



# Chapter 13: Query Optimization

**Database System Concepts, 6<sup>th</sup> Ed.**

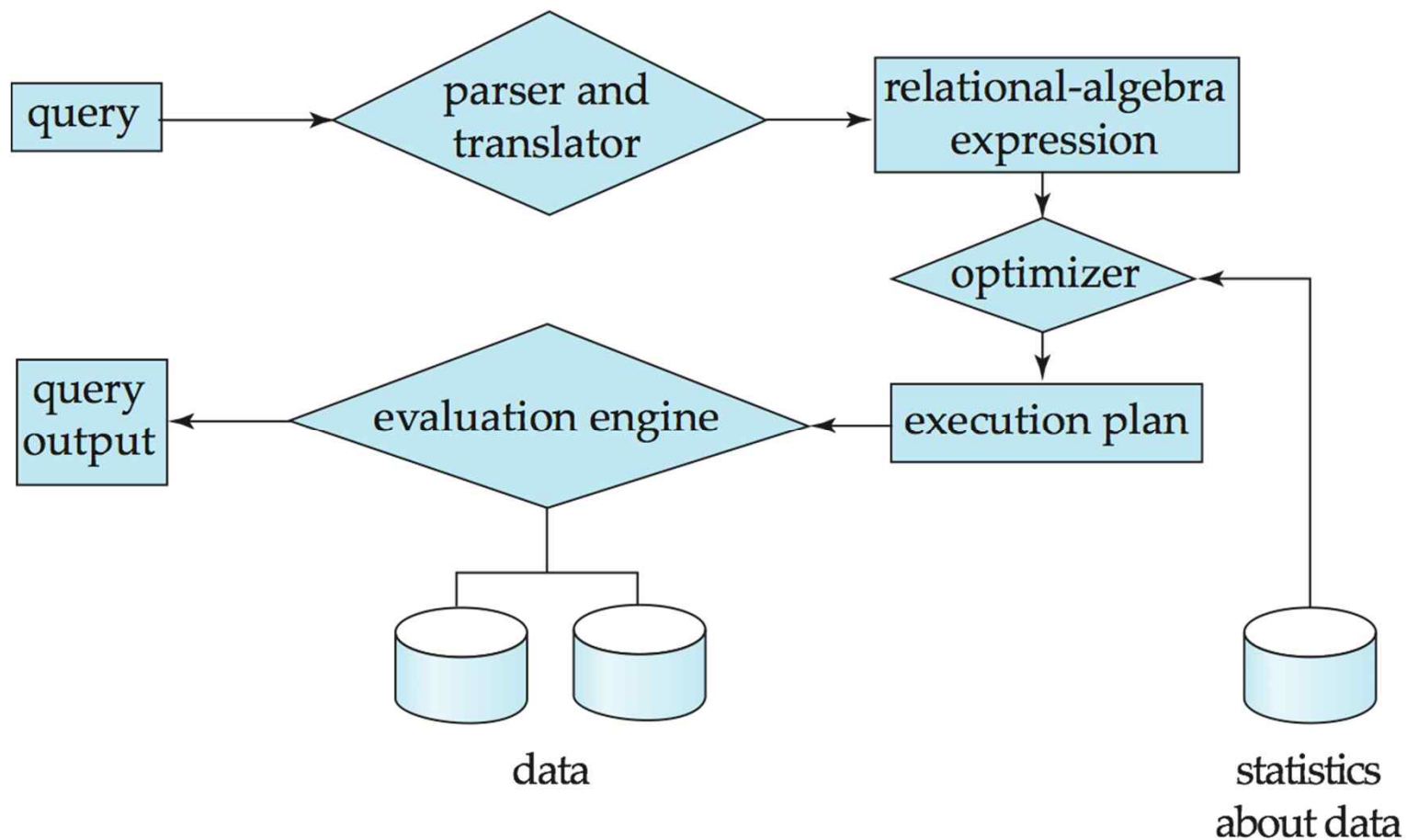
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# Basic Steps in Query Processing

1. Parsing and translation
2. Optimization
3. Evaluation





# Query Optimization

- **Alternative ways** of evaluating a given query
  - Equivalent expressions
    - ▶ E.g.,  $\sigma_{salary < 75000}(\Pi_{salary}(instructor))$  is equivalent to  $\Pi_{salary}(\sigma_{salary < 75000}(instructor))$
  - Different algorithms for each operation
    - ▶ E.g., to find instructors with salary < 75000,
      - can use an index on *salary*,
      - or can perform complete relation scan and discard instructors with salary  $\geq 75000$
- **Query optimization**
  - The process of selecting the most efficient strategies (query evaluation plan) for processing a given query



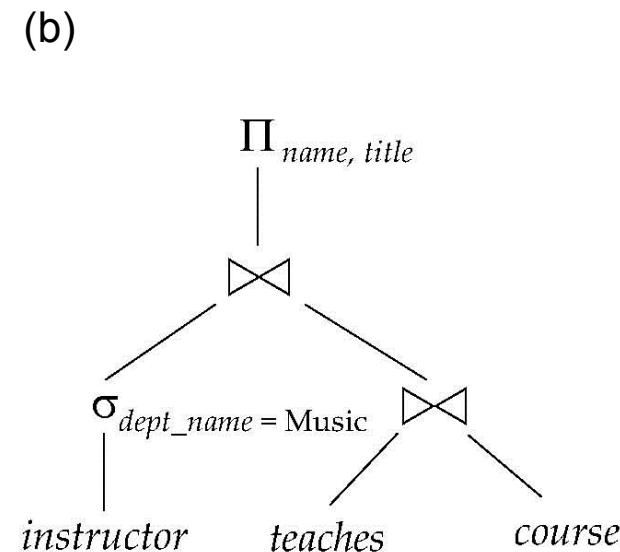
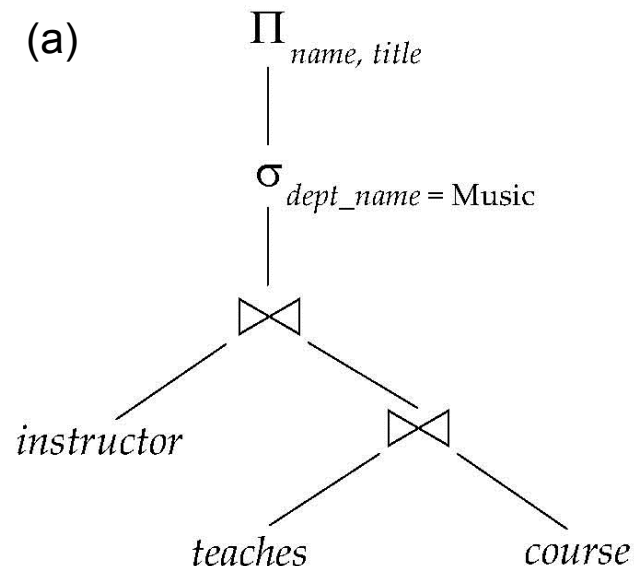
# Equivalent Expression

- Two relational-algebra expressions are **equivalent** if, on every legal database instance, the two expressions generate the same (multi)set of tuples
  - Discussion in this chapter is based on the **set** version of the relation algebra
    - ▶ In SQL, the inputs and outputs are *multisets* of tuples, and the *multiset* version of the relational algebra is used for evaluating SQL queries

