Mathematics Question Bank

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Contents

Introduction

This paper is a question bank on many different pre-university and university mathematics topics. Answers for each question is given. At the end of each subsection.

1 Sequences and Series

1. In a geometric series, $t_5 + t_7 = 1500$ and $t_{11} + t_{13} = 187500$. Find all possible values for the first three terms.

2. Given that a, b, and c are consecutive terms in an arithmetic sequence that has distinct terms, calculate x if

$$(b-c)x^2 + (c-a)x + (a-b) = 0$$

3. Three different numbers, whose product is 125, are 3 consecutive terms in a geometric sequence. At the same time they are the first, third and sixth terms of an arithmetic sequence. Find these three numbers.

1.1 Solutions

1. In a geometric series, $t_5 + t_7 = 1500$ and $t_{11} + t_{13} = 187500$. Find all possible values for the first three terms.

$$\text{Geometric Series: } \sum_{n=1}^{\infty} ar^{n-1}$$

$$\begin{aligned} ar^4 + ar^6 &= 1500 \\ ar^4(1+r^2) &= 1500 \qquad (1) \\ ar^{10} + ar^{12} &= 187500 \\ ar^{10}(1+r^2) &= 187500 \qquad (2) \end{aligned}$$

Divide Equation (1) from Equation (2)

$$\frac{ar^{10}(1+r^2)}{ar^4(1+r^2)} = \frac{187500}{1500}$$
$$r^6 = 125$$
$$r = \sqrt{5}$$

$$a = \frac{1500}{r^4 + r^6}$$
$$r = \sqrt{5} \Rightarrow a = \frac{1500}{25 + 125}$$
$$= 10$$

$$t_1 = 10$$
$$t_2 = 10\sqrt{5}$$
$$t_3 = 10 \cdot 5 = 50$$

2. Given that a, b, and c are consecutive terms in an arithmetic sequence that has distinct terms, calculate x if

$$(b-c)x^2 + (c-a)x + (a-b) = 0$$

$$b = a + d$$
$$c = a + 2d$$

$$(a + d - a - 2d)x^2 + (a + 2d - a)x + (a - a - d) = 0$$
$$-dx^2 + 2dx - d = 0$$
$$d(-x^2 + 2x - 1) = 0$$
$$d(x^2 - 2x + 1) = 0$$
$$(x^2 - 1) = 0$$
$$x = 1$$

3. Three different numbers, whose product is 125, are 3 consecutive terms in a geometric sequence. At the same time they are the first, third and sixth terms of an arithmetic

sequence. Find these three numbers.

$$a \cdot ar \cdot ar^2 = 125$$
$$a^3r^3 = 125$$
$$ar = 5$$
$$r = \frac{5}{a}$$
$$ar = a + 2d$$
$$ar^2 = a + 5d$$
$$ar = 5 \Rightarrow a + 2d = 5 \Rightarrow d = \frac{5-a}{2} \tag{1}$$
$$(ar)^2 = a^2 + 5ad$$
$$a^2 + 5ad = 25 \tag{2}$$
$$\text{Sub Equation (1) into Equation (2)}$$
$$a^2 + \frac{25a - 5a^2}{2} = 25$$
$$\frac{3}{2}a^2 - \frac{25}{2}a + 25 = 0$$
$$a = 5, \text{ or } a = \frac{10}{3}$$
$$\text{Equation (2)} \Rightarrow d = \frac{25 - a^2}{5a} \tag{3}$$
$$\text{Check } a = 5,$$
$$d = \frac{25 - 25}{25}$$
$$= 0 \text{ (reject, } d = 0)$$
$$\text{Check } a = \frac{10}{3}$$
$$d = \frac{25 - \left(\frac{10}{3}\right)^2}{5\left(\frac{10}{3}\right)}$$
$$= \frac{5}{6}$$
$$\text{First Term: } \frac{10}{3}$$
$$\text{Second Term: } \frac{10}{3} \cdot \frac{5}{\frac{10}{3}} = 5$$
$$\text{Third Term: } \frac{10}{3} \cdot \left(\frac{5}{\frac{10}{3}}\right)^2 = \frac{15}{2}$$

2 Integration

2.1 Pre-University

1. Evaluate the following integrals:

(a) $\displaystyle\int \frac{1}{\sqrt{x}(1+x)}\,dx$ (b) $\displaystyle\int x\tan(x^2)\,dx$ (c) $\displaystyle\int \frac{\cos(2x)}{\cos(x)}\,dx$

(d) $\displaystyle\int \ln(x+1)\,dx$ (e) $\displaystyle\int \frac{1}{x^2-x+1}\,dx$ (f) $\displaystyle\int \frac{12}{4t^2+8t-5}\,dx$

2.2 Solutions

1(a) $\displaystyle\int \frac{1}{\sqrt{x}(1+x)}\,dx$

Sub $x = u^2$

$$dx = 2u\,du$$
$$\frac{1}{u}dx = 2\,du$$
$$x = u^2 \Rightarrow \sqrt{x} = u$$

$$2\int \frac{1}{1+u^2}\,dx$$
$$=2\arctan(u) + c$$
$$=2\arctan(\sqrt{x}) + c$$

1(b) $\displaystyle\int x\tan(x^2)\,dx$

Sub $u = x^2$

$$du = 2x\,dx$$

$$\frac{1}{2}\int \tan(u)\;du = \frac{1}{2}\int \frac{\sin(u)}{\cos(u)}\;du$$
$$= -\frac{1}{2}\int \frac{-\sin(u)}{\cos(u)}\;du$$
$$= -\frac{1}{2}\ln|\cos(u)| + c$$
$$= \frac{1}{2}\ln|\sec(x^2)| + c$$

1(c) $\int \frac{\cos(2x)}{\cos(x)}\;dx$

$$= \int \frac{2\cos^2(x)-1}{\cos(x)}\;dx$$
$$= \int (2\cos(x) - \sec(x))\;\;dx$$

$\int sec(x)\;dx = \text{Sub } u$ $= \cos(x)$

$du = -\sin(u)\;dx$

$$\int \frac{\sin^2(x)}{\cos(x)}\;dx = -\int \frac{\sin^2(x)}{\cos(x)}\;du$$

1(d) $\int \ln(x+1)\;dx$

Sub $u = x + 1$

$du = dx$

$$\int \ln(u)\;du = u\ln(u) - \int 1\;du$$
$$= u\ln(u) - u$$
$$= (x+1)\ln(x+1) - (x+1) + c$$
$$= (x+1)\ln(x+1) - x + c$$