Pure mathematics

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Contents

1 Number theory

The number theory is primarily devoted to the study of the integers and integer-valued functions.

1.1 Arithmetic

Defines the basic mathematical rules.

$$1 + 2 = 3$$

1.1.1 Operations

1. Parenthesis: $(3 + 2)^2 \times 2 = 50$

2. Exponentiation: $3^2 = 3 \times 3 = 9$

   - Commutative $(2^2)^3 = (2^3)^2$
   - Associative $(2^2)^3 = 2^6$

3. Multiplication: $3 \times 2 = 2 + 2 + 2 = 3 + 3 = 6$

   - Identity element: 1
   - Commutative
   - Associative

4. Division: $6 \div 3 = 2$

   - Inverse operation of Multiplication

5. Addition: $1 + 2 = 3$

   - Identity element: 0
   - Commutative
   - Associative

6. Subtraction: $3 - 2 = 1$

   - Inverse operation of Addition

1.1.2 Fundamental theorem of arithmetic
Prime factorization theorem

Every integer greater than 1 can be represented as a **unique product of prime numbers**, up to the order of the factors.

$$n = p_1^{n_1} p_2^{n_2} \dots p_k^{n_k} = \prod_{i=1}^{k} p_i^{n_i}$$

1.2 Examples

1.2.1 Prime factorization

The easiest way to do a prime factorization is to start with the smallest primes.

$$88 \div 2 = 44$$
$$44 \div 2 = 22$$
$$22 \div 2 = 11$$
$$88 = 2 \times 2 \times 2 \times 11 = 2^3 \times 11$$

1.2.2 Sum of natural numbers

$$S = \sum_{a=1}^{n} a = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

To get this formula just write down a few examples and find out the pattern.

$$1 = 1 = \frac{1 * 2}{2}$$
$$3 = 1 + 2 = \frac{2 * 3}{2}$$
$$6 = 1 + 2 + 3 = \frac{3 * 4}{2}$$

2　Algebra

Algebra uses Arithmetic to solve equations i.e. finding the unknown elements.

$$1 + 2 = x$$
$$3 = x$$

2.1　Operations

Are the same as in 1.1 but Multiplication is the default operation.

$$10ab = 10 \times a \times b$$

3　Calculus and analysis

4　Geometry and topology

5　Combinatorics

6　Logic