Chapters of Data Structure Book

- Linear List
- Array and Matrix
- Linked List
- Stack
- Queue
- Dictionary and Hashing
- Tree
  - Binary Tree
  - Priority Queue
  - Binary Search Tree
  - Balanced Tree
- Graph
  - Directed Graph
  - Undirected Graph

Linear List Examples

Students in COP3530 = (Jack, Jill, Abe, Henry, Mary, …, Judy)

Exams in COP3530 = (exam1, exam2, exam3)

Grades in COP3530 = (“Jack A+”, “Jill B-”, “Abe D”, … “Judy F”)

Days of Week = (S, M, T, W, Th, F, Sa)

Months = (Jan, Feb, Mar, Apr, …, Nov, Dec)

LinearList Operations: *Suppose L = (a, b, c, d, e, f, g)*

- size() : Determine list size
  - *L.size() = 7*
- get(index) : Get element with given index
  - *get(0) = a* *get(2) = c* *get(4) = e* *get(-1) =* error *get(9) =* error
- indexOf(element) : Determine the index of an element
  - *indexOf(d) = 2* *indexOf(a) = 0* *indexOf(z) = -1*
- remove(index) : Remove and return element with given index.
  - *remove(2)* returns *c, L* becomes *(a,b,d,e,f,g),* indices of *d,e,f* and *g are decreased* by 1
  - *remove*(-1) → error *remove*(20) → error
- add(index, element) : Add an element so that the new element has a specified index
  - *add(0,h)* → *L = (h,a,b,c,d,e,f,g) //* indices of *a,b,c,d,e,f,* and *g* are increased by *1*
  - *add(2,h)* → *L = (a,b,h,c,d,e,f,g) //* indices of *c,d,e,f,* and *g* are increased by *1*
  - *add(10,h)* → error *add(-6,h)* → error

Python for Linear List?

- List (built-in data type in Python)

```
>>> L = [3, 7, 1]
>>> L.append(5)
>>> L
[3, 7, 1, 5]
```

- Array module in Python (not popular)

```
>>> from array import *
>>> A = array(‘i’, [4, 3, 6])
>>> A
Array(‘i’, [4,3,6])
>>> A.append(9)
>>>A
array(‘i', [4,3,6,9])
```

- Python에서 Vector는 List로 cover된다고 볼수 있음

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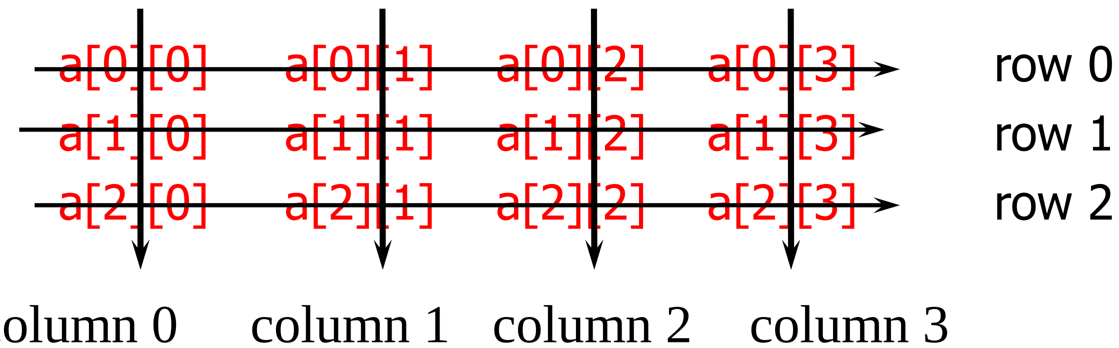
2-D Array or Matrix

- 2D-Array

a[0][0]	a[0][1]	a[0][2]	a[0][3]
a[1][0]	a[1][1]	a[1][2]	a[1][3]
a[2][0]	a[2][1]	a[2][2]	a[2][3]



- Matrix: Table of values
  - has as rows and columns like 2-D array
  - but numbering begins at 1 rather than 0

$$\begin{array}{cl} \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} & \begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{array} \end{array}$$

The Abstract Data Type: Matrix

```
AbstractDataType Matrix  {

  instances

  operations
    clone()
    copy (Matrix m)
    get (int i, int j)  : return the value of the pair with this index
    set (int i, int j, Object newValue) : overwrite existing one (if any)
                                           with the same index
    add (Matrix m)
    multiply(Matrix m)
}
```

Python for Array and Matrix?