## ■ Example

(a)  $\Pi_{name, title} ( \sigma_{dept\_name = \text{Music}} (instructor \bowtie (teaches \bowtie course)) )$

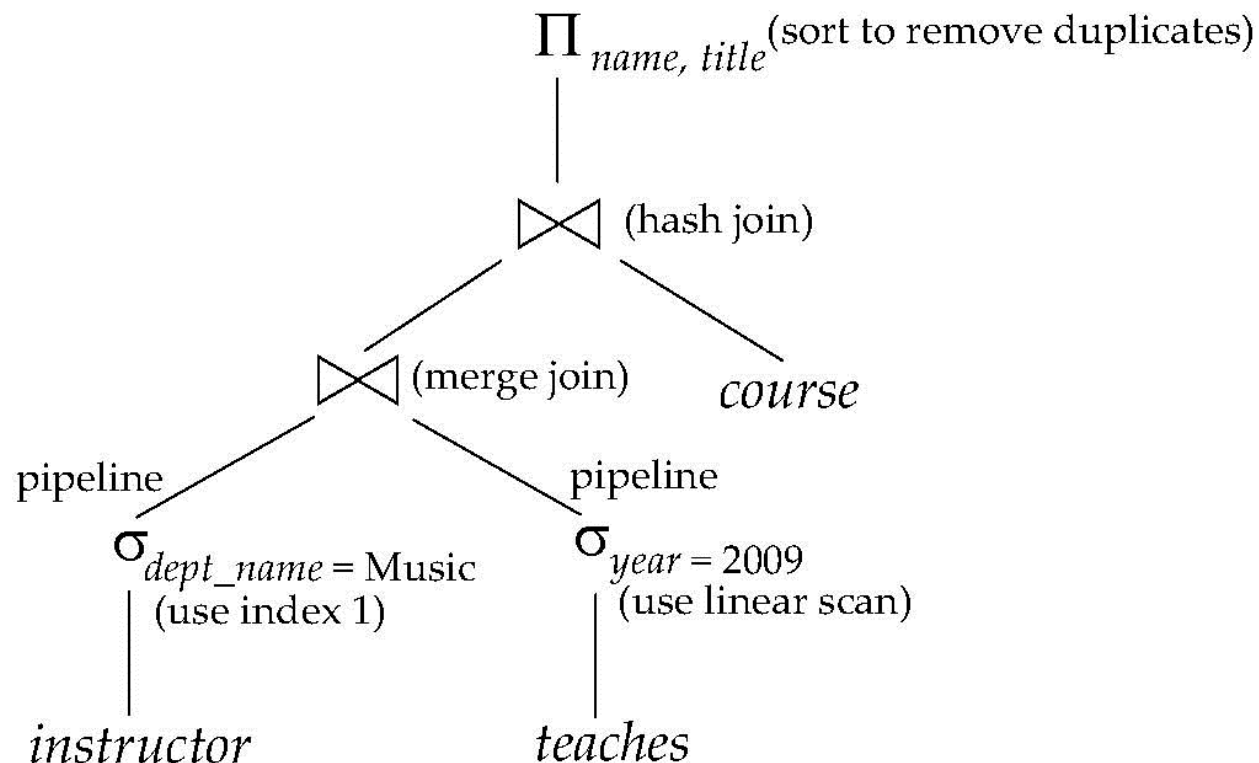
(b)  $\Pi_{name, title} ( (\sigma_{dept\_name = \text{Music}} (instructor)) \bowtie (teaches \bowtie course) )$





# Query Evaluation Plan

- An **evaluation plan** defines exactly what algorithm is used for each operation, and how the execution of the operations is coordinated





# Cost-Based Query Optimization

## ■ Cost-based query optimization

- Amongst all equivalent evaluation plans choose the one with lowest cost

## ■ Generating query evaluation plan in cost-based query optimization

1. Generate logically equivalent expressions using **equivalence rules**
2. Annotate resultant expressions to get alternative query plans
3. Choose the cheapest plan based on **estimated cost**

## ■ Estimation of plan cost based on:

- **Statistical information about relations.**
  - ▶ Examples: number of tuples, number of distinct values for an attribute
- **Statistics estimation for intermediate results**
  - ▶ to compute cost of complex expressions
- Cost formulae for algorithms, computed using statistics



# Equivalence Rules #1~4

1. **Conjunctive selection operations** can be deconstructed into a sequence of individual selections.

$$\sigma_{\theta_1 \wedge \theta_2}(E) = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

2. **Selection operations** are commutative.

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

3. Only the last in a sequence of **projection operations** is needed, the others can be omitted.

$$\Pi_{L_1}(\Pi_{L_2}(\dots(\Pi_{L_n}(E))\dots)) = \Pi_{L_1}(E)$$

- $L_i$  = lists of attributes

4. Selections can be combined with **Cartesian products and theta joins**.

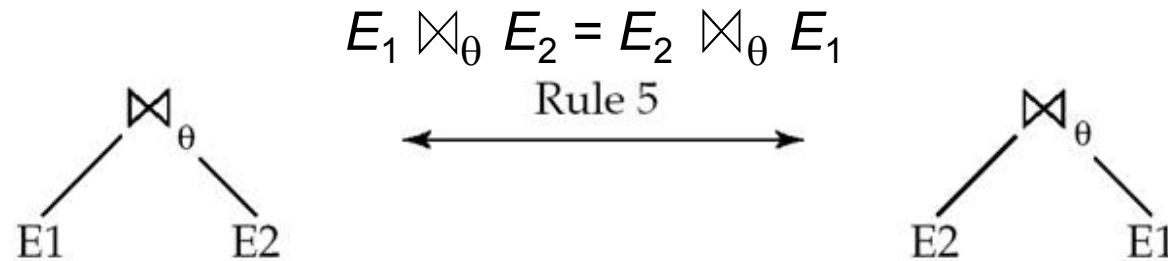
a.  $\sigma_{\theta}(E_1 \times E_2) = E_1 \bowtie_{\theta} E_2$

b.  $\sigma_{\theta_1}(E_1 \bowtie_{\theta_2} E_2) = E_1 \bowtie_{\theta_1 \wedge \theta_2} E_2$

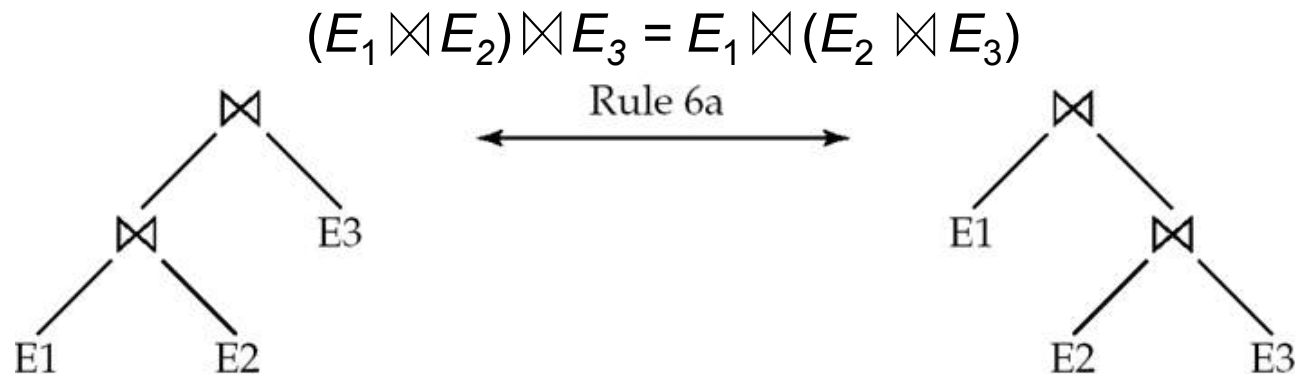


## Equivalence Rules #5~6

5. Theta-join operations (and natural joins) are commutative.



6. (a) Natural join operations are associative:



- (b) Theta joins are associative in the following manner:

$$(E_1 \bowtie_{\theta_1} E_2) \bowtie_{\theta_2 \wedge \theta_3} E_3 = E_1 \bowtie_{\theta_1 \wedge \theta_3} (E_2 \bowtie_{\theta_2} E_3)$$

where  $\theta_2$  involves attributes from only  $E_2$  and  $E_3$ .





