Matrix Factorization and Collaborative Filtering

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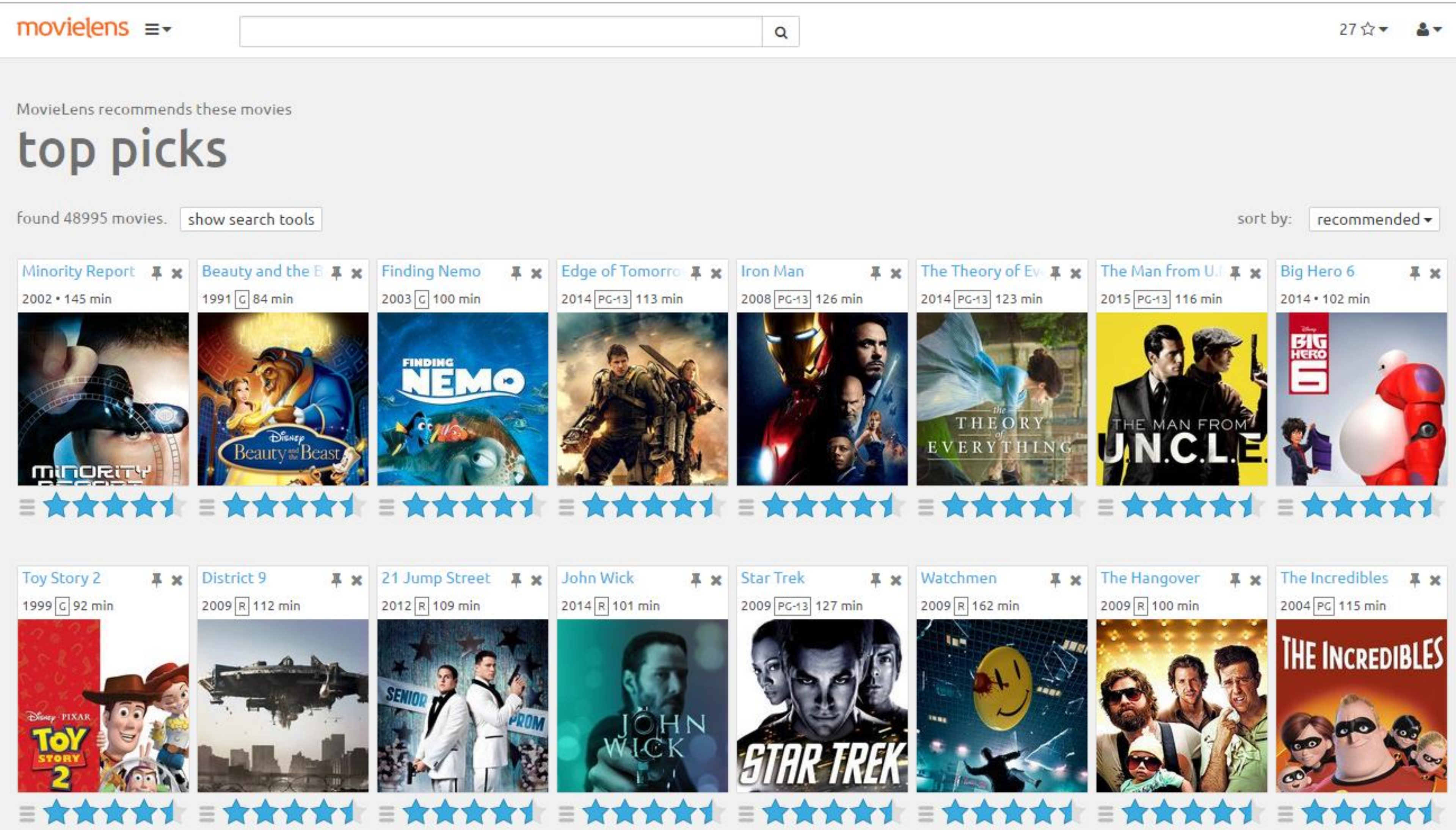
Outline

