Pre-University Mathematics

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Introduction

This paper consists of many different pre-university topics primarily from Cambridge A-Level H2 and H3 mathematics such as Fourier, Maclaurin Series, Conics, Equation of Planes etc. It covers the derivation and applications of the different topics. Some examples are also given.

1 Sequences and Series

Consider the following infinite series of elements:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{2^i}, \ldots$$

Can we assign this infinite sum to a numerical value? Indeed, we can. We must first understand what are **series** and **sequences**.

1.1 Sequence

A sequence is any number of elements arranged in a specific order.

Example 1. An infinite sequence of ascending odd numbers.

$$1, 3, 5, 7, \ldots$$

Sequences can be both infinite and finite. An example of a finite sequence is:

$$2, 4, 8, 16, \ldots, n$$

where n is the final element in the sequence.

More formally, the algebraic notation for a sequence is expressed as:

$$a_1, a_2, a_3, \ldots, a_n$$

where a_1 is the first term, a_2 is the second term, and a_n is the n^{th} term.

1.2 Series

A series is the total sum of all elements in a sequence. In the first section, we posed the question of obtaining a numerical value from an infinite sum. What we are really asking is: *How do we evaluate a series?*

With the initial sequence in **Section 1**, let's first change it to a finite sequence. We can rewrite it into this series:

$$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2^n}$$

We can also write the above in summation notation:

$$S_n = \sum_{k=1}^{n} \frac{1}{2^k}$$

This describes the sum of elements of $\frac{1}{2^n}$ where $k = 1$ to $k = n$.

Suppose we want to find the sum of 10 elements in the series. We can write it as:

$$\begin{aligned} S_{10} &= \sum_{k=1}^{10} \frac{1}{2^k} \\ &= \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{10}} \\ &= 0.99902 \end{aligned}$$

In the next sections, we will explore the different types of series.

1.3 Arithmetic Progression

1.4 Geometric Progression