# Example Relations for Equivalence Rules

*instructor*

ID	name	dept_name	salary
10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

*course*

course_id	title	dept_name	credits
BIO-101	Intro. to Biology	Biology	4
BIO-301	Genetics	Biology	4
BIO-399	Computational Biology	Biology	3
CS-101	Intro. to Computer Science	Comp. Sci.	4
CS-190	Game Design	Comp. Sci.	4
CS-315	Robotics	Comp. Sci.	3
CS-319	Image Processing	Comp. Sci.	3
CS-347	Database System Concepts	Comp. Sci.	3
EE-181	Intro. to Digital Systems	Elec. Eng.	3
FIN-201	Investment Banking	Finance	3
HIS-351	World History	History	3
MU-199	Music Video Production	Music	3
PHY-101	Physical Principles	Physics	4

*teaches*

ID	course_id	sec_id	semester	year
10101	CS-101	1	Fall	2009
10101	CS-315	1	Spring	2010
10101	CS-347	1	Fall	2009
12121	FIN-201	1	Spring	2010
15151	MU-199	1	Spring	2010
22222	PHY-101	1	Fall	2009
32343	HIS-351	1	Spring	2010
45565	CS-101	1	Spring	2010
45565	CS-319	1	Spring	2010
76766	BIO-101	1	Summer	2009
76766	BIO-301	1	Summer	2010
83821	CS-190	1	Spring	2009
83821	CS-190	2	Spring	2009
83821	CS-319	2	Spring	2010
98345	EE-181	1	Spring	2009



# Example for Equivalence Rule #6

■ Example:  $(instructor \bowtie teaches) \bowtie course$

$(instructor \bowtie teaches)$

ID	name	dept_name	salary	course_id	sec_id	semester	year
10101	Srinivasan	Comp. Sci.	65000	CS-101	1	Fall	2009
10101	Srinivasan	Comp. Sci.	65000	CS-315	1	Spring	2010
10101	Srinivasan	Comp. Sci.	65000	CS-347	1	Fall	2009
12121	Wu	Finance	90000	FIN-201	1	Spring	2010
15151	Mozart	Music	40000	MU-199	1	Spring	2010
22222	Einstein	Physics	95000	PHY-101	1	Fall	2009
32343	El Said	History	60000	HIS-351	1	Spring	2010
45565	Katz	Comp. Sci.	75000	CS-101	1	Spring	2010
45565	Katz	Comp. Sci.	75000	CS-319	1	Spring	2010
76766	Crick	Biology	72000	BIO-101	1	Summer	2009
76766	Crick	Biology	72000	BIO-301	1	Summer	2010
83821	Brandt	Comp. Sci.	92000	CS-190	1	Spring	2009
83821	Brandt	Comp. Sci.	92000	CS-190	2	Spring	2009
83821	Brandt	Comp. Sci.	92000	CS-319	2	Spring	2010
98345	Kim	Elec. Eng.	80000	EE-181	1	Spring	2009

$(instructor \bowtie teaches)$

$\bowtie course$

ID	name	dept_name	salary	course_id	sec_id	semester	year	title	credits
10101	Srinivasan	Comp. Sci.	65000	CS-101	1	Fall	2009	Intro. to Computer Science	4
10101	Srinivasan	Comp. Sci.	65000	CS-315	1	Spring	2010	Robotics	3
10101	Srinivasan	Comp. Sci.	65000	CS-347	1	Fall	2009	Database System Concepts	3
12121	Wu	Finance	90000	FIN-201	1	Spring	2010	Investment Banking	3
15151	Mozart	Music	40000	MU-199	1	Spring	2010	Music Video Production	3
22222	Einstein	Physics	95000	PHY-101	1	Fall	2009	Physical Principles	4
32343	El Said	History	60000	HIS-351	1	Spring	2010	World History	3
45565	Katz	Comp. Sci.	75000	CS-101	1	Spring	2010	Intro. to Computer Science	4
45565	Katz	Comp. Sci.	75000	CS-319	1	Spring	2010	Image Processing	3
76766	Crick	Biology	72000	BIO-101	1	Summer	2009	Intro. to Biology	4
76766	Crick	Biology	72000	BIO-301	1	Summer	2010	Genetics	4
83821	Brandt	Comp. Sci.	92000	CS-190	1	Spring	2009	Game Design	4
83821	Brandt	Comp. Sci.	92000	CS-190	2	Spring	2009	Game Design	4
83821	Brandt	Comp. Sci.	92000	CS-319	2	Spring	2010	Image Processing	3
98345	Kim	Elec. Eng.	80000	EE-181	1	Spring	2009	Intro. to Digital Systems	3



# Example for Equivalence Rule #6 (Cont.)

■ Example: *instructor* ⋈ (*teaches* ⋈ *course*)

(*teaches* ⋈ *course*)

ID	course_id	sec_id	semester	year	title	dept_name	credits
10101	CS-101	1	Fall	2009	Intro. to Computer Science	Comp. Sci.	4
10101	CS-315	1	Spring	2010	Robotics	Comp. Sci.	3
10101	CS-347	1	Fall	2009	Database System Concepts	Comp. Sci.	3
12121	FIN-201	1	Spring	2010	Investment Banking	Finance	3
15151	MU-199	1	Spring	2010	Music Video Production	Music	3
22222	PHY-101	1	Fall	2009	Physical Principles	Physics	4
32343	HIS-351	1	Spring	2010	World History	History	3
45565	CS-101	1	Spring	2010	Intro. to Computer Science	Comp. Sci.	4
45565	CS-319	1	Spring	2010	Image Processing	Comp. Sci.	3
76766	BIO-101	1	Summer	2009	Intro. to Biology	Biology	4
76766	BIO-301	1	Summer	2010	Genetics	Biology	4
83821	CS-190	1	Spring	2009	Game Design	Comp. Sci.	4
83821	CS-190	2	Spring	2009	Game Design	Comp. Sci.	4
83821	CS-319	2	Spring	2010	Image Processing	Comp. Sci.	3
98345	EE-181	1	Spring	2009	Intro. to Digital Systems	Elec. Eng.	3

*instructor* ⋈

(*teaches* ⋈ *course*)

