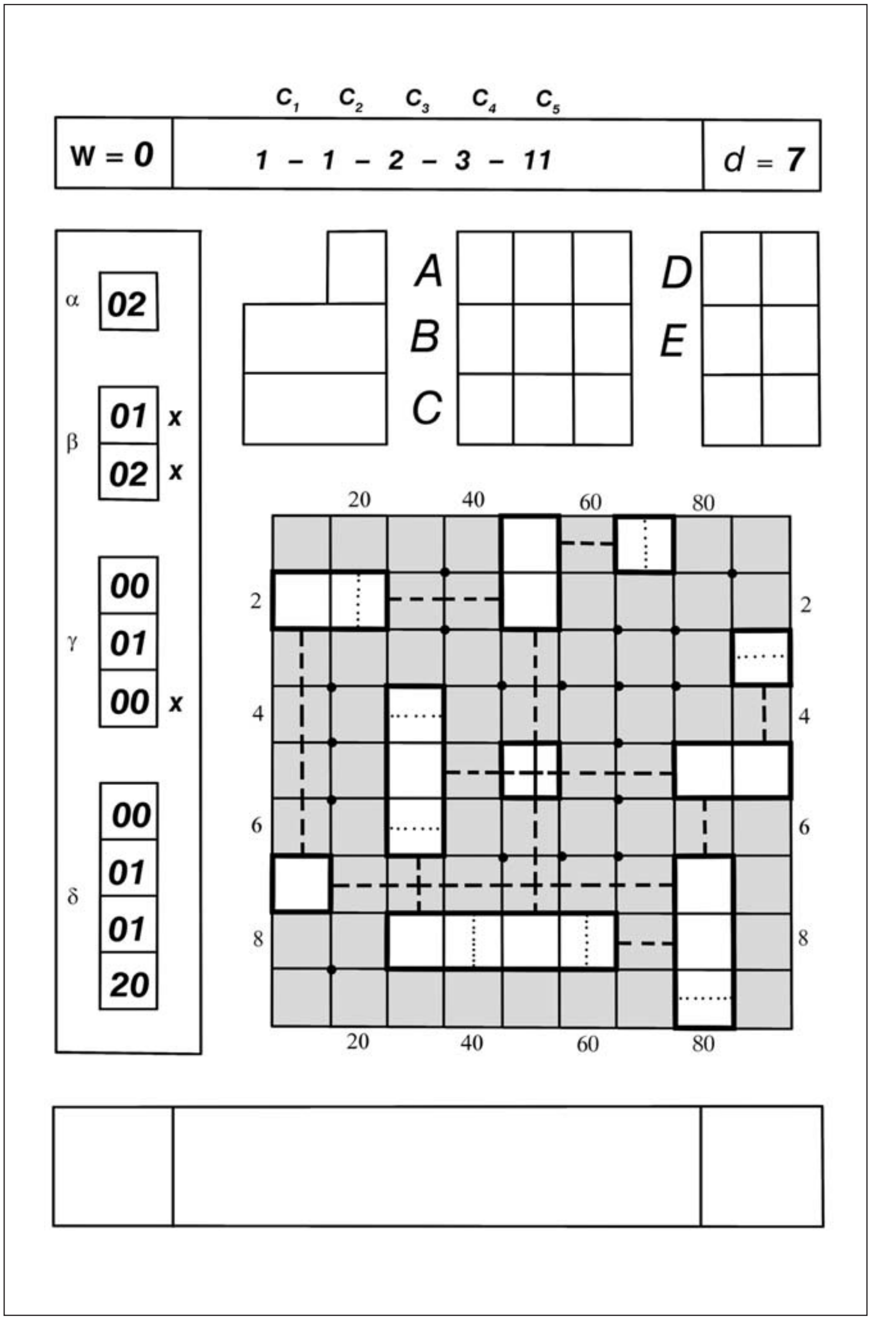


Figure 1.



c_1 c_2 c_3 c_4 c_5
W = 0
1 – 1 – 2 – 3 – 11
d = 7
α
02
β
01 x
02 x
γ
00
01
00 x
δ
00
01
01
20
A
B
C
D
E
20
40
60
80
2
4
6
8

Figure 2.

digit stands for the number ofborken (or red) lines (r_j), the second digit stands for the number of dotted (or green) lines (g_j), Obviously, j = 1, 2, ... , I 0. It can be proved that

$$\sum_{j=1}^{10} (g_j - r_j) = 6-2w \tag{2}$$

Now comes the more difficult part of the game. She must find all the ships on the basis of the declared information. She may ask for additional information by placing letters A, B, C, D, and E into any cell on the grid. To get more information, the same letter may be placed into up to three different cells. He must declare the situation in each cell chosen by Her. However, if a letter is placed in more than one cell, the answer will not specify which data belong to which cell. For example, if the letter A is placed in two cells, there will be two different answers but without the coordinates of the chosen cells.

