CS5691: Programming Assignment 1

Team 20

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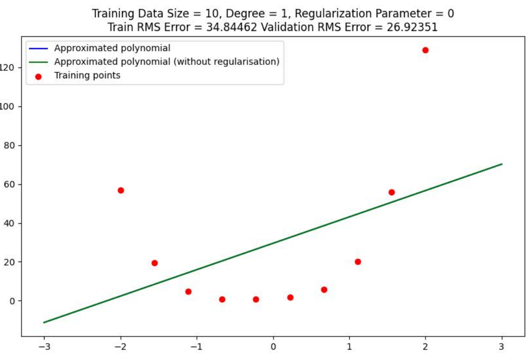
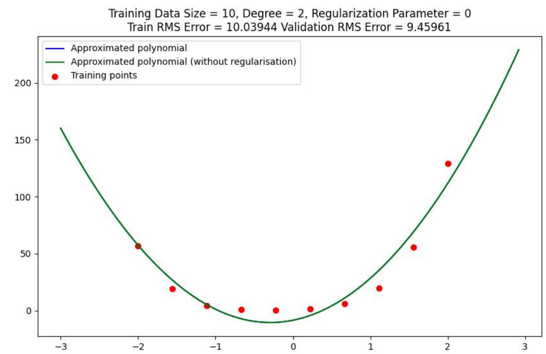
1.1 Training Dataset of size 10

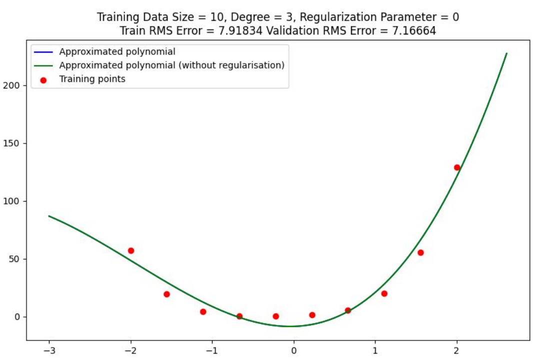
1.1.1 Plots of Approximated Functions without Regularization



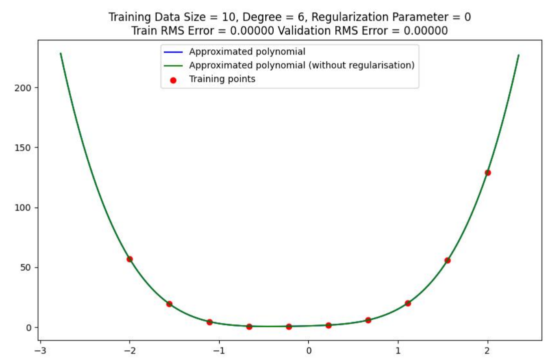
(1) $m = 1$



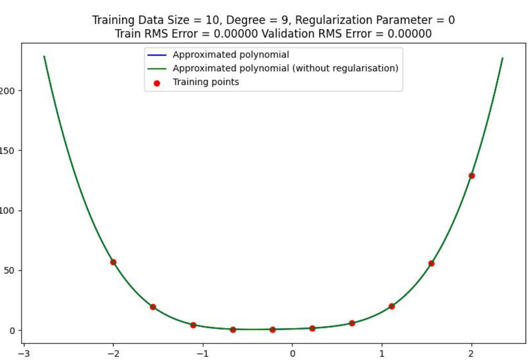
(2) $m = 2$



(3) $m = 3$



(4) $m = 6$



(5) $m = 9$

Figure 1: Approximated Functions without Regularization for training dataset of size 10

1.1.2 Inferences

1. With small values of degree m such as $1, 2$ and 3, the approximated model is unable to fit to the training data accurately. Both the training and validation errors are large. This is a case of underfitting. This indicates that the underlying distribution of the data has a higher complexity than expected.

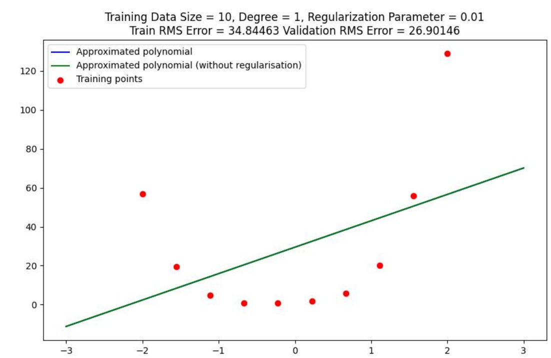
2. With large value of degree $m = 9$, the approximated model fits exactly onto the training dataset. But this can be a case of overfitting as there are only 10 data points available. The number of parameters to estimate is given by

$$D = \frac{(m+d)!}{m! \cdot d!}, m = \text{degree of polynomial}, \; d = \text{dimensionality of input vector} \tag{1}$$

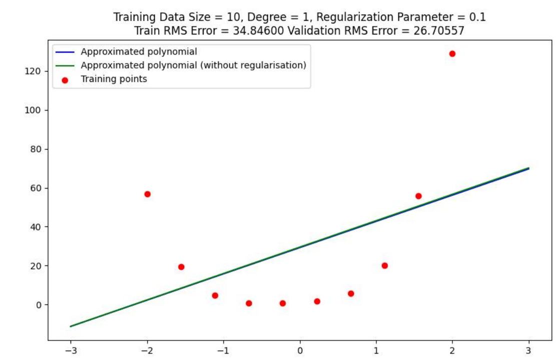
   which with $m = 9$ and $d = 1$, gives 10 parameters. So, we need much more than 10 datapoints to train this model effectively. Hence, this is not the best model possible.

3. With degree $m = 6$, the approximated model fits exactly onto the training data and the number of datapoints is sufficient to train this model. Hence, this model gives the best result with the given training points. Since this model has nearly zero training and validation error, we don't require the use of regularization to improve the model.

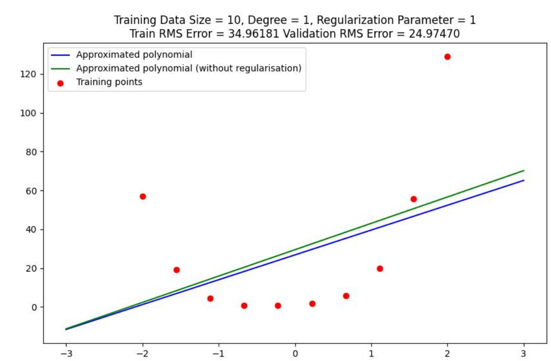
1.1.3 Plots of Approximated Functions with Regularization



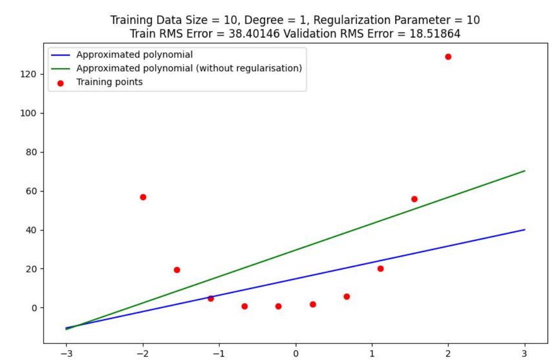
(1) $m = 1, \lambda = 0.01$



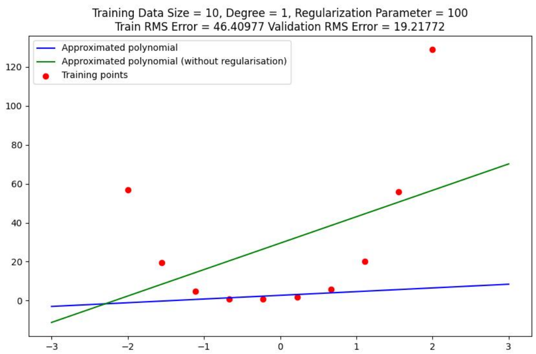
(2) $m = 1, \lambda = 0.1$



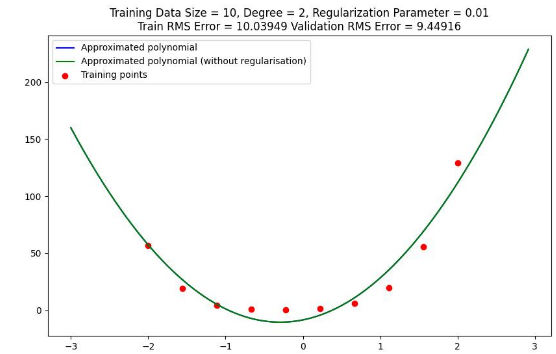
(3) $m = 1, \lambda = 1$



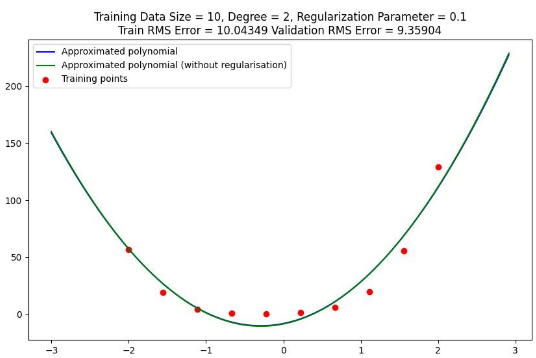
(4) $m = 1, \lambda = 10$



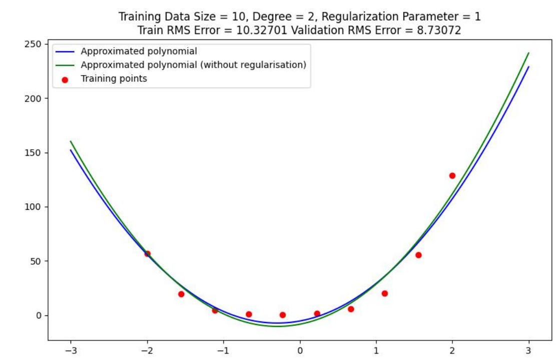
(5) $m = 1, \lambda = 100$



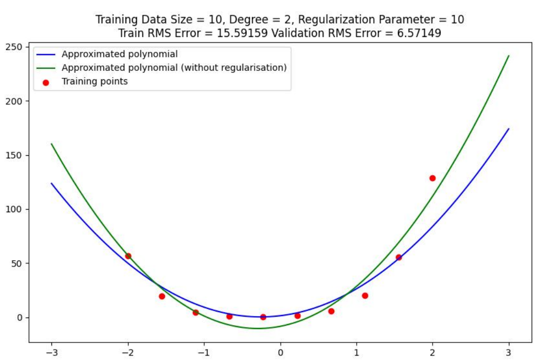
(6) $m = 2, \lambda = 0.01$



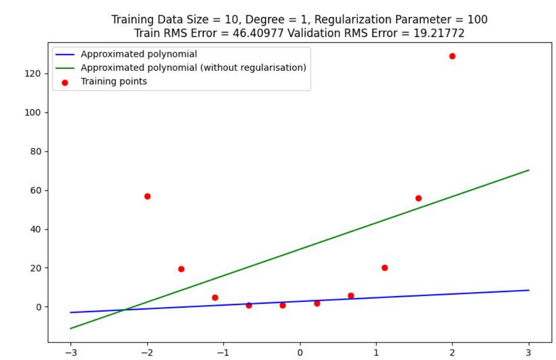
(7) $m = 2, \lambda = 0.1$



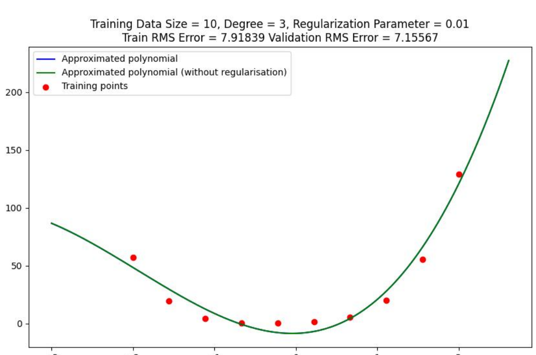
(8) $m = 2, \lambda = 1$



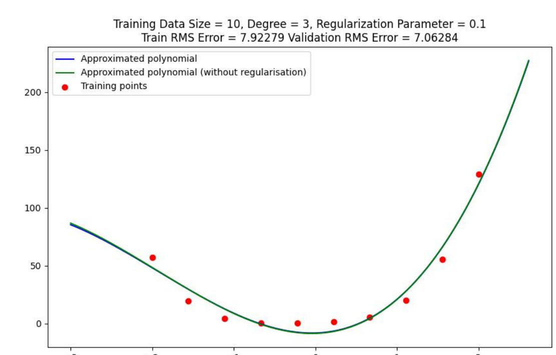
(9) $m = 2, \lambda = 10$



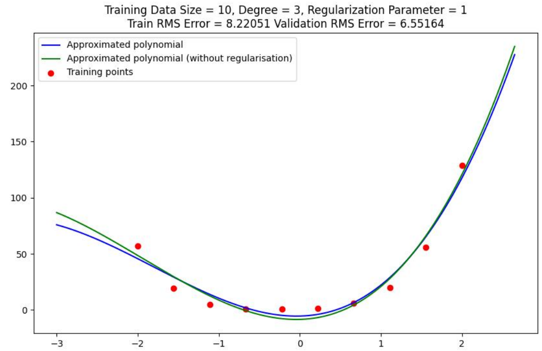
(10) $m = 2, \lambda = 100$



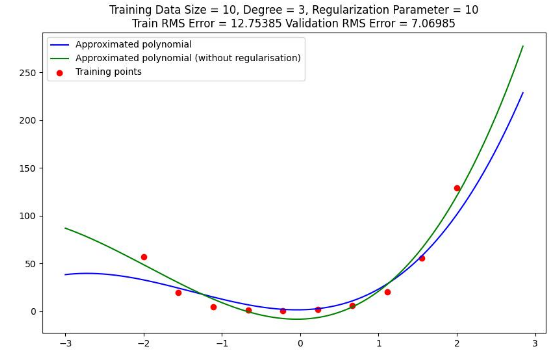
(11) $m = 3, \lambda = 0.01$



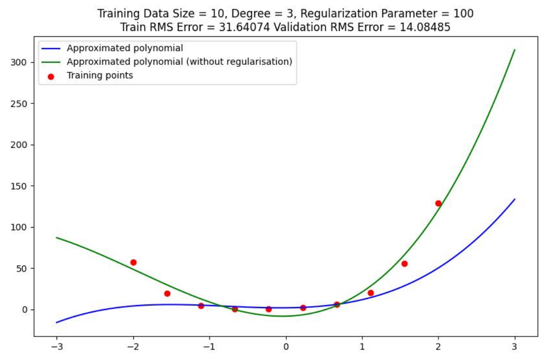
(12) $m = 3, \lambda = 0.1$



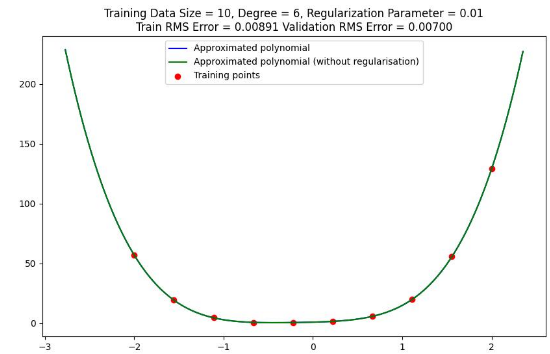
(13) $m = 3, \lambda = 1$



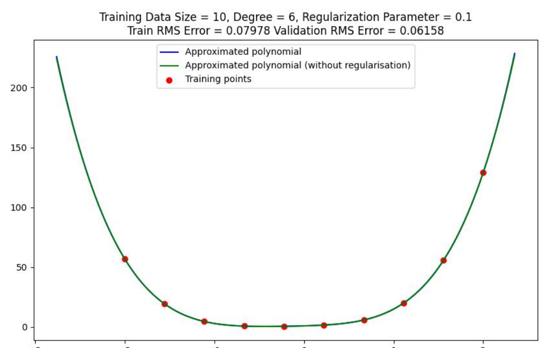
(14) $m = 3, \lambda = 10$



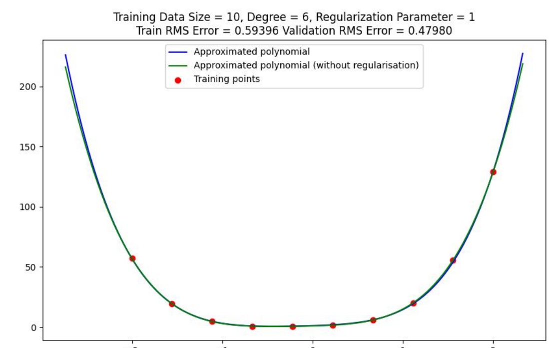
(15) $m = 3, \lambda = 100$



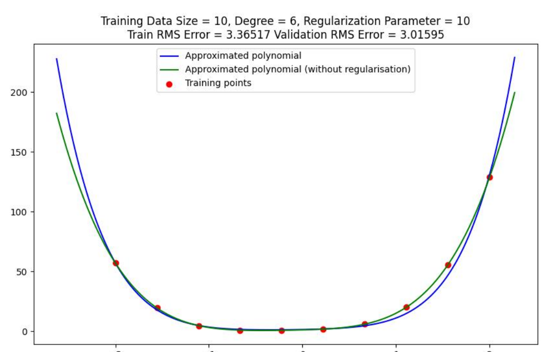
(16) $m = 6, \lambda = 0.01$



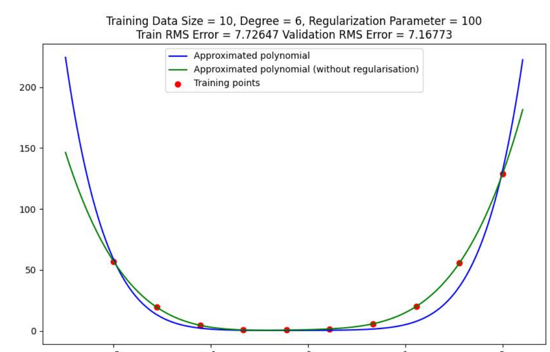
(17) $m = 6, \lambda = 0.1$



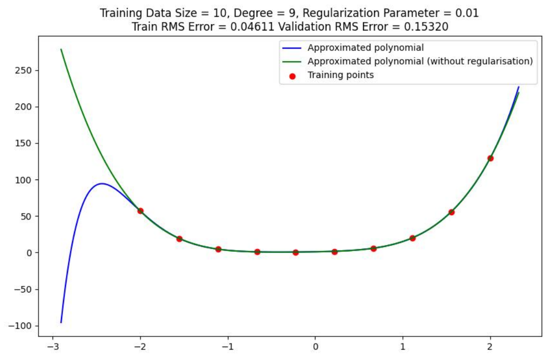
(18) $m = 6, \lambda = 1$



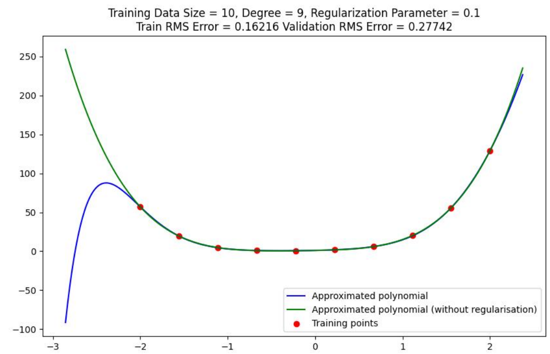
(19) $m = 6, \lambda = 10$



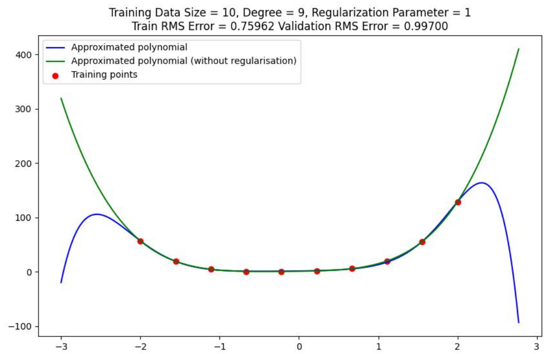
(20) $m = 6, \lambda = 100$



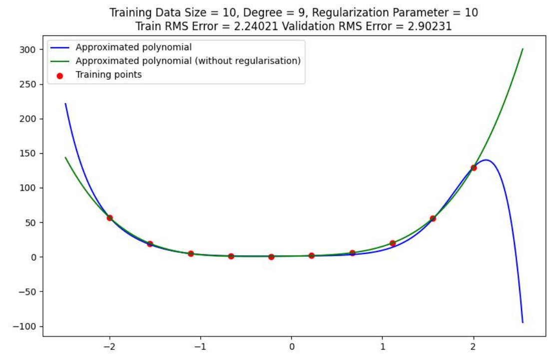
(21) $m = 9, \lambda = 0.01$



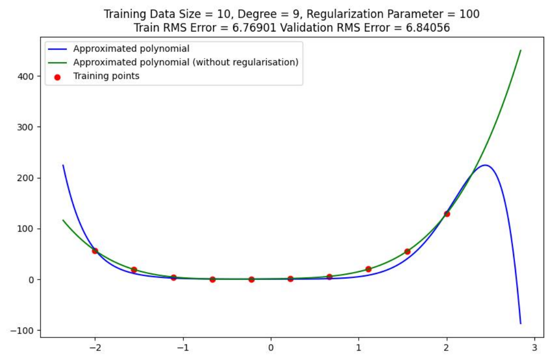
(22) $m = 9, \lambda = 0.1$



(23) $m = 9, \lambda = 1$



(24) $m = 9, \lambda = 10$



(25) $m = 9, \lambda = 100$

Figure 2: Approximated Functions with Regularization for training dataset of size 10

1.1.4 Inferences

1. For small values of $m = 1, 2, 3$, the model is still unable to fit to the underlying data. This is a case of underfitting and regularization will not be of help in these models.

