SNEWS Word Problems

1 Exercises

Problem 1: *How Big is a Neutrino Detector?*

The sizes of neutrino detectors are often measured in *kilotons.* One kiloton is a thousand tons or one million kilograms, so a ten-kiloton water-based detector contains one million kilograms of water in its tank (when it is completely filled). (The infamous sea-going vessel HMS *Titanic* had a mass of about 26 kilotons.) For the following, it may be useful to know that the density of water is about 1 gram per cubic centimeter, or 1 kilogram per liter. Also, according to the way the metric system was defined, it takes 1000 (10^3) liters to make up one cubic meter.

(a) Suppose we want to build a 50-kiloton water-based neutrino detector. As a first estimate to the size our tank must be, imagine that the tank will be built in the shape of a cube. How many meters long must the side of the cube be to hold 50 kilotons of H_2O when it is completely filled?

(b)

Problem 2: *How Strong is the Signal?*

We measure *intensity* in terms of the energy falling on a unit area, like one square meter, in a small unit of time, like one second.

$$\text{intensity} = \frac{\text{amountofenergy}}{(\text{area})(\text{time})}.$$

As we move away from a source of light, sound or other "radiant energy", it becomes dimmer. The *inverse square law* says that intensity drops off with the square of the distance: if the source moves to twice its original distance, it appears four times as dim. We can write this rule as an equation; if we measure an intensity I_0 at an original distance R_0, then the intensity at some other radius R is given by

$$I(R) = I_0 \left(\frac{R_0}{R}\right)^2 . \quad (1)$$

(a) Suppose that a certain lamp radiates 1000 joules (1 kilojoule, or 1 kJ) of energy per square meter per second, when measured a distance of 10 m away. What will the intensity be at a distance of 30 m?

(b) Astronomers like to find objects they can use as "standard candles". If we know how bright a star would be at some reference distance, say one light-year away, and we measure how bright the star appears to be from Earth, we can work out how far away the star must be. One type of standard candle, useful for measuring distances to far-away galaxies, is a Type Ia supernova. (Technically speaking, they aren't exactly "standard candles", but scientists have learned to correct for the differences, making them "standardizeable". For this problem, we can pretend that all Type Ia supernovae have about the same intrinsic brightness.) Suppose that all Type Ia supernovae emit at an intensity I_0 when viewed from a standard distance of $R_0 = 1$ megaparsec (3.26 million light-years). One day, an astronomer sees a new Type Ia supernova in a distant galaxy, and its observed brightness is only 1% of the intrinsic brightness I_0. How far away is the galaxy?

Problem 3: *Temperature Scales*

Scientists like to use the *Kelvin scale* for measuring temperatures. It is convenient for many scientific purposes, because zero on the Kelvin scale is the coldest temperature possible in the Universe, also known as *absolute zero.* Before 1967, scientists said "degrees Kelvin", just as we say "degrees Celsius" or "degrees Fahrenheit", but since that year, it has been agreed to say "kelvins" instead. Therefore, we say that one kelvin is the same size as one degree Celsius.

On the Fahrenheit scale, water freezes at 32° and boils at 212°, so there are 180° Fahrenheit between the two points. On the scale invented by Celsius, water freezes at 0° and boils at 100°, for a difference of 100°, meaning that there are 1.8 Fahrenheit degrees for every degree Celsius (and for every kelvin). Absolute zero is −273.15° C, so we can write formulas for a kelvin temperature in terms of either Fahrenheit or Celsius degrees.

$$K = C + 273.15 = \frac{F + 459.67}{1.8}. \tag{2}$$

(a) What is a comfortable room temperature in degrees Celsius and in kelvins? What is human body temperature in kelvins?

(b) For very large temperatures, it doesn't matter too much whether they are reported in Celsius degrees or in kelvins. Georg Stefan estimated the surface of the Sun to have a temperature around 5500°C. What is this temperature in kelvins?

