Chapter 11: Indexing and Hashing

Database System Concepts, 6th Ed.

Basic Concepts

- Indexing mechanisms used to speed up access to desired data
  - e.g., author catalog in library, term index at the end of a book

- **Search key** - attribute to set of attributes used to look up records in a file
- An **index file** consists of records (called **index entries**) of the form

search-key	pointer

- Index files are typically much smaller than the original file

- Two basic kinds of indices:
  - **Ordered indices:** search keys are stored in sorted order
  - **Hash indices:** search keys are distributed uniformly across “buckets” using a “hash function”

Index Evaluation Metrics

- Access types supported efficiently
  - **Point query**: records with a specified value in the attribute
  - **Range query**: records with an attribute value falling in a specified range of values
- Access time
- Insertion time
- Deletion time
- Space overhead

Ordered Indices

- **Ordered index**: index entries are stored sorted on the search key value
  - e.g., author catalog in library

- **Primary index**: an index whose search key specifies the sequential order of the file
  - Also called clustering index
  - The search key of a primary index is usually (but not necessarily) the primary key
- **Secondary index**: an index whose search key specifies an order different from the sequential order of the file
  - Also called non-clustering index

- Index-sequential file**:** ordered sequential file with a primary index

Dense Index Files

- **Dense index**
  - Index record appears for every search-key value in the file

10101	
12121	
15151	
22222	
32343	
33456	
45565	
58583	
76543	
76766	
83821	
98345	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
32343	El Said	History	60000	
33456	Gold	Physics	87000	
45565	Katz	Comp. Sci.	75000	
58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

Primary Indices Example

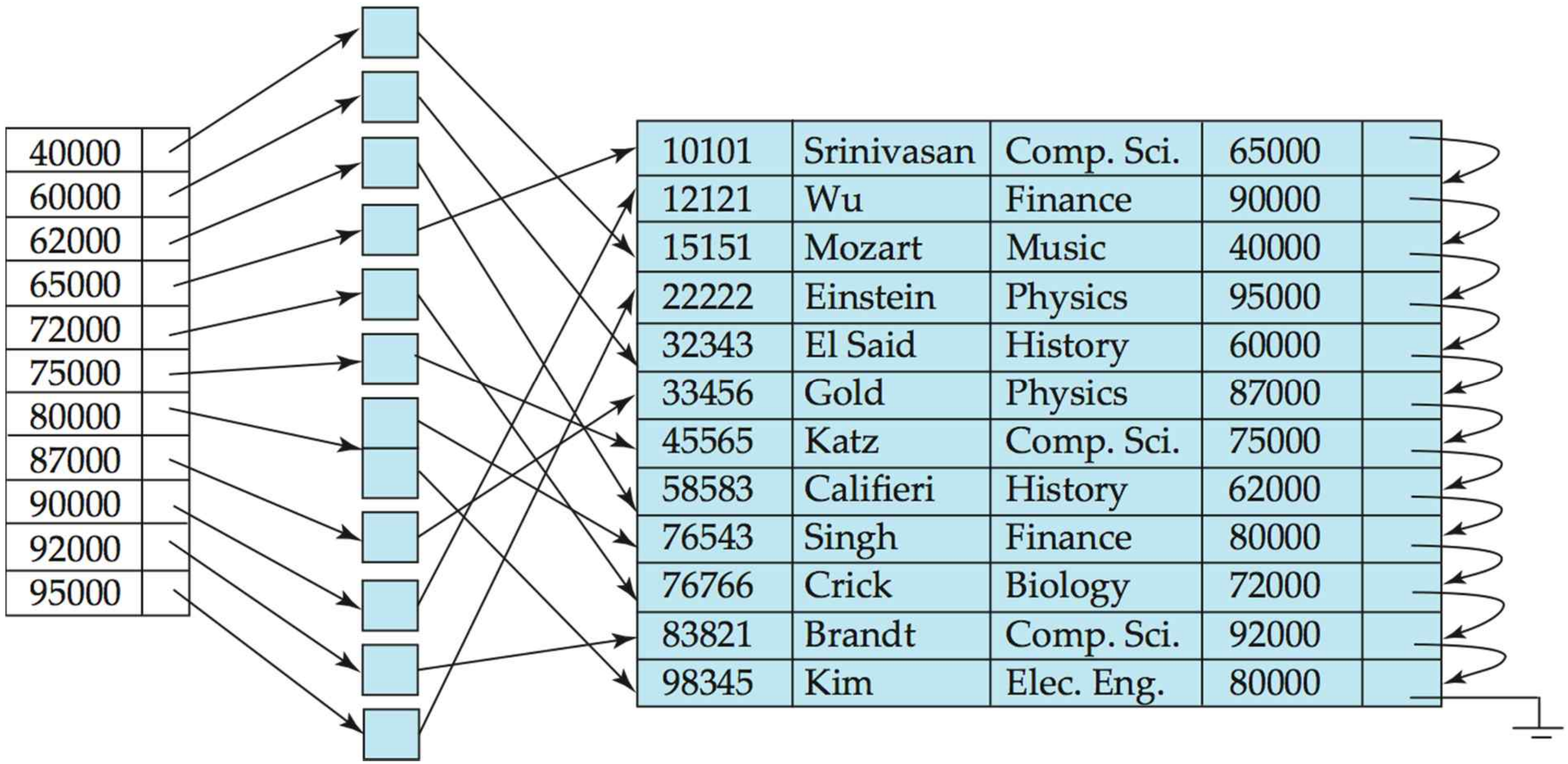
- Example: dense index on *dept_name*, with *instructor* file sorted on *dept_name*

Biology	
Comp. Sci.	
Elec. Eng.	
Finance	
History	
Music	
Physics	

76766	Crick	Biology	72000	
10101	Srinivasan	Comp. Sci.	65000	
45565	Katz	Comp. Sci.	75000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	
12121	Wu	Finance	90000	
76543	Singh	Finance	80000	
32343	El Said	History	60000	
58583	Califieri	History	62000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
33465	Gold	Physics	87000	

Secondary Indices Example

- Index record points to a bucket that contains pointers to all the actual records with that particular search-key value.
- Secondary indices have to be dense



Secondary index on *salary* field of *instructor*

Sparse Index Files

- **Sparse Index**: contains index records for only some search-key values.
  - Applicable when records are sequentially ordered on search-key
- Compared to dense indices:
  - Less space and less maintenance overhead for insertions and deletions
  - Generally slower than dense index for locating records

10101	
32343	
76766	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
32343	El Said	History	60000	
33456	Gold	Physics	87000	
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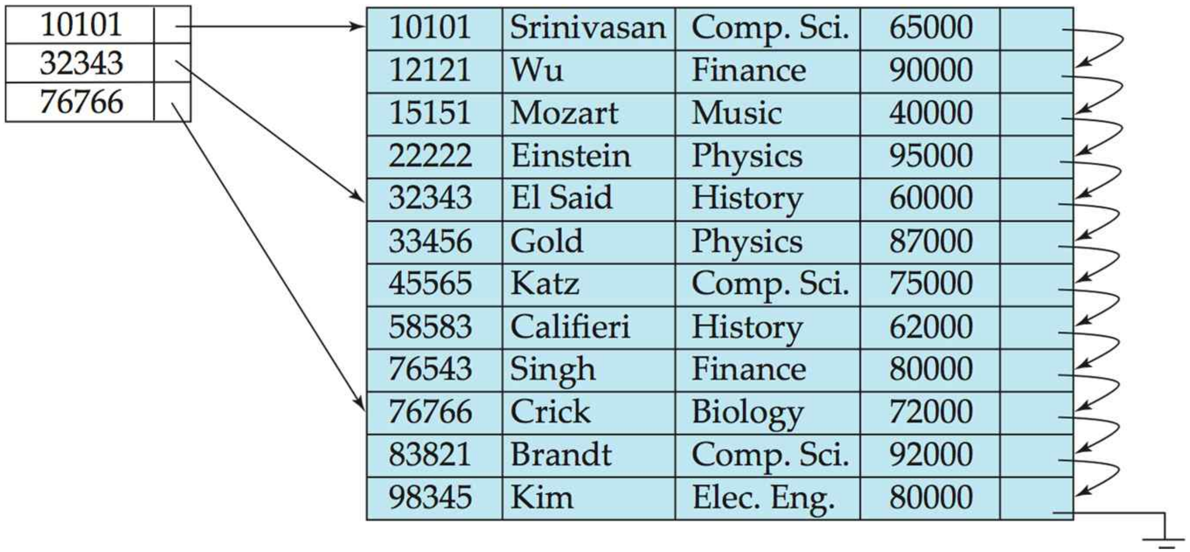
Index Update: Deletion