2. For large values of $m = 6, 9$, even a small amount of regularization causes the model to deviate a lot from the training data. This can be seen in the marked increase in the training error from $\lambda = 0$ to $\lambda = 0.01$ for $m = 6, 9$. This is likely due to the extremely small size of the training dataset.

3. For all values of the degree m, as the regularization parameter λ increases, the training error increases and the validation error decreases.

4. Regularization forces the model to slightly deviate from the training points, hence reducing possible overfitting. However, with large values of λ such as 100.0, the model deviates too much from the training data and does not capture the underlying distribution. This can be seen by the increase in validation error. Hence, large values of λ can lead to underfitting.

5. We can see that the use of regularization did not improve the model's performance, hence proving validity our initial assertion regarding the same.

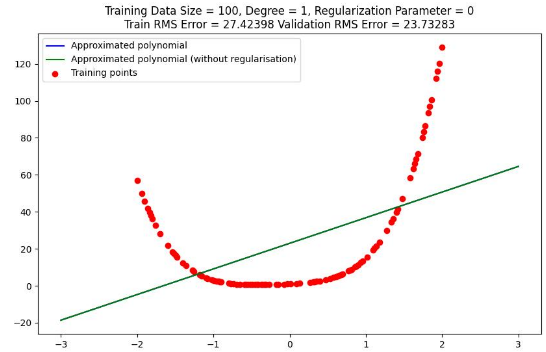
1.1.5 Hyperparameters v/s Training, Validation and Testing Errors

Degree	Regularization Parameter	Training Error	Validation Error	Testing Error
1	0.0	34.844 62	26.923 51	24.777 53
1	0.01	34.844 63	26.901 46	24.774 97
1	0.1	34.846 00	26.705 57	24.753 86
1	1.0	34.961 81	24.974 70	24.705 09
1	**10.0**	**38.40146**	**18.51864**	**28.38081**
1	100.0	46.409 77	19.217 72	37.623 10
2	0.0	10.039 44	9.459 61	8.968 14
2	0.01	10.039 49	9.449 16	8.959 79
2	0.1	10.043 49	9.359 04	8.888 79
2	1.0	10.327 01	8.730 72	8.484 40
2	**10.0**	**15.59159**	**6.57149**	**10.95333**
2	100.0	34.874 18	11.308 98	27.616 02
3	0.0	7.918 34	7.166 64	8.262 92
3	0.01	7.918 39	7.155 67	8.256 67
3	0.1	7.922 79	7.062 84	8.203 53
3	**1.0**	**8.22051**	**6.55164**	**7.88404**
3	10.0	12.753 85	7.069 85	8.206 95
3	100.0	31.640 74	14.084 85	23.444 66
6	**0.0**	$\mathbf{8.13774 \times 10^{-12}}$	$\mathbf{5.24059 \times 10^{-12}}$	$\mathbf{7.29628 \times 10^{-12}}$
6	0.01	0.008 91	0.006 99	0.010 73
6	0.1	0.079 78	0.061 58	0.097 18
6	1.0	0.593 96	0.479 80	0.724 80
6	10.0	3.365 17	3.015 95	3.951 49
6	100.0	7.726 47	7.167 73	9.182 62
9	**0.0**	$\mathbf{3.48263 \times 10^{-9}}$	$\mathbf{3.43536 \times 10^{-9}}$	$\mathbf{3.57539 \times 10^{-9}}$
9	0.01	0.046 11	0.153 20	0.173 41
9	0.1	0.162 16	0.277 42	0.314 50
9	1.0	0.759 62	0.997 00	1.123 76
9	10.0	2.240 21	2.902 31	2.578 38
9	100.0	6.769 01	6.840 56	7.822 74

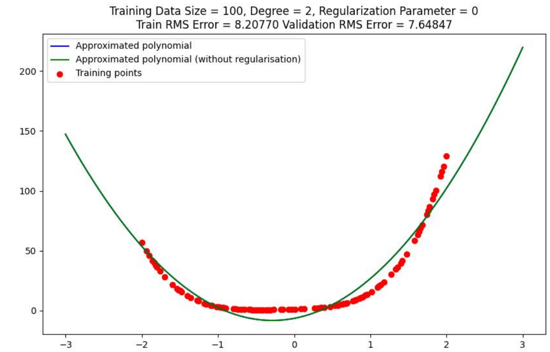
Table 1: Training, Validation and Testing Errors for different m, λ for training dataset of size 10. The best model for each m is taken to be the one with the lowest validation error and are shown in boldface.

1.2 Training Dataset of size 100

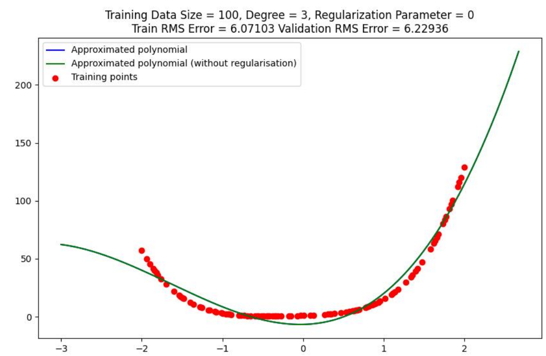
1.2.1 Plots of Approximated Functions without Regularization



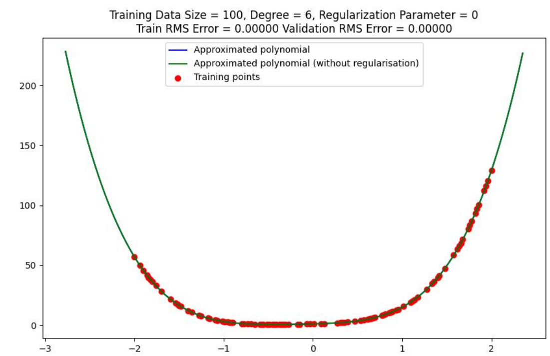
(1) $m = 1$



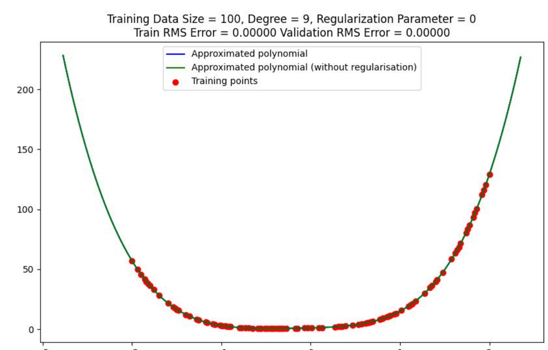
(2) $m = 2$



(3) $m = 3$



(4) $m = 6$



(5) $m = 9$

Figure 3: Approximated Functions without Regularization for training dataset of size 100

1.2.2 Inferences

1. For small values of m such as $1, 2$ and 3, the model does not fit well with the training data and there is underfitting. This is reflected in the high training and validation errors. This indicated that the underlying distribution of data has a much higher complexity than these models.

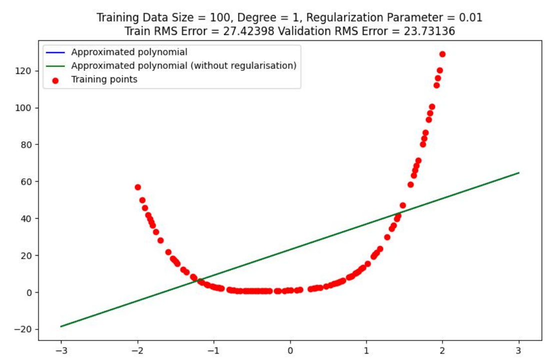
2. As the model degree increases, the complexity of the model increases and the model fits well with the training data. This is reflected in the lowering of the training and validation errors.

3. However, with higher complexity, the model may overfit with the data. This is reflected in the validation errors for $m = 6$ and $m = 9$, which are 2.4329×10^{-12} and 1.3962×10^{-10} respectively, which is an increase by an order of two.

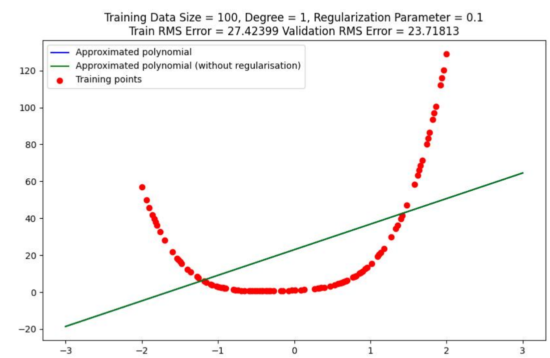
4. We also find that the performance of the model with $m = 9$, has improved by an order of 1 in comparison to the model trained with just 10 data points. This can be attributed to the increase in the size of the training dataset which gives us a better estimation of the model parameters.

5. The best model among these, will be $m = 6$ as it achieves the least validation error. Since, this model has nearly zero training and testing data, we don't require the use of regularization to improve the model.

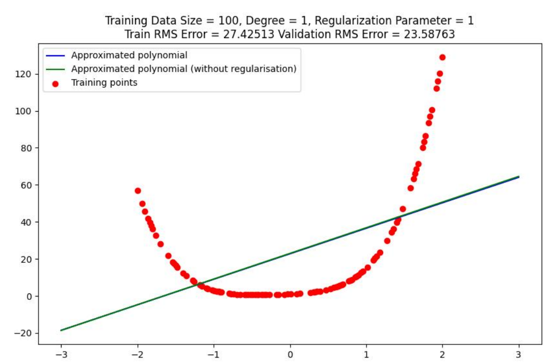
1.2.3 Plots of Approximated Functions with Regularization



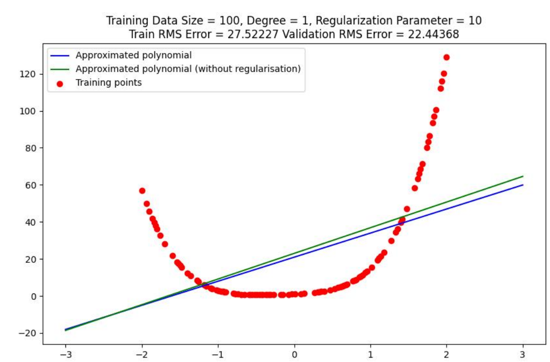
(1) $m = 1, \lambda = 0.01$



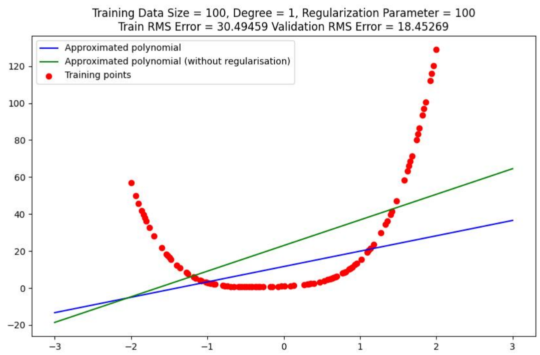
(2) $m = 1, \lambda = 0.1$



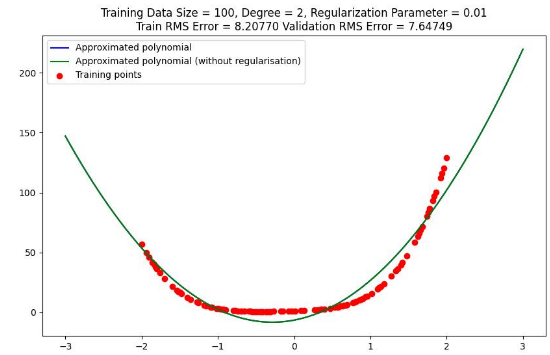
(3) $m = 1, \lambda = 1$



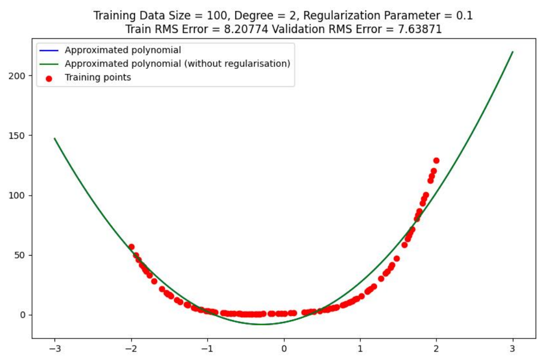
(4) $m = 1, \lambda = 10$



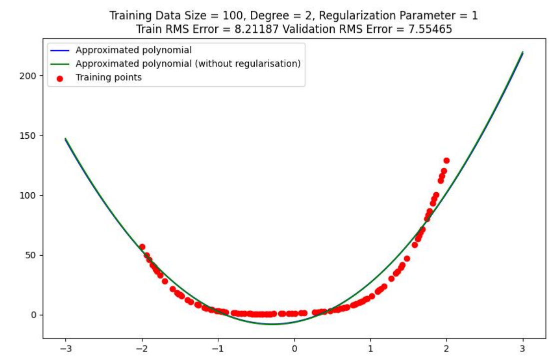
(5) $m = 1, \lambda = 100$



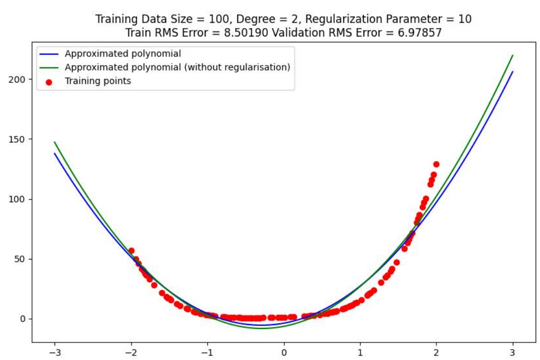
(6) $m = 2, \lambda = 0.01$



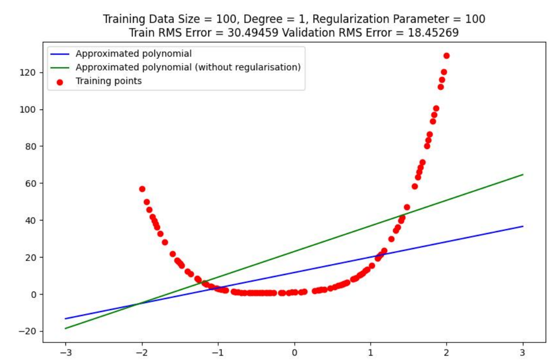
(7) $m = 2, \lambda = 0.1$



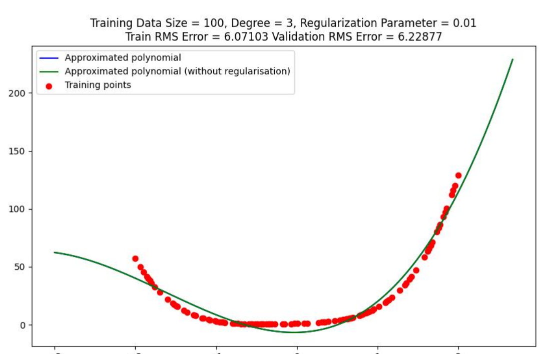
(8) $m = 2, \lambda = 1$



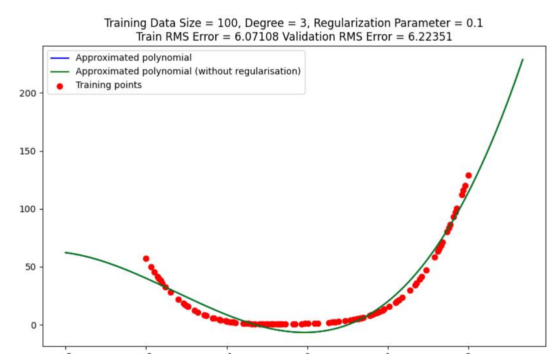
(9) $m = 2, \lambda = 10$



(10) $m = 2, \lambda = 100$



(11) $m = 3, \lambda = 0.01$



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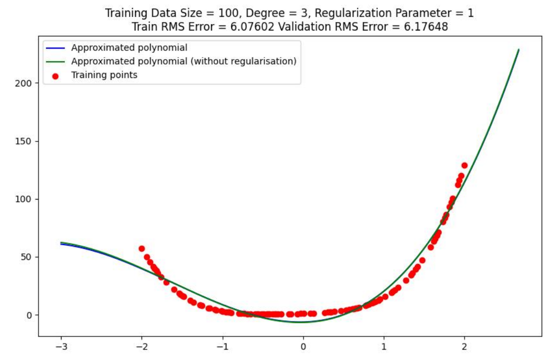
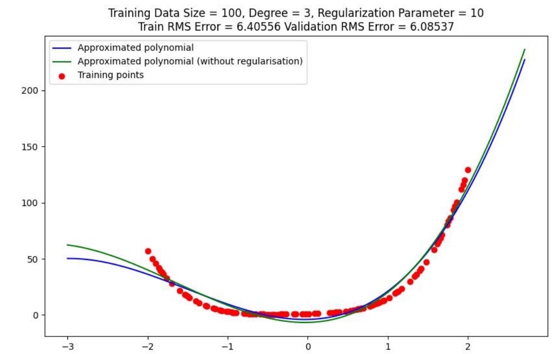
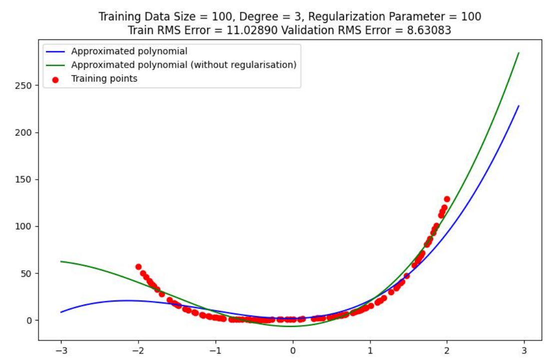