Problem 4: *Stellar Masses*

Over the last several decades, scientists have discovered that the mass of a star is the single most important factor in determining how long it will "live" and in what fashion it will "die". Frequently, masses for distant stars are given as multiples of our Sun's mass.

(a) Scientists believe that the star Betelgeuse (pronounced "beetle-juice") is a likely candidate to go supernova relatively soon—perhaps in the next few thousand years. Betelgeuse, the red star marking Orion's left shoulder, is estimated to have a mass about 15 times that of the Sun. What is this value in (i) kilograms and (ii) Earth masses?

(b) It is estimated that if the planet Jupiter were about 70 times more massive, its core would begin fusion reactions and the planet would be a small star. Estimate this mass, and give the result in (i) kilograms and (ii) Solar masses. Stars in this mass range (below about 75 Jupiter masses) are known as *brown dwarfs.* They do not sustain normal hydrogen fusion, although for the first portion of their lifespan they do fuse the heavier hydrogen isotope *deuterium.*

Problem 5: *Signal Delay*

Problem 6: *Nuclear Burning*

Problem 7: *Half-Lives*

Many phenomena relating to radioactivity can be described in terms of half-lives. Individual atoms decay randomly; we can't say when a particular atom in a piece of radioactive mineral will decay, but we can calculate how many atoms will have decayed after some amount of time has passed. The *half-life* for a substance is the amount of time it takes for half of the material to break down into radioactive decay products. We use the Greek letter τ (*tau*) to represent the half-life, which may be billions of years (for some uranium isotopes) or a tiny fraction of a second. If we begin with N_0 atoms of radioactive material, the half-life equation tells us how many atoms will still be present t seconds later, a quantity we write $N(t)$.

$$N(t) = N_0 \cdot \left(\frac{1}{2}\right)^{t/\tau}. \tag{3}$$

(a) *Radium decay.* As Pierre Curie first determined in the early years of the twentieth century, radium has a half-life of around sixteen centuries. More precisely, the most stable isotope of radium (the one found in nature) has a half-life of 1602 years. This isotope, ^{226}Ra, decays by emitting an alpha particle (a helium nucleus), turning the radium atom into an atom of the radioactive gas radon:

$$^{226}\text{Ra} \rightarrow ^4\text{He} + ^{222}\text{Rn}. \tag{4}$$

Suppose we begin with 10 grams of pure radium-226. How much will be left after 1000 years? If you have studied molar masses and atomic weights, try computing what volume of radon gas will be emitted during this time.

(b) *Carbon-14 dating.* The radioactive isotope carbon-14 is produced by cosmic rays impacting the Earth's atmosphere. It decays to nitrogen-14 with a half-life of 5,730 years. All living organisms take carbon-14 into their bodies, maintaining their carbon-14 level at a fairly constant percentage of the total carbon in their systems. Once they die, they no longer take in carbon-14, so radioactive decay means that the carbon-14 level will drop. Suppose an archaeologist finds a wooden box full of treasure buried in ancient Roman ruins; the gold coins in the treasure chest claim to be from the reign of Augustus Caesar, about two thousand years ago. She takes the box to a carbon-dating lab and finds that its carbon-14 content is 80% $\pm$ 2% of what it had been when the trees for the box were chopped down. Using the half-life decay equation, tell whether or not this measurement is consistent with the information from the coins.

(c) *Supernova light curve.* Scientists believe that most of the light from a Type Ia supernova doesn't come from the original explosion. Instead, it is believed that the supernova's shock wave creates heavy elements like uranium, which the normal nuclear fusion processes in stars cannot create. Some of these elements are radioactive, breaking down to release secondary energy as heat and light. Suppose that SNEWS catches a supernova early in its development, and a clever amateur astronomer takes a spectrum with his backyard telescope. The spectrum reveals the presence of nickel-56, which has a half-life of 6.077 days. How many days before the nickel-56 has decayed to $\frac{1}{64}$ of the original amount?

(d) *Bonus:* If the supernova is still half its original brightness 80 days after it first exploded, can nickel-56 be its only source of light? If not, guess what other elements might be responsible.