- If deleted record was the only record in the file with its particular search-key value, the search-key is deleted from the index also.
- **Dense indices** – deletion of search-key is similar to file record deletion
- **Sparse indices**
  - If an entry for the search key exists in the index,
    - Replace the entry in the index with the next search-key value in the file (in search-key order)
  - If the next search-key value already has an index entry
    - Delete the entry



Index Update: Insertion

- Perform a lookup using the search-key value appearing in the record to be inserted

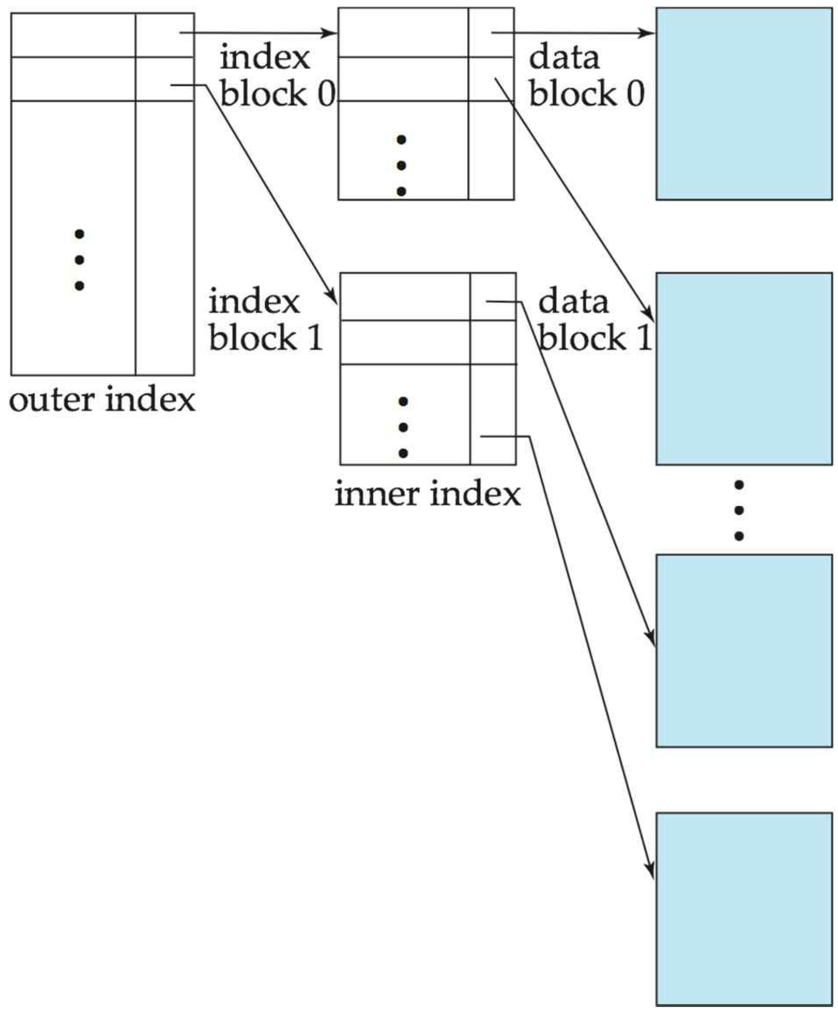
- **Dense indices**
  - If the search-key value does not appear in the index, insert it.
- **Sparse indices**
  - If index stores an entry for each block of the file
    - No change needs to be made to the index, unless a new block is created
    - If a new block is created, the first search-key value appearing in the new block is inserted into the index

Primary and Secondary Indices

- Indices offer substantial benefits when searching for records
- When a file is modified, every index on the file must be updated
  - Updating indices imposes overhead on database modification
- Sequential scan using primary index is efficient, but a sequential scan using a secondary index is expensive
  - Each record access may fetch a new block from disk
  - Block fetch requires about 5 to 10 milliseconds, versus about 100 nanoseconds for memory access

Multilevel Index

- If primary index does not fit in memory, access becomes expensive
- Solution: treat primary index kept on disk as a sequential file and construct a sparse index on it
  - outer index – a sparse index of primary index
  - inner index – the primary index file
- If even outer index is too large to fit in main memory, yet another level of index can be created, and so on
- Indices at all levels must be updated on insertion or deletion from the file