- List
- numpy library
  - matrix class

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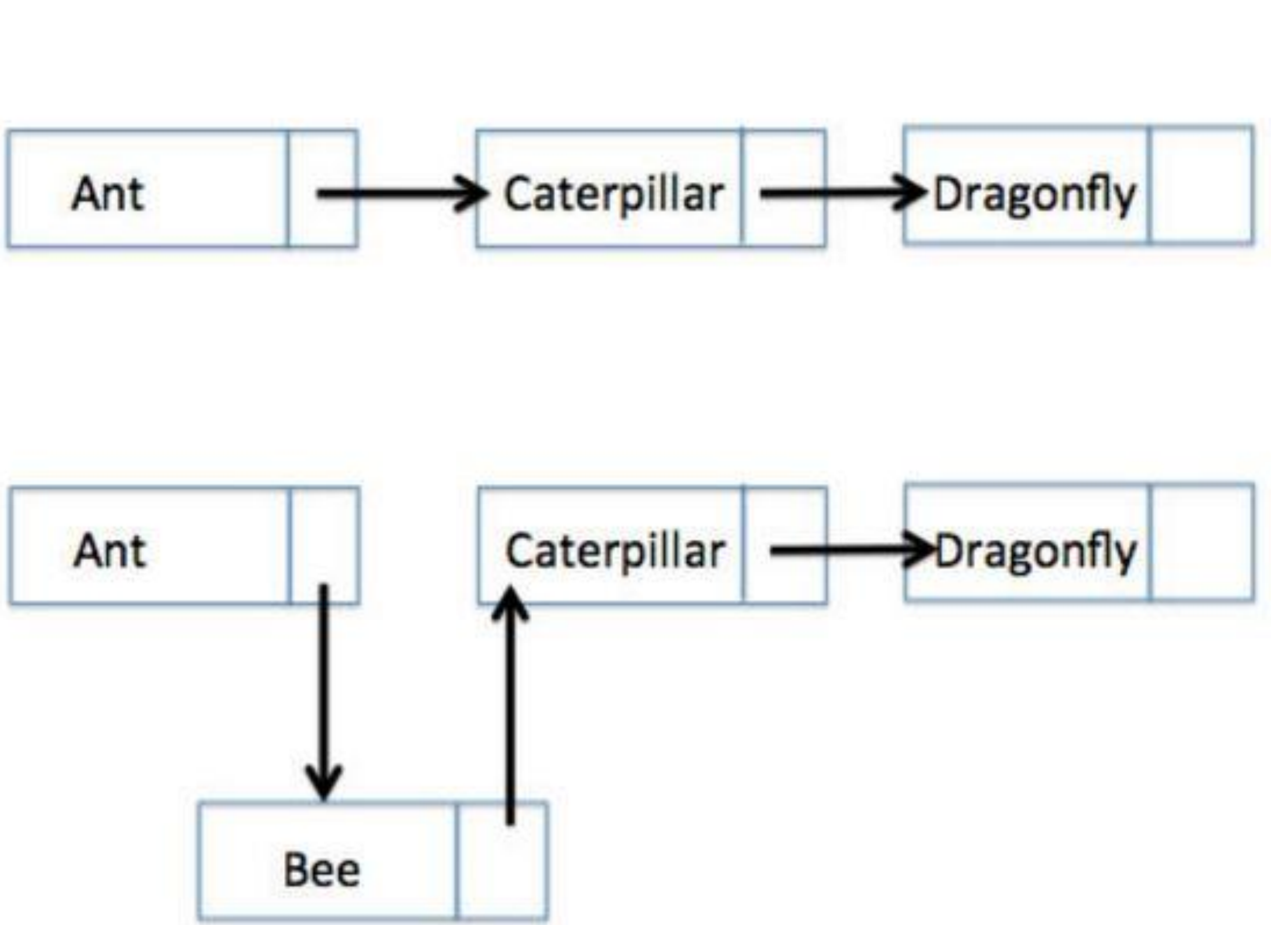
Linked Lists