ID	name	dept_name	salary	course_id	sec_id	semester	year	title	credits
10101	Srinivasan	Comp. Sci.	65000	CS-101	1	Fall	2009	Intro. to Computer Science	4
10101	Srinivasan	Comp. Sci.	65000	CS-315	1	Spring	2010	Robotics	3
10101	Srinivasan	Comp. Sci.	65000	CS-347	1	Fall	2009	Database System Concepts	3
12121	Wu	Finance	90000	FIN-201	1	Spring	2010	Investment Banking	3
15151	Mozart	Music	40000	MU-199	1	Spring	2010	Music Video Production	3
22222	Einstein	Physics	95000	PHY-101	1	Fall	2009	Physical Principles	4
32343	El Said	History	60000	HIS-351	1	Spring	2010	World History	3
45565	Katz	Comp. Sci.	75000	CS-101	1	Spring	2010	Intro. to Computer Science	4
45565	Katz	Comp. Sci.	75000	CS-319	1	Spring	2010	Image Processing	3
76766	Crick	Biology	72000	BIO-101	1	Summer	2009	Intro. to Biology	4
76766	Crick	Biology	72000	BIO-301	1	Summer	2010	Genetics	4
83821	Brandt	Comp. Sci.	92000	CS-190	1	Spring	2009	Game Design	4
83821	Brandt	Comp. Sci.	92000	CS-190	2	Spring	2009	Game Design	4
83821	Brandt	Comp. Sci.	92000	CS-319	2	Spring	2010	Image Processing	3
98345	Kim	Elec. Eng.	80000	EE-181	1	Spring	2009	Intro. to Digital Systems	3



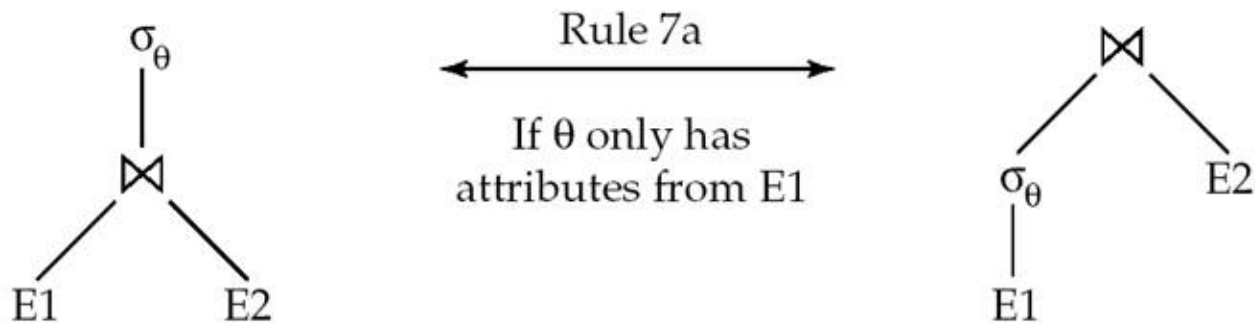


## Equivalence Rules #7

7. The selection operation distributes over the theta join operation under the following two conditions:

(a) When all the attributes in  $\theta_0$  involve only the attributes of one of the expressions ( $E_1$ ) being joined.

$$\sigma_{\theta_0}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta_0}(E_1)) \bowtie_{\theta} E_2$$



(b) When  $\theta_1$  involves only the attributes of  $E_1$  and  $\theta_2$  involves only the attributes of  $E_2$ .

$$\sigma_{\theta_1 \wedge \theta_2}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta_1}(E_1)) \bowtie_{\theta} (\sigma_{\theta_2}(E_2))$$



# Example for Equivalence Rule #7

■ Example:  $\sigma_{dept\_name = \text{"Music"}}(instructor \bowtie (teaches \bowtie course))$

$(teaches \bowtie course)$

ID	course_id	sec_id	semester	year	title	dept_name	credits
10101	CS-101	1	Fall	2009	Intro. to Computer Science	Comp. Sci.	4
10101	CS-315	1	Spring	2010	Robotics	Comp. Sci.	3
10101	CS-347	1	Fall	2009	Database System Concepts	Comp. Sci.	3
12121	FIN-201	1	Spring	2010	Investment Banking	Finance	3
15151	MU-199	1	Spring	2010	Music Video Production	Music	3
22222	PHY-101	1	Fall	2009	Physical Principles	Physics	4
32343	HIS-351	1	Spring	2010	World History	History	3
45565	CS-101	1	Spring	2010	Intro. to Computer Science	Comp. Sci.	4
45565	CS-319	1	Spring	2010	Image Processing	Comp. Sci.	3
76766	BIO-101	1	Summer	2009	Intro. to Biology	Biology	4
76766	BIO-301	1	Summer	2010	Genetics	Biology	4
83821	CS-190	1	Spring	2009	Game Design	Comp. Sci.	4
83821	CS-190	2	Spring	2009	Game Design	Comp. Sci.	4
83821	CS-319	2	Spring	2010	Image Processing	Comp. Sci.	3
98345	EE-181	1	Spring	2009	Intro. to Digital Systems	Elec. Eng.	3

$instructor \bowtie$   
 $(teaches \bowtie course)$

ID	name	dept_name	salary	course_id	sec_id	semester	year	title	credits
10101	Srinivasan	Comp. Sci.	65000	CS-101	1	Fall	2009	Intro. to Computer Science	4
10101	Srinivasan	Comp. Sci.	65000	CS-315	1	Spring	2010	Robotics	3
10101	Srinivasan	Comp. Sci.	65000	CS-347	1	Fall	2009	Database System Concepts	3
12121	Wu	Finance	90000	FIN-201	1	Spring	2010	Investment Banking	3
15151	Mozart	Music	40000	MU-199	1	Spring	2010	Music Video Production	3
22222	Einstein	Physics	95000	PHY-101	1	Fall	2009	Physical Principles	4
32343	El Said	History	60000	HIS-351	1	Spring	2010	World History	3
45565	Katz	Comp. Sci.	75000	CS-101	1	Spring	2010	Intro. to Computer Science	4
45565	Katz	Comp. Sci.	75000	CS-319	1	Spring	2010	Image Processing	3
76766	Crick	Biology	72000	BIO-101	1	Summer	2009	Intro. to Biology	4
76766	Crick	Biology	72000	BIO-301	1	Summer	2010	Genetics	4
83821	Brandt	Comp. Sci.	92000	CS-190	1	Spring	2009	Game Design	4
83821	Brandt	Comp. Sci.	92000	CS-190	2	Spring	2009	Game Design	4
83821	Brandt	Comp. Sci.	92000	CS-319	2	Spring	2010	Image Processing	3
98345	Kim	Elec. Eng.	80000	EE-181	1	Spring	2009	Intro. to Digital Systems	3

$\sigma_{dept\_name = \text{"Music"}}(instructor \bowtie (teaches \bowtie course))$

ID	name	dept_name	salary	course_id	sec_id	semester	year	title	credits
15151	Mozart	Music	40000	MU-199	1	Spring	2010	Music Video Production	3



# Example for Equivalence Rule #7 (Cont.)

- Example:  $(\sigma_{dept\_name = \text{"Music"}}(instructor)) \bowtie (teaches \bowtie course)$