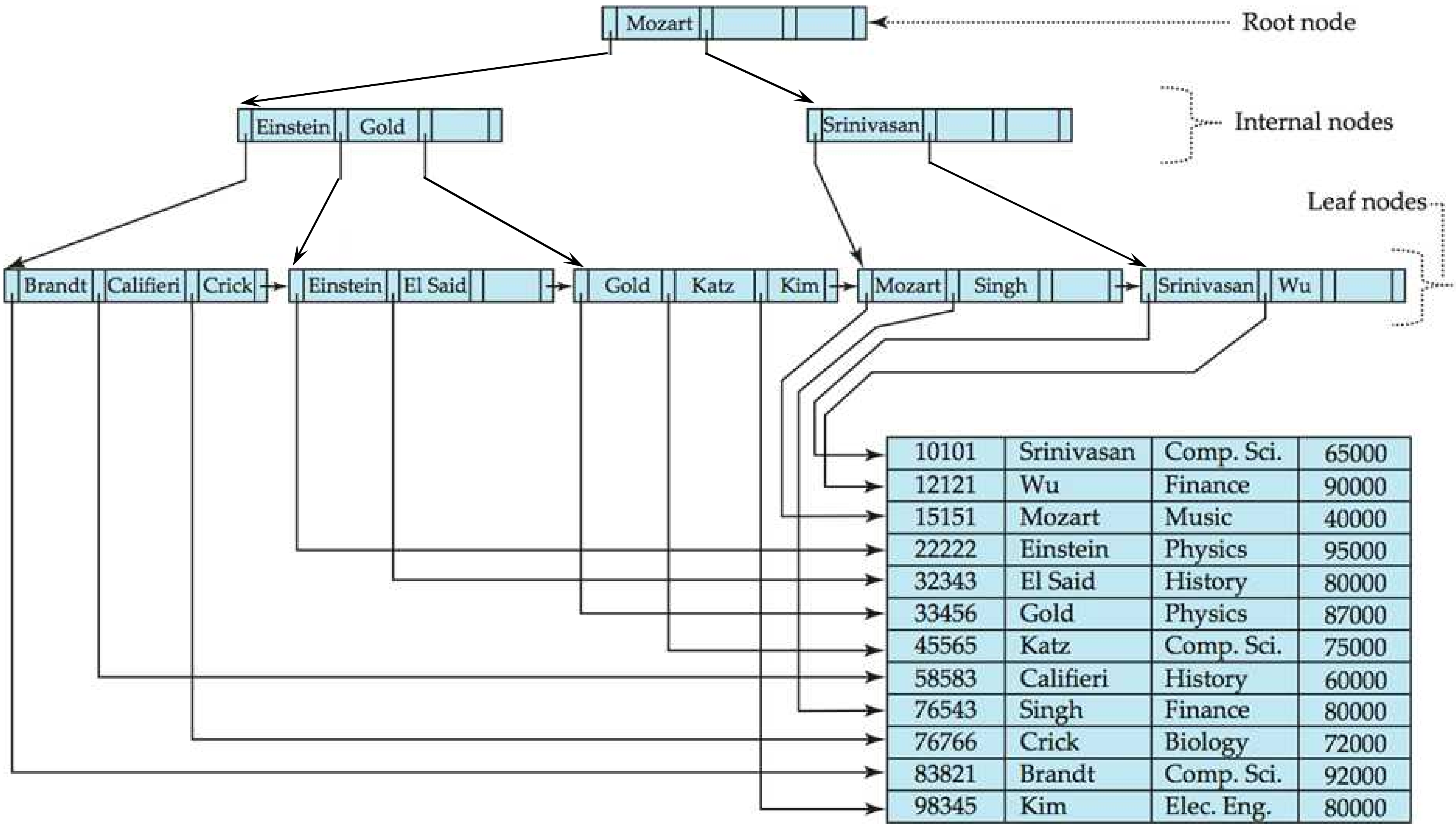


B^+-Tree Index Files

- Advantage of B^+-tree index files:
  - Automatically reorganizes itself with small local changes, in the face of insertions and deletions
  - Reorganization of entire file is not required to maintain performance
- (Minor) disadvantage of B^+-trees:
  - Extra insertion and deletion overhead, space overhead

- B^+-trees are used extensively
  - Advantages of B^+-trees outweigh disadvantages

Example of B+-Tree



B+-Tree Index Files (Cont.)

A B+-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children
- A leaf node has between $\lceil (n–1)/2 \rceil$ and n–1 values
- Special cases: root node
  - If the root is not a leaf, it has at least 2 children
  - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and (n–1) values

B^+-Tree Node Structure

- Typical node (non-leaf node)

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes)
    or pointers to records (for leaf nodes)

- The search-keys in a node are ordered

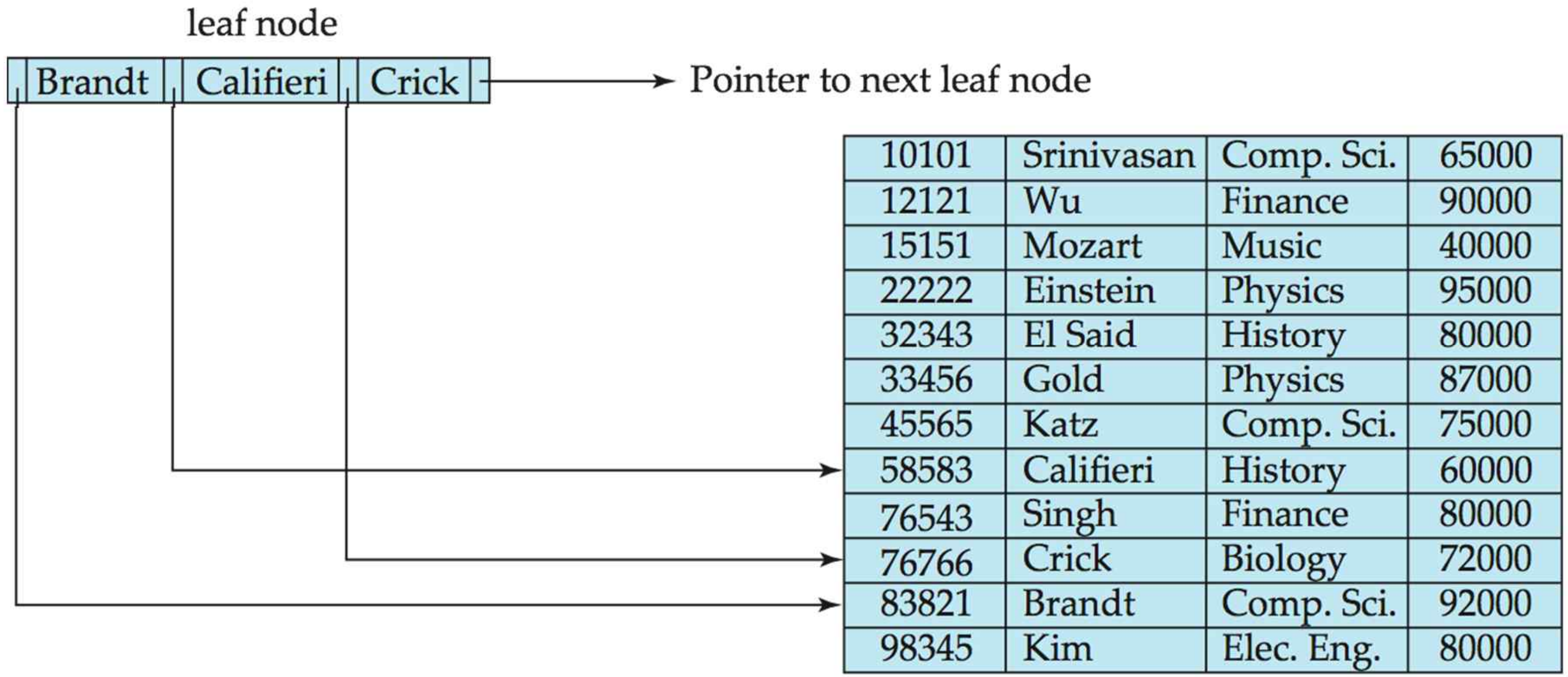
$$K_1 < K_2 < K_3 < . . . < K_{n-1}$$

  - We assume no duplicate keys

Leaf Nodes in B^+-Trees

- For $i = 1, 2, \ldots, n–1$, pointer P_i points to a file record with search-key value K_i
- P_n points to next leaf node in search-key order

- If L_i, L_j are leaf nodes and $i < j$, L_i's search-key values are less than or equal to L_j's search-key values



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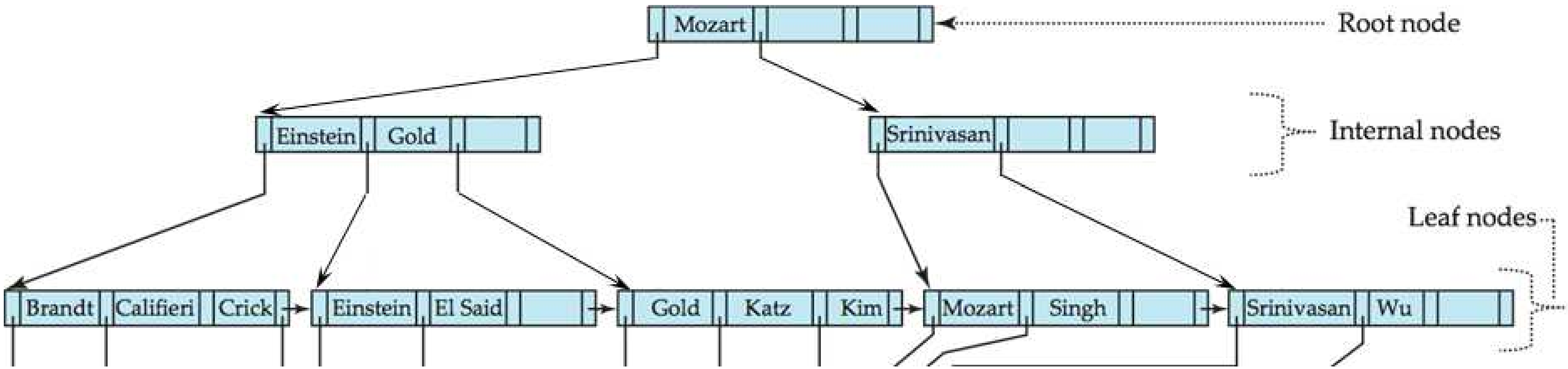
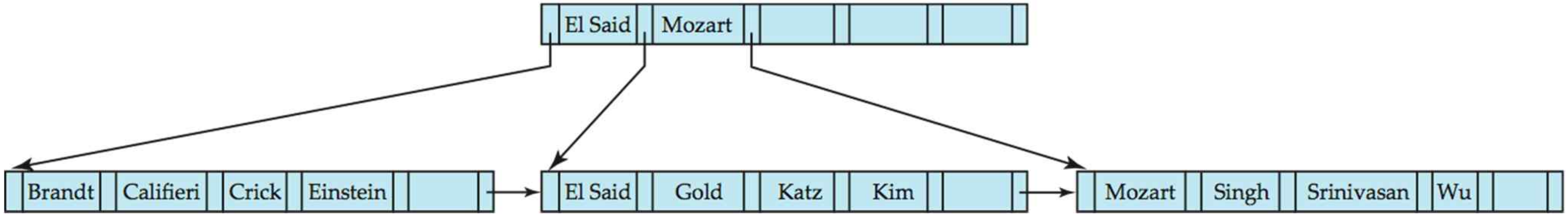
Non-Leaf Nodes in B$^+$-Trees