- **Recommender System**
- Matrix Factorization
- Reference

Recommender System



Recommender System Strategies

1. Content Filtering
   - Create a profile for each user or product to characterize its nature

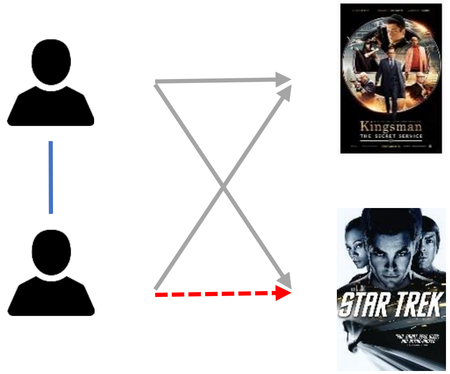
Genres: Crime, Comedy, Action, Adventure
Directors: Matthew Vaughn
Cast: Taron Egerton, Colin Firth, Samuel L. Jackson, …
Distributor: Fox
Box Office Popularity: …

Gender
Region
…
Answers provided on a suitable questionnaire

Recommender System Strategies

2. Collaborative filtering
   - Rely only on past user behavior
   - Everyday examples
     - Bestseller lists
     - Top 40 music lists
     - Unmarked but well-used paths thru the woods
     - The “recent returns” shelf at the library

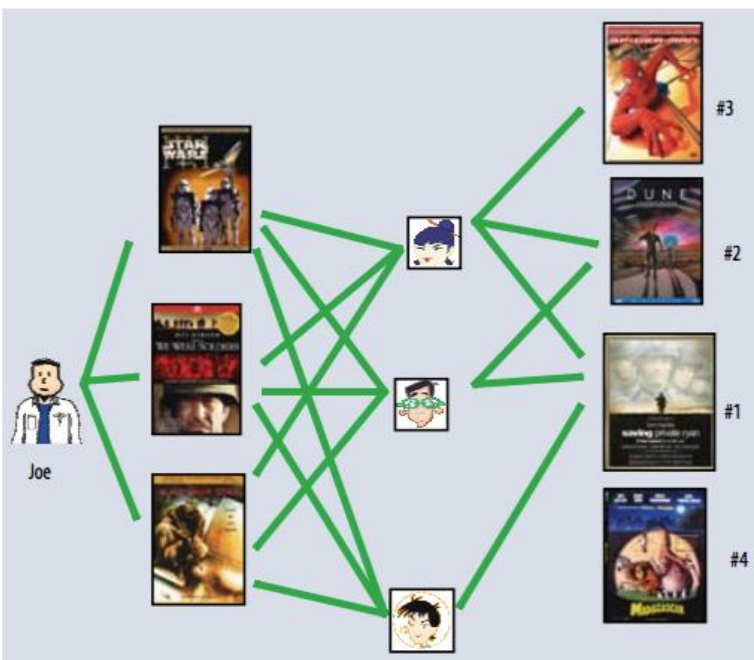
   - Common insight: **personal tastes are correlated**