$\sigma_{dept\_name = \text{"Music"}}(instructor)$

ID	name	dept_name	salary
15151	Mozart	Music	40000

$(teaches \bowtie course)$

ID	course_id	sec_id	semester	year	title	dept_name	credits
10101	CS-101	1	Fall	2009	Intro. to Computer Science	Comp. Sci.	4
10101	CS-315	1	Spring	2010	Robotics	Comp. Sci.	3
10101	CS-347	1	Fall	2009	Database System Concepts	Comp. Sci.	3
12121	FIN-201	1	Spring	2010	Investment Banking	Finance	3
15151	MU-199	1	Spring	2010	Music Video Production	Music	3
22222	PHY-101	1	Fall	2009	Physical Principles	Physics	4
32343	HIS-351	1	Spring	2010	World History	History	3
45565	CS-101	1	Spring	2010	Intro. to Computer Science	Comp. Sci.	4
45565	CS-319	1	Spring	2010	Image Processing	Comp. Sci.	3
76766	BIO-101	1	Summer	2009	Intro. to Biology	Biology	4
76766	BIO-301	1	Summer	2010	Genetics	Biology	4
83821	CS-190	1	Spring	2009	Game Design	Comp. Sci.	4
83821	CS-190	2	Spring	2009	Game Design	Comp. Sci.	4
83821	CS-319	2	Spring	2010	Image Processing	Comp. Sci.	3
98345	EE-181	1	Spring	2009	Intro. to Digital Systems	Elec. Eng.	3

$(\sigma_{dept\_name = \text{"Music"}}(instructor)) \bowtie (teaches \bowtie course)$

ID	name	dept_name	salary	course_id	sec_id	semester	year	title	credits
15151	Mozart	Music	40000	MU-199	1	Spring	2010	Music Video Production	3



## Equivalence Rules #8

8. The projection operation distributes over the theta join operation as follows:

(a) if  $\theta$  involves only attributes from  $L_1 \cup L_2$ :

$$\Pi_{L_1 \cup L_2} (E_1 \bowtie_{\theta} E_2) = (\Pi_{L_1} (E_1)) \bowtie_{\theta} (\Pi_{L_2} (E_2))$$

(b) Consider a join  $E_1 \bowtie_{\theta} E_2$ .

- Let  $L_1$  and  $L_2$  be sets of attributes from  $E_1$  and  $E_2$ , respectively.
- Let  $L_3$  be attributes of  $E_1$  that are involved in join condition  $\theta$ , but are not in  $L_1 \cup L_2$ , and
- let  $L_4$  be attributes of  $E_2$  that are involved in join condition  $\theta$ , but are not in  $L_1 \cup L_2$ .

$$\Pi_{L_1 \cup L_2} (E_1 \bowtie_{\theta} E_2) = \Pi_{L_1 \cup L_2} ((\Pi_{L_1 \cup L_3} (E_1)) \bowtie_{\theta} (\Pi_{L_2 \cup L_4} (E_2)))$$



# Example for Equivalence Rule #8

■ Example:  $\Pi_{name, title}((\sigma_{dept\_name = \text{"Music"}}(instructor) \bowtie teaches) \bowtie course)$

$\sigma_{dept\_name = \text{"Music"}}(instructor)$

ID	name	dept_name	salary
15151	Mozart	Music	40000

$\sigma_{dept\_name = \text{"Music"}}(instructor) \bowtie teaches$

ID	name	dept_name	salary	course_id	sec_id	semester	year
15151	Mozart	Music	40000	MU-199	1	Spring	2010

$(\sigma_{dept\_name = \text{"Music"}}(instructor) \bowtie teaches) \bowtie course$

ID	name	dept_name	salary	course_id	sec_id	semester	year	title	credits
15151	Mozart	Music	40000	MU-199	1	Spring	2010	Music Video Production	3

$\Pi_{name, title}((\sigma_{dept\_name = \text{"Music"}}(instructor) \bowtie teaches) \bowtie course)$

name	title
Mozart	Music Video Production





# Example for Equivalence Rule #8 (Cont.)

■ Example:  $\Pi_{name, title}((\Pi_{name, course\_id}(\sigma_{dept\_name = \text{"Music"}}(instructor)) \bowtie teaches) \bowtie \Pi_{course\_id, title}(course))$

$\sigma_{dept\_name = \text{"Music"}}(instructor)$

ID	name	dept_name	salary
15151	Mozart	Music	40000

$(\sigma_{dept\_name = \text{"Music"}}(instructor)) \bowtie teaches$

ID	name	dept_name	salary	course_id	sec_id	semester	year
15151	Mozart	Music	40000	MU-199	1	Spring	2010

$\Pi_{name, course\_id}(\sigma_{dept\_name = \text{"Music"}}(instructor)) \bowtie teaches$

name	course_id
Mozart	MU-199

$\Pi_{course\_id, title}(course)$

course_id	title
BIO-101	Intro. to Biology
BIO-301	Genetics
BIO-399	Computational Biology
CS-101	Intro. to Computer Science
CS-190	Game Design
CS-315	Robotics
CS-319	Image Processing
CS-347	Database System Concepts
EE-181	Intro. to Digital Systems
FIN-201	Investment Banking
HIS-351	World History
MU-199	Music Video Production
PHY-101	Physical Principles

$\Pi_{name, title}((\Pi_{name, course\_id}(\sigma_{dept\_name = \text{"Music"}}(instructor)) \bowtie teaches) \bowtie \Pi_{course\_id, title}(course))$

name	title
Mozart	Music Video Production



# Equivalence Rules for Set Operations

9. The set operations **union** and **intersection** are commutative

$$E_1 \cup E_2 = E_2 \cup E_1$$

$$E_1 \cap E_2 = E_2 \cap E_1$$

■ (set difference is not commutative).

10. Set **union** and **intersection** are associative.

$$(E_1 \cup E_2) \cup E_3 = E_1 \cup (E_2 \cup E_3)$$

$$(E_1 \cap E_2) \cap E_3 = E_1 \cap (E_2 \cap E_3)$$

11. The **selection** operation distributes over  $\cup$ ,  $\cap$  and  $-$ .

$$\sigma_{\theta} (E_1 - E_2) = \sigma_{\theta} (E_1) - \sigma_{\theta}(E_2)$$

and similarly for  $\cup$  and  $\cap$  in place of  $-$

Also: 
$$\sigma_{\theta} (E_1 - E_2) = \sigma_{\theta}(E_1) - E_2$$

and similarly for  $\cap$  in place of  $-$ , but not for  $\cup$

12. The **projection** operation distributes over **union**

$$\Pi_L(E_1 \cup E_2) = (\Pi_L(E_1)) \cup (\Pi_L(E_2))$$



## Transformation Example: Pushing Selections

- Performing the selection as early as possible reduces the size of the relation to be joined
- Example query: Find the names of all instructors in the Music department, along with the titles of the courses that they teach

$\Pi_{name, title}(\sigma_{dept\_name = \text{"Music"}}(instructor \bowtie (teaches \bowtie \Pi_{course\_id, title}(course))))$

- Transformation using rule 7a

$\Pi_{name, title}((\sigma_{dept\_name = \text{"Music"}}(instructor)) \bowtie (teaches \bowtie \Pi_{course\_id, title}(course)))$



# Transformation Example: Pushing Projections

- Performing the projection as early as possible reduces the size of the relation to be joined

- Example query:

$\Pi_{name, title}((\sigma_{dept\_name = \text{"Music"}}(instructor) \bowtie teaches) \bowtie \Pi_{course\_id, title}(course))$