- Non leaf nodes form a multi-level sparse index on the leaf nodes
- For a non-leaf node with m pointers ($m \leq n$):
  - All the search-keys in the subtree to which P_1 points are less than K_1
  - For $2 \leq i \leq m - 1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
  - All the search-keys in the subtree to which P_m points have values greater than or equal to K_{m-1}



Example of B+-tree



B+-tree for *instructor* file (n = 6)

- Leaf nodes must have between 3 and 5 values
  - $\lceil (n-1)/2 \rceil$ and $n-1$, with $n = 6$
- Non-leaf nodes other than root must have between 3 and 6 children
  - $\lceil (n/2) \rceil$ and n, with $n = 6$
- Root must have at least 2 children

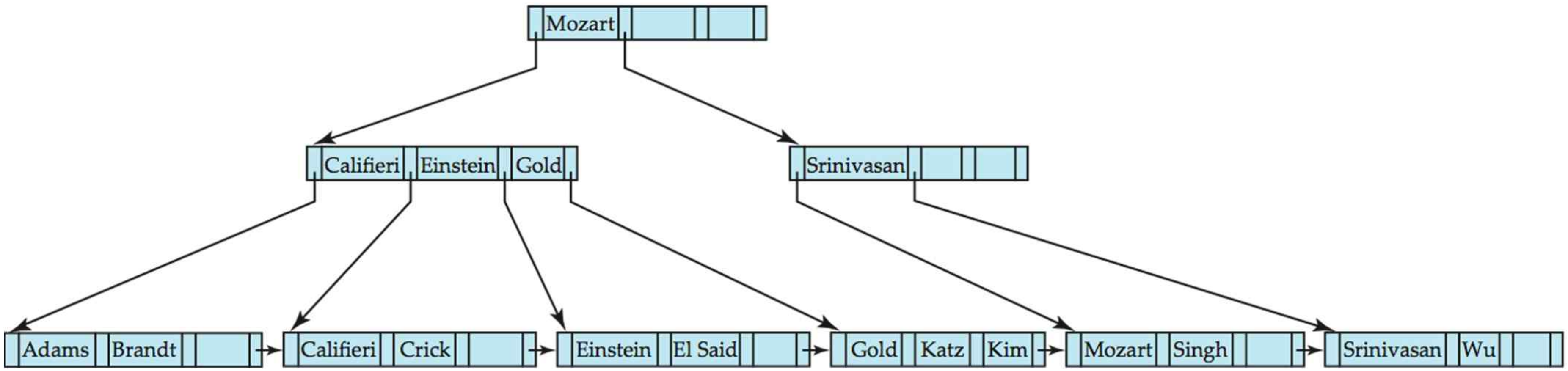
Observations about B$^+$-trees

- Since the inter-node connections are done by pointers, "logically" close blocks need not be "physically" close
- The non-leaf levels of the B$^+$-tree form a hierarchy of sparse indices
- The B$^+$-tree contains a relatively small number of levels
    - Level below root has at least 2 * $\lceil n/2 \rceil$ values
    - Next level has at least 2 * $\lceil n/2 \rceil$ * $\lceil n/2 \rceil$ values
    - .. etc.
  - If there are K search-key values in the file, the tree height is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$
  - Thus searches can be conducted efficiently
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).

Queries on B^+-Trees

- Find record with search-key value *V*
  1. *C = root*
  2. While C is not a leaf node {
     1. Let *i* be least value s.t. $V \leq K_i$.
     2. If no such exists, set *C = last non-null pointer in C*
     3. Else { if ($V = K_i$) set C = P_{i+1}
        else set $C = P_i$ }

     }
  3. Let *i* be least value s.t. $K_i = V$
  4. If there is such a value *i*, follow pointer P_i to the desired record
  5. Else no record with search-key value *k* exists



Queries on B^{+}-Trees (Cont.)

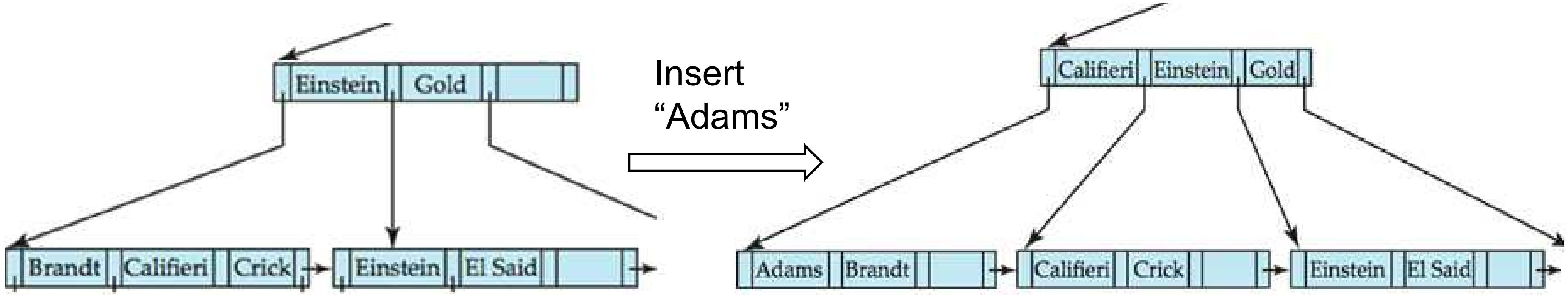
- If there are K search-key values in the file, the height of the tree is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$
- A node is generally the same size as a disk block, typically 4 kilobytes
  - n is typically around 100 (40 bytes per index entry)
- With 1 million search key values and $n = 100$
  - At most $log_{50}(1{,}000{,}000) = 4$ nodes are accessed in a lookup
- Contrast this with a balanced binary tree with 1 million search key values
  - Around 20 nodes are accessed in a lookup
  - Difference is significant since every node access may need a disk I/O, costing around 20 milliseconds

Updates on B$^+$-Trees: Insertion

1. Find the leaf node in which the search-key value would appear
2. Add the record to the file
3. If there is room in the leaf node, insert (key-value, pointer) pair in the leaf node
4. Otherwise, split the node (along with the new (key-value, pointer) entry)

