Note on oblique parameters, JH, April 15, 2021

I have used 2HDMC [1] in the IDM mode as well as the formulas from [2] and [3] to compute the S and T parameters (oblique or Peskin-Takeuchi parameter). For T the expressions from [2] [eq. (27)] and [3] [eq. (5.26)] are the same while for the S parameter the are different. The simple expression in [2] [eq. (65)] can just be integrated numerically[1] while the one in [3] [eq. (5.25)] requires the evaluation of Passarino-Veltman integrals using *e.g.* LoopTools. For the T parameter I found agreement on the per-mille level between 2HDMC and the analytic expression. For the S parameter the deviations are a bit larger, see table 1.

m_{H^0}	m_{A^0}	$m_{H^\pm}$	S(2HDMC)	S(Barbieri)	S(Haber)
106.2	1040.4	873.9	-0.0119	-0.01233	-0.01235
411.8	2362.8	2655.8	-0.0259	-0.02687	-0.02685
754.4	815.9	860.8	-0.00468	-0.00486	-0.00487
187.6	277.3	261.2	-0.00584	-0.00561	-0.00569
246.9	1112.8	1287.7	-0.02667	-0.02763	-0.02766
1255.2	1687.2	1860.8	-0.01166	-0.01212	-0.01211
349.5	1483.7	1171.2	-0.0068	-0.00708	-0.0071
175.8	1658.	1275.8	-0.00734	-0.00764	-0.00773
351.7	834.7	996.3	-0.02405	-0.0249	-0.02494
783.3	2749.8	3248.5	-0.02651	-0.0275	-0.02731
624.3	2405.2	2018.5	-0.00948	-0.00987	-0.01013
1357.4	2005.9	2459.5	-0.01886	-0.01958	-0.01958
307.4	1646.9	1352.4	-0.00965	-0.01003	-0.00999
294.	915.5	892.3	-0.01595	-0.01652	-0.01654
380.1	2241.3	1779.4	-0.00816	-0.00849	-0.00846
305.3	786.2	607.6	-0.00258	-0.00265	-0.00266
1140.9	1225.7	1335.	-0.00612	-0.00638	-0.00638
180.9	699.1	661.4	-0.01574	-0.01624	-0.01629
859.7	2116.6	2616.8	-0.02617	-0.02714	-0.02716
1089.2	1197.8	1385.4	-0.00976	-0.01014	-0.01015

Table 1: Comparison of the S oblique parameter from using 2HDMC [1] and the analytic expressions from Barbieri *et al.* [2] and Haber *et al.* [3] for 20 random parameter points.

In our study, we want to use the analytic expressions [2, 4] (see footnote 1):

$$S = \frac{1}{2\pi}\left(\frac{1}{6}\ln\frac{{M_{H^0}}^2}{{M_{H^\pm}}^2} - \frac{5}{36} + \frac{{M_{H^0}}^2{M_{A^0}}^2}{3({M_{A^0}}^2-{M_{H^0}}^2)^2} + \frac{{M_{A^0}}^4({M_{A^0}}^2-3{M_{H^0}}^2)}{6({M_{A^0}}^2-{M_{H^0}}^2)^3}\ln\frac{{M_{A^0}}^2}{{M_{H^0}}^2}\right) \quad (1)$$

$$T = \frac{1}{16\pi^2\alpha_{\rm ew}v^2}\Big(F(M_{H^\pm},M_{H^0}) + F(M_{H^\pm},M_{A^0}) - F(M_{A^0},M_{H^0})\Big)\,, \quad (2)$$

where $F(m_1,m_2) = (m_1^2+m_2^2)/2 - m_1^2 m_2^2/(m_1^2-m_2^2)\ln(m_1^2/m_2^2)$. I used $\alpha_{\rm ew} = 1/127.9$. Furthermore, eq. (2) is written for $v \simeq 246\,\mathrm{GeV}$ (note the difference in the pre-factor with respect to [4]).

References

[1] D. Eriksson, J. Rathsman, and O. Stal, "2HDMC: Two-Higgs-Doublet Model Calculator Physics and Manual," *Comput. Phys. Commun.* **181** (2010) 189–205, arXiv:0902.0851 [hep-ph].

[1] BTW, it seems it can actually be integrated numerically, as [4] give a fully algebraic expression that numerically resembles the one from [2] to machine precision.

[2] R. Barbieri, L. J. Hall, and V. S. Rychkov, "Improved naturalness with a heavy Higgs: An Alternative road to LHC physics," *Phys. Rev. D* **74** (2006) 015007, `arXiv:hep-ph/0603188`.

[3] H. E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model III: The CP-conserving limit, custodial symmetry, and the oblique parameters S, T, U," *Phys. Rev. D* **83** (2011) 055017, `arXiv:1011.6188 [hep-ph]`.

[4] M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Ludwig, K. Moenig, M. Schott, and J. Stelzer, "Updated Status of the Global Electroweak Fit and Constraints on New Physics," *Eur. Phys. J. C* **72** (2012) 2003, `arXiv:1107.0975 [hep-ph]`.