- When we compute  $(\sigma_{dept\_name = \text{"Music"}}(instructor) \bowtie teaches)$ , we obtain a relation whose schema is:  
 $(ID, name, dept\_name, salary, course\_id, sec\_id, semester, year)$

- Push projections using equivalence rules 8a and 8b;  
eliminate unneeded attributes from intermediate results to get:

$\Pi_{name, title}((\Pi_{name, course\_id}(\sigma_{dept\_name = \text{"Music"}}(instructor)) \bowtie teaches) \bowtie \Pi_{course\_id, title}(course))$



## Example with Multiple Transformations

- Query: Find the names of all instructors in the Music department who have taught a course in 2009, along with the titles of the courses that they taught

$$\Pi_{name, title}(\sigma_{dept\_name = \text{"Music"} \wedge year = 2009}(instructor \bowtie (teaches \bowtie \Pi_{course\_id, title}(course))))$$

- Transformation using **join associatively** (Rule 6a):

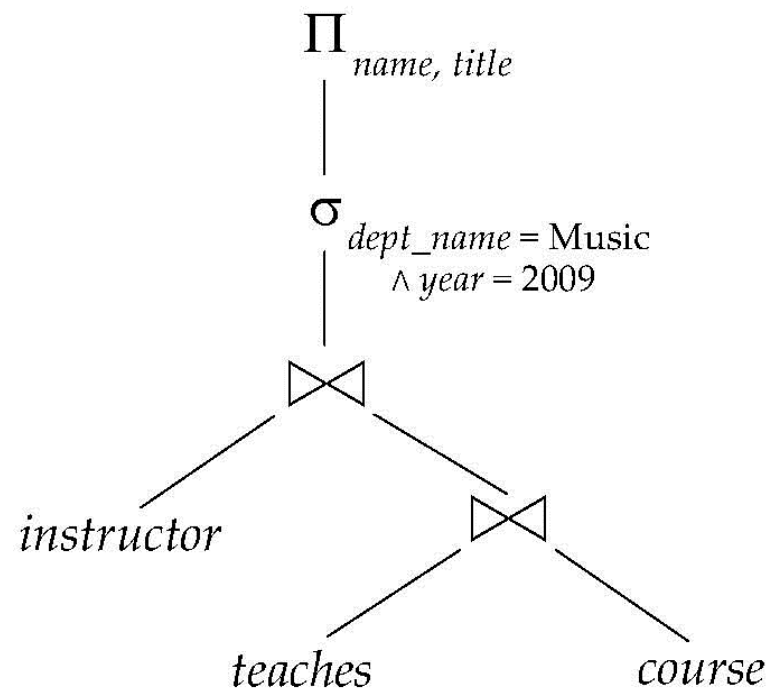
$$\Pi_{name, title}(\sigma_{dept\_name = \text{"Music"} \wedge year = 2009}((instructor \bowtie teaches) \bowtie \Pi_{course\_id, title}(course)))$$

- Second form provides an opportunity to apply the “**perform selections early**” rule

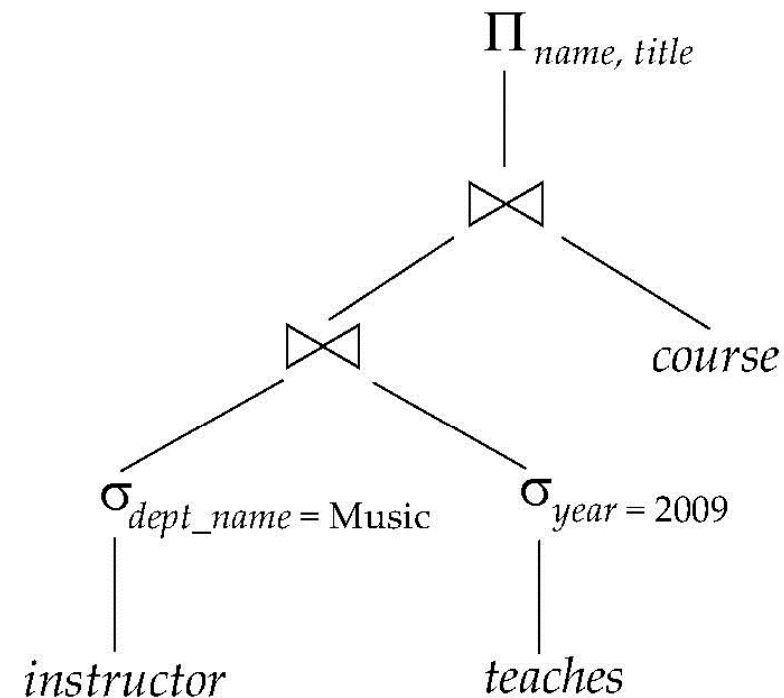
$$\Pi_{name, title}((\sigma_{dept\_name = \text{"Music"}}(instructor) \bowtie \sigma_{year = 2009}(teaches)) \bowtie \Pi_{course\_id, title}(course))$$



## Multiple Transformations (Cont.)



(a) Initial expression tree



(b) Tree after multiple transformations



# Join Ordering

- For all relations  $r_1, r_2$ , and  $r_3$ ,  $(r_1 \bowtie r_2) \bowtie r_3 = r_1 \bowtie (r_2 \bowtie r_3)$  (Rule 6a)
- Choose the expression that will yield **smaller temporary result**
  - If  $r_2 \bowtie r_3$  is quite large and  $r_1 \bowtie r_2$  is small, we choose
$$(r_1 \bowtie r_2) \bowtie r_3$$
so that we compute and store a smaller temporary relation

- Example

$\Pi_{name, title}((\sigma_{dept\_name = \text{“Music”}}(instructor) \bowtie teaches) \bowtie \Pi_{course\_id, title}(course))$

- Which join expression is it better to compute first?
  1. Compute  $teaches \bowtie \Pi_{course\_id, title}(course)$  first,  
and join result with  $\sigma_{dept\_name = \text{“Music”}}(instructor)$ 
    - ▶ The result of the first join is likely to be a large relation
  2. Compute  $\sigma_{dept\_name = \text{“Music”}}(instructor) \bowtie teaches$  first
    - ▶ Only a small fraction of the university's instructors are likely to be from the Music department – This would be better



# Enumeration of Equivalent Expressions

- Query optimizers use **equivalence rules** to systematically generate expressions equivalent to the given expression
- Can generate **all equivalent expressions** as follows:
  - Repeat
    - ▶ apply **all applicable equivalence rules** on every subexpression of every equivalent expression found so far
    - ▶ add newly generated expressions to the set of equivalent expressionsUntil no new equivalent expressions are generated above
- The above approach is very **expensive in space and time**
  - Two approaches
    - ▶ Optimized plan generation based on transformation rules – avoid examining some of the expressions by considering the estimated cost
    - ▶ Heuristic-based transformation: special case approach for queries with only selections, projections and joins





# Cost Estimation

- Cost of each operator computer as described in Chapter 12
  - Need statistics of input relations
    - ▶ E.g. number of tuples, sizes of tuples
- Inputs can be results of sub-expressions
  - Need to estimate statistics of expression results
  - To do so, we require additional statistics
    - ▶ E.g. number of distinct values for an attribute