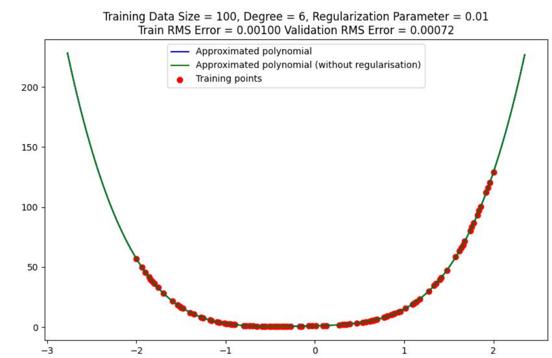
(13) $m = 3, \lambda = 1$



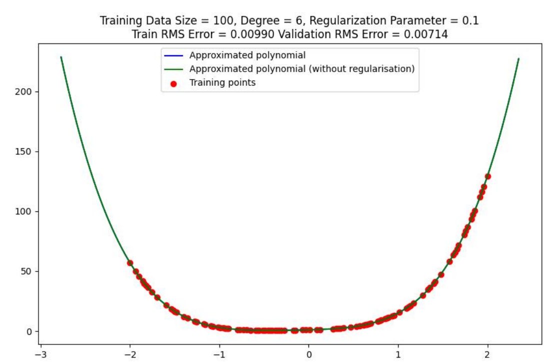
(14) $m = 3, \lambda = 10$



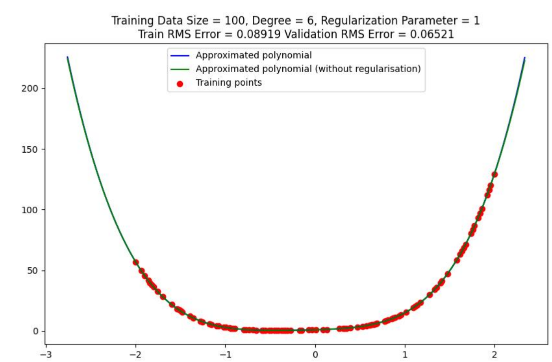
(15) $m = 3, \lambda = 100$



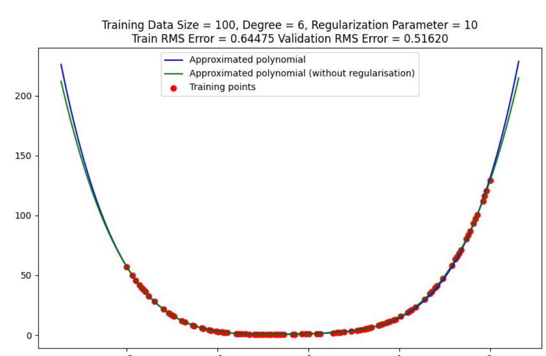
(16) $m = 6, \lambda = 0.01$



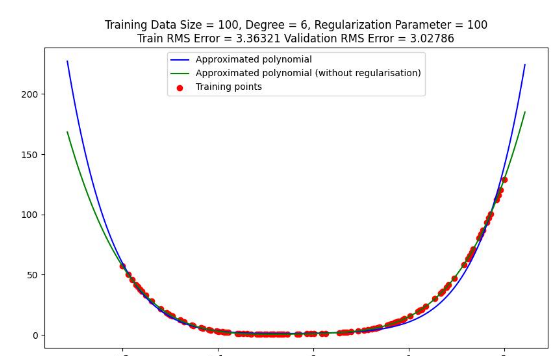
(17) $m = 6, \lambda = 0.1$



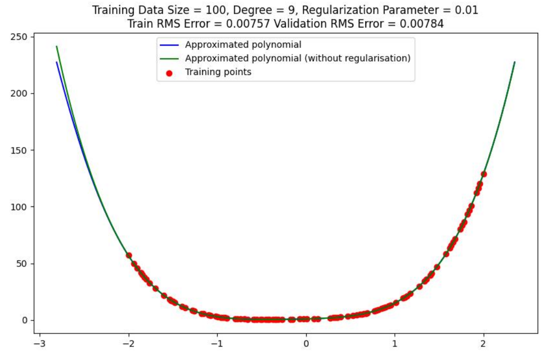
(18) $m = 6, \lambda = 1$



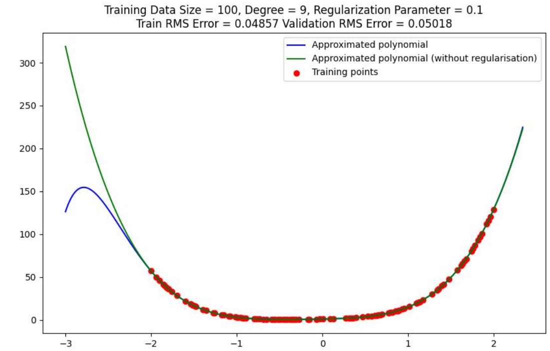
(19) $m = 6, \lambda = 10$



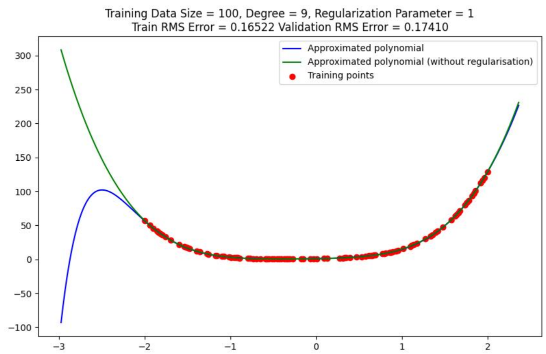
(20) $m = 6, \lambda = 100$



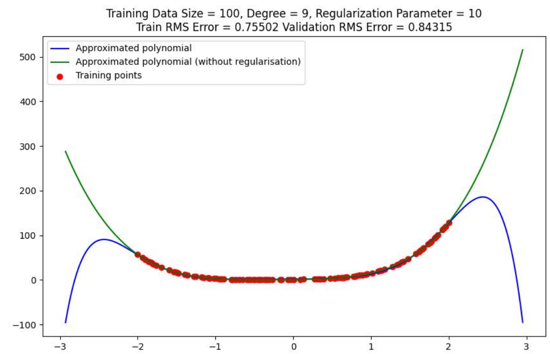
(21) $m = 9, \lambda = 0.01$



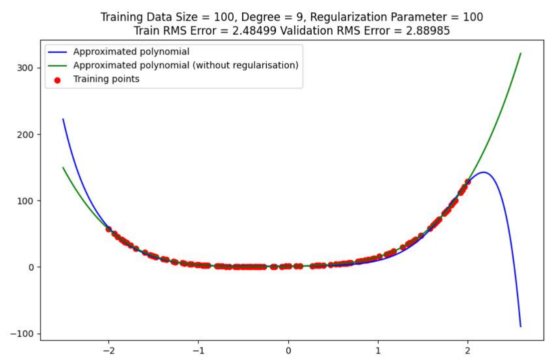
(22) $m = 9, \lambda = 0.1$



(23) $m = 9, \lambda = 1$



(24) $m = 9, \lambda = 10$



(25) $m = 9, \lambda = 100$

Figure 4: Approximated Functions with Regularization for training dataset of size 100

1.2.4 Inferences

1. The models with $m = 1, 2, 3$ continue to inadequately fit to the underlying data. This is a case of underfitting and indicates that the data has a much more complex distribution. Regularization is not of help here, as it enhances validation performance at the expense of training performance.

2. In parallel with the training dataset of size 10, as the regularization parameter λ increases, the training error increases and the validation error decreases.

3. As λ increases, it forces the model to slightly deviate from the training points which reduces overfitting. This can also been seen in increasing training error and reducing validation error as λ increases.

4. However when λ is too large, the model begins to deviate too much from the training data and the training and validation errors increase. This can be seen with $m = 3, 6, 9$ and $\lambda = 100$. Hence, large regularization has led to underfitting of the model in these cases.

5. Once again, we see that the use of regularization has not improved the model's performance, proving the validity of our initial assertion.

6. The dataset appears less sensitive to regularization, as varying regularization parameters doesn't significantly affect the model's performance.

7. Higher polynomial degrees (e.g., 9) don't contribute significantly to model improvement. The dataset seems to have a more linear or lower-degree relationship between features and the target variable.

8. The optimal model complexity seems to lie around degree 6, striking a balance between capturing essential patterns and avoiding unnecessary complexity. This suggests that a polynomial of degree 6 adequately represents the underlying structure of the dataset.

1.2.5 Hyperparameters v/s Training, Validation and Testing Errors

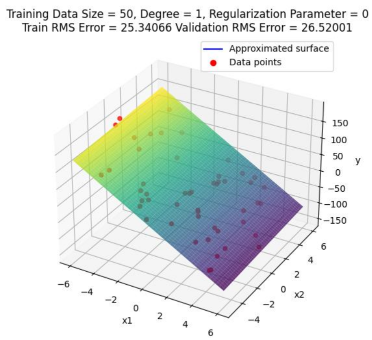
Degree	Regularization Parameter	Training Error	Validation Error	Testing Error
1	0.0	27.423 98	23.732 83	24.773 74
1	0.01	27.423 98	23.731 36	24.774 14
1	0.1	27.423 99	23.718 13	24.777 76
1	1.0	27.425 13	23.587 63	24.814 69
1	10.0	27.522 27	22.443 68	25.240 80
1	**100.0**	**30.49459**	**18.45269**	**29.78836**
2	0.0	8.207 70	7.648 47	7.958 20
2	0.01	8.207 70	7.647 49	7.958 01
2	0.1	8.207 74	7.638 71	7.956 37
2	1.0	8.211 87	7.554 65	7.943 85
2	10.0	8.501 90	6.978 57	8.098 85
2	**100.0**	**13.65707**	**5.64613**	**13.24083**
3	0.0	6.071 03	6.229 36	6.707 46
3	0.01	6.071 03	6.228 77	6.707 04
3	0.1	6.071 08	6.223 51	6.703 32
3	1.0	6.076 02	6.176 48	6.669 90
3	**10.0**	**6.40556**	**6.08537**	**6.59956**
3	100.0	11.028 90	8.630 83	9.782 65
6	**0.0**	$\mathbf{2.19356 \times 10^{-12}}$	$\mathbf{2.43291 \times 10^{-12}}$	$\mathbf{1.86658 \times 10^{-12}}$
6	0.01	0.001 00	0.000 72	0.001 10
6	0.1	0.009 90	0.007 14	0.010 85
6	1.0	0.089 19	0.065 21	0.100 07
6	10.0	0.644 75	0.516 20	0.743 94
6	100.0	3.363 21	3.027 86	3.755 94
9	**0.0**	$\mathbf{1.60555 \times 10^{-10}}$	$\mathbf{1.39628 \times 10^{-10}}$	$\mathbf{1.88667 \times 10^{-10}}$
9	0.01	0.007 57	0.007 83	0.006 64
9	0.1	0.048 57	0.050 18	0.043 58
9	1.0	0.165 22	0.174 10	0.163 59
9	10.0	0.755 02	0.843 15	0.741 37
9	100.0	2.484 99	2.889 85	2.388 83

Table 2: Training, Validation and Testing Errors for different m, λ for training dataset of size 100. The best model is taken to be the one with the lowest validation error and are shown in boldface.

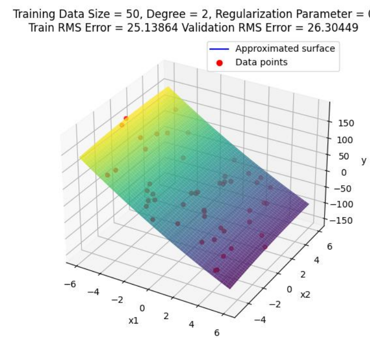
2 Task 2: Linear Model for Regression using Polynomial Basis Functions for Dataset 2

2.1 Training Dataset of size 50

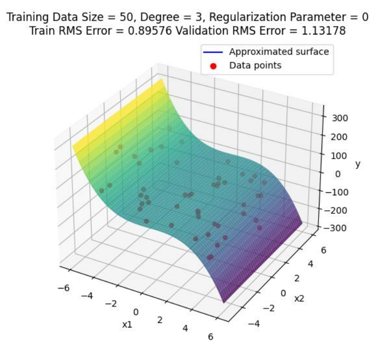
2.1.1 Surface Plots of Approximated Functions without Regularization



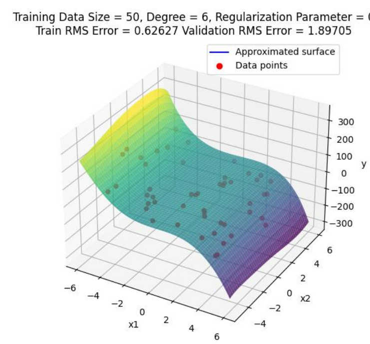
(1) $m = 1$



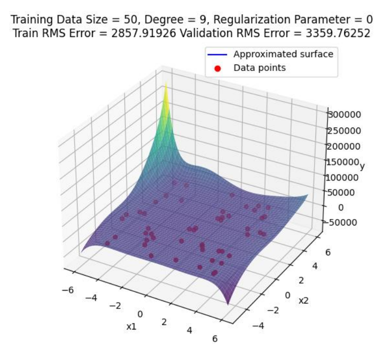
(2) $m = 2$



(3) $m = 3$



(4) $m = 6$



(5) $m = 9$

Figure 5: Approximated Functions without Regularization for training dataset of size 50

2.1.2 Inferences

1. **Underfitting (Degree 1-2):** For degrees 1 and 2, the model exhibits underfitting with relatively high training and validation losses. These degrees are too simplistic to capture the complexity of the underlying patterns in the data.

2. **Effective Fitting (Degree 3):** Degree 3 performs well, achieving low training loss (0.89) and validation loss (1.13). This indicates an effective balance between model complexity and fitting the data.
   The number of parameters (D) for degree m and input dimensionality d is given by

$$D = \frac{(m+d)!}{m! \cdot d!}.$$

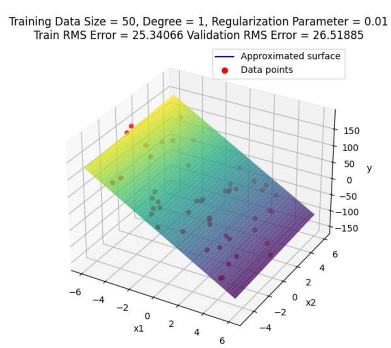
3. **Appropriate Complexity (Degree 6):** Degree 6 shows low training loss (0.62) and a slightly higher validation loss (1.89). The increase in complexity compared to degree 3 is acceptable, and further investigation, such as regularization techniques, could be explored to optimize performance.

4. **Lack of Training Samples (Degree 9):** The extremely high training loss (2857.91) and even higher validation loss (3359.76) for degree 9 indicate that the model is too complex for the limited number of training samples (50). The number of parameters (D) for degree $m = 9$ and input dimensionality $d = 2$ is given by

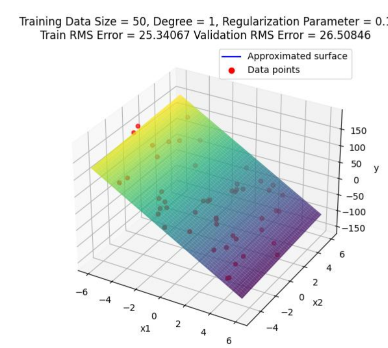
$$D = \frac{(9+2)!}{9! \cdot 2!} = 55.$$

   With only 50 samples, the model lacks sufficient data to effectively learn and generalize, resulting in poor performance.