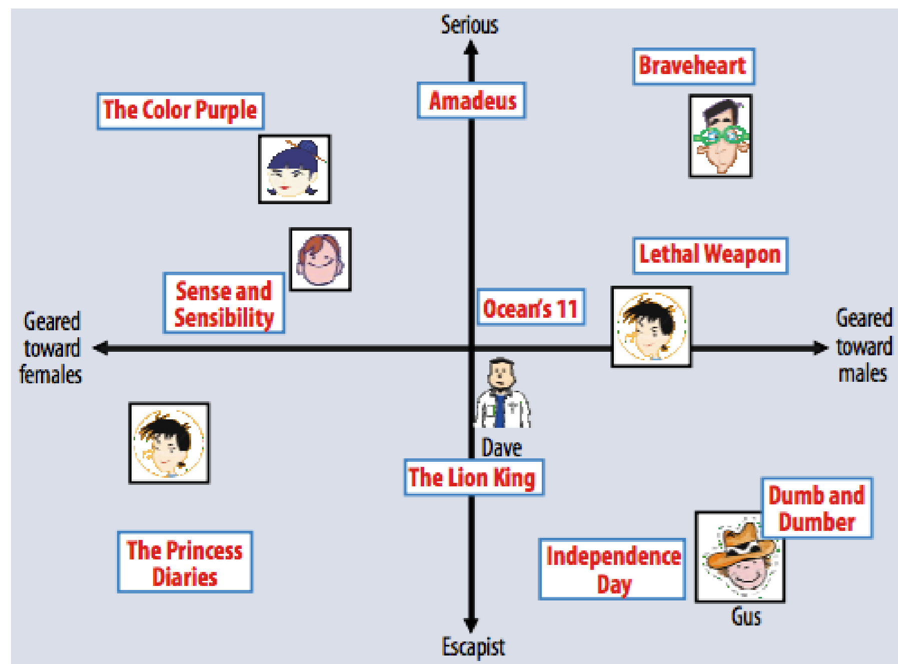


Types of Collaborative Filtering

a. Neighborhood Methods
- Find neighbors based on similarity of movie preferences
- Recommend movies that those neighbors watched

b. Latent Factor Methods
- Characterize both items and users
- Recommend a movie based on its proximity to the user in the latent space





* Figures in [3]

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- **Matrix Factorization**
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Netflix Prize