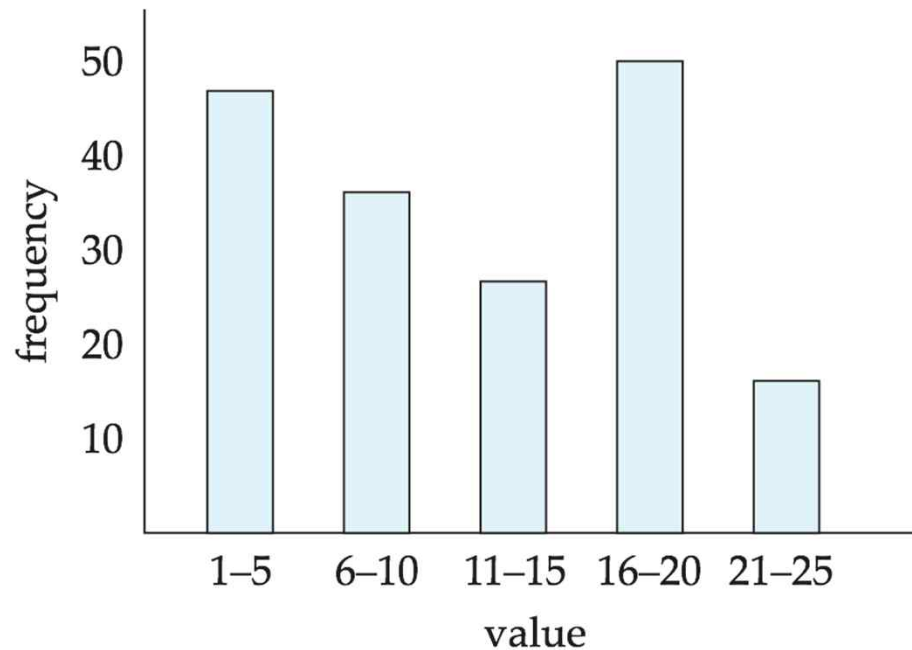
# Statistical Information for Cost Estimation

- $n_r$ : number of tuples in a relation  $r$
- $b_r$ : number of blocks containing tuples of  $r$ 
  - $b_r = \lceil n_r / f_r \rceil$ , if tuples of  $r$  are stored together physically in a file
- $l_r$ : size of a tuple of  $r$
- $f_r$ : blocking factor of  $r$  — i.e., the number of tuples of  $r$  that fit into one block
  
- $V(A, r)$ : number of distinct values that appear in  $r$  for attribute  $A$  (= size of  $\Pi_A(r)$ )
- $SC(A, r)$ : selection cardinality of attribute  $A$  of relation  $r$ 
  - Average number of records that satisfy equality on  $A$
  
- $f_i$ : average fan-out of internal nodes of index  $i$ , for B<sup>+</sup>-trees
- $HT_i$ : number of levels in index  $i$  ( i.e., the height of  $i$  & on attribute  $A$  of relation  $r$ )
  - For a B<sup>+</sup>-tree index,  $HT_i = \lceil \log_{f_i}(V(A, r)) \rceil$
  - For a hash index,  $HT_i = 1$
- $LB_i$ : number of lowest-level index blocks in  $i$  (i.e, the # of blocks at the leaf level)



# Histograms

- Histogram on attribute *age* of relation *person*



- **Equi-width** histograms – the size of each range is equal
- **Equi-depth** histograms – each range has the same number of values



# Selection Size Estimation

## ■ Equality selection $\sigma_{A=v}(r)$

- $SC(A, r)$ : number of records that will satisfy the selection  
= 1, if A is a key attribute  
=  $n_r / V(A, r)$ , otherwise

## ■ $\sigma_{A \leq v}(r)$ (case of $\sigma_{A \geq v}(r)$ is symmetric)

- Let c denote the estimated number of tuples satisfying the condition
- If  $\min(A, r)$  and  $\max(A, r)$  are available in catalog
  - ▶  $c = 0$  if  $v < \min(A, r)$
  - ▶  $c = n_r \cdot \frac{v - \min(A, r)}{\max(A, r) - \min(A, r)}$
- If histograms available, can refine above estimate
- In absence of statistical information c is assumed to be  $n_r / 2$



# Size Estimation of Complex Selections

- **Selectivity** of a condition  $\theta_i$ : the probability that a tuple in the relation  $r$  satisfies  $\theta_i$ 
  - If  $s_i$  is the number of satisfying tuples in  $r$ , the selectivity of  $\theta_i$  is given by  $s_i / n_r$

- **Conjunction**:  $\sigma_{\theta_1 \wedge \theta_2 \wedge \dots \wedge \theta_n}(r)$ .

Assuming independence, estimate of tuples in the result is:

$$n_r * \frac{s_1 * s_2 * \dots * s_n}{n_r^n}$$

- **Disjunction**:  $\sigma_{\theta_1 \vee \theta_2 \vee \dots \vee \theta_n}(r)$ .

Estimated number of tuples:

$$n_r * \left( 1 - \left( 1 - \frac{s_1}{n_r} \right) * \left( 1 - \frac{s_2}{n_r} \right) * \dots * \left( 1 - \frac{s_n}{n_r} \right) \right)$$

- **Negation**:  $\sigma_{\neg \theta}(r)$ .

Estimated number of tuples:  $n_r - \text{size}(\sigma_{\theta}(r))$



# Join Operation: Running Example

Running example: *student* ⋈ *takes*

Catalog information for join examples:

- $n_{student} = 5,000$
- $f_{student} = 50$ , which implies that  $b_{student} = 5000/50 = 100$
- $n_{takes} = 10,000$
- $f_{takes} = 25$ , which implies that  $b_{takes} = 10000/25 = 400$
- $V(ID, takes) = 2500$ , which implies that on average, each student who has taken a course has taken 4 courses.
  - Attribute *ID* in *takes* is a foreign key referencing *student*.
- $V(ID, student) = 5000$  (primary key!)



# Join Size Estimation

- If  $R \cap S = \emptyset$ ,  $r \bowtie s = r \times s$ 
  - $r \times s$  contains  $n_r \cdot n_s$  tuples
  - Each tuple occupies  $s_r + s_s$  bytes
  
- If  $R \cap S$  is a key for  $R$ ,
  - A tuple of  $s$  will join with at most one tuple from  $r$
  - The number of tuples in  $r \bowtie s$  is no greater than the number of tuples in  $s$
  - If  $R \cap S$  in  $S$  is a foreign key in  $S$  referencing  $R$ , then the number of tuples in  $r \bowtie s$  is exactly the same as the number of tuples in  $s$ .
  