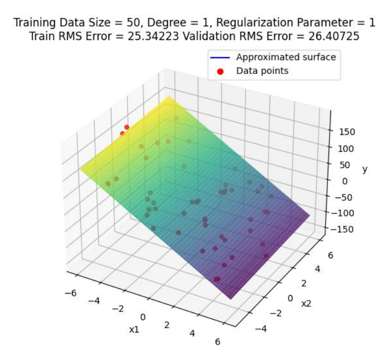
2.1.3 Surface Plots of Approximated Functions with Regularization



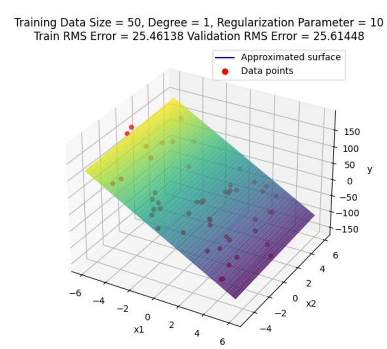
(1) $m = 1, \lambda = 0.01$



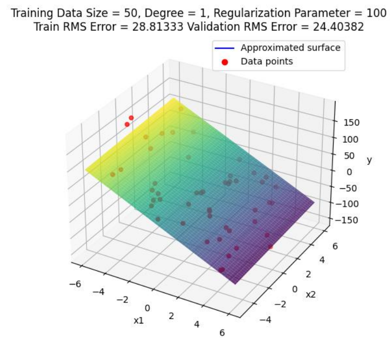
(2) $m = 1, \lambda = 0.1$



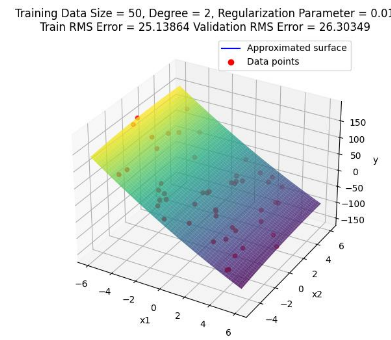
(3) $m = 1, \lambda = 1$



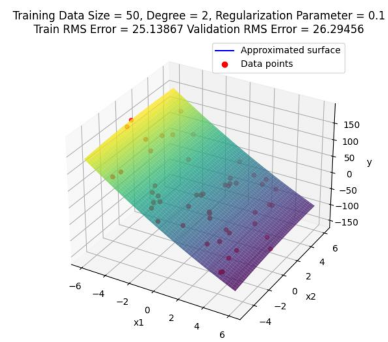
(4) $m = 1, \lambda = 10$



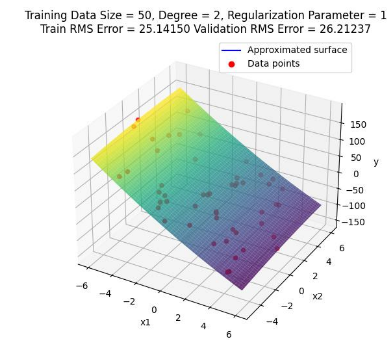
(5) $m = 1, \lambda = 100$



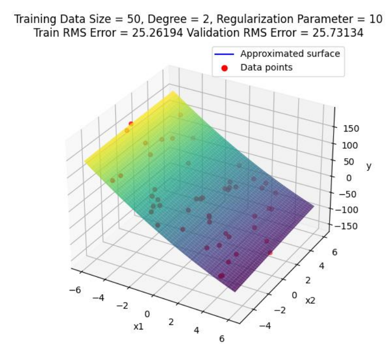
(6) $m = 2, \lambda = 0.01$



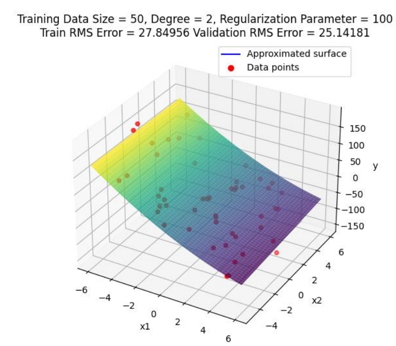
(7) $m = 2, \lambda = 0.1$



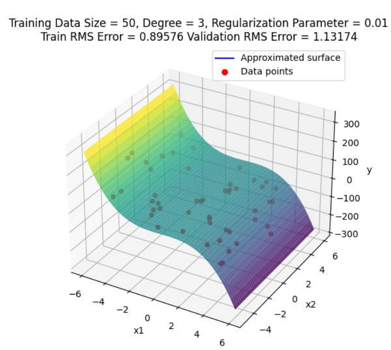
(8) $m = 2, \lambda = 1$



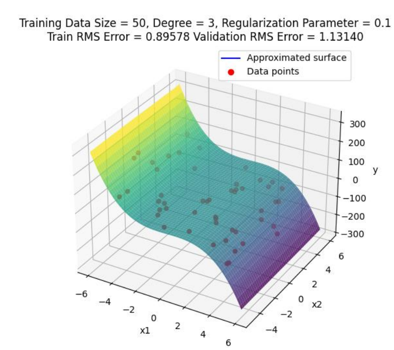
(9) $m = 2, \lambda = 10$



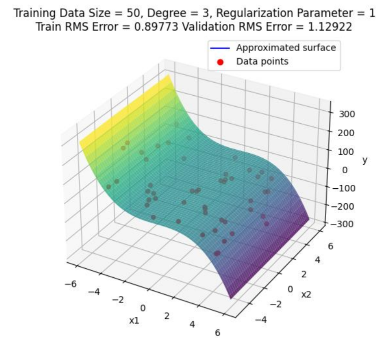
(10) $m = 2, \lambda = 100$



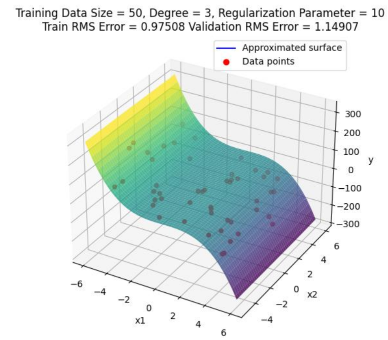
(11) $m = 3, \lambda = 0.01$



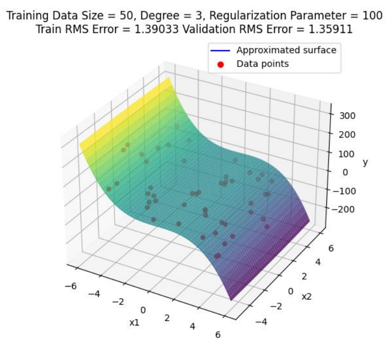
(12) $m = 3, \lambda = 0.1$



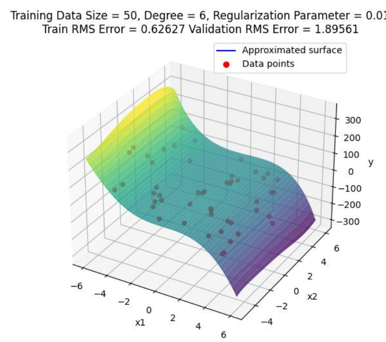
(13) $m = 3, \lambda = 1$



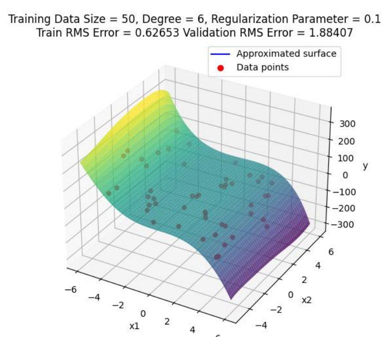
(14) $m = 3, \lambda = 10$



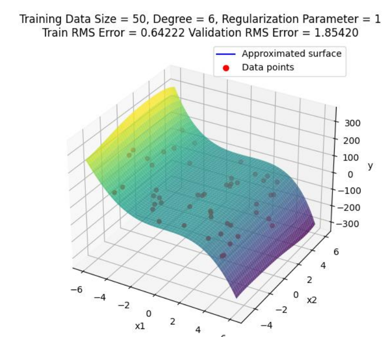
(15) $m = 3, \lambda = 100$



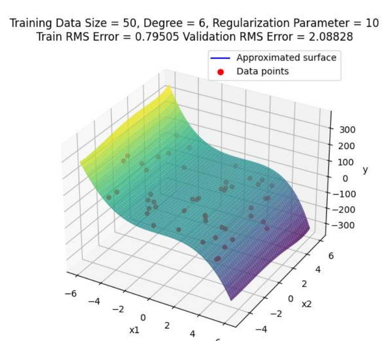
(16) $m = 6, \lambda = 0.01$



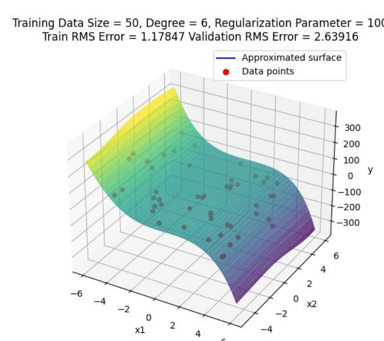
(17) $m = 6, \lambda = 0.1$



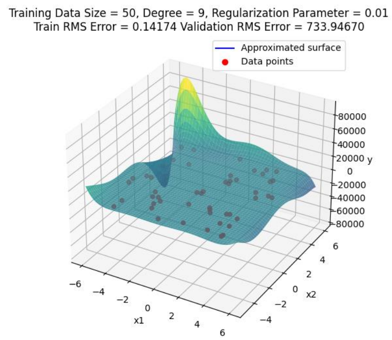
(18) $m = 6, \lambda = 1$



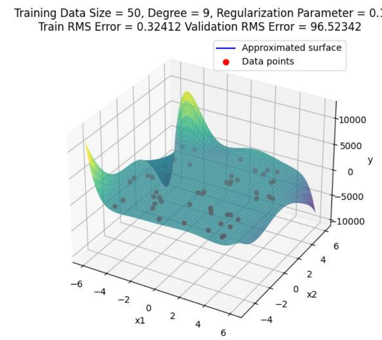
(19) $m = 6, \lambda = 10$



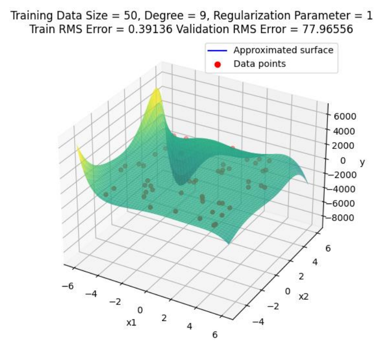
(20) $m = 6, \lambda = 100$



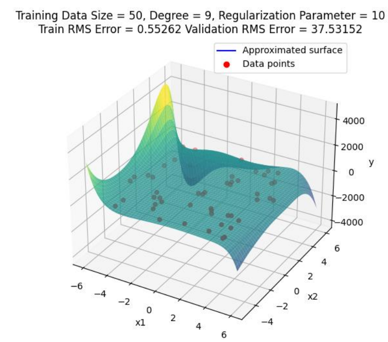
(21) $m = 9, \lambda = 0.01$



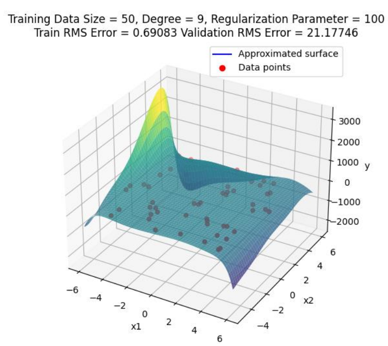
(22) $m = 9, \lambda = 0.1$



(23) $m = 9, \lambda = 1$



(24) $m = 9, \lambda = 10$



(25) $m = 9, \lambda = 100$

Figure 6: Approximated Functions with Regularization for training dataset of size 50

2.1.4 Inferences

1. **Degree 1:** As the regularization parameter λ increases, both training and validation errors increase consistently. This aligns with the expected behavior of regularization, which tends to prevent overfitting. The increase in errors indicates a constraint on the model's complexity, preventing it from fitting the training data too closely.

2. **Degree 2:** Similar to degree 1, increasing λ leads to higher training and validation errors, indicating consistent regularization effects. The model, being of higher degree, exhibits a more pronounced sensitivity to regularization, emphasizing the need for appropriate parameter tuning.

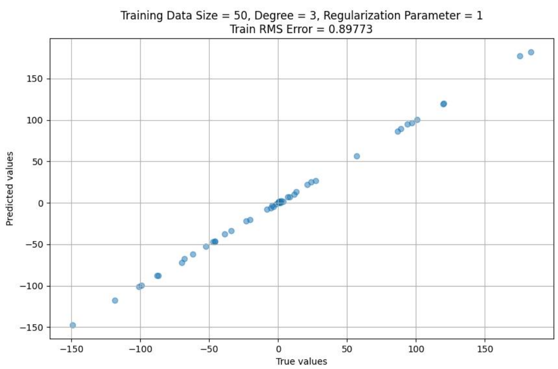
3. **Degree 3:** For degree 3, the pattern of increasing errors with higher λ continues, but the impact is less severe compared to degrees 1 and 2. This suggests that the model with degree 3 is more robust to regularization, potentially due to its moderate complexity.

4. **Degree 6:** With degree 6, the trend of increasing errors with higher λ persists. The model appears sensitive to regularization, particularly with $\lambda = 100.0$. This indicates that, at this level of complexity, too much regularization can negatively impact performance.

5. **Degree 9:** Degree 9 exhibits a distinct behavior. With $\lambda = 0.0$, the model has an extremely high training error, suggesting potential underfitting. However, as λ increases, both training and validation errors decrease. This unexpected behaviour could be a result of numerical instability and a lack of input samples to properly train the model.

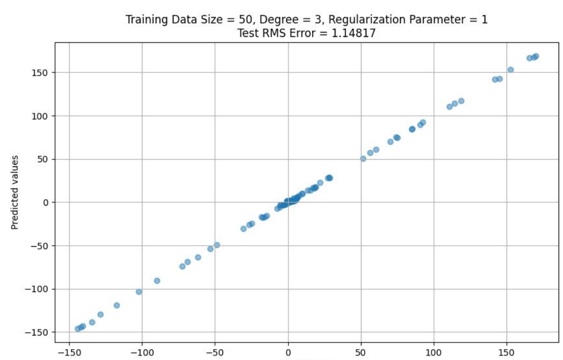
6. Generally, increasing λ has a regularizing effect, helping to prevent overfitting. However, the impact of regularization varies with the degree of the polynomial and the sample size used to train the model. Polynomial basis functions of degree 3 and 6, tend to have a better overall performance in the train, test and validation dataset.

2.1.5 Scatter Plots of the Best Model



(1) Best model ($m = 3, \lambda = 1$) performance in Training Data

(2) Best model ($m = 3, \lambda = 1$) performance in Validation Data



(3) Best model ($m = 3, \lambda = 1$) performance in Testing Data

Figure 7: Best model performance with Training, Validation and Testing Data

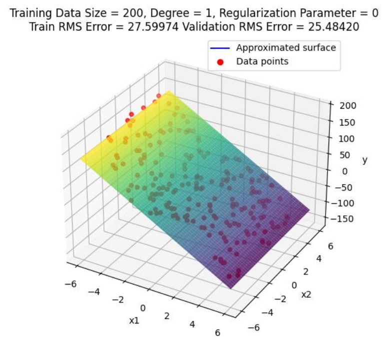
2.1.6 Hyperparameters v/s Training, Validation and Testing Errors

Degree	Regularization Parameter	Training Error	Validation Error	Testing Error
1	0.0	25.34066	26.52001	27.88623
1	0.01	25.34066	26.51885	27.88566
1	0.1	25.34067	26.50846	27.88051
1	1.0	25.34223	26.40725	27.83135
1	10.0	25.46138	25.61448	27.51958
1	**100.0**	**28.81333**	**24.40382**	**29.12246**
2	0.0	25.13864	26.30449	27.30884
2	0.01	25.13864	26.30349	27.30837
2	0.1	25.13867	26.29456	27.30411
2	1.0	25.14150	26.21237	27.26680
2	10.0	25.26194	25.73134	27.12393
2	**100.0**	**27.84956**	**25.14181**	**28.27266**
3	0.0	0.89576	1.13178	1.15301
3	0.01	0.89576	1.13174	1.15294
3	0.1	0.89578	1.13140	1.15237
3	**1.0**	**0.89773**	**1.12922**	**1.14817**
3	10.0	0.97508	1.14907	1.16326
3	100.0	1.39033	1.35911	1.43895
6	0.0	0.62627	1.89705	2.25723
6	0.01	0.62627	1.89561	2.25213
6	0.1	0.62653	1.88407	2.20832
6	**1.0**	**0.64222**	**1.85420**	**1.91613**
6	10.0	0.79505	2.08828	1.60729
6	100.0	1.17847	2.63916	2.58578
9	0.0	2857.91926	3359.76252	3154.95808
9	0.01	0.14174	733.94670	691.73234
9	0.1	0.32412	96.52342	99.82646
9	1.0	0.39136	77.96556	89.14619
9	10.0	0.55262	37.53152	45.30810
9	**100.0**	**0.69083**	**21.17746**	**8.48779**

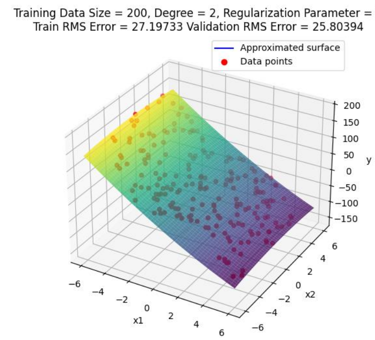
Table 3: Training, Validation and Testing Errors for different m, λ for training dataset of size 50. The best model is taken to be the one with the lowest validation error and are shown in boldface.

2.2 Training Dataset of size 200

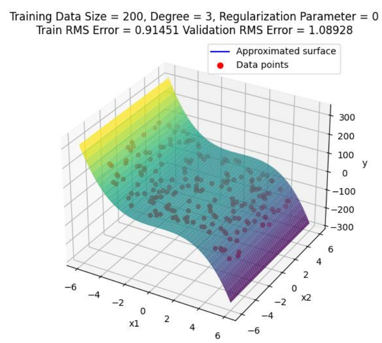
2.2.1 Plots of Approximated Functions without Regularization



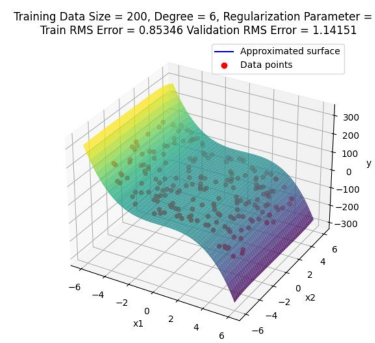
(1) $m = 1$



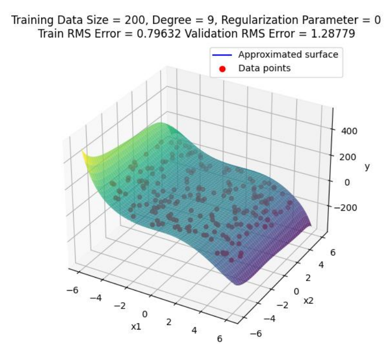
(2) $m = 2$



(3) $m = 3$



(4) $m = 6$



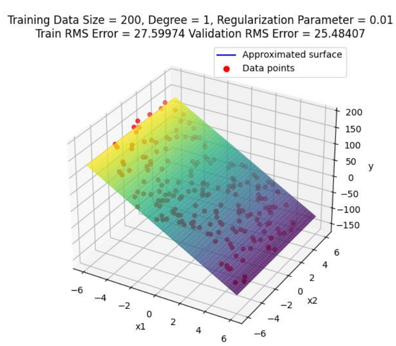
(5) $m = 9$

Figure 8: Approximated Functions without Regularization for training dataset of size 200

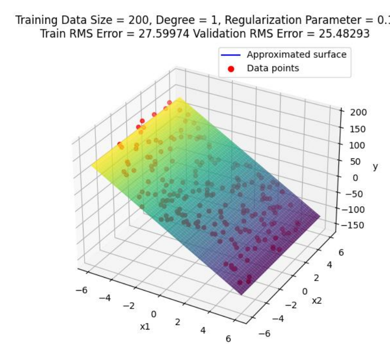
2.2.2 Inferences

1. For Degree 1 and Degree 2, errors are relatively high across all datasets, indicating potential underfitting and suggesting the need for regularization.
2. Degree 3 exhibits a substantial decrease in errors, indicating improved model fit, especially in the testing dataset.
3. Degrees 6 and 9 continue the trend of decreasing errors, showcasing the benefits of higher degrees. Though with higher degree, we face the issue of overfitting.
4. Generally, there is a need for regularization across all degrees to avoid overfitting. Higher degrees (6 and 9) show a risk of overfitting, particularly when regularization is absent.
5. With 200 data points and no regularization, underfitting is not as pronounced, especially in the case of higher degrees like Degree 6 and 9.
6. The larger number of training samples seems to mitigate the underfitting effect, emphasizing the impact of dataset size on regularization outcomes.
7. The models with higher degrees such as $m = 6, 9$ have much lower training error than validation error. Hence, this may be a case where regularization might improve the generalizability of the model.