Leaf Node Split in B+-Trees

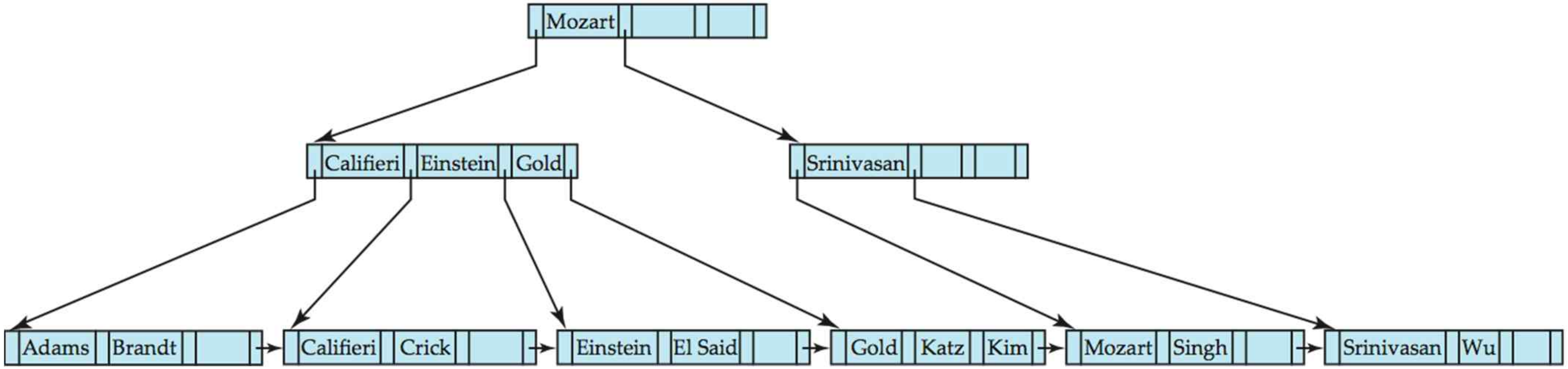
- Splitting a leaf node:
  - Take the *n* (search-key value, pointer) pairs (including the one being inserted) in sorted order
    - Place the first $\lceil n/2 \rceil$ in the original node, and the rest in a new node
    - Make the original node point to the new node
  - Let the new node be *p,* and let *k* be the least key value in *p*
    - Insert (*k, p*) in the parent of the node being split
  - If the parent is full, split it and **propagate** the split further up
- Splitting of nodes proceeds upwards till a node that is not full is found



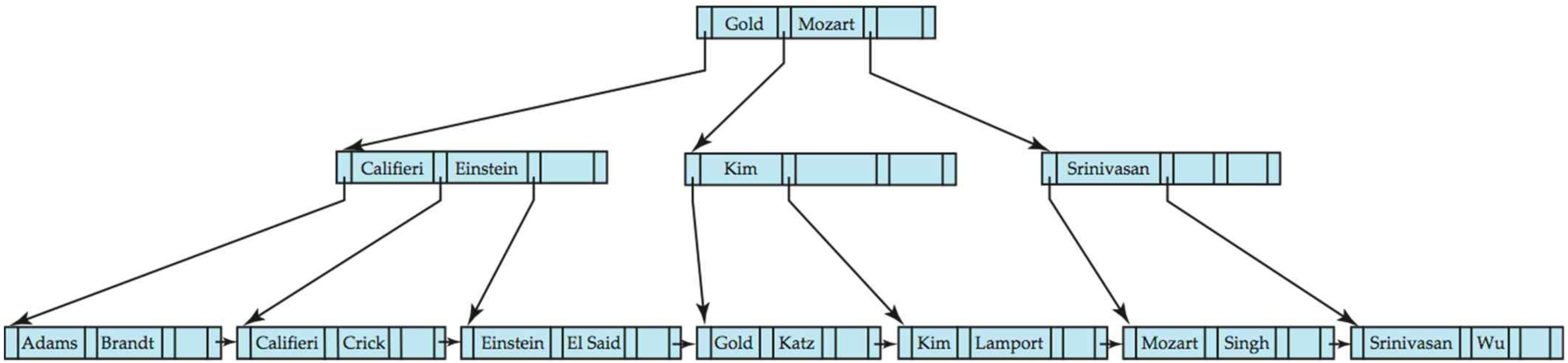
Non-Leaf Node Split in B$^+$-Trees

- Splitting a non-leaf node: when inserting (*k,p*) into an already full internal node N
  - Copy N to an in-memory area M with space for n+1 pointers and n keys
  - Insert (*k,p*) into M
  - Copy $P_1, K_1, \ldots, K_{\lceil n/2 \rceil -1}, P_{\lceil n/2 \rceil}$ from M back into node N
  - Copy $P_{\lceil n/2 \rceil+1}, K_{\lceil n/2 \rceil+1}, \ldots, K_n, P_{n+1}$ from M into newly allocated node N'
  - Insert $(K_{\lceil n/2 \rceil}, N')$ into parent N

B$^+$-Tree Non-Leaf Node Split Example



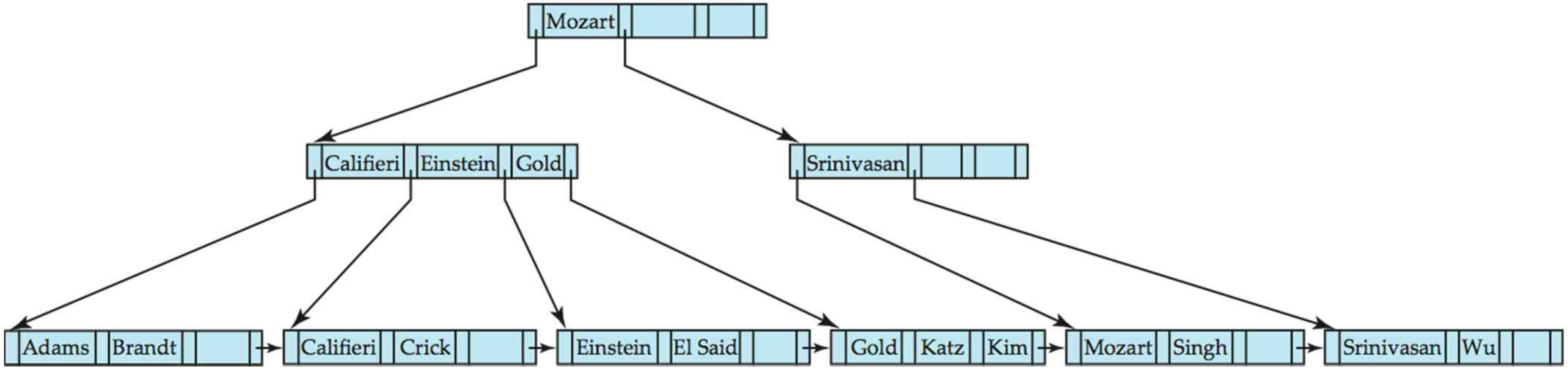
After insertion of “Lamport”