The answers must satisfy the following rules. If a chosen cell is a ship cell, He declares the category of the ship. If it is a water cell, He declares the number of water points bordering that cell. The number of broken (or red) lines inside the cell is declared by drawing as many horizontal lines *above* the Greek letter or the number as the number of broken (or red) lines in that cell. The number of dotted (or green) lines inside the cell is given similarly but the horizontal lines are drawn *below* the Greek letter or number.

For an illustration, see Figure 3 where all the information is provided for the unique solution of the problem presented in Figure 2.

At first glance, the game seems to be too complicated, but remember that this is a mathematical game that replaces guessing with logic. Simplification of the rules would lead to the re-emergence of guessing. After some practice, the feeling of undue complication will disappear.

The real challenge is to construct SMYOZ problems with unique solutions providing as little amount of information as possible. An example is given in Figure 4 where additional information is provided for one cell only. The unique solution is given in Figure 5. The Reader is invited to construct such problems and also to work on the mathematical theory of the game.

Acknowledgment

The authors are indebted to Dr. Martin Gardner for his kind interest in this work.

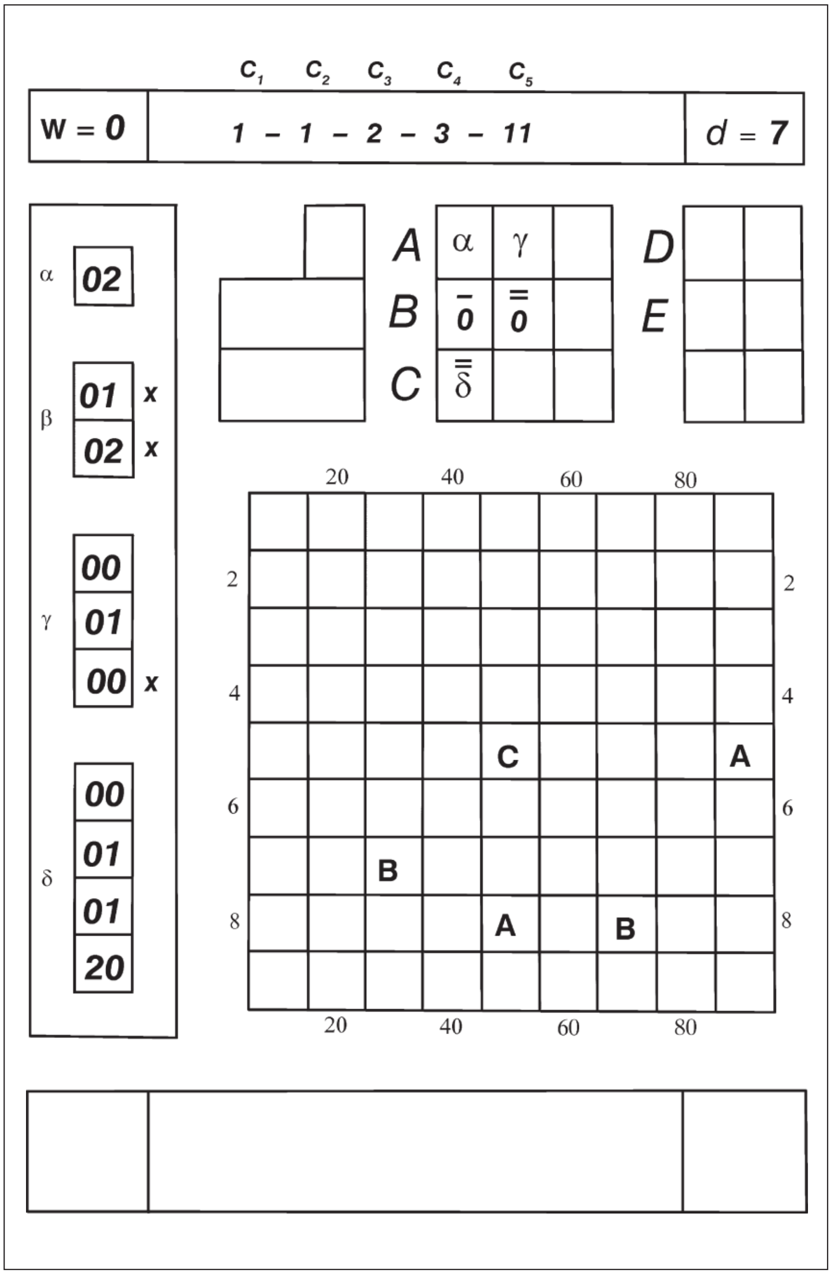
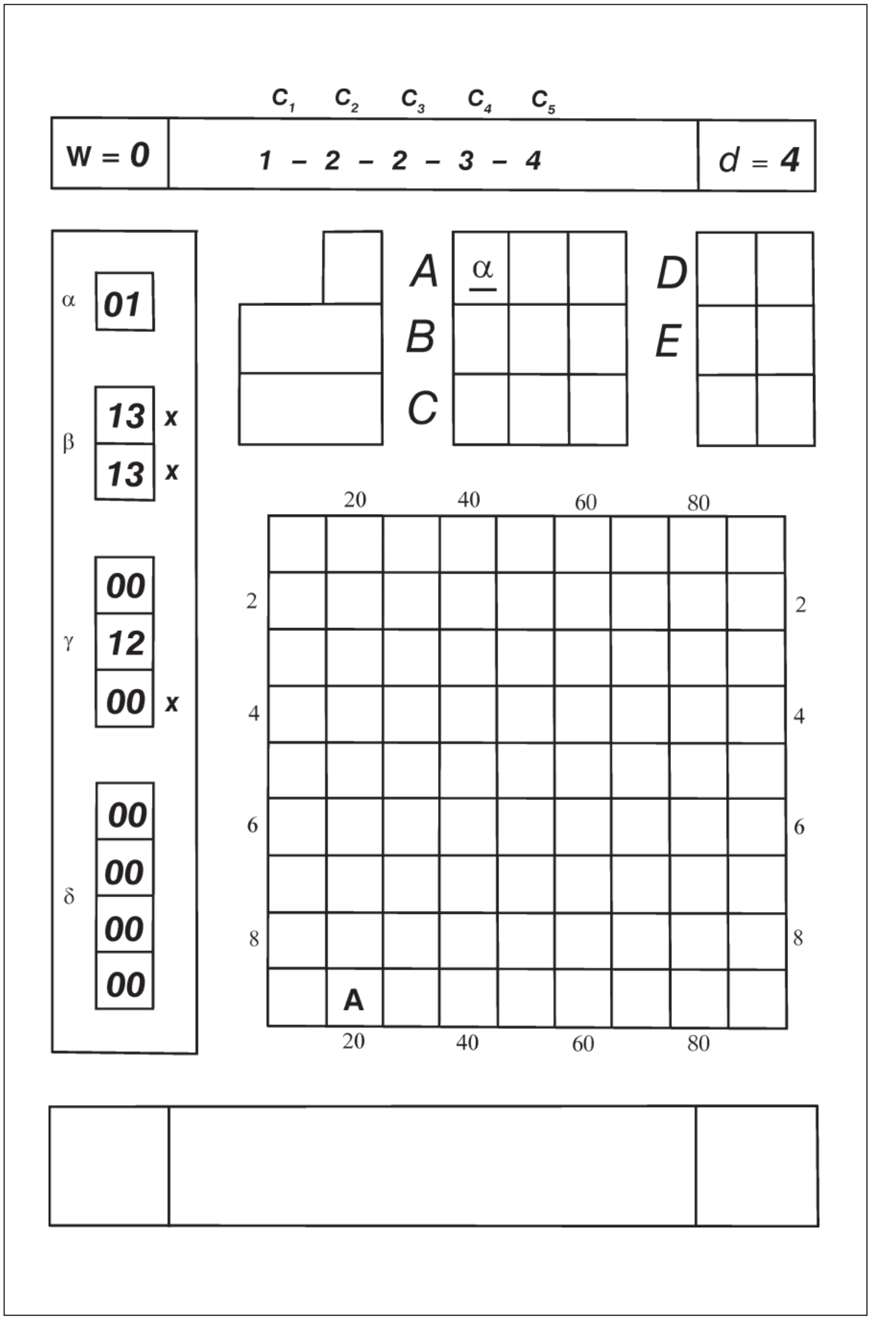
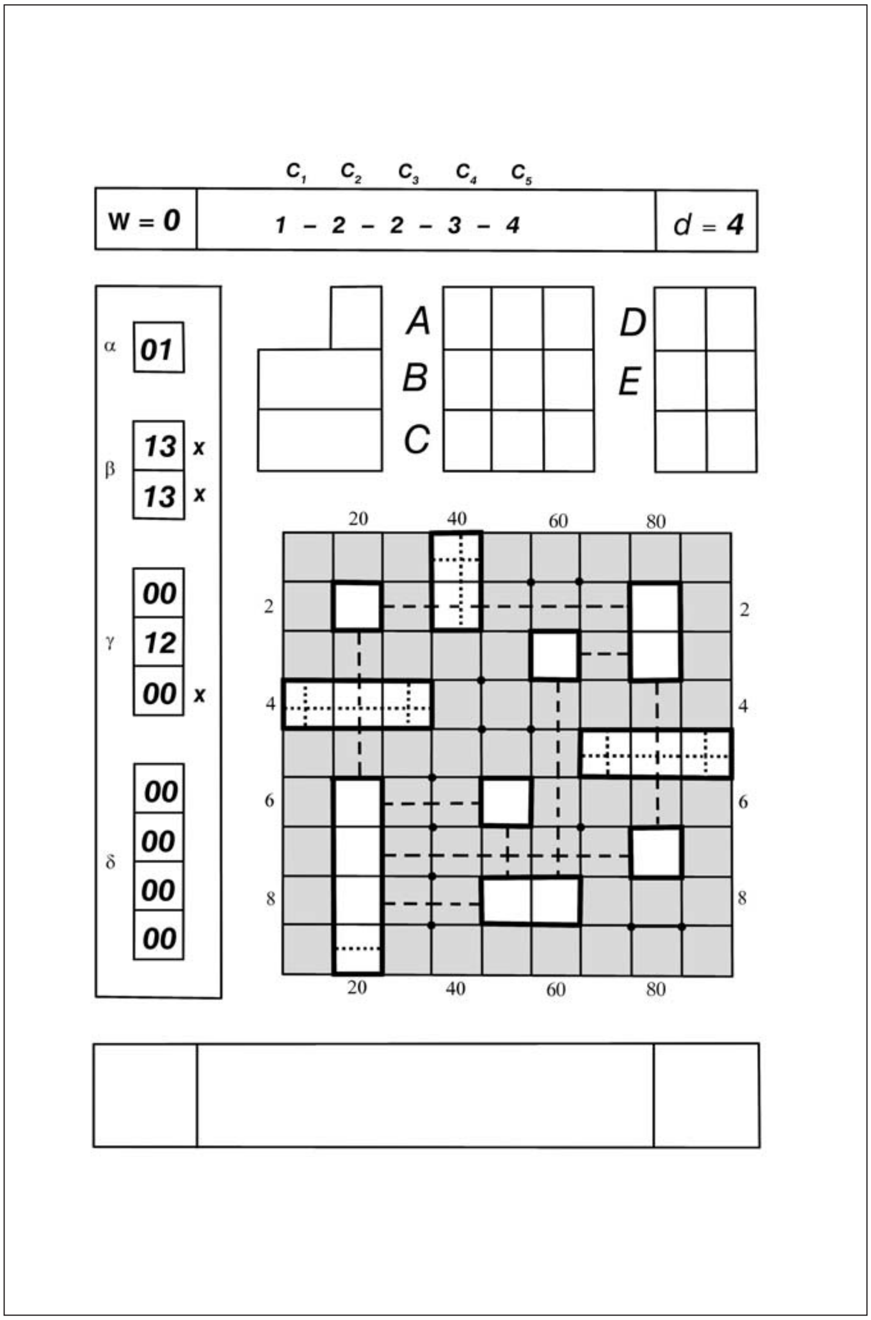

C1 C2 C3 C4 C5
W = 0
1 – 1 – 2 – 3 – 11
d = 7
α 02
β 01 x 02 x
γ 00 01 00 x
δ 00 01 01 20
A B C D E
20 40 60 80
2 4 6 8

Figure 3.



C1 C2 C3 C4 C5
W = 0
1 – 2 – 2 – 3 – 4
d = 4
α 01
β 13 x
13 x
γ 00
12
00 x
δ 00
00
00
00
A α
B
C
D
E
20 40 60 80
2 4 6 8
A

Figure 4.



c_1 c_2 c_3 c_4 c_5
w = 0
1 – 2 – 2 – 3 – 4
d = 4
α 01
β 13 x 13 x
γ 00 12 00 x
δ 00 00 00 00
A
B
C
D
E
20 40 60 80
2 4 6 8

Figure 5.