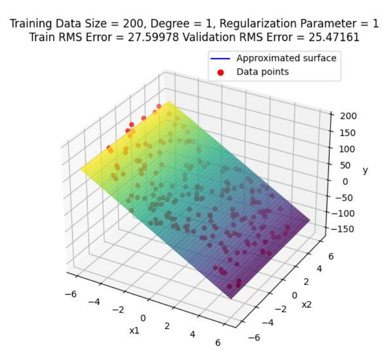
2.2.3 Plots of Approximated Functions with Regularization



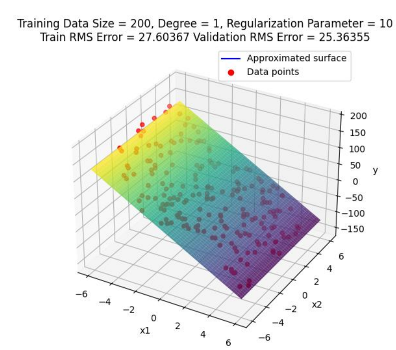
(1) $m = 1, \lambda = 0.01$



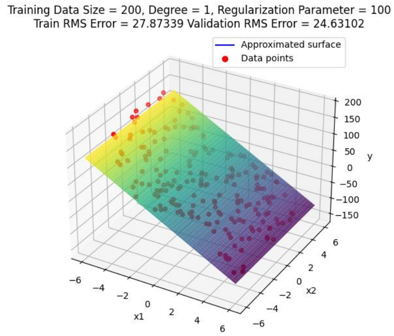
(2) $m = 1, \lambda = 0.1$



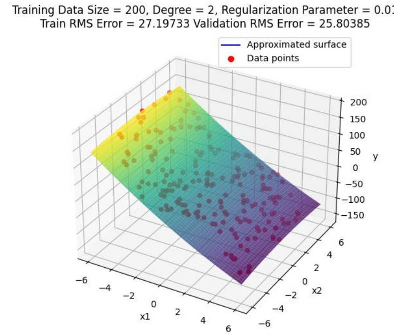
(3) $m = 1, \lambda = 1$



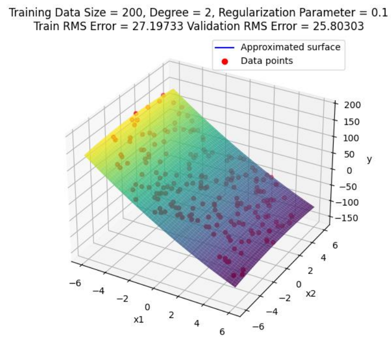
(4) $m = 1, \lambda = 10$



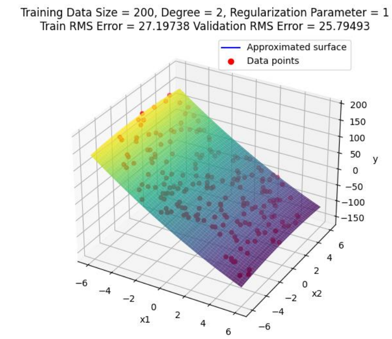
(5) $m = 1, \lambda = 100$



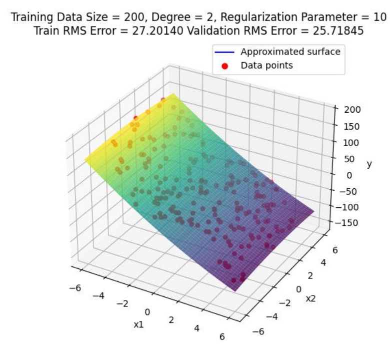
(6) $m = 2, \lambda = 0.01$



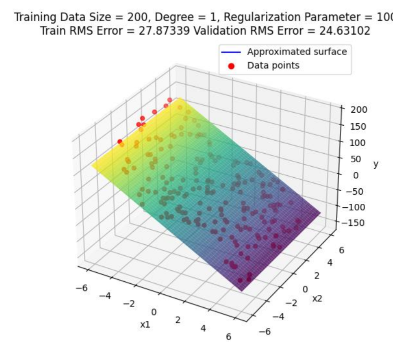
(7) $m = 2, \lambda = 0.1$



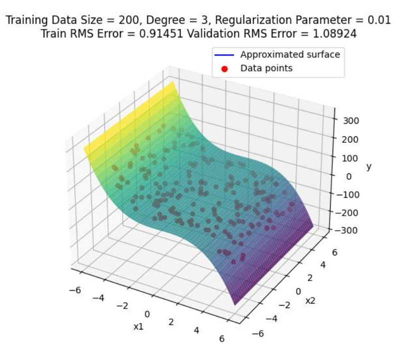
(8) $m = 2, \lambda = 1$



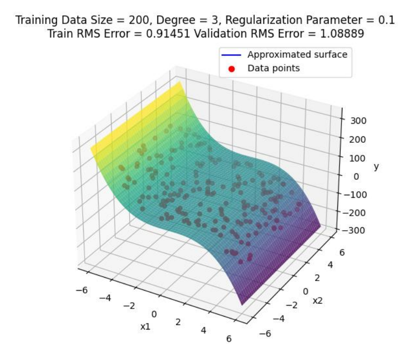
(9) $m = 2, \lambda = 10$



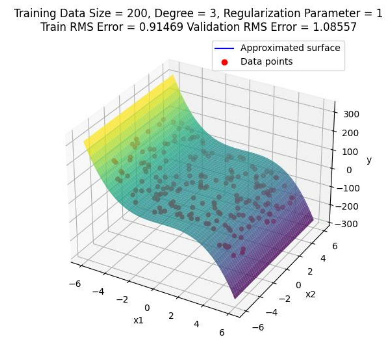
(10) $m = 2, \lambda = 100$



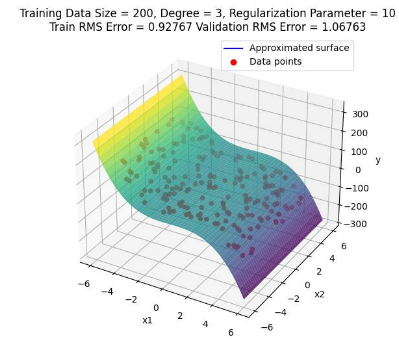
(11) $m = 3, \lambda = 0.01$



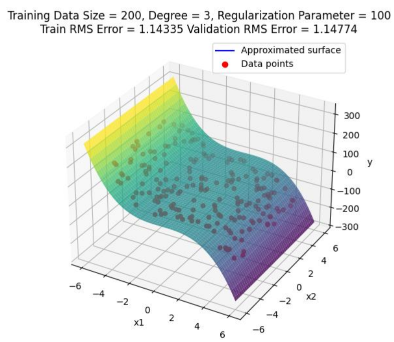
(12) $m = 3, \lambda = 0.1$



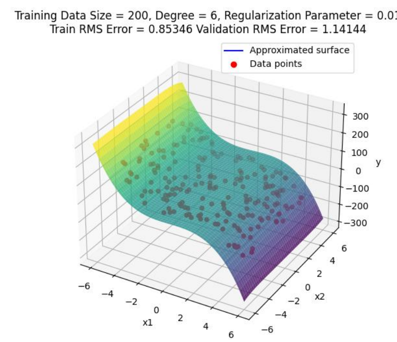
(13) $m = 3, \lambda = 1$



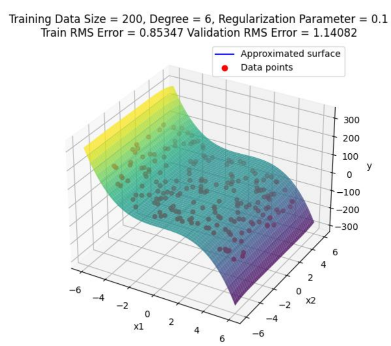
(14) $m = 3, \lambda = 10$



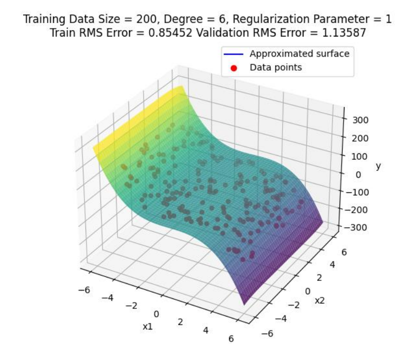
(15) $m = 3, \lambda = 100$



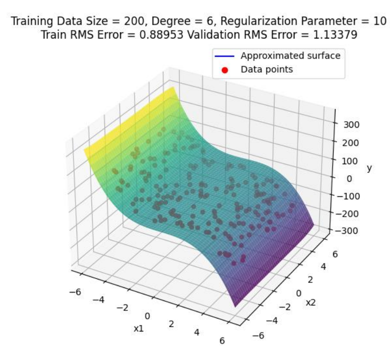
(16) $m = 6, \lambda = 0.01$



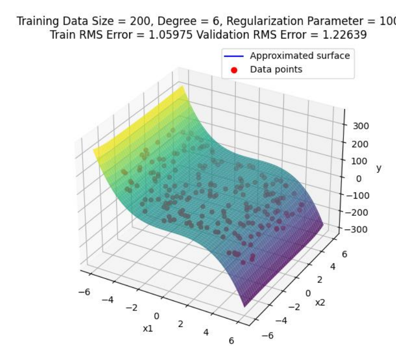
(17) $m = 6, \lambda = 0.1$



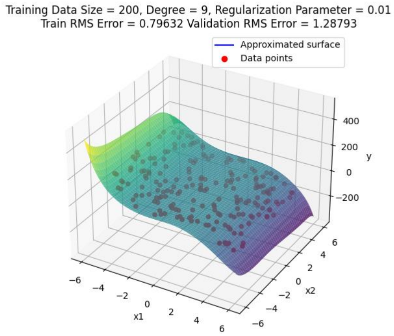
(18) $m = 6, \lambda = 1$



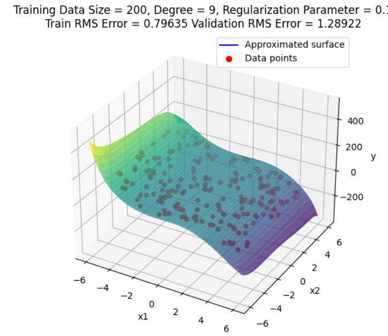
(19) $m = 6, \lambda = 10$



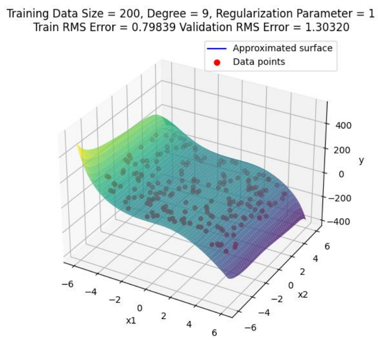
(20) $m = 6, \lambda = 100$



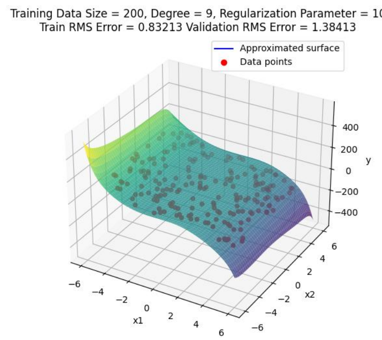
(21) $m = 9, \lambda = 0.01$



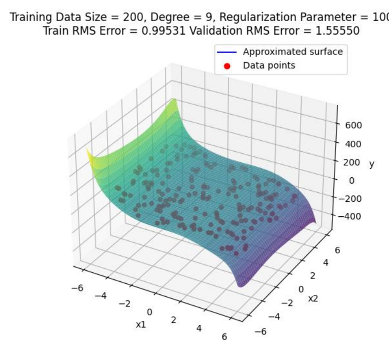
(22) $m = 9, \lambda = 0.1$



(23) $m = 9, \lambda = 1$



(24) $m = 9, \lambda = 10$



(25) $m = 9, \lambda = 100$

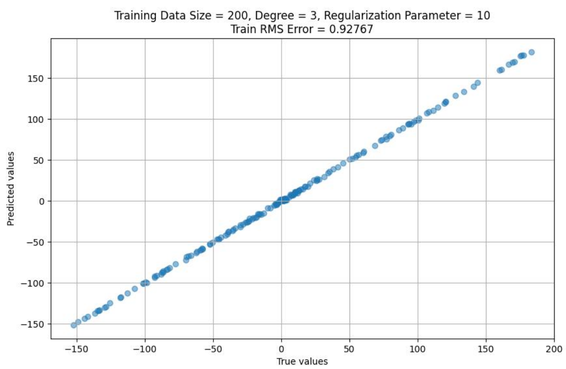
Figure 9: Approximated Functions with Regularization for training dataset of size 200

2.2.4 Inferences

1. Higher degrees generally reduce the training error. But similar to the 50 training sample case, we observe overfitting in these cases.
2. Low degrees $m = 1, 2$ exhibit underfitting, with validation errors decreasing as regularization increases.
3. For Degree 3, a moderate regularization parameter $\lambda = 1$ yields good results, but higher degrees show overfitting tendencies.
4. Degrees 6 and 9 follow similar trends, with higher regularization improving performance, but overfitting is more pronounced.
5. Low regularization $\lambda = 0, 0.01$ leads to high errors due to overfitting in case of higher degrees $m = 6, 9$.

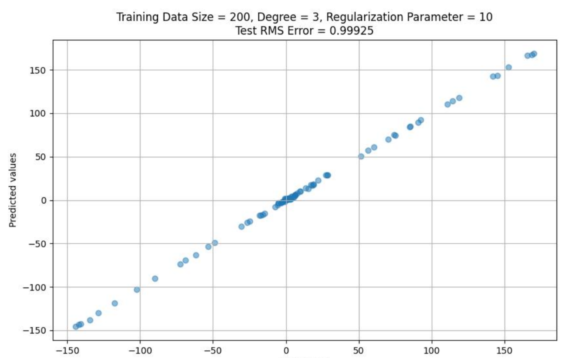
6. Moderate regularization $\lambda = 0.1, 1$ strikes a balance, preventing overfitting and providing good generalization.
7. High regularization $\lambda = 10$ is effective, but very high $\lambda = 100$ leads to underfitting.
8. Underfitting is observed with simple models i.e, low degrees and high regularization.
9. Overfitting is evident with higher degrees and low regularization, leading to a decrease in testing performance.
10. Beyond polynomial degree 6, the performance improvement becomes marginal, suggesting diminishing returns with increasing model complexity.
11. Polynomial degrees 6 and 3 outperform degree 9, indicating an optimal complexity level that balances bias and variance for this dataset.
12. Regularization has improved the performance of the model, thus validating our original assertion about the same.

2.2.5 Scatter Plots of the Best Model



(1) Best model ($m = 3, \lambda = 10$) performance in Training Data

(2) Best model ($m = 3, \lambda = 10$) performance in Validation Data



(3) Best model ($m = 3, \lambda = 10$) performance in Testing Data

Figure 10: Best model performance with Training, Validation and Testing Data

2.2.6 Hyperparameters v/s Training, Validation and Testing Errors

Degree	Regularization Parameter	Training Error	Validation Error	Testing Error
1	0.0	27.599 74	25.484 20	27.372 36
1	0.01	27.599 74	25.484 07	27.372 33
1	0.1	27.599 74	25.482 93	27.372 12
1	1.0	27.599 78	25.471 61	27.370 05
1	10.0	27.603 67	25.363 55	27.353 14
1	**100.0**	**27.87339**	**24.63102**	**27.42702**
2	0.0	27.197 33	25.803 94	26.946 46
2	0.01	27.197 33	25.803 85	26.946 49
2	0.1	27.197 33	25.803 03	26.946 75
2	1.0	27.197 38	25.794 93	26.949 24
2	10.0	27.201 40	25.718 45	26.970 62
2	**100.0**	**27.41277**	**25.14321**	**27.09778**
3	0.0	0.914 51	1.089 28	1.029 57
3	0.01	0.914 51	1.089 24	1.029 52
3	0.1	0.914 51	1.088 89	1.029 04
3	1.0	0.914 69	1.085 57	1.024 50
3	**10.0**	**0.92767**	**1.06763**	**0.99925**
3	100.0	1.143 35	1.147 74	1.119 98
6	0.0	0.853 46	1.141 51	1.185 74
6	0.01	0.853 46	1.141 44	1.185 52
6	0.1	0.853 47	1.140 82	1.183 55
6	1.0	0.854 52	1.135 87	1.166 50
6	**10.0**	**0.88953**	**1.13379**	**1.11094**
6	100.0	1.059 75	1.226 39	1.255 13
9	0.0	0.796 32	1.287 79	1.217 48
9	**0.01**	**0.79632**	**1.28793**	**1.21703**
9	0.1	0.796 35	1.289 22	1.213 25
9	1.0	0.798 39	1.303 20	1.189 97
9	10.0	0.832 13	1.384 13	1.191 77
9	100.0	0.995 31	1.555 50	1.460 03

Table 4: Training, Validation and Testing Errors for different m, λ for training dataset of size 200. The best model is taken to be the one with the lowest validation error and are shown in boldface.

