# Pre-University Mathematics

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June 2025

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# Introduction

This paper consists of many different pre-university topics primarily from Cambridge A-Level H2 and H3 mathematics such as Fourier, Maclaurin Series, Conics, Equation of Planes etc. It covers the derivation and applications of the different topics. Some examples are also given.

# 1 Sequences and Series

Consider the following infinite series of elements:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{2^i}, \ldots$$

Can we assign this infinite sum to a numerical value? Indeed, we can. We must first understand what are **series** and **sequences**.

## 1.1 Sequence

A sequence is any number of elements arranged in a specific order.

**Example 1.** An infinite sequence of ascending odd numbers.

$$1, 3, 5, 7, \ldots$$

Sequences can be both infinite and finite. An example of a finite sequence is:

$$2, 4, 8, 16, \ldots, n$$

where $n$ is the final element in the sequence.

More formally, the algebraic notation for a sequence is expressed as:

$$a_1, a_2, a_3, \ldots, a_n$$

where $a_1$ is the first term, $a_2$ is the second term, and $a_n$ is the $n$-th term.

## 1.2 Series

A series is the total sum of all elements in a sequence. In the first section, we posed the question of obtaining a numerical value from an infinite sum. What we are really asking is: *How do we evaluate a series?*

With the initial sequence in **Section 1**, let's first change it to a finite sequence. We can rewrite it into this series:

$$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2^n}$$

We can also write the above in summation notation:

$$S_n = \sum_{k=1}^{n} \frac{1}{2^k}$$

This describes the sum of elements of $\frac{1}{2^n}$ where $k = 1$ to $k = n$.

Suppose we want to find the sum of 10 elements in the series. We can write it as:

$$\begin{aligned} S_{10} &= \sum_{k=1}^{10} \frac{1}{2^k} \\ &= \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{10}} \\ &= 0.99902 \end{aligned}$$

In the next sections, we will explore the different types of sequences and series.

## 1.3 Arithmetic Progression

The first type of sequence is known as an **arithmetic progression**.

**Example 2.** Consider the following sequence:

$$2, 5, 8, 11, \ldots$$

To obtain the above sequence, we start with the first term and add a fixed value to each term successively.

$$2, 2 + 3 = 5, 5 + 3 = 8, 8 + 3 = 11, \ldots$$

This fixed value is called the **common difference**. In the above example, the common difference would be 3.

More formally, an **arithmetic progression** or **AP** is a sequence whereby the difference between the preceding and succeeding terms is common. In algebraic notation, it can be written as:

$$a, a+d, a+2d, a+3d, \ldots, a+(n-1)d$$

where $a$ is the first term, $d$ is the common difference, and $n$ is the number of terms in the sequence.

From the above, we can observe that the $n$-th term is:

$$a_n = a + (n-1)d$$

### 1.3.1 Sum of AP Series

An AP series is simply the total sum of terms in a given arithmetic progression. For a general arithmetic progression:

$$a, (a+d), (a+2d), \ldots, (\ell-2d), (\ell-d), \ell$$

where $a$ is the first term, $d$ is the common difference and $\ell$ is the $n$-th term.

The AP series is written as:

$$S_n = a + (a+d) + (a+2d) + \cdots + (\ell-2d) + (\ell-d) + \ell$$

To derive a general formula for the AP series, let's first reverse the order of $S_n$:

$$S_n = \ell + (\ell-d) + (\ell-2d) + \cdots + (a+2d) + (a+d) + a$$

Then, let's add the original AP series to the reverse AP series by vertically summing each term.

$$2S_n = (a+\ell) + (a+\ell) + (a+\ell) + \cdots + (a+\ell) + (a+\ell) + (a+\ell)$$

Notice how all the terms have become $(a+\ell)$. There are now $n$ number of $(a+\ell)$ terms. Now, we can make $S_n$ the subject:

$$S_n = \frac{1}{2}n(a+\ell)$$

As seen in the previous section, the $n-th$ term, $\ell$, can also be written as $a+(n-1)d$. **Derivation.** Hence, the general formula for the sum of any AP series is:

$$S_n = \frac{1}{2}n(2a+(n-1)d)$$

## 1.4 Geometric Progression

The second type of sequence is known as a **geometric progression**.

**Example 3.** Consider the following sequence:

$$2, 6, 18, 54, \ldots$$

To obtain the above sequence, we start with the first term and multiply it by a fixed value to each term successively.

$$2, 2 \times 3 = 6, 6 \times 3 = 18, 18 \times 3 = 54, \ldots$$

This fixed value is called the **common ratio**. In the above example, the common ratio would be 3.

More formally, a **geometric progression** or **GP** is a sequence whereby each term is produced by multiplying each preceding term by a constant value. In algebraic notation, it can be written as:

$$a, ar, ar^2, ar^3, \ldots, ar^{n-1}$$

where $a$ is the first term, $r$ is the common ratio, and $n$ is the number of terms in the sequence.

From the above, we can observe that the $n$-th term is:

$$a_n = ar^{n-1}$$

### 1.4.1 Sum of GP Series

Similar to a series, a GP Series is simply the total sum of terms in a given geometric progression. For a general geometric progression, its GP series can be written as:

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$