NETFLIX

Netflix Prize

COMPLETED

Home | Rules | Leaderboard | Update

500,000 users
20,000 movies
100M ratings

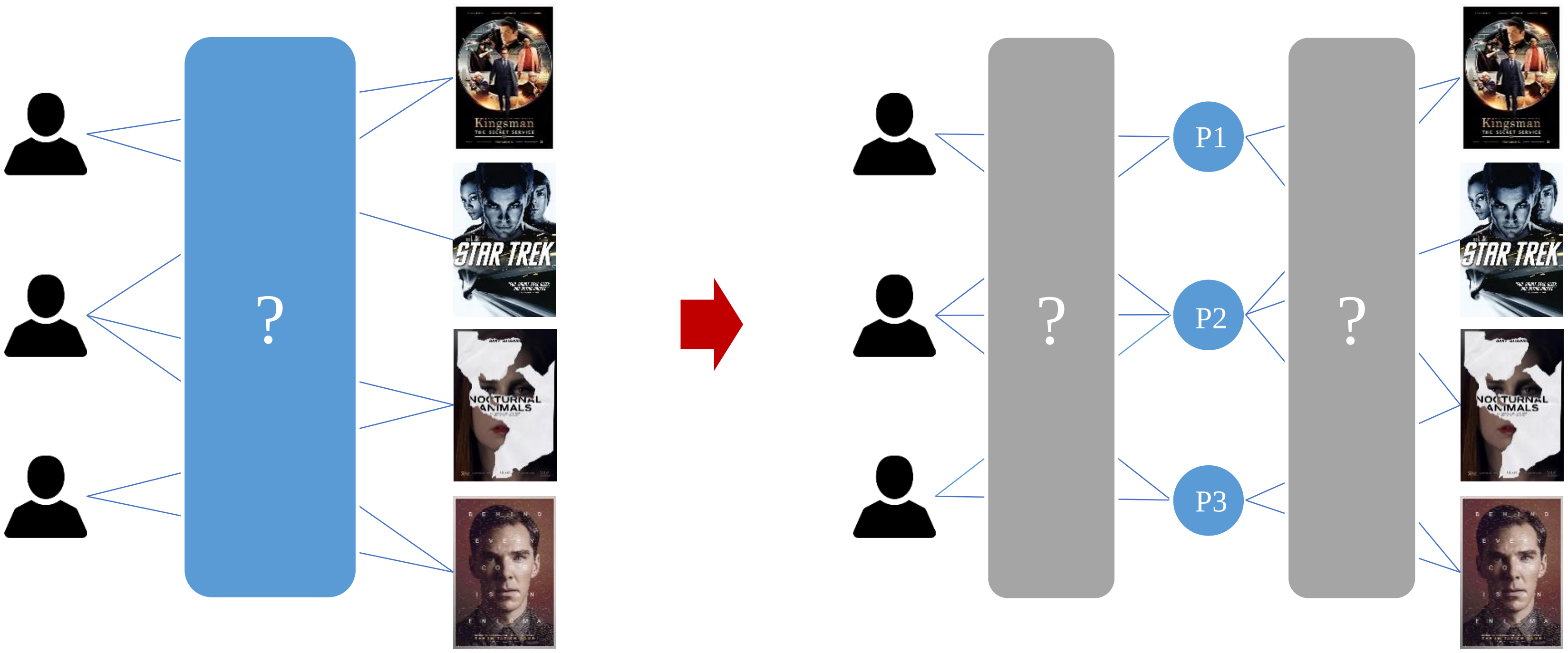
Leaderboard

Showing Test Score. Click here to show quiz score

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	0.8582	9.90	2009-07-10 21:24:40
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace_	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11

Matrix Factorization

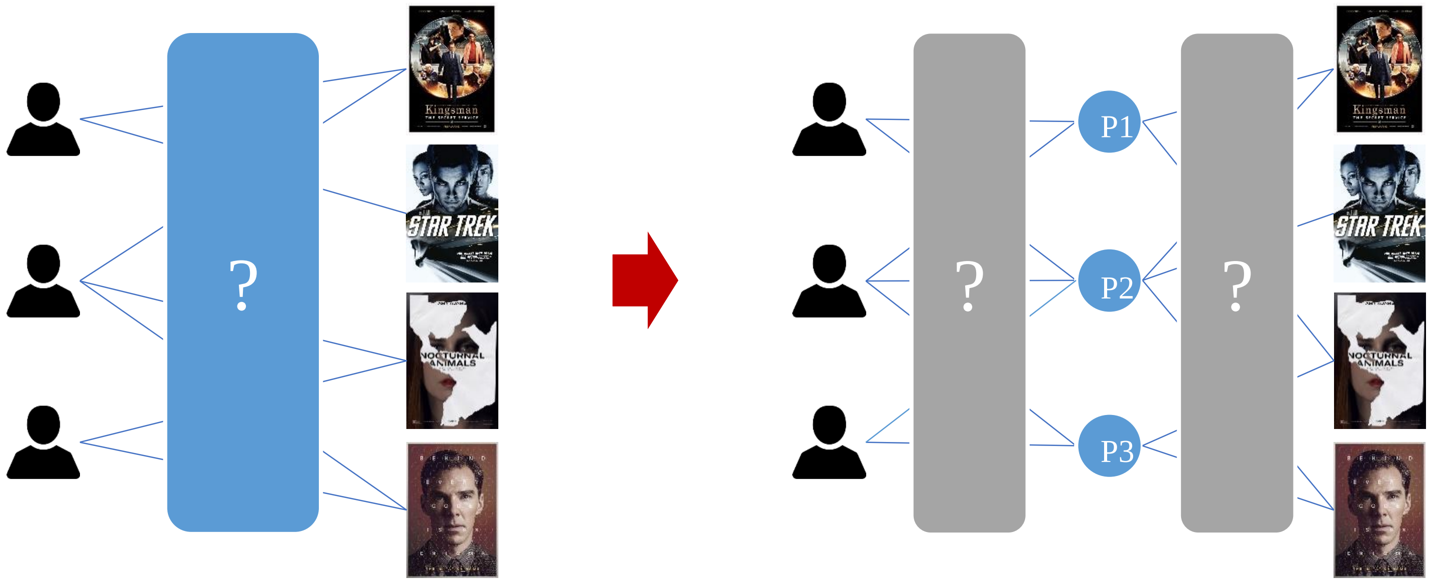
- Assume latent factors in user preference