3 Task 3.1: Linear Model for Regression using Gaussian Basis Functions for Dataset 2

3.1 Training Dataset of size 50

3.1.1 Hyperparameters v/s Training, Validation and Testing Errors

width parameter

Clusters	Covariance	Regularization Parameter	Training Error	Validation Error	Testing Error
5	5	0	35.273 06	36.402 88	39.326 88
5	5	0.01	35.662 19	37.195 47	40.757 77
5	5	0.1	36.017 30	37.429 22	41.299 28
5	**5**	**1**	**35.51236**	**34.34175**	**38.17339**
5	5	10	47.830 11	38.069 93	45.261 82
5	5	100	65.215 37	52.063 07	60.162 15
5	10	0	28.086 64	29.910 68	31.782 25
5	10	0.01	28.126 03	29.780 30	31.619 11
5	10	0.1	28.281 30	29.452 46	31.702 75
5	**10**	**1**	**31.66447**	**28.41727**	**33.07450**
5	10	10	53.795 90	42.487 39	49.947 23
5	10	100	67.416 66	53.995 61	61.701 13
5	15	0	26.530 99	27.961 72	29.473 38
5	15	0.01	26.581 31	27.901 42	29.571 23
5	**15**	**0.1**	**27.10568**	**27.00792**	**29.54564**
5	15	1	44.118 60	34.592 18	41.576 21
5	15	10	63.118 73	50.335 65	57.885 77
5	15	100	68.664 24	55.106 72	62.638 23
5	20	0	25.896 91	27.217 46	28.440 12
5	20	0.01	26.010 19	27.036 01	28.735 01
5	**20**	**0.1**	**29.32086**	**26.02244**	**30.20199**
5	20	1	53.103 54	41.863 07	49.099 26
5	20	10	66.863 01	53.571 28	61.071 60
5	20	100	69.045 22	55.419 98	63.084 28
10	**5**	**0**	**9.78331**	**14.81851**	**15.41025**
10	5	0.01	24.348 73	30.847 36	34.439 38
10	5	0.1	29.235 12	33.097 72	36.741 51
10	5	1	33.389 20	33.764 48	37.482 36
10	5	10	42.443 71	34.397 77	41.354 92
10	5	100	61.767 64	49.007 86	56.615 64
10	**10**	**0**	**6.33656**	**9.21987**	**6.98774**
10	10	0.01	27.818 66	29.621 02	31.339 42
10	10	0.1	27.979 84	29.462 16	31.365 40
10	10	1	29.418 51	27.811 33	31.142 77
10	10	10	44.327 61	35.476 66	42.032 02
10	10	100	66.015 04	52.743 45	60.725 92
10	**15**	**0**	**2.69528**	**2.53537**	**2.89432**
10	15	0.01	26.502 52	27.868 94	29.431 10
10	15	0.1	26.671 73	27.449 14	29.476 11
10	15	1	33.664 20	27.966 71	33.317 65
10	15	10	59.107 91	46.866 90	54.368 10
10	15	100	67.953 58	54.493 56	62.038 27
10	**20**	**0**	**2.03270**	**3.00227**	**3.01889**
10	20	0.01	26.087 29	27.199 46	28.956 31
10	20	0.1	26.777 83	26.230 96	28.822 34
10	20	1	44.197 11	34.753 17	41.573 78

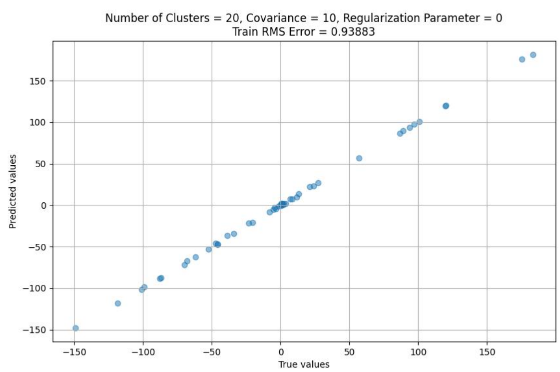
Clusters	Covariance	Regularization Parameter	Training Error	Validation Error	Testing Error
10	20	10	64.877 75	51.790 70	59.340 01
10	20	100	68.780 92	55.195 47	62.796 04
15	**5**	**0**	**8.01771**	**10.01373**	**11.22802**
15	5	0.01	13.321 50	17.555 16	18.774 83
15	5	0.1	25.072 80	28.799 90	31.603 46
15	5	1	33.408 76	34.677 99	38.302 22
15	5	10	39.457 71	32.723 74	39.279 26
15	5	100	58.914 56	46.506 18	54.465 83
15	**10**	**0**	**1.70331**	**3.32245**	**3.24209**
15	10	0.01	27.716 68	29.555 17	31.317 12
15	10	0.1	27.973 38	29.479 54	31.442 20
15	10	1	28.741 18	28.309 75	31.027 95
15	10	10	41.319 34	33.016 06	39.386 97
15	10	100	63.666 20	50.743 22	58.410 65
15	**15**	**0**	**1.66608**	**2.65128**	**2.83373**
15	15	0.01	26.522 79	27.906 70	29.468 58
15	15	0.1	26.651 68	27.655 00	29.631 11
15	15	1	31.542 01	27.096 25	31.885 38
15	15	10	54.599 39	43.184 30	50.316 19
15	15	100	67.524 68	54.079 45	61.751 83
15	**20**	**0**	**5.80971**	**5.51535**	**5.51729**
15	20	0.01	26.019 61	27.250 59	28.797 71
15	20	0.1	26.462 20	26.210 44	28.576 25
15	20	1	40.489 36	32.051 42	38.567 13
15	20	10	62.776 63	50.044 02	57.477 87
15	20	100	68.315 96	54.805 67	62.342 14
20	**5**	**0**	**3.10027**	**7.55743**	**7.22285**
20	5	0.01	11.398 58	15.914 33	16.031 20
20	5	0.1	23.673 99	28.156 36	30.685 77
20	5	1	32.327 32	34.248 39	37.942 77
20	5	10	39.560 77	32.683 18	38.779 47
20	5	100	56.832 47	44.433 28	52.134 06
20	**10**	**0**	**0.93883**	**1.79379**	**1.78864**
20	10	0.01	27.505 91	29.426 91	31.232 08
20	10	0.1	28.039 43	29.426 41	31.233 09
20	10	1	28.486 98	28.244 55	30.742 72
20	10	10	38.118 47	30.884 01	37.000 21
20	10	100	63.623 59	50.638 96	58.446 36
20	**15**	**0**	**13.77664**	**13.54697**	**13.53529**
20	15	0.01	26.442 17	27.875 78	29.350 38
20	15	0.1	26.683 31	27.394 21	29.135 52
20	15	1	29.192 67	26.420 54	30.380 21
20	15	10	53.621 04	42.150 80	49.635 01
20	15	100	66.626 54	53.317 17	60.871 35
20	20	0	37.772 26	38.311 06	38.366 92
20	20	0.01	26.038 50	27.330 75	28.933 32
20	**20**	**0.1**	**26.25120**	**26.59861**	**28.82540**
20	20	1	34.593 78	28.776 72	34.035 47
20	20	10	62.247 02	49.654 85	57.090 16
20	20	100	68.071 75	54.596 34	62.150 12

Table 5: Training, Validation and Testing Errors for different K, σ, λ for training dataset of size 50. The best model is taken to be the one with the lowest validation error and are shown in boldface.

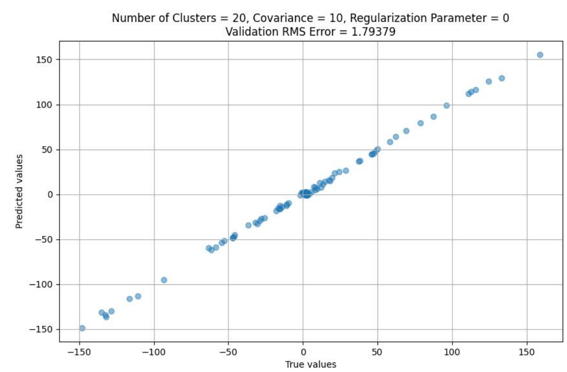
3.1.2 Inferences

1. Generally, increasing clusters tends to decrease training errors, possibly leading to overfitting.
2. Configuration with 20 clusters, $\lambda = 0$, and covariance of 10 exhibits the least training error at 0.9388. It also performs well on the validation and test data.
3. Increasing regularization parameterλ raises the training and reduces the validation errors. However we observe an increase in the validation error for large values of λ $(10, 100)$.
4. Non-zero $\lambda = 0.01, 0.1$ values show a decrease in validation errors, suggesting a reduction in overfitting.
5. The number of clusters influences training errors. An increase in clusters generally reduces the errors.
6. Configuration with $\lambda = 100$ consistently shows higher errors across all datasets, indicating potential oversensitivity to regularization.
7. The error is also dependent on covariance. A general observed trend is that the error drops as the covariance increases. But above a certain threshold, the error starts rising with a rise in covariance. The optimal value of covariance depends on number of clusters chosen for the data. Typically, we can observe a moderate value of covariance $\sigma = 10, 15$, which seems to be the optimal value.
8. The model has roughly similar performance on train, test and validation data, given that the optimal parameters are chosen.
9. The model performs optimally with $\lambda = 0$, covariance $= 10$, and 20 clusters.

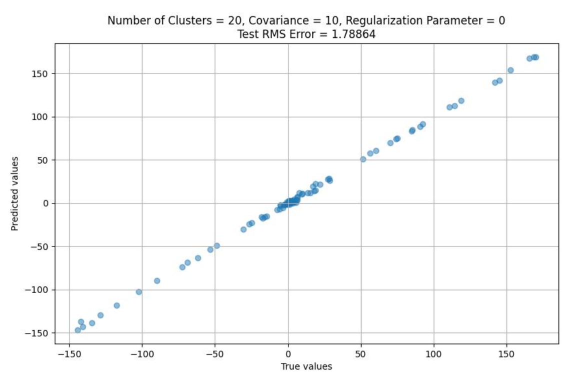
3.1.3 Scatter Plots of the Best Model



(1) Best model $(K = 20, \sigma = 10, \lambda = 0)$ performance in Training Data



(2) Best model $(K = 20, \sigma = 10, \lambda = 0)$ performance in Validation Data



(3) Best model $(K = 20, \sigma = 10, \lambda = 0)$ performance in Testing Data

Figure 11: Best model performance with Training, Validation and Testing Data

3.2 Training dataset of size 200

3.2.1 Hyperparameters v/s Training, Validation and Testing Errors

Clusters	Covariance	Regularization Parameter	Training Error	Validation Error	Testing Error
5	5	0	40.640 18	35.862 42	39.960 56
5	5	0.01	39.063 53	35.150 61	39.696 44
5	5	0.1	39.066 89	35.045 01	39.600 92
5	5	1	40.145 58	34.767 84	39.609 70
5	**5**	**10**	**41.42510**	**32.88108**	**38.93777**
5	5	100	59.090 50	44.155 42	52.214 15
5	10	0	31.468 82	28.752 14	30.895 21
5	10	0.01	31.603 62	28.571 43	31.269 16
5	10	0.1	31.662 34	28.552 36	31.212 75
5	**10**	**1**	**31.84037**	**27.79075**	**31.25499**
5	10	10	41.112 52	30.708 98	37.283 74
5	10	100	64.658 12	48.857 58	56.357 48
5	15	0	29.268 93	26.998 04	28.730 17
5	15	0.01	29.328 68	26.872 98	29.005 99
5	15	0.1	29.398 36	26.578 80	29.095 30
5	**15**	**1**	**31.91692**	**25.74882**	**30.47151**
5	15	10	54.532 52	40.662 65	47.942 13
5	15	100	70.135 44	53.516 17	61.060 98
5	20	0	28.582 94	26.240 94	28.205 88
5	20	0.01	28.547 26	26.169 77	28.243 77
5	**20**	**0.1**	**28.89758**	**25.43993**	**28.38748**
5	20	1	38.366 52	28.673 93	34.989 81
5	20	10	64.494 18	48.826 59	56.232 59
5	20	100	71.691 89	54.838 27	62.354 84
10	5	0	24.491 47	22.512 41	24.501 72
10	**5**	**0.01**	**19.87771**	**17.69592**	**17.98640**
10	5	0.1	35.333 69	31.691 05	35.025 74
10	5	1	37.262 93	32.703 40	37.243 84
10	5	10	39.950 33	32.777 68	38.162 86
10	5	100	52.665 87	38.970 96	47.058 18
10	**10**	**0**	**4.71791**	**4.30419**	**3.74850**
10	10	0.01	30.678 25	28.534 03	30.699 82
10	10	0.1	31.118 71	28.545 52	30.844 56
10	10	1	31.420 10	27.994 70	30.831 35
10	10	10	35.251 07	27.656 55	32.992 69
10	10	100	58.586 30	43.957 51	51.250 08
10	**15**	**0**	**2.20590**	**1.84296**	**2.07088**
10	15	0.01	29.087 43	26.909 81	28.808 13
10	15	0.1	29.274 59	26.670 68	28.867 35
10	15	1	30.015 63	25.706 83	29.185 21
10	15	10	44.223 69	32.677 04	39.594 59
10	15	100	67.194 67	51.078 77	58.564 54
10	**20**	**0**	**4.81130**	**5.03836**	**4.42378**
10	20	0.01	28.496 44	26.241 69	28.247 20
10	20	0.1	28.615 20	25.820 12	28.201 92
10	20	1	32.043 04	25.369 08	30.244 51
10	20	10	57.172 37	42.786 35	49.930 28
10	20	100	70.509 48	53.866 52	61.307 50

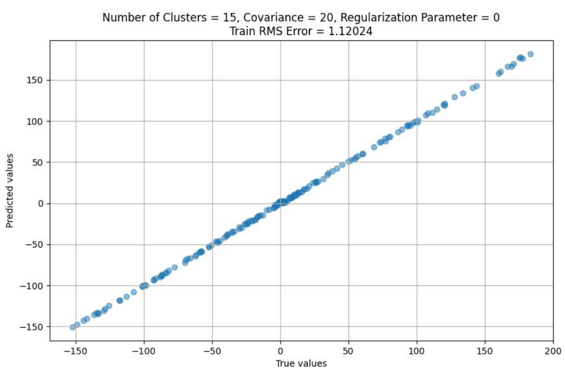
Clusters	Covariance	Regularization Parameter	Training Error	Validation Error	Testing Error
15	**5**	**0**	**8.37685**	**7.66738**	**8.59318**
15	5	0.01	10.641 33	9.136 97	10.131 70
15	5	0.1	21.214 37	19.043 04	21.290 36
15	5	1	34.378 82	30.665 66	34.085 19
15	5	10	39.260 76	32.822 60	38.067 10
15	5	100	49.172 41	36.725 89	44.343 94
15	**10**	**0**	**2.70458**	**2.68651**	**3.33026**
15	10	0.01	30.307 48	28.259 43	30.435 47
15	10	0.1	31.044 07	28.470 48	30.812 57
15	10	1	31.259 51	28.202 97	30.843 38
15	10	10	33.766 07	27.304 18	32.372 65
15	10	100	54.934 97	40.754 10	48.461 01
15	**15**	**0**	**1.45429**	**1.46338**	**1.65193**
15	15	0.01	28.970 72	26.950 58	28.694 94
15	15	0.1	29.191 81	26.766 84	28.883 67
15	15	1	29.633 92	25.927 28	28.986 01
15	15	10	39.635 51	29.469 10	35.752 67
15	15	100	64.542 02	48.820 60	56.182 10
15	**20**	**0**	**1.12024**	**1.20874**	**1.27879**
15	20	0.01	28.489 55	26.238 74	28.210 82
15	20	0.1	28.592 11	26.018 17	28.331 61
15	20	1	30.782 69	25.016 92	29.408 24
15	20	10	51.218 29	38.081 95	45.386 90
15	20	100	69.499 93	52.974 35	60.552 05
20	**5**	**0**	**6.69271**	**6.65142**	**7.27503**
20	5	0.01	8.537 01	7.323 87	8.496 01
20	5	0.1	19.270 48	17.516 89	18.184 28
20	5	1	33.357 60	29.613 30	33.021 24
20	5	10	38.712 46	32.488 89	37.251 33
20	5	100	46.224 24	35.026 03	42.575 46
20	**10**	**0**	**2.03519**	**2.17390**	**2.44225**
20	10	0.01	30.107 14	28.051 97	30.128 22
20	10	0.1	30.871 45	28.526 02	30.697 10
20	10	1	31.322 84	28.233 47	30.896 66
20	10	10	32.539 92	27.291 30	31.546 49
20	10	100	51.748 68	38.305 28	45.723 73
20	**15**	**0**	**1.83906**	**2.24362**	**2.42230**
20	15	0.01	28.935 64	26.970 73	28.682 73
20	15	0.1	29.204 68	26.794 78	28.903 53
20	15	1	29.522 57	26.085 19	28.912 51
20	15	10	36.742 44	27.835 36	33.748 77
20	15	100	62.902 49	47.405 23	55.106 58
20	**20**	**0**	**14.65931**	**14.82575**	**14.77602**
20	20	0.01	28.439 02	26.232 38	28.139 48
20	20	0.1	28.571 21	26.029 46	28.219 59
20	20	1	29.639 08	25.163 23	28.857 96
20	20	10	46.707 80	34.479 75	41.399 73
20	20	100	68.242 46	51.964 59	59.457 32

Table 6: Training, Validation and Testing Errors for different K, σ, λ for training dataset 2 of size 200. The best model is taken to be the one with the lowest validation error and are shown in boldface.

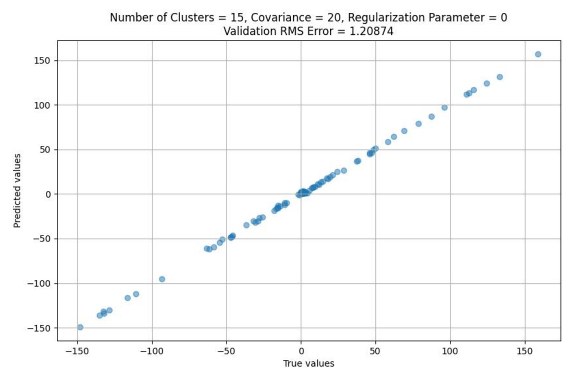
3.2.2 Inferences

1. Increasing the number of clusters generally reduces training errors but can lead to overfitting.
2. Effect of regularization on the errors is similar to what was observed in case of 50 data points.
3. With 200 samples, the model consistently exhibits a similar magnitude of the training and validation errors, indicating less overfitting and generalization.
4. The relation between covariance and model performance is similar to the case of 50 sample test inputs. Here a moderate covariance value of $\sigma = 15$, seems to be the optimal value for different cluster sizes.
5. The model has a similar performance on the training, validation and testing data when optimal parameters are chosen.
6. The model performs optimally with $\lambda = 0$, covariance = 20, and 15 clusters.
7. The dataset exhibits resilience against overfitting, as evidenced by the optimal configuration's preference for a regularization parameter of 0. This suggests that the model generalizes well without the need for strong regularization
8. The model's performance appears to be sensitive to changes in covariance values. The fact that the optimal configuration includes a covariance of 10 indicates that the dataset responds favorably to a moderate level of covariance.