Problem 8: *Einstein's Equation*

According to Einstein's Special Theory of Relativity, matter and energy are interchangeable. It is possible to convert an amount of mass into pure energy, which may take the form of light or other electromagnetic radiation. The exact rule is given by Einstein's famous equation,

$$E = mc^2. \tag{5}$$

Here, c is the speed of light, roughly 3×10^8 meters per second. If m is given in kilograms and c in meters per second, then E will have units of joules.

(a) How much energy would be released if one kilogram of matter (any kind of matter) were turned entirely into energy?

(b) A key characteristic of *antimatter* is that when it comes into contact with regular matter, both objects vanish, releasing their mass as a flood of energy. One example of such an *annihilation* reaction happens when an electron meets its antiparticle, called a *positron* because it has the same mass as an electron but a positive electric charge. Typically, when an electron and a positron annihilate, they produce two photons of equal energy. What is the energy of each photon in MeV? Convert this number to joules.

(c) According to the quantum theory Max Planck helped found, the wavelength of a photon is inversely proportional to its energy. Planck's equation says that

$$E = \frac{hc}{\lambda} \tag{6}$$

where h is *Planck's constant,* a number which experiments show is roughly 6.626×10^{-34} joule-seconds. (Don't worry about the units too much for this problem.) What is the wavelength of the photon whose energy you calculated in part (b)? In which part of the electromagnetic spectrum does this photon fall—can you see it with the naked eye?

(d) Astrophysicists estimate that a Type II supernova can release 10^{44} joules of energy. Using Einstein's equation, find how many kilograms lighter the supernova remnant must be than the original star. Express this number as a percentage, assuming that the original star was twenty times as massive as the Sun.

2 Useful Information

Shape	Characteristic Length	Surface Area	Volume
Cube	Side a	$8a^2$	a^3
Sphere	Radius r	$4\pi r^2$	$\frac{4\pi}{3}r^3$
Cylinder	Height h, Radius r	$2\pi r^2 + 2\pi rh$	$\pi r^2 h$

Table 1: Useful geometric formulas for common 3D shapes.

Particle	Symbol	Mass (kg)	Mass (eV/c^2)	Charge	Spin
Electron	e^-	9.109×10^{-31}	5.11×10^5	-1	$\frac{1}{2}$
Proton	p	1.673×10^{-27}	9.38×10^8	$+1$	$\frac{1}{2}$
Neutron	n	1.675×10^{-27}	9.40×10^8	$+1$	$\frac{1}{2}$
Photon	γ	0	0	0	0
Electron Neutrino	ν_e	≈ 0	< 2.5	0	$\frac{1}{2}$
Muon Neutrino	ν_μ	$< 3 \times 10^{-31}$	$< 1.7 \times 10^5$	0	$\frac{1}{2}$
Tau Neutrino	ν_τ	$< 3 \times 10^{-29}$	$< 1.8 \times 10^7$	0	$\frac{1}{2}$

Table 2: Basic properties of common particles. Charges are given in multiples of the fundamental charge unit, 1.602×10^{-19} coulombs.

Object	Mass (kg)	Radius (km)	Distance (km)
Earth	5.9736×10^{24}	6,400	—
Moon	7.348×10^{22}	1738	384,400
Jupiter	1.899×10^{27}	$71,492$	620 to 920 million
Sun	1.9891×10^{30}	6.960×10^5	149.6 million

Table 3: Masses and distances from Earth for various astronomical bodies.

Fuel	Main Product	T (10^9 K)	Duration (yr)
H	He	0.037	8.1×10^6
He	O, C	0.19	1.2×10^6
C	Ne, Mg	0.87	9.8×10^2
O	Si, S	2.0	1.3
Si	Fe	3.3	0.031 (11 days)

Table 4: Nuclear "burning" stages for a star of 20 solar masses. Source: "Massive Star Evolution Through the Ages" (`http://arxiv.org/abs/astro-ph/0211062`).