* Redraw figures in [2]

Matrix Factorization

- Assume latent factors in user preference



	I_1	I_2	I_3	I_4
U_1				
U_2				
U_3				

	P_1	P_2	P_3
U_1			
U_2			
U_3			

	I_1	I_2	I_3	I_4
P_1				
P_2				
P_3				

* Redraw figures in [2]

Singular Value Decomposition

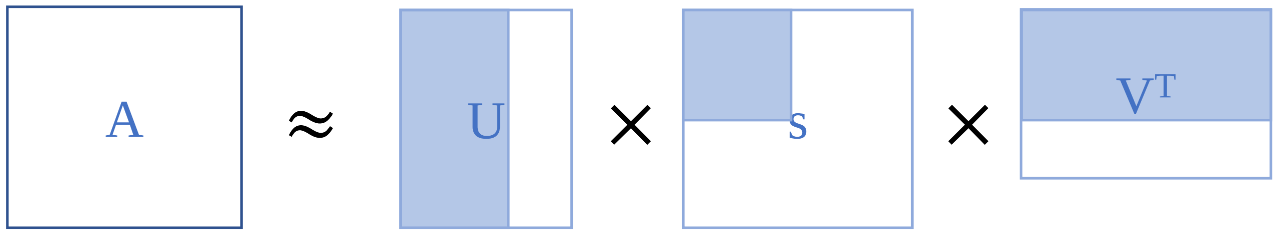
$$A = U \times s \times V^T$$

```
[ 4.   2.   3.   5.   1.]
[ 0.   3.   0.   4.   2.]
[ 5.   4.   3.   3.   0.]  =
[ 0.   0.   5.   5.   2.]
[ 5.   0.   0.   5.   0.]
```

```
[-0.54    0.03   -0.021  0.099 -0.835]     [ 13.707   0.       0.       0.       0.    ]     [-0.503 -0.29   -0.381 -0.705 -0.143]
[-0.29   -0.225   0.393 -0.84    0.07 ]     [  0.      5.607    0.       0.       0.    ]     [ 0.738  0.156 -0.489 -0.254 -0.357]
[-0.506   0.372   0.574  0.374   0.371]  ×  [  0.      0.       3.791    0.       0.    ]  ×  [-0.169  0.905  0.15   -0.35    0.087]
[-0.417  -0.79   -0.217  0.277   0.279]     [  0.      0.       0.       3.645    0.    ]     [ 0.265 -0.227  0.769 -0.455 -0.282]
[-0.441   0.432  -0.685 -0.26    0.287]     [  0.      0.       0.       0.       0.155]     [-0.321  0.147  0.029  0.33   -0.875]
```