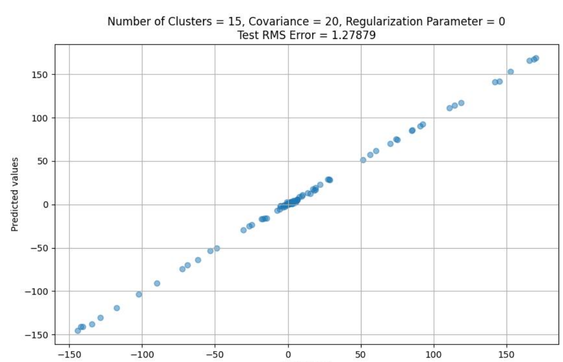
3.2.3 Scatter Plots of the Best Model



(1) Best model ($K = 15, \sigma = 20, \lambda = 0$) performance in Training Data



(2) Best model ($K = 15, \sigma = 20, \lambda = 0$) performance in Validation Data



(3) Best model ($K = 15, \sigma = 20, \lambda = 0$) performance in Testing Data

Figure 12: Best model performance with Training, Validation and Testing Data

4 Task 3.2: Linear Model for Regression using Gaussian Basis Functions for Dataset 3

4.1 Hyperparameters v/s Training, Validation and Testing Errors

Clusters	Covariance	Regularization Parameter	Training Error	Validation Error	Testing Error
70	30	0	4.859 90	5.860 90	4.884 40
70	30	0.01	4.547 01	5.758 11	4.375 50
70	**30**	**0.1**	**4.97758**	**5.69835**	**4.97815**
70	30	1	5.454 79	6.044 21	5.148 35
70	30	10	6.604 19	6.780 48	5.832 49
70	30	100	7.588 41	7.694 55	6.444 55
70	35	0	4.492 46	5.550 18	4.308 94
70	**35**	**0.01**	**4.44613**	**5.13883**	**4.13445**
70	35	0.1	4.642 19	5.534 98	4.565 31
70	35	1	5.195 98	5.673 02	4.623 87
70	35	10	6.545 91	6.694 33	5.666 87
70	35	100	7.467 05	7.594 60	6.411 17
70	40	0	4.179 18	7.960 01	10.644 99
70	**40**	**0.01**	**4.26652**	**4.94447**	**4.23797**
70	40	0.1	4.492 84	5.031 10	4.224 03
70	40	1	4.919 02	5.403 96	4.316 62
70	40	10	6.292 91	6.459 31	5.217 22
70	40	100	7.371 95	7.530 51	6.353 26
70	45	0	4.057 28	5.010 81	3.630 40
70	**45**	**0.01**	**4.03433**	**4.83836**	**4.55810**
70	45	0.1	4.490 24	4.948 23	3.766 78
70	45	1	4.799 52	5.357 22	4.113 64
70	45	10	6.152 42	6.306 33	5.284 58
70	45	100	7.379 11	7.518 80	6.305 01
80	30	0	4.790 50	6.589 12	5.314 81
80	30	0.01	4.529 92	5.563 67	4.749 55
80	**30**	**0.1**	**4.82768**	**5.44171**	**4.94848**
80	30	1	5.441 19	6.019 45	5.330 29
80	30	10	6.551 03	6.703 06	5.807 79
80	30	100	7.507 56	7.590 85	6.401 70
80	35	0	3.993 17	5.274 19	3.811 21
80	35	0.01	4.166 27	5.014 27	4.186 11
80	**35**	**0.1**	**4.29566**	**4.98269**	**4.42704**
80	35	1	5.179 35	5.852 10	4.820 69
80	35	10	6.424 90	6.562 28	5.509 46
80	35	100	7.354 18	7.475 00	6.302 53
80	40	0	4.286 54	5.161 95	4.546 75
80	**40**	**0.01**	**3.96875**	**4.80560**	**4.07899**
80	40	0.1	4.314 02	5.064 04	4.439 76
80	40	1	4.808 23	5.548 80	4.428 92
80	40	10	6.157 46	6.323 72	5.217 49
80	40	100	7.391 10	7.505 59	6.308 61
80	45	0	3.692 79	5.050 54	4.402 99
80	**45**	**0.01**	**4.06057**	**4.61357**	**3.90831**
80	45	0.1	3.778 29	4.822 18	4.141 81
80	45	1	4.771 64	5.274 37	3.949 63
80	45	10	6.031 36	6.213 91	4.999 07

Clusters	Covariance	Regularization Parameter	Training Error	Validation Error	Testing Error
80	45	100	7.200 76	7.281 94	6.107 75
90	30	0	3.941 69	6.145 66	4.813 93
90	**30**	**0.01**	**4.37316**	**6.13613**	**5.00570**
90	30	0.1	4.360 28	5.680 26	4.620 16
90	30	1	5.147 67	5.780 47	4.841 31
90	30	10	6.550 92	6.751 44	5.688 54
90	30	100	7.446 56	7.546 54	6.407 53
90	35	0	4.019 00	5.367 29	4.204 06
90	**35**	**0.01**	**4.06997**	**5.04099**	**4.85758**
90	35	0.1	4.176 59	5.192 02	4.935 16
90	35	1	4.886 31	5.532 47	4.488 77
90	35	10	6.258 47	6.437 67	5.486 50
90	35	100	7.350 40	7.502 49	6.294 98
90	40	0	4.118 35	5.410 62	4.360 57
90	40	0.01	4.167 99	4.936 26	4.369 52
90	**40**	**0.1**	**4.17786**	**4.85045**	**3.76992**
90	40	1	4.582 11	5.356 71	4.291 08
90	40	10	6.221 07	6.364 06	5.335 81
90	40	100	7.261 67	7.410 75	6.284 28
90	45	0	3.629 36	5.263 19	4.299 99
90	**45**	**0.01**	**4.00408**	**4.92861**	**4.12769**
90	45	0.1	4.158 19	5.007 33	3.903 64
90	45	1	4.608 48	5.221 12	4.268 25
90	45	10	6.003 92	6.120 66	5.076 66
90	45	100	7.178 24	7.285 04	6.138 11
100	30	0	4.029 23	5.997 53	4.539 48
100	30	0.01	3.969 63	6.088 81	4.668 52
100	**30**	**0.1**	**3.93209**	**5.07671**	**4.91263**
100	30	1	5.016 64	5.691 45	5.089 15
100	30	10	6.437 87	6.666 13	5.632 27
100	30	100	7.450 83	7.567 68	6.381 36
100	35	0	4.074 09	5.849 14	4.217 43
100	35	0.01	3.972 32	5.689 57	4.741 08
100	**35**	**0.1**	**3.84848**	**5.06922**	**4.58001**
100	35	1	4.778 13	5.361 12	4.572 75
100	35	10	6.262 18	6.385 83	5.540 69
100	35	100	7.301 44	7.365 32	6.238 57
100	40	0	3.672 63	4.945 74	4.373 85
100	**40**	**0.01**	**3.62957**	**4.90307**	**4.01890**
100	40	0.1	3.612 08	4.958 22	3.737 03
100	40	1	4.708 01	5.573 62	4.623 02
100	40	10	6.065 52	6.266 21	5.000 80
100	40	100	7.268 61	7.387 39	6.231 37
100	**45**	**0**	**3.24214**	**4.61831**	**3.62015**
100	45	0.01	3.776 04	4.761 22	4.057 99
100	45	0.1	3.701 40	4.771 69	4.152 95
100	45	1	4.448 08	5.083 10	3.840 32
100	45	10	5.934 19	6.145 46	4.888 25
100	45	100	7.111 78	7.219 86	6.085 91

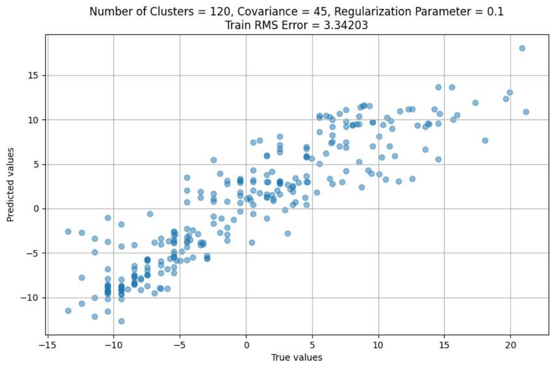
Clusters	Covariance	Regularization Parameter	Training Error	Validation Error	Testing Error
120	30	0	3.727 69	5.175 59	4.669 18
120	30	0.01	3.808 74	5.393 89	4.507 66
120	**30**	**0.1**	**3.87249**	**5.11061**	**4.68030**
120	30	1	4.811 72	5.649 88	4.968 79
120	30	10	6.351 81	6.617 74	5.604 94
120	30	100	7.376 18	7.452 49	6.309 66
120	**35**	**0**	**3.57649**	**4.73260**	**3.53759**
120	35	0.01	3.340 96	4.953 57	3.948 92
120	35	0.1	3.645 43	5.076 57	4.426 74
120	35	1	4.639 30	5.514 31	4.709 62
120	35	10	6.195 91	6.390 44	5.394 81
120	35	100	7.250 15	7.351 05	6.225 25
120	**40**	**0**	**3.24904**	**4.72630**	**3.76178**
120	40	0.01	3.489 87	5.117 46	4.402 99
120	40	0.1	3.621 17	4.808 67	3.665 93
120	40	1	4.336 36	5.083 81	4.080 22
120	40	10	5.938 80	6.160 45	4.978 75
120	40	100	7.149 89	7.263 03	6.100 04
120	45	0	3.553 32	4.587 58	3.995 85
120	45	0.01	3.243 45	4.706 85	3.818 02
120	**45**	**0.1**	**3.34203**	**4.57870**	**3.97437**
120	45	1	4.505 99	4.937 47	4.039 46
120	45	10	5.850 53	6.027 41	4.735 81
120	45	100	6.980 48	7.017 48	6.001 87

Table 7: Training, Validation and Testing Errors for different K, σ, λ for training dataset 2 of size 274. The best model is taken to be the one with the lowest validation error and are shown in boldface.

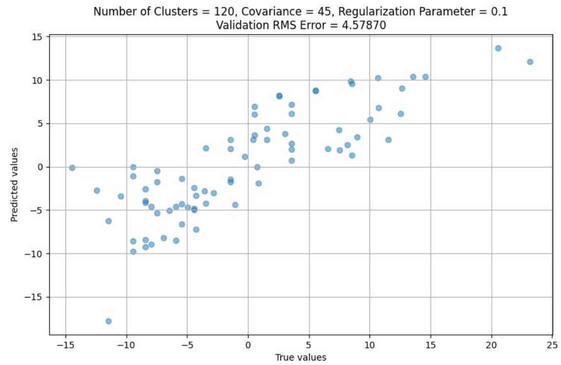
4.2 Inferences

1. Higher λ values lead to increased training errors. Validation and testing errors similarly rise with higher λ.
2. Increasing regularization leads to higher errors ($\lambda = 100$, generally has large errors for all the datasets).
3. Changes in covariance can affect errors. We can roughly observe that increasing covariance reduces the errors.
4. Higher cluster counts tend to reduce training errors; e.g., higher counts like 70 or 80 clusters exhibit lower training errors. However, higher cluster counts, as seen in the 120 clusters, 40 covariance, and regularization parameter 0 configuration, might lead to overfitting.
5. The model has roughly similar performance on the training, validation and testing data when optimal hyper-parameters are chosen.
6. Model with 120 clusters, 45 covariance, and regularization parameter 0.1 performs optimally with both training and validation datasets.
7. The dataset exhibits sensitivity to regularization, emphasizing the importance of controlling model complexity to avoid overfitting.
8. Higher cluster counts (e.g., 120 clusters) contribute to improved model performance, suggesting that the underlying patterns in the dataset benefit from a more intricate representation.
9. The dataset benefits from a higher covariance (45), indicating that a broader spread of data is essential for capturing the nuanced relationships within the dataset.

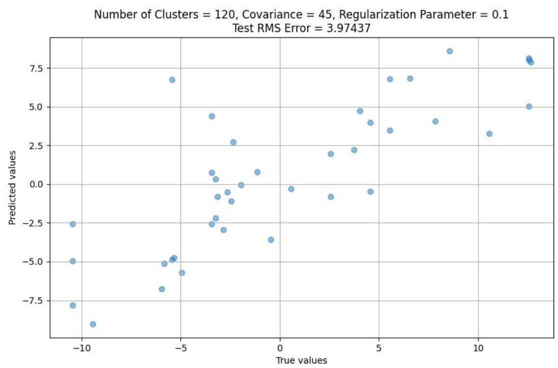
4.3 Scatter Plots of the Best Model



(1) Best model ($K = 120, \sigma = 45, \lambda = 0.1$) performance in Training Data



(2) Best model ($K = 120, \sigma = 45, \lambda = 0.1$) performance in Validation Data



(3) Best model ($K = 120, \sigma = 45, \lambda = 0.1$) performance in Testing Data

Figure 13: Best model performance with Training, Validation and Testing Data

5 Task 4: Linear Model for Regression using Polynomial Basis Functions for Dataset 4

5.1 Description of Dataset

We have chosen the **California Real Estate Dataset**. The input features pertain to several features of the a typical house in a Californian district, which are listed below and an output feature that is the median housing value in that Californian district. The entire dataset has a total of 17,000 data points. The random sample of 500 data points were chosen for the purpose of this assignment. Out of this, 60% of the data was taken for the training split, 20% for the validation split and 20% for the testing split.

Input Features:

1. Latitude
2. Longitude
3. Median Housing Age
4. Total Rooms
5. Total Bedrooms
6. Population
7. Number of households
8. Median Income

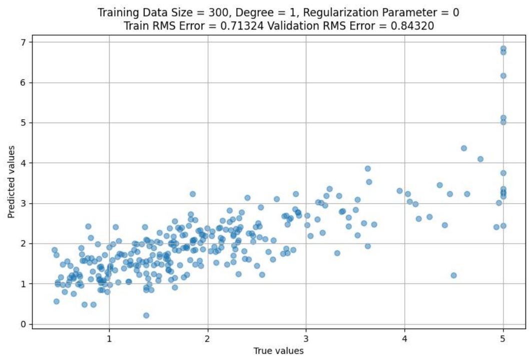
Output Feature:

1. Median House Value

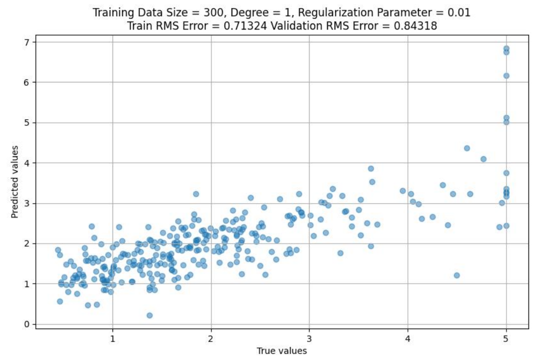
5.2 Procedure

- We performed a linear regression using polynomial basis functions for the dataset.
- The input features have varying scales and hence each feature was divided by a factor to make all the data similar in scale.
- Several different combinations of the degree of the polynomial, $m = 1, 2, 3, 4, 5, 8$, and the regularization parameter, $\lambda = 0, 0.01, 0.1, 1, 10, 100$ were tested.
- The model was trained on the training data and tested using validation data.
- The best model was chosen using the Cross-Validation method and the final model was tested on the testing data.
- All findings are detailed below.

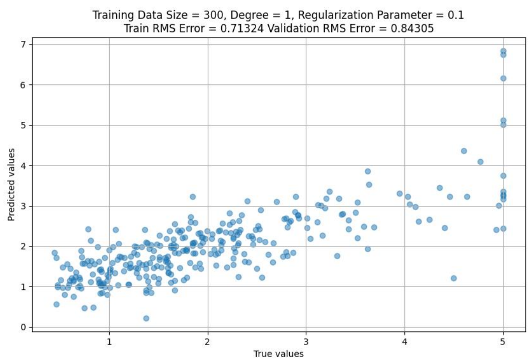
5.3 Scatter Plots of Approximated Functions