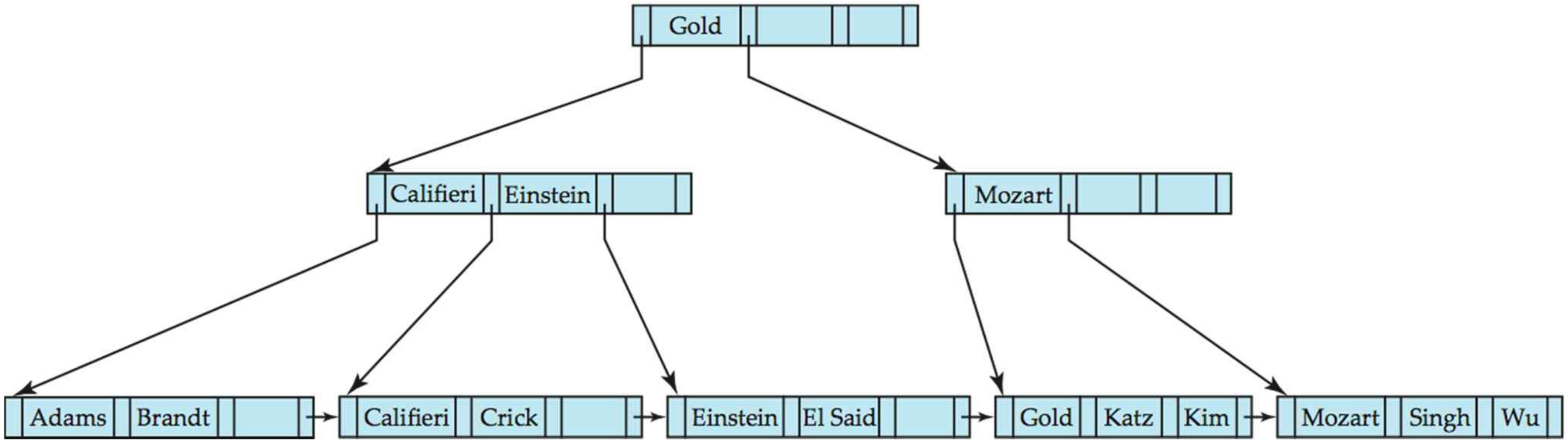
Updates on B$^+$-Trees: Deletion

1. Find the record to be deleted, and remove it from the main file
2. Remove (search-key value, pointer) from the leaf node
3. If the node has too few entries due to the removal,
    1. The entries in the node and a sibling fit into a single node – ***merge siblings:***
        - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node
        - Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure
    2. The entries in the node and a sibling do not fit into a single node – **redistribute pointers**:
        - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries
        - Update the corresponding search-key value in the parent of the node
4. The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found
5. If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root

Examples of B$^+$-Tree Deletion

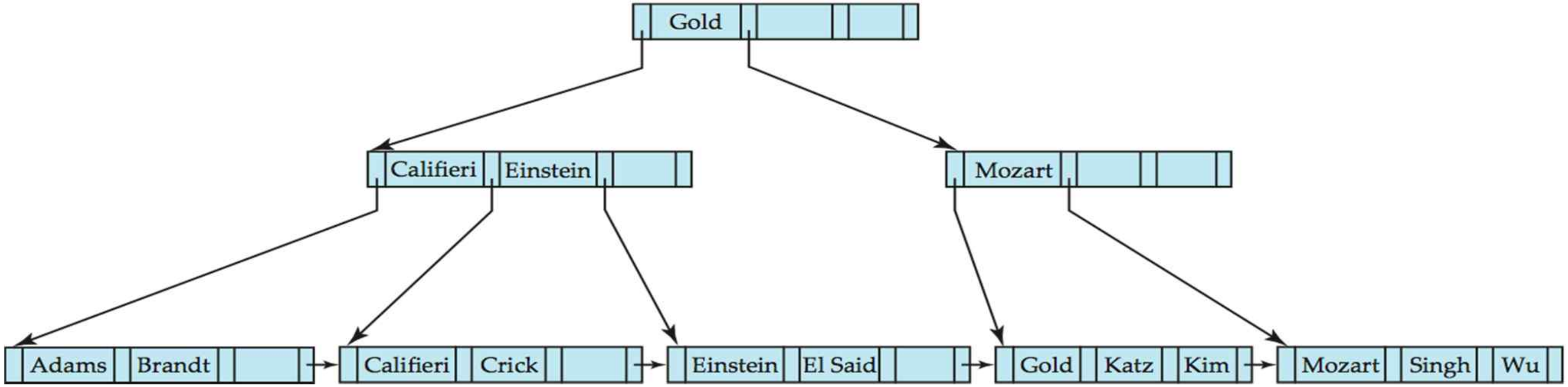


After deletion of "Srinivasan"

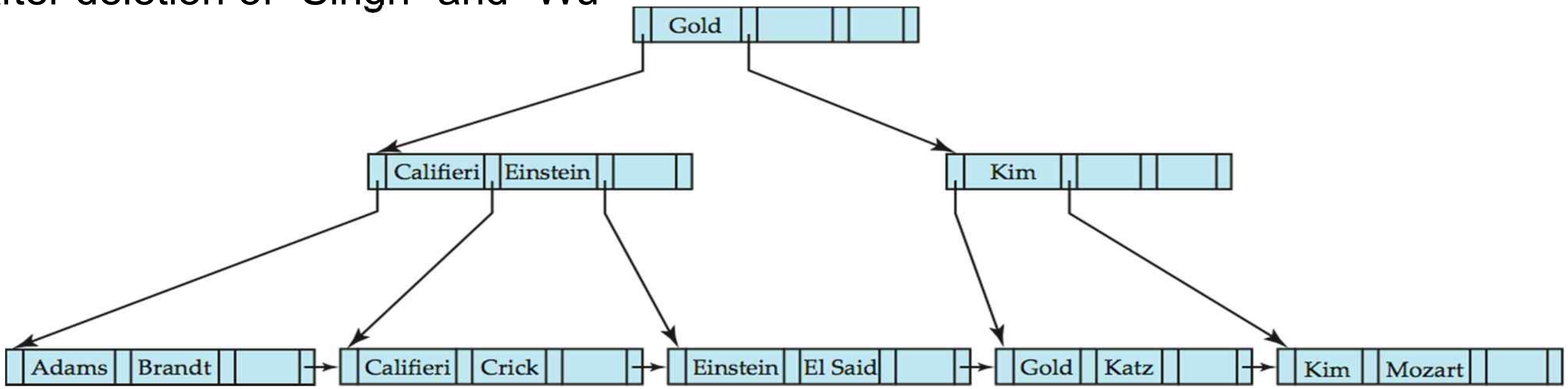


- Deleting "Srinivasan" causes merging of under-full leaves

Examples of B$^+$-Tree Deletion (Cont.)

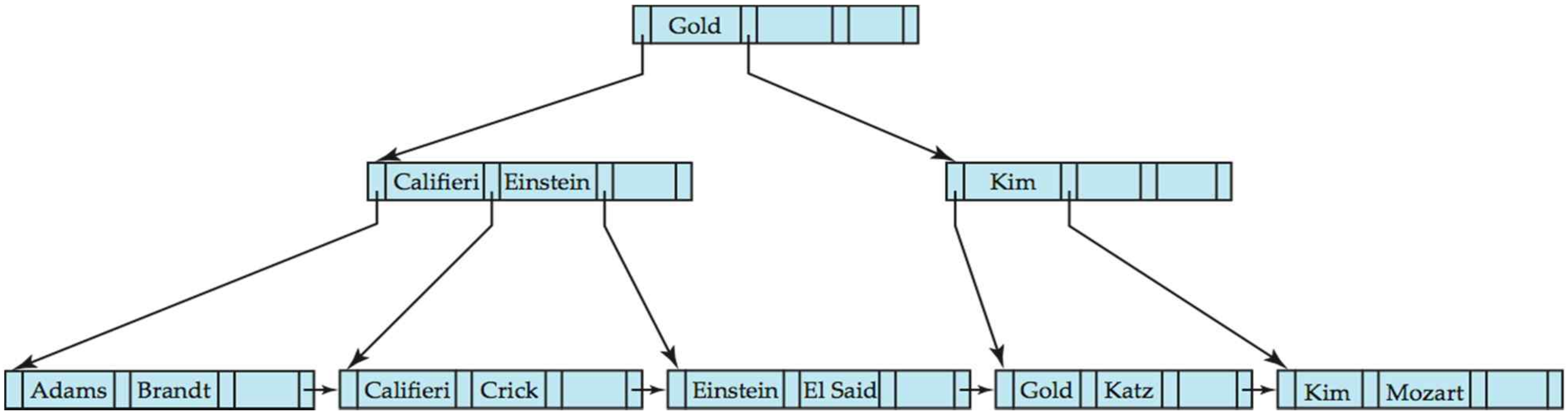


After deletion of "Singh" and "Wu"