We can drop less important information

Singular Value Decomposition




```
[-0.54    0.03   -0.021  0.099 -0.835]        [ 13.707    0.        0.        0.        0.    ]        [-0.503 -0.29   -0.381 -0.705 -0.143]
[-0.29   -0.225   0.393 -0.84    0.07 ]        [  0.       5.607    0.        0.        0.    ]        [ 0.738   0.156 -0.489 -0.254 -0.357]
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[-0.54    0.03   -0.021  0.099 -0.835]        [ 13.707    0.        0.        0.        0.    ]        [-0.503 -0.29   -0.381 -0.705 -0.143]
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[-0.417 -0.79   -0.217  0.277  0.279]        [  0.       0.       0.       0.        0.    ]        [ 0.265 -0.227   0.769 -0.455 -0.282]
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```
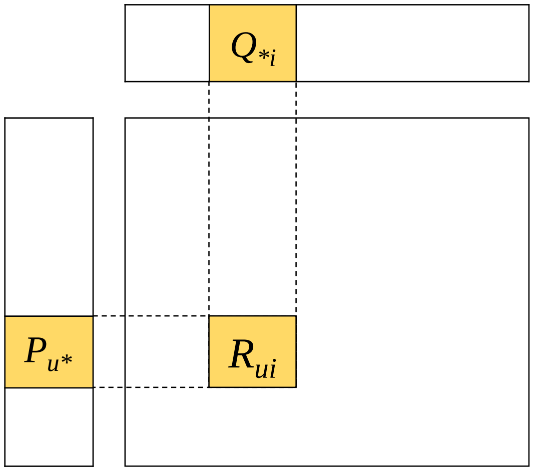

Matrix Factorization

- Matrices
  - User vector
    - $(P_{u*})^T \in \mathbb{R}^f$
  - Item vectors:
    - $(Q_{*i}) \in \mathbb{R}^f$
  - Rating prediction
    - $R_{ui} = P_{u*}Q_{*i} = [PQ]_{ui}$



- Vectors
  - User vector
    - $p_u \in \mathbb{R}^r$
  - Item vectors:
    - $q_i \in \mathbb{R}^r$
  - Rating prediction
    - $\hat{r}_{ui} = q_i^T p_u$
  - Set of non-zero entries
    - $\kappa = \{(u,i) \colon r_{ui} \neq 0\}$
  - Objective
    - $\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2$

* Figures in [1]

Matrix Factorization

$$R_{ui} \approx \hat{R}_{ui} = U \times I$$

- Minimize the error between R and $\hat{\mathrm{R}}$

$$\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2$$

$$\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2 + \boxed{\lambda(\|q_i\|^2 + \|p_u\|^2)}$$

Regularization factor
- avoid overfitting
- make simple model

* Figures in [2]

Matrix Factorization

- How to deal with empty cells in matrix

	I_1	I_2	I_3	I_4
U_1		3	4	2