- In the example query  $student \bowtie takes$ ,
  - $ID$  in  $takes$  is a foreign key referencing  $student$
  - hence, the result has exactly  $n_{takes}$  tuples, which is 10,000



## Estimation of the Size of Joins (Cont.)

- If  $R \cap S = \{A\}$  is not a key for  $R$  or  $S$ ,

- If we assume that every tuple  $t$  in  $r$  produces tuples in  $r \bowtie s$ , the number of tuples in  $r \bowtie s$  is estimated to be:

$$\frac{n_r * n_s}{V(A, s)}$$

- If the reverse is true, the estimate obtained will be:

$$\frac{n_r * n_s}{V(A, r)}$$

- The lower of these two estimates is probably the more accurate one

- Can improve on above if histograms are available

- Use formula similar to above, for each cell of histograms on the two relations

- Example: *students*  $\bowtie$  *takes* without using information about foreign keys

- $V(ID, takes) = 2500$ , and  $V(ID, student) = 5000$

- The two estimates are  $5000 * 10000/2500 = 20,000$   
and  $5000 * 10000/5000 = 10,000$





# Size Estimation for Other Operations

- Projection: estimated size of  $\Pi_A(r) = V(A, r)$ 
  - Projection eliminates duplicates
- Aggregation: estimated size of  $Agg_F(r) = V(A, r)$ 
  - There is one tuple for each distinct value of A
- Set operations
  - For operations on different relations:
    - ▶ estimated size of  $r \cup s$  = size of  $r$  + size of  $s$
    - ▶ estimated size of  $r \cap s$  = minimum size of  $r$  and size of  $s$
    - ▶ estimated size of  $r - s = r$
    - ▶ All the three estimates may be quite inaccurate, but provide upper bounds on the sizes
  - For unions/intersections of selections on the same relation: rewrite and use size estimate for selections
    - ▶ E.g.  $\sigma_{\theta_1}(r) \cup \sigma_{\theta_2}(r)$  can be rewritten as  $\sigma_{\theta_1 \vee \theta_2}(r)$



# Estimation of Number of Distinct Values

Selections:  $\sigma_{\theta}(r)$

- If  $\theta$  forces  $A$  to take a specified value:  $V(A, \sigma_{\theta}(r)) = 1$ 
  - e.g.,  $A = 3$
- If  $\theta$  forces  $A$  to take on one of a specified set of values:  
 $V(A, \sigma_{\theta}(r)) = \text{number of specified values}$ 
  - e.g.,  $(A = 1 \vee A = 3 \vee A = 4)$
- If the selection condition  $\theta$  is of the form  $A \text{ op } r$   
estimated  $V(A, \sigma_{\theta}(r)) = V(A, r) * s$ , where  $s$  is the selectivity of the selection
- In all the other cases: use approximate estimate of  
 $\min(V(A, r), n_{\sigma_{\theta}(r)})$ 
  - More accurate estimate can be got using probability theory, but this one works fine generally



# Size Estimation of Distinct Values (Cont.)

Joins:  $r \bowtie s$

- If all attributes in  $A$  are from  $r$   
estimated  $V(A, r \bowtie s) = \min(V(A, r), n_{r \bowtie s})$
- If  $A$  contains attributes  $A1$  from  $r$  and  $A2$  from  $s$ ,  
estimated  $V(A, r \bowtie s) = \min(V(A1, r) * V(A2 - A1, s), V(A1 - A2, r) * V(A2, s), n_{r \bowtie s})$ 
  - More accurate estimate can be got using probability theory, but this one works fine generally
- **Projection**: Estimation of distinct values are straightforward for projections
  - They are the same in  $\Pi_A(r)$  as in  $r$
- **Aggregation**: The same holds for grouping attributes of aggregation
  - For aggregated values
    - ▶ For  $\min(A)$  and  $\max(A)$ , the number of distinct values can be estimated as  $\min(V(A, r), V(G, r))$  where  $G$  denotes grouping attributes
    - ▶ For other aggregates, assume all values are distinct, and use  $V(G, r)$



# Choice of Evaluation Plans

- Must consider the interaction of evaluation techniques when choosing evaluation plans
  - Choosing the cheapest algorithm for each operation independently may not yield best overall algorithm, e.g.,
    - ▶ Merge-join may be costlier than hash-join, but may provide a sorted output which reduces the cost for an outer level aggregation
    - ▶ Nested-loop join may provide opportunity for pipelining
- Practical query optimizers incorporate elements of the following two broad approaches:
  1. Search all the plans and choose the best plan in a cost-based fashion
  2. Uses heuristics to choose a plan



# Cost-Based Optimization

- Consider finding the best join-order for  $r_1 \bowtie r_2 \bowtie \dots \bowtie r_n$
- There are  $(2(n-1))!/(n-1)!$  different join orders for above expression (see Practice Exercise 13.10)
  - with  $n = 7$ , the number is 665280
  - with  $n = 10$ , the number is greater than 176 billion!
- Can reduce search space using dynamic programming
  - Using dynamic programming, the least-cost join order for any subset of  $\{r_1, r_2, \dots, r_n\}$  is computed only once and stored for future use
  - Time complexity:  $O(3^n)$ , with bushy trees (see Practice Exercise 13.11)
    - ▶ with  $n = 10$ , the number is 59,000 (instead of 176 billion!)
  - Space complexity:  $O(2^n)$



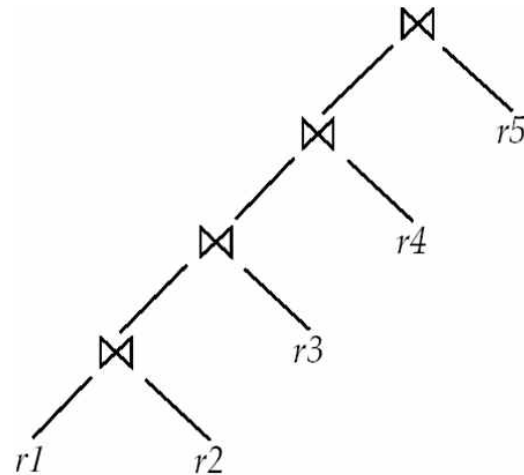
# Heuristic Optimization

- Cost-based optimization is expensive, even with dynamic programming
- Heuristic optimization transforms the query-tree by using a set of rules that typically (but not in all cases) improve execution performance:
  - Perform selection early (reduces the number of tuples)
  - Perform projection early (reduces the number of attributes)
  - Perform most restrictive selection and join operations (i.e. with smallest result size) before other similar operations
- Some systems use only heuristics, others combine heuristics with partial cost-based optimization

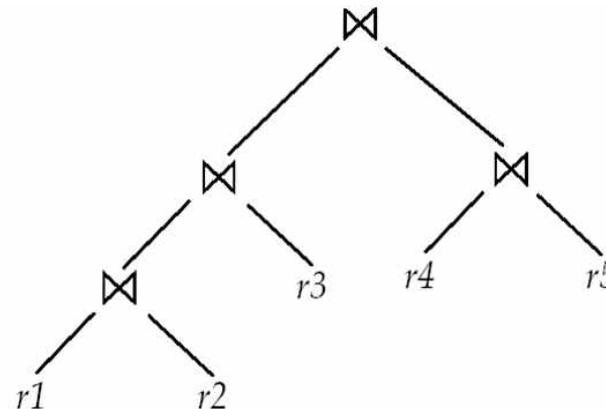


# Left Deep Join Trees

- In **left-deep join trees**, the right-hand-side input for each join is a relation, not the result of an intermediate join
- If **only left-deep trees are considered**, time complexity of finding best join order is  $O(n 2^n)$  (see Practice Exercise 13.12)
  - with  $n = 10$ , the number of join orders is 10,000 (c.f., 59,000 or 176 billion)
  - Space complexity remains at  $O(2^n)$
- Left-deep join orders are convenient for pipelined evaluation: the right operand is a stored relation and only one input to each join is pipelined
- Many **optimizers** considers **only left-deep join orders**



(a) Left-deep join tree



(b) Non-left-deep join tree



# End of Chapter 13

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