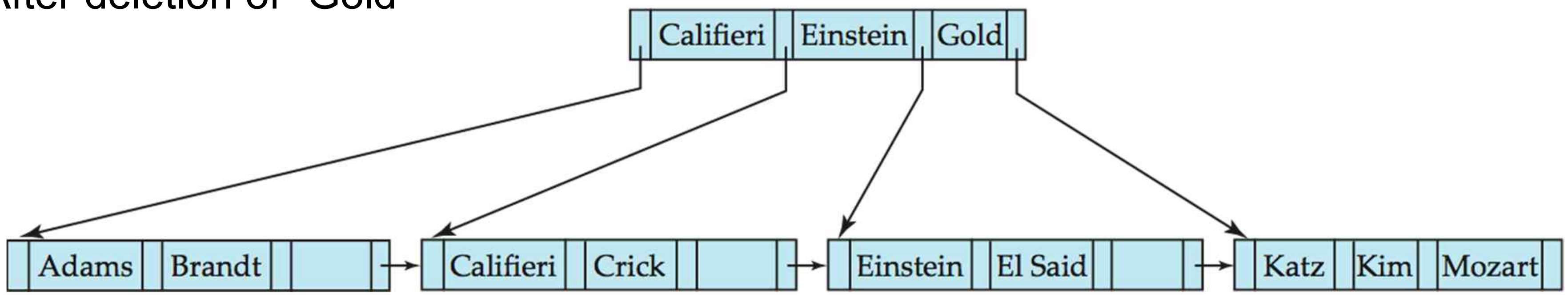


- Leaf containing Singh and Wu became underfull, and borrowed a value Kim from its left sibling
- Search-key value in the parent changes as a result

Example of B+-tree Deletion (Cont.)



After deletion of "Gold"



- Node with Gold and Katz became underfull, and was merged with its sibling
- Parent node becomes underfull, and is merged with its sibling
  - Value separating two nodes (at the parent) is pulled down when merging
- Root node then has only one child, and is deleted

Handling Non-Unique Search Keys

- Buckets on separate block (bad idea)
- List of tuple pointers with each key
  - Extra code to handle long lists
  - Deletion of a tuple can be expensive if there are many duplicates on search key
  - Low space overhead, no extra cost for queries
- Make search key unique by adding a record-identifier
  - Extra storage overhead for keys
  - Simpler code for insertion/deletion
  - Widely used

Bulk Loading and Bottom-Up Build

- Inserting entries one-at-a-time into a B$^+$-tree requires $\geq$ 1 IO per entry
  - assuming leaf level does not fit in memory
  - can be very inefficient for loading a large number of entries at a time (**bulk loading**)
- Efficient alternative 1:
  - Sort entries first (using efficient external-memory sort algorithms discussed later in Section 12.4)
  - Insert in sorted order
    - insertion will go to existing page (or cause a split)
    - much improved IO performance, but most leaf nodes half full
- Efficient alternative 2: **Bottom-up B$^+$-tree construction**
  - As before sort entries
  - And then create tree layer-by-layer, starting with leaf level
    - details as an exercise
  - Implemented as part of bulk-load utility by most database systems

Multiple-Key Access

- Use multiple indices for certain types of queries.
- Example:

  select *ID*

  from *instructor*

  where *dept_name* = "Finance" **and** *salary* = 80000

- Possible strategies for processing query using indices on single attributes:
  1. Use index on *dept_name* to find instructors with department name Finance; test *salary = 80000*
  2. Use index on *salary* to find instructors with a salary of $80000; test *dept_name* = "Finance".
  3. Use *dept_name* index to find pointers to all records pertaining to the "Finance" department. Similarly use index on *salary*. Take intersection of both sets of pointers obtained.

Indices on Multiple Keys

- **Composite search keys** are search keys containing more than one attribute
  - E.g. (*dept_name, salary*)
- Lexicographic ordering: $(a_1, a_2) < (b_1, b_2)$ if either
  - $a_1 < b_1$, or
  - $a_1 = b_1$ and $a_2 < b_2$

- Suppose we have an index on combined search-key (*dept_name, salary*).
- Can efficiently handle
  where *dept_name* = "Finance" **and** *salary* = 80000
  - Fetch only records that satisfy both conditions
- Can also efficiently handle
  where *dept_name* = "Finance" **and** *salary* < 80000
- But cannot efficiently handle
  where *dept_name* < "Finance" **and** *balance* = 80000
  - May fetch many records that satisfy the first but not the second condition

Static Hashing

- **Bucket**
  - A unit of storage containing one or more records
  - Typically a disk block
- **Hash file organization**
  - The bucket of a record is directly obtained from its search-key value using a **hash function**
- Hash function *h*
  - A function from the set of all search-key values *K* to the set of all bucket addresses *B*
  - is used to locate records for access, insertion as well as deletion.
- Records with different search-key values may be mapped to the same bucket
  - Thus entire bucket has to be searched sequentially to locate a record

Example of Hash File Organization

Hash file organization of *instructor* file, using *dept_name* as key

- # of buckets = 10
- The binary representation of the *i*th character is assumed to be the integer *i*
- The hash function returns the sum of the binary representations of the characters modulo 10
  - E.g. h(Music) = 1
    h(History) = 2
    h(Physics) = 3
    h(Elec. Eng.) = 3

bucket 0

bucket 1

15151	Mozart	Music	40000

bucket 2

32343	El Said	History	80000
58583	Califieri	History	60000

bucket 3

22222	Einstein	Physics	95000
33456	Gold	Physics	87000
98345	Kim	Elec. Eng.	80000

bucket 4

12121	Wu	Finance	90000
76543	Singh	Finance	80000

bucket 5

76766	Crick	Biology	72000

bucket 6

10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000

bucket 7

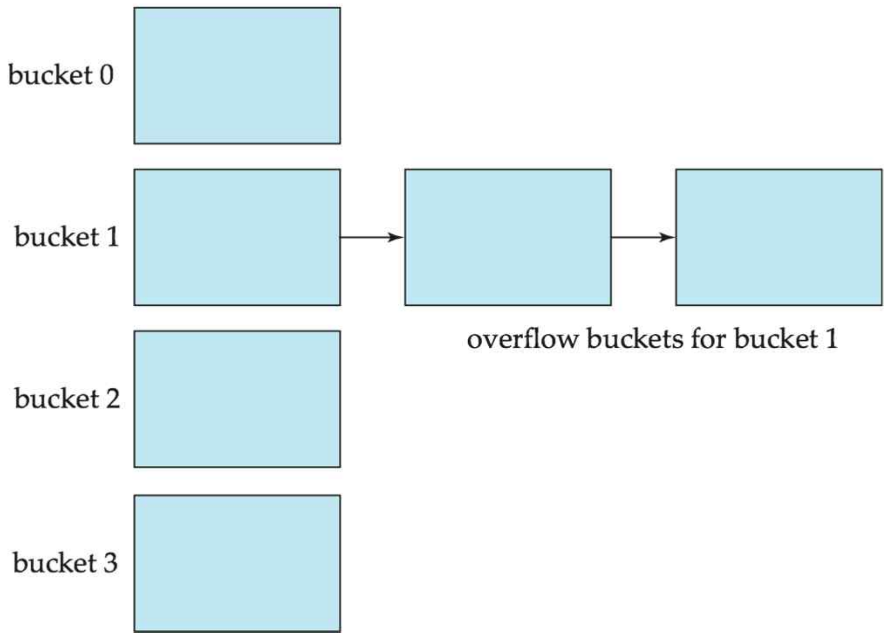
Hash Functions

- Worst hash function
  - All search-key values are mapped to the same bucket
  - Access time is proportional to the number of search-key values in the file
- Ideal hash function
  - **Uniform**: each bucket is assigned the same number of search-key values from the set of *all* possible values
  - **Random**: each bucket will have the same number of records assigned to it irrespective of the *actual distribution* of search-key values in the file

- Typical hash functions perform computation on the internal binary representation of the search-key