U_2	5			
U_3	3		2	

$$\boldsymbol{b_{ui} = \mu + b_i + b_u}$$

μ: average of the whole users

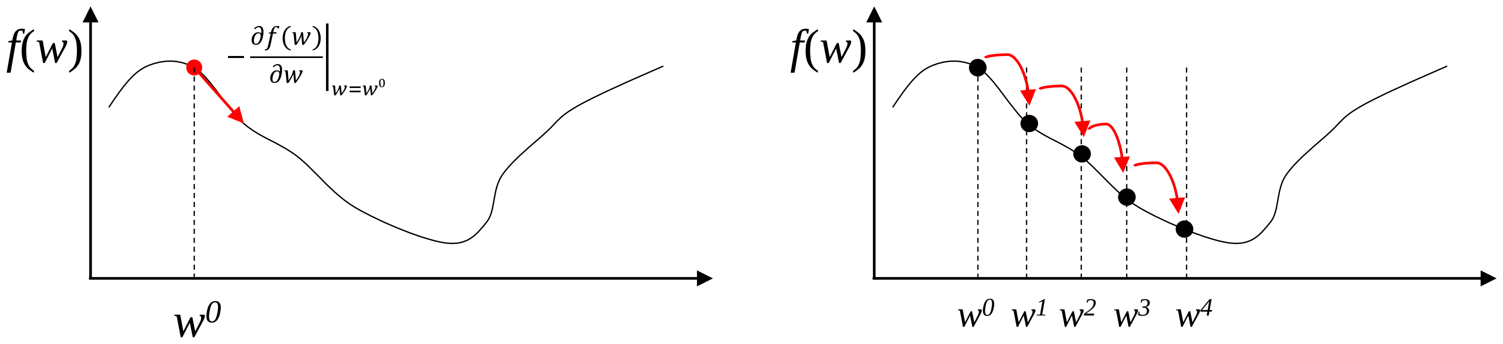
b_i, b_u: the observed deviations of u and i

$$\hat{r}_{ui} = \mu + b_i + b_u + q_i^T p_u$$

$$\min_{q*,p*,b*} \sum_{(u,i)\in\kappa} (r_{ui} - \mu - b_i - b_u - p_u^T q_i)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2 + b_u^2 + b_i^2)$$

Matrix Factorization

- Approaches to minimizing $\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2)$
  1. Stochastic gradient descent



- Associated prediction error e_{ui}
  - $q_i \leftarrow q_i + \gamma(e_{ui} \cdot p_u - \lambda \cdot q_i)$
  - $p_u \leftarrow p_u + \gamma(e_{ui} \cdot q_i - \lambda \cdot p_u)$

* Figures in [2]

Matrix Factorization

- Approaches to minimizing $\min_{q*,p*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2)$
  2. Alternating least squares
     - Rotate between fixing the q_i's and fixing the p_u's
       – When all p_u's are fixed, the system recomputes the q_i's by solving a least-squares problems, and vice versa
     - Stochastic gradient descent is easier and faster than ALS in general, ALS is favorable in at least two cases
       – When the system can use parallelization
       – For systems centered on implicit data

HOSVD [4]

- SVD on each matrix

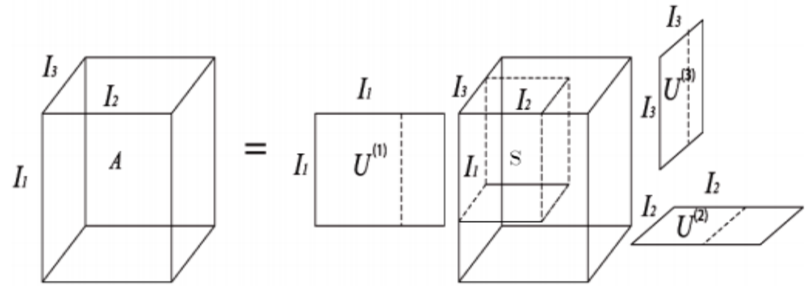
$$A_1 = U^{(1)} \cdot S_1 \cdot V_1^T$$
$$A_2 = U^{(2)} \cdot S_2 \cdot V_2^T$$
$$A_3 = U^{(3)} \cdot S_3 \cdot V_3^T$$

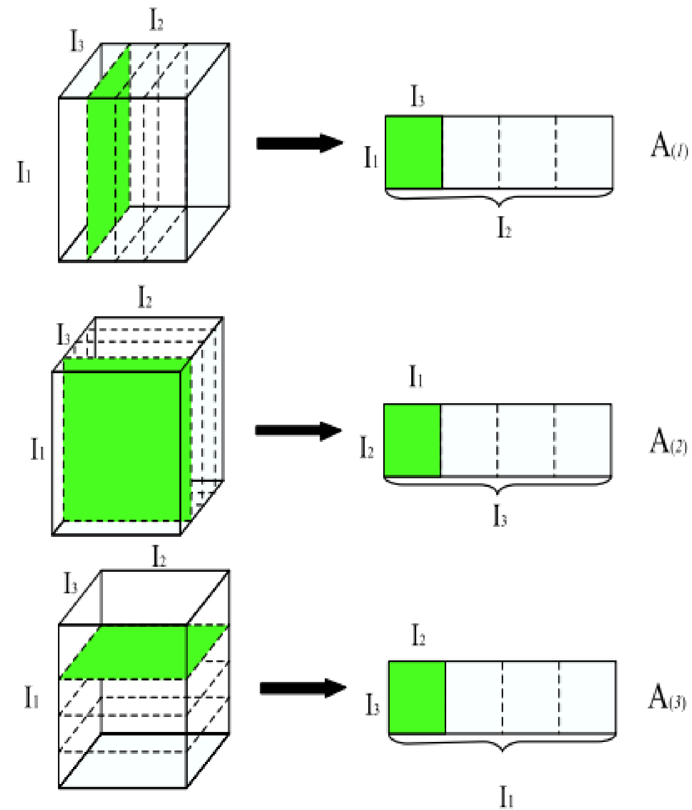
- Construction of core tensor

$$\mathcal{S} = \mathcal{A} \times_1 {U_{c_1}^{(1)}}^T \times_2 {U_{c_2}^{(2)}}^T \times_3 {U_{c_3}^{(3)}}^T$$

- Construction of tensor $\hat{\mathcal{A}}$

$$\hat{\mathcal{A}} = \mathcal{S} \times_1 U_{c_1}^{(1)} \times_2 U_{c_2}^{(2)} \times_3 U_{c_3}^{(3)}$$





Example

- MF in Python