A linked list is a data structure that uses pointers to point to the next item in the list. A linked list can be implemented using an array or using a class.



```python
class Node:
    def __init__(self, contents=None, next=None):
        self.contents = contents
        self.next  = next

    def getContents(self):
        return self.contents

    def __str__(self):
        return str(self.contents)

def print_list(node):
    while node:
        print(node.getContents())
        node = node.next
    print()

def testList():
    node1 = Node("car")
    node2 = Node("bus")
    node3 = Node("lorry")
    node1.next = node2
    node2.next = node3
    print_list(node1)
```

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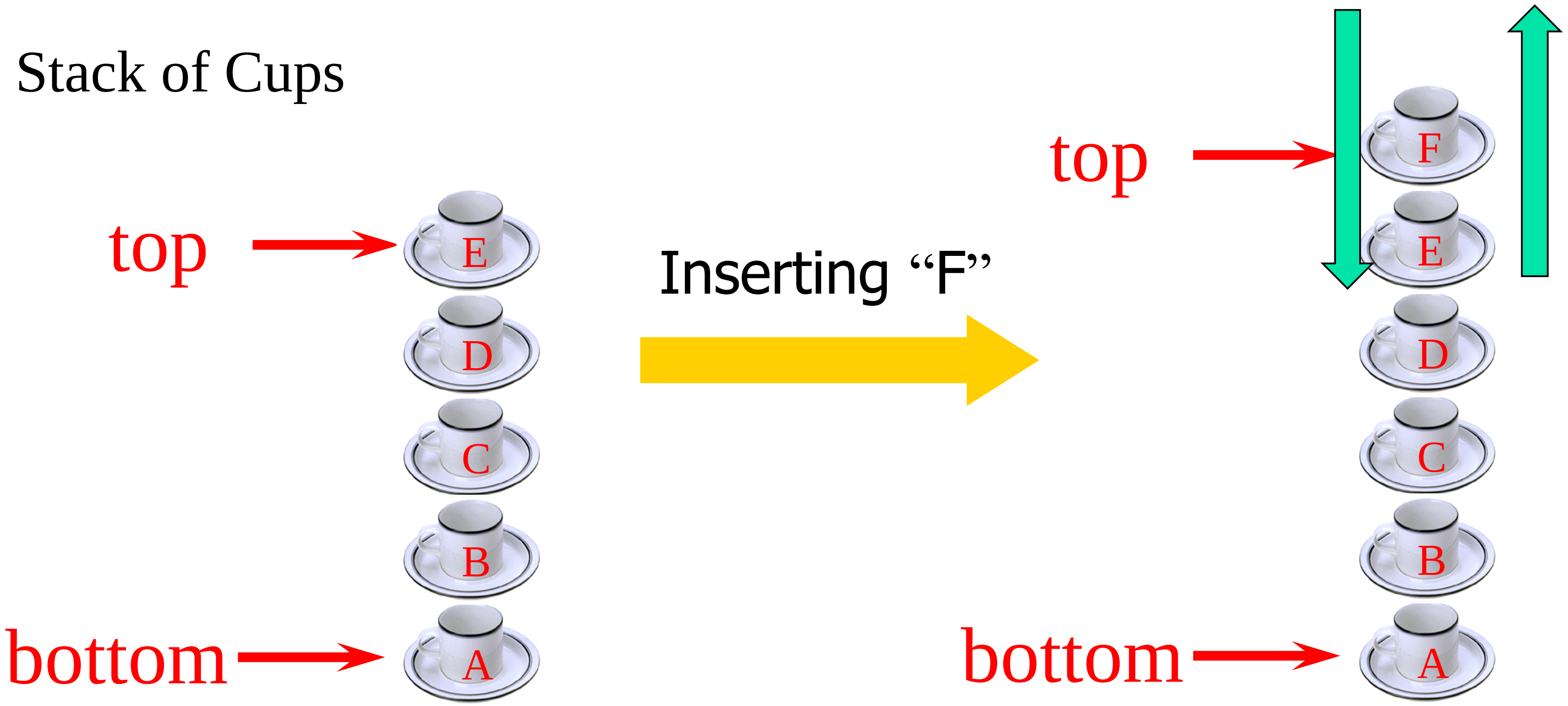
Stack