Handling of Bucket Overflows

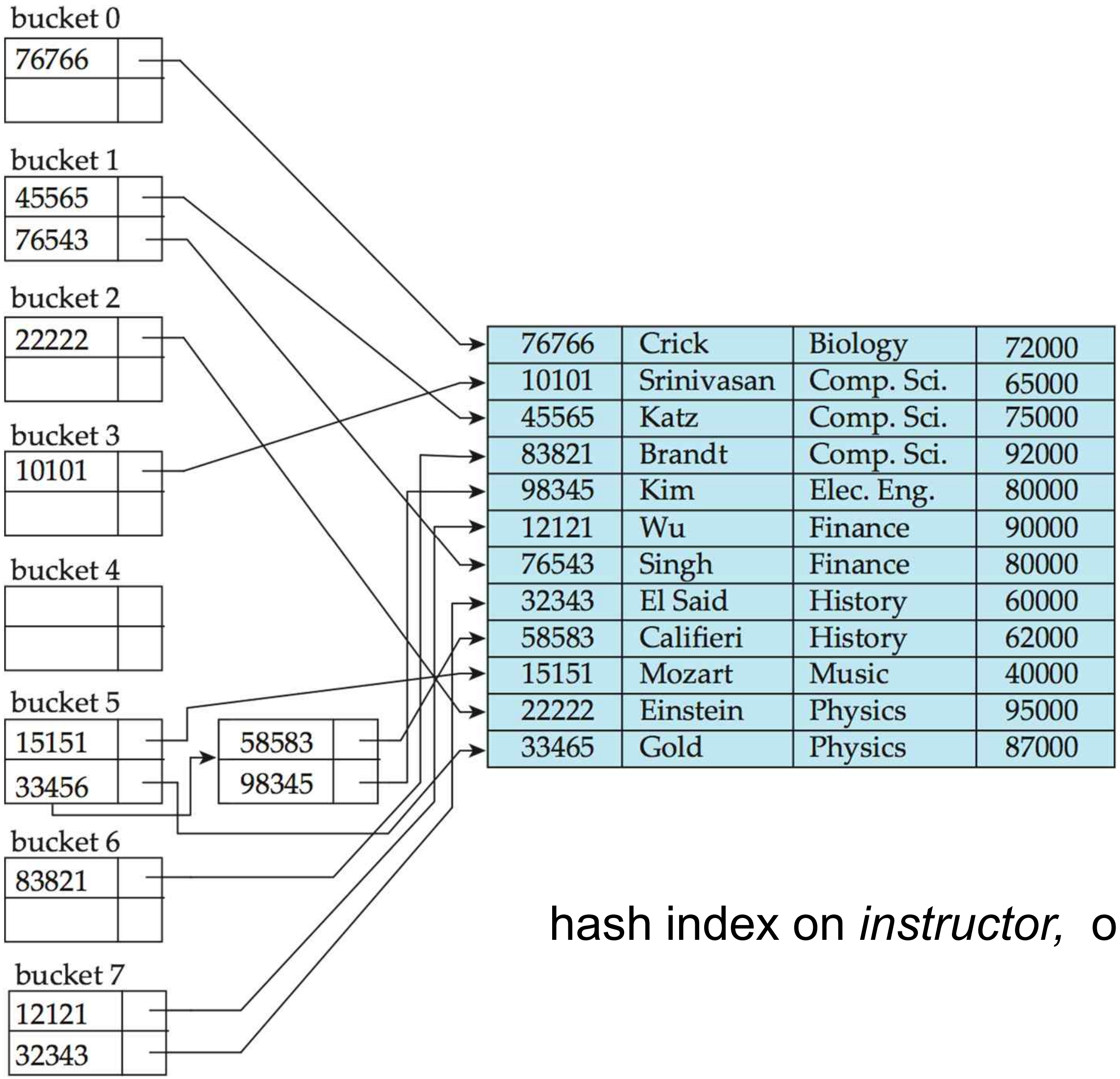
- Bucket overflow can occur because of
  - Insufficient buckets
  - Skew in distribution of records. This can occur due to two reasons:
    - multiple records have same search-key value
    - chosen hash function produces non-uniform distribution of key values
- Although the probability of bucket overflow can be reduced, it cannot be eliminated; it is handled by using ***overflow buckets.***
- **Overflow chaining**
  - The overflow buckets of a given bucket are chained together in a linked list



Hash Indices

- A **hash index** organizes the search keys, with their associated record pointers, into a hash file structure.
- Strictly speaking, hash indices are always secondary indices
  - If the file itself is organized using hashing, a separate primary hash index on it using the same search-key is unnecessary
  - However, we use the term hash index to refer to both secondary index structures and hash organized files

Example of Hash Index



hash index on *instructor,* on attribute *ID*

Deficiencies of Static Hashing

- In static hashing, function *h* maps search-key values to a fixed set of *B* of bucket addresses
  - Databases grow or shrink with time
  - If initial number of buckets is too small, and file grows, performance will degrade due to too much overflows
  - If space is allocated for anticipated growth, a significant amount of space will be wasted initially (and buckets will be underfull)
  - If database shrinks, again space will be wasted
- One solution: periodic re-organization of the file with a new hash function
  - Expensive, disrupts normal operations
- Better solution: allow the number of buckets to be modified dynamically
  - Dynamic hashing is not covered in this class

Comparison of Ordered Indexing and Hashing

- Cost of periodic re-organization
- Relative frequency of insertions and deletions
- Is it desirable to optimize average access time at the expense of worst-case access time?
- Expected type of queries:
  - Hashing is generally better at retrieving records having a specified value of the key (point query)
  - If range queries are common, ordered indices are to be preferred

- In practice:
  - PostgreSQL supports hash indices, but discourages use due to poor performance
  - Oracle supports static hash organization, but not hash indices
  - SQLServer supports only B^+-trees

Bitmap Indices

- **Bitmap**: simply an array of bits
- **Bitmap index**: a specialized type of index designed for efficient querying on multiple keys
- In the simplest form, a bitmap index on an attribute has a bitmap for each value of the attribute
  - Bitmap has as many bits as records
  - In a bitmap for value v, the bit for a record is 1 if the record has the value v for the attribute, and is 0 otherwise

record number	*ID*	*gender*	*income_level*
0	76766	m	L1
1	22222	f	L2
2	12121	f	L1
3	15151	m	L4
4	58583	f	L3

Bitmaps for *gender*

m	10010
f	01101

Bitmaps for *income_level*

L1	10100
L2	01000
L3	00001
L4	00010
L5	00000

Bitmap Indices (Cont.)

- Applicable on attributes that take on a relatively small number of distinct values
  - E.g. gender, country, state, …
  - E.g. income-level (income broken up into a small number of levels such as 0-9999, 10000-19999, 20000-50000, 50000- infinity)
- Bitmap indices generally very small compared with relation size
  - E.g. if record is 100 bytes, space for a single bitmap is 1/800 of space used by relation
    - If # of distinct attribute values is 8, bitmap is only 1% of relation size

Bitmap Indices (Cont.)

- Bitmap indices are useful for queries on multiple attributes
  - not particularly useful for single attribute queries
- Queries are answered using bitmap operations
  - Intersection (and): e.g., 100110 AND 110011 = 100010
  - Union (or): e.g., 100110 OR 110011 = 110111
  - Complementation (not): e.g., 100110 NOT 100110 = 011001
- Each operation takes two bitmaps of the same size and applies the operation on corresponding bits to get the result bitmap
  - E.g. males with income level L1:   10010 AND 10100 = 10000
    - Can then retrieve required tuples
    - Counting number of matching tuples is even faster

Index Definition in SQL

- Create an index

  create index <index-name> **on** <relation-name>
  (<attribute-list>)

  E.g.: **create index** *dept_index* **on** *instructor (dept_name)*

- Use **create unique index** to indirectly specify and enforce the condition that the search key is a candidate key
  - Not really required if SQL **unique** integrity constraint is supported

- To drop an index

  drop index <index-name>

- Most database systems allow specification of type of index, and clustering

End of Chapter 11

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