```python
#!/usr/bin/python

import numpy as np

np.set_printoptions(precision = 3) # set decimal display

# Matrix
A = np.zeros((5, 5))

A[0, 4] = 1
A[0, 1] = A[1, 4] = A[3, 4] = 2
A[0, 2] = A[1, 1] = A[2, 2] = A[2, 3] = 3
A[0, 0] = A[1, 3] = A[2, 1] = 4
A[0, 3] = A[2, 0] = A[3, 2] = A[3, 3] = A[4, 0] = A[4, 3] = 5

# SVD
U, s, V = np.linalg.svd(A, full_matrices = True)

# Reconstruction
S = np.diag(s)

P = np.dot(U, np.dot(S, V))
```

```
[ 4.  2.  3.  5.  1.]
[ 0.  3.  0.  4.  2.]
[ 5.  4.  3.  3.  0.]
[ 0.  0.  5.  5.  2.]
[ 5.  0.  0.  5.  0.]
```

```
[   4e+00    2e+00    3e+00    5e+00    1e+00]
[  -4e-16    3e+00    3e-15    4e+00    2e+00]
[   5e+00    4e+00    3e+00    3e+00    7e-16]
[  -4e-15    4e-15    5e+00    5e+00    2e+00]
[   5e+00    1e-15   -2e-16    5e+00   -5e-16]
```

Example

- MF in Python with r

```
#!/usr/bin/python

import rpy2.robjects as robjects

r = robjects.r

r('''
    rsvd <- function() {
        # MATRIX
        A <- matrix(c(4, 2, 3, 5, 1, 0, 3, 0, 4, 2, 5, 4, 3, 3, 0, 0, 0, 5, 5, 2, 5, 0, 0, 5, 0), nrow = 5, ncol = 5, byrow = TRUE)

        # SVD
        result <- svd(A)

        # RECONSTRUCTION
        U <- result$u
        s <- result$d
        V <- result$v

        ApproxA <- U %*% diag(s) %*% t(V)
    }
    ''')

svd = r['rsvd']

result = svd()

print result
```

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	4	2	3	5	1
[2,]	0	3	0	4	2
[3,]	5	4	3	3	0
[4,]	0	0	5	5	2
[5,]	5	0	0	5	0

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	4.000000e+00	2.000000e+00	3.000000e+00	5	1.000000e+00
[2,]	-1.955901e-16	3.000000e+00	5.999975e-16	4	2.000000e+00
[3,]	5.000000e+00	4.000000e+00	3.000000e+00	3	-1.110223e-16
[4,]	-6.366435e-16	3.387048e-15	5.000000e+00	5	2.000000e+00
[5,]	5.000000e+00	1.949829e-15	-6.570265e-16	5	1.408595e-15

Reference

1. Slides in “Matrix Factorization and Collaborative Filtering”
   – By Matt Gormley (Carnegie Mellon Univ.)

2. Slides in “Recommender Systems”
   – By Jee-Hyong Lee (Sungkyunkwan Univ.)

3. Y. Koren *et al*., “Matrix Factorization Techniques for Recommender Systems,” Journal Computer, 42(8), 2009

4. P. Symeonidis *et.al*, “Tag Recommendations based on Tensor Dimensionality Reduction,” Recsys’08