

Pure mathematics

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April 16, 2023

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1 Number theory

The number theory is primarily devoted to the study of the integers and integer-valued functions.

1.1 Arithmetic

Defines the basic mathematical rules.

$$1 + 2 = 3$$

1.1.1 Operations

1. Parenthesis: $(3 + 2)^2 \times 2 = 50$
2. Exponentiation: $3^2 = 3 \times 3 = 9$
 - Commutative $(2^2)^3 = (2^3)^2$
 - Associative $(2^2)^3 = 2^6$
3. Multiplication: $3 \times 2 = 2 + 2 + 2 = 3 + 3 = 6$
 - Identity element: 1

- Commutative
- Associative

4. Division: $6 \div 3 = 2$

- Inverse operation of Multiplication

5. Addition: $1 + 2 = 3$

- Identity element: 0
- Commutative
- Associative

6. Subtraction: $3 - 2 = 1$

- Inverse operation of Addition

1.1.2 Fundamental theorem of arithmetic Prime factorization theorem

Every integer greater than 1 can be represented as a **unique product of prime numbers**, up to the order of the factors.

$$n = p_1^{n_1} p_2^{n_2} \dots p_k^{n_k} = \prod_{i=1}^k p_i^{n_i}$$

1.2 Examples

1.2.1 Prime factorization

The easiest way to do a prime factorization is to start with the smallest primes.

$$88 \div 2 = 44$$

$$44 \div 2 = 22$$

$$22 \div 2 = 11$$

$$88 = 2 \times 2 \times 2 \times 11 = 2^3 \times 11$$

1.2.2 Sum of natural numbers

$$S = \sum_{a=1}^n a = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

To get this formula just write down a few examples and find out the pattern.

$$1 = 1 = \frac{1 * 2}{2}$$

$$3 = 1 + 2 = \frac{2 * 3}{2}$$

$$6 = 1 + 2 + 3 = \frac{3 * 4}{2}$$

2 Algebra

Algebra uses arithmetic to solve equations i.e. finding the unknown elements.

$$1 + 2 = x$$

$$3 = x$$

2.1 Operations

Are the same as in arithmetic but Multiplication is the default operation.

$$10ab = 10 \times a \times b$$

2.1.1 Exponent rules

- 0 rule: $x^0 = 1$
- Product rule: $x^n \times x^m = x^{n+m}$
- Quotient rule: $x^n \div x^m = x^{n-m}$
- Power rule: $(x^n)^m = x^{n \times m}$
- Negative exponent: $x^{-n} = 1 \div x^n$
- Fraction exponent: $x^{\frac{1}{2}} = \sqrt{x}$
- Distribution: $(x \times y)^n = x^n \times y^n$

2.1.2 Inequalities

- Greater: $x > 1$
 - Sign turns when multiplying with a negative number: $-x < -1$
- Greater equal: $x \geq 1$
 - Sign turns when multiplying with a negative number: $-x \leq -1$
- Less: $x < 1$
 - Sign turns when multiplying with a negative number: $-x > -1$
- Less equal: $x \leq 1$
 - Sign turns when multiplying with a negative number: $-x \geq -1$
- Unequal: $x \neq 1$

3 Calculus and analysis

4 Geometry and topology

5 Combinatorics

6 Logic