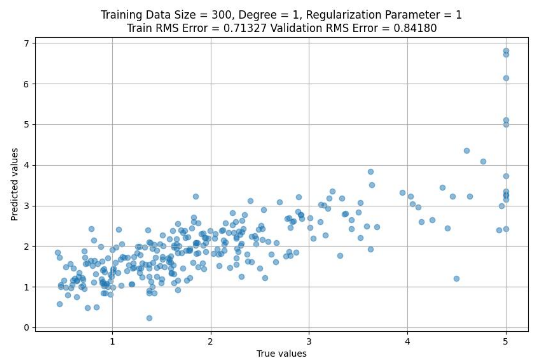
(1) $m = 1, \lambda = 0$



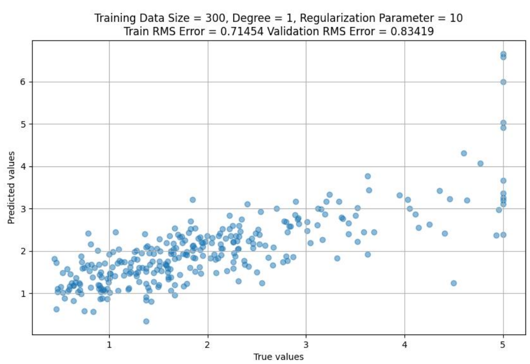
(2) $m = 1, \lambda = 0.01$



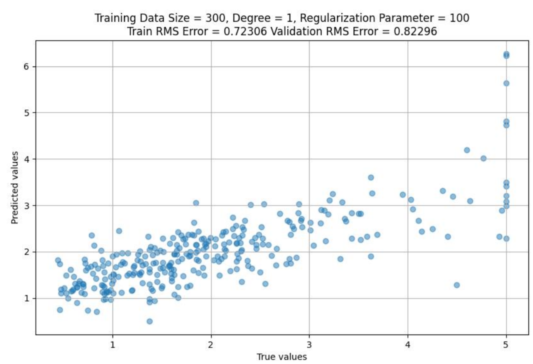
(3) $m = 1, \lambda = 0.1$



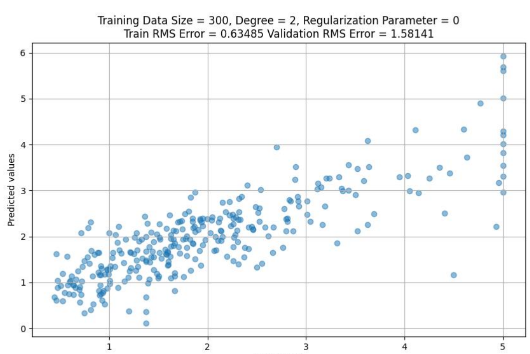
(4) $m = 1, \lambda = 1$



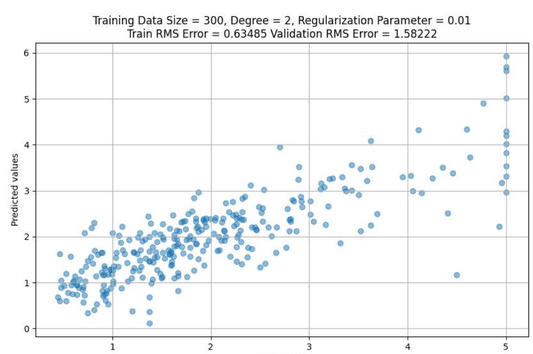
(5) $m = 1, \lambda = 10$



(6) $m = 1, \lambda = 100$



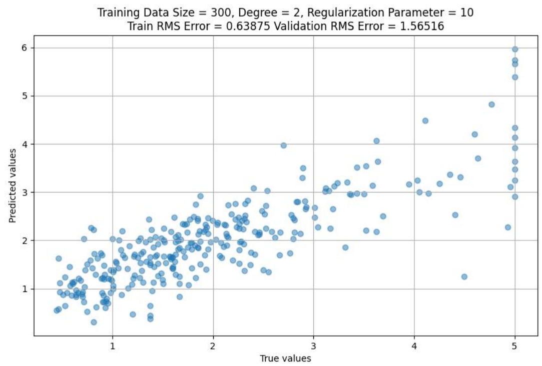
(7) $m = 2, \lambda = 0$



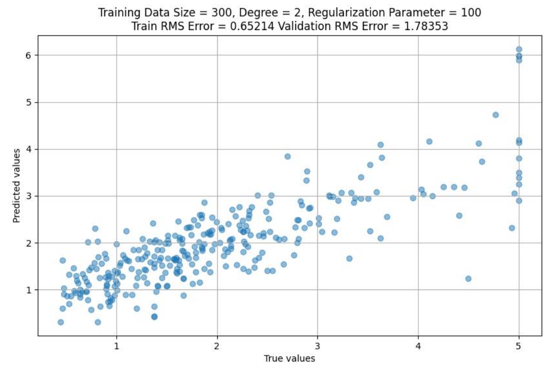
(8) $m = 2, \lambda = 0.01$

(9) $m = 2, \lambda = 0.1$

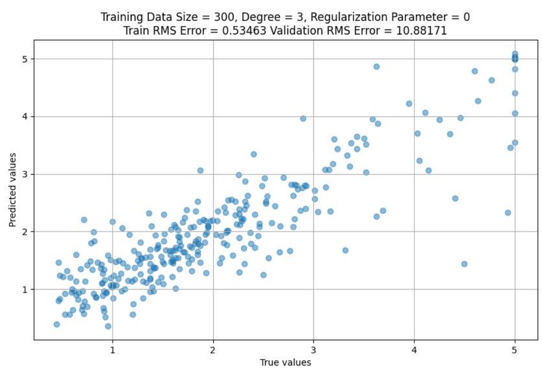
(10) $m = 2, \lambda = 1$



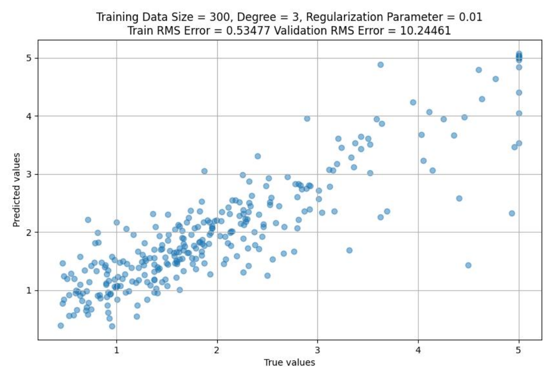
(11) $m = 2, \lambda = 10$



(12) $m = 2, \lambda = 100$

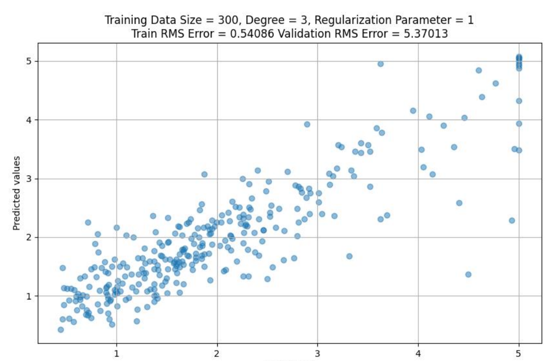


(13) $m = 3, \lambda = 0$



(14) $m = 3, \lambda = 0.01$

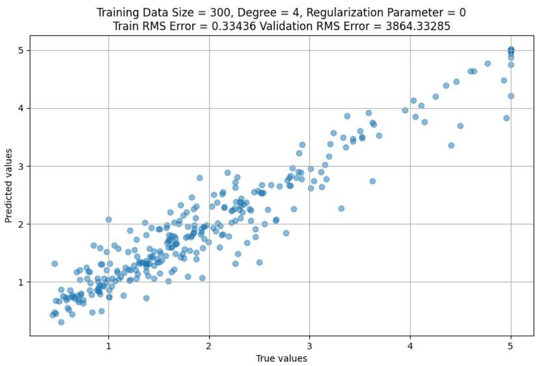
(15) $m = 3, \lambda = 0.1$



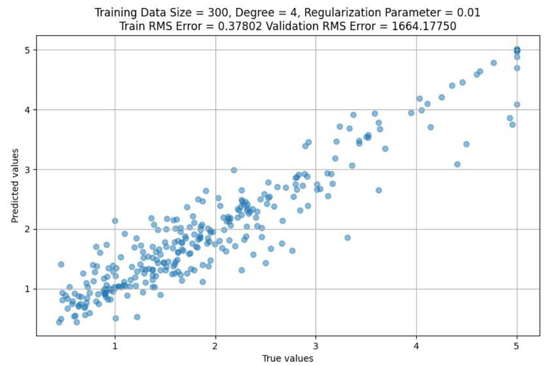
(16) $m = 3, \lambda = 1$

(17) $m = 3, \lambda = 10$

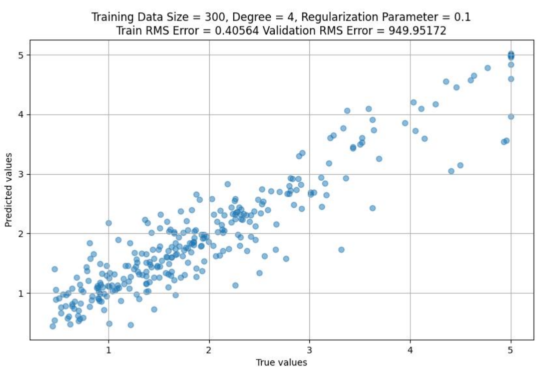
(18) $m = 3, \lambda = 100$



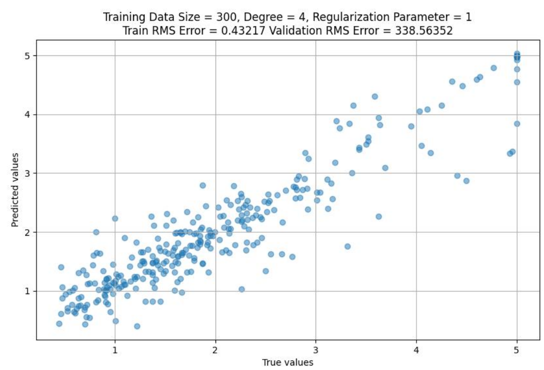
(19) $m = 4, \lambda = 0$



(20) $m = 4, \lambda = 0.01$



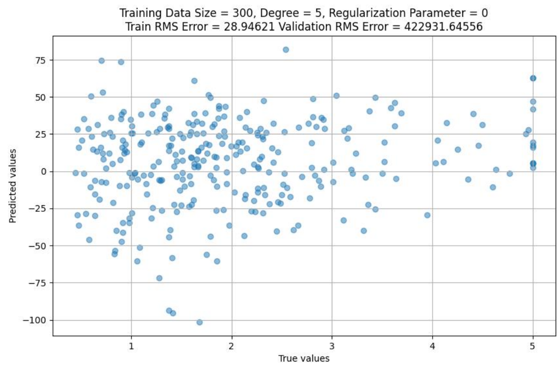
(21) $m = 4, \lambda = 0.1$



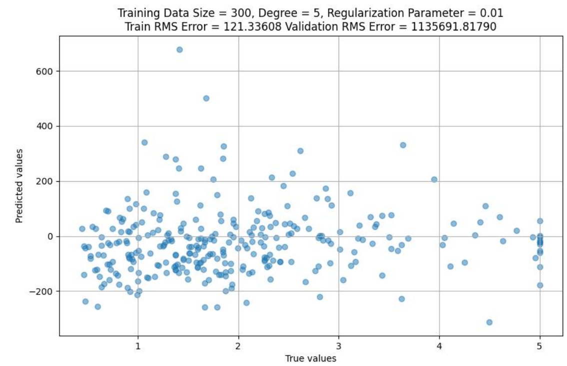
(22) $m = 4, \lambda = 1$

(23) $m = 4, \lambda = 10$

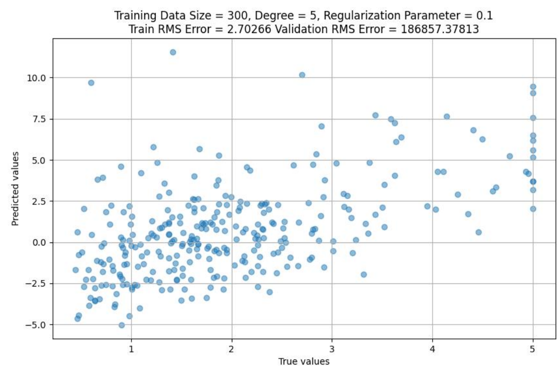
(24) $m = 4, \lambda = 100$



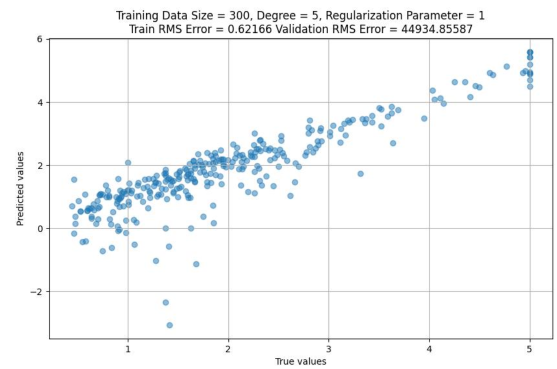
(25) $m = 5, \lambda = 0$



(26) $m = 5, \lambda = 0.01$

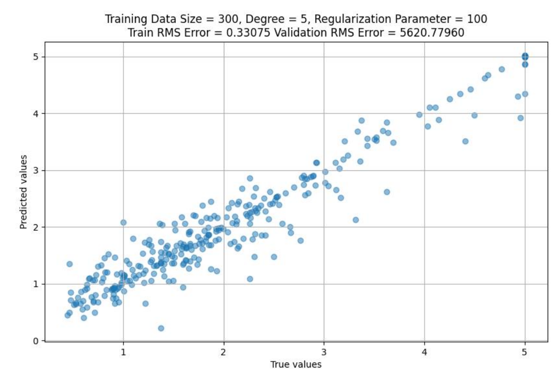


(27) $m = 5, \lambda = 0.1$

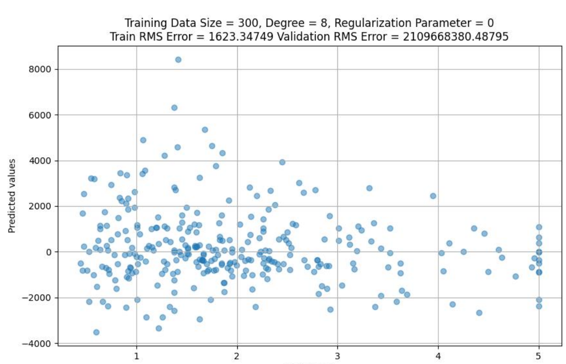


(28) $m = 5, \lambda = 1$

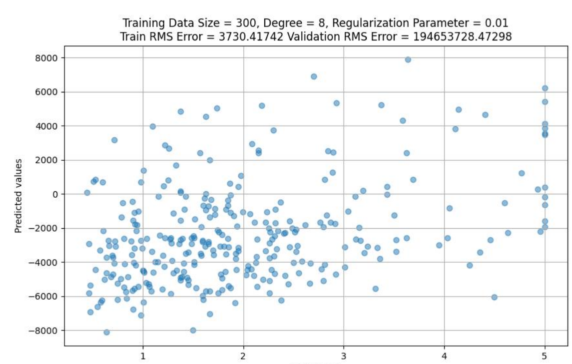
(29) $m = 5, \lambda = 10$



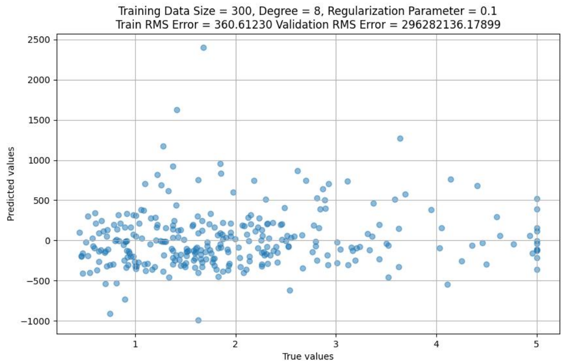
(30) $m = 5, \lambda = 100$



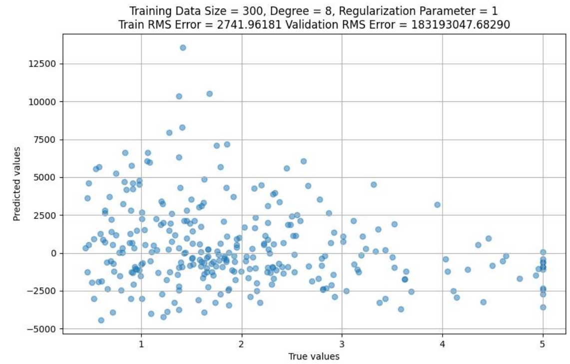
(31) $m = 8, \lambda = 0$



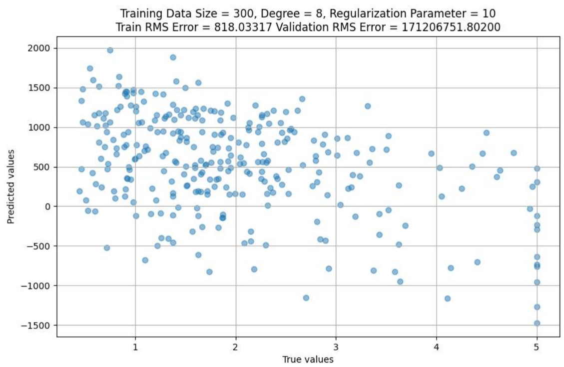
(32) $m = 8, \lambda = 0.01$



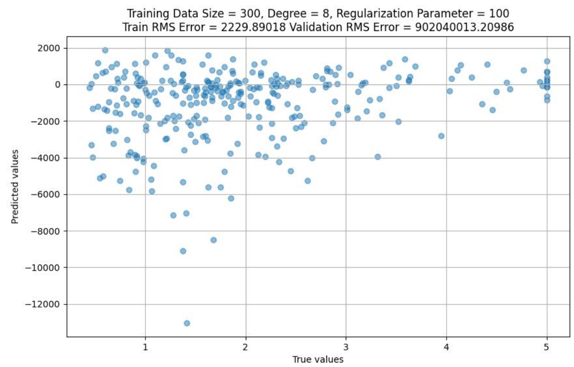
(33) $m = 8, \lambda = 0.1$



(34) $m = 8, \lambda = 1$



(35) $m = 8, \lambda = 10$



(36) $m = 8, \lambda = 100$

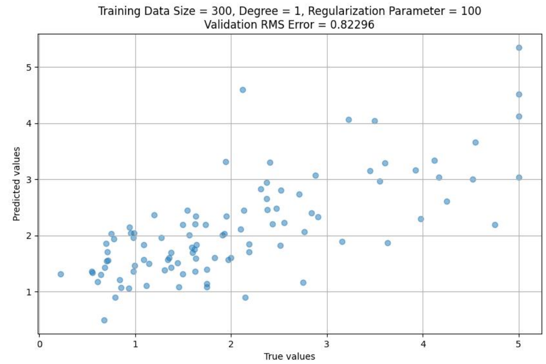
Figure 14: Scatter Plots of Approximated Functions for different m, λ

5.4 Inferences

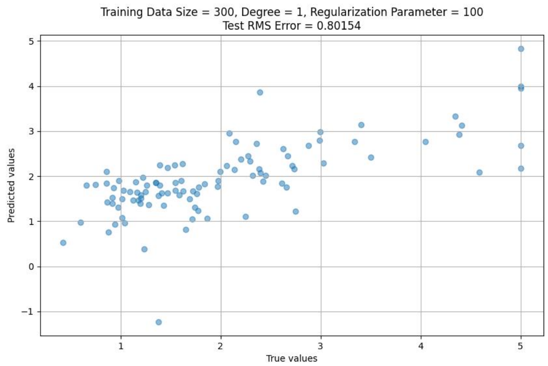
1. For $m = 1$, the model fits well with the training data and also performs well with the validation data.
2. With regularization, the model performance on the validation data further increases with minimal drop in performance in the training data.
3. As the degree increases, the model begins to overfit on the training data. This can be seen with the decrease in the training error and increase in the validation error.
4. At degree $m = 4$, with no regularization the model has the lowest training error, but the validation error is huge.
5. The validation error improves with the usage of regularization but the model, still, is highly overfitted on the training data.
6. For degrees, $m = 5, 8$ the model begins to fall apart and both the training and validation errors shoot up. This is due to the insufficient number of samples for the training of the model. For $m = 5$, we require atleast 462 samples and for $m = 8$, we require atleast 3003 samples for a good approximation. These values are given by equation (1).
7. The best model is identified to be one with parameters of $m = 1, \lambda = 100$.

5.5 Scatter Plots of the Best Model

(1) Best model ($m = 1, \lambda = 100$) performance in Training Data



(2) Best model ($m = 1, \lambda = 100$) performance in Validation Data



(1) Best model ($m = 1, \lambda = 100$) performance in Testing Data

Figure 15: Best model performance with Training, Validation and Testing Data

5.6 Inferences

1. The model has similar errors for training, validation and testing data. This shows that the model has not overfitted with the training data and is generalizable.
2. The usage of regularization has improved the performance of the model marginally. The training error has increases by 0.009 82 while the validation error has reduced by 0.020 24.
3. So, the usage of regularization is not necessary in this situation.
4. High errors on validation and testing datasets when higher degrees are used, even with regularization, suggest a substantial overfitting problem. The model struggles to generalize to unseen data when larger degree polynomials are chosen $m = 4, 5, 8$.
5. The observed increase in errors with higher polynomial degrees indicates that the dataset might not have complex relationships that require a polynomial of higher degree. Overfitting becomes more pronounced as the model tries to fit noise in the data.
6. The fact that the best performance is observed with a high regularization parameter (100) suggests that the dataset is particularly sensitive to overfitting. Strong regularization is required to control model complexity and prevent fitting noise.
7. The observed trend of increasing errors with higher degrees and the necessity for strong regularization implies that the dataset may have inherent noise or is inherently less complex.

5.7 Hyperparameters v/s Training, Validation and Testing Errors

Degree	Regularization Parameter	Training Error	Validation Error	Testing Error
1	0.0	0.713 24	0.843 20	0.815 70
1	0.01	0.713 24	0.843 18	0.815 70
1	0.1	0.713 24	0.843 05	0.815 70
1	1.0	0.713 27	0.841 80	0.815 73
1	10.0	0.714 54	0.834 19	0.815 56
1	**100.0**	**0.72306**	**0.82296**	**0.80154**
2	0.0	0.634 85	1.581 41	1.074 04
2	0.01	0.634 85	1.582 22	1.073 27
2	0.1	0.634 87	1.588 82	1.066 79
2	1.0	0.635 49	1.617 83	1.027 31
2	**10.0**	**0.63875**	**1.56516**	**0.93007**
2	100.0	0.652 14	1.783 53	0.933 89
3	0.0	0.534 63	10.881 71	8.562 20
3	0.01	0.534 77	10.244 61	7.198 08
3	0.1	0.536 41	8.241 80	3.448 51
3	1.0	0.540 86	5.370 13	1.172 56
3	**10.0**	**0.55471**	**3.69018**	**1.05909**
3	100.0	0.571 22	4.633 24	1.239 05
4	0.0	0.334 36	3864.332 85	3628.205 62
4	0.01	0.378 02	1664.177 50	2366.817 40
4	0.1	0.405 64	949.951 72	1054.380 88
4	1.0	0.432 17	338.563 52	164.148 53
4	10.0	0.453 52	110.676 12	313.247 51
4	**100.0**	**0.48415**	**61.81093**	**347.27459**
5	0.0	28.946 21	422 931.645 56	268 060.953 15
5	0.01	121.336 08	1 135 691.817 90	856 210.757 88
5	0.1	2.702 66	186 857.378 13	148 532.377 73
5	1.0	0.621 66	44 934.855 87	60 520.811 26
5	10.0	0.270 57	9941.526 58	16 778.371 84
5	**100.0**	**0.33075**	**5620.77960**	**5049.82310**
8	0.0	1623.347 49	2 109 668 380.487 95	2 269 890 415.068 18
8	0.01	3730.417 42	194 653 728.472 98	15 490 863.695 70
8	0.1	360.612 30	296 282 136.178 99	29 393 537.967 09
8	1.0	2741.961 81	183 193 047.682 90	58 460 107.929 91
8	**10.0**	**818.03317**	**171206751.80200**	**44590205.69076**
8	100.0	2229.890 18	902 040 013.209 86	49 619 018.591 18

Table 8: Training, Validation and Testing Errors for different m, λ for training dataset of size 200. The best model is taken to be the one with the lowest validation error and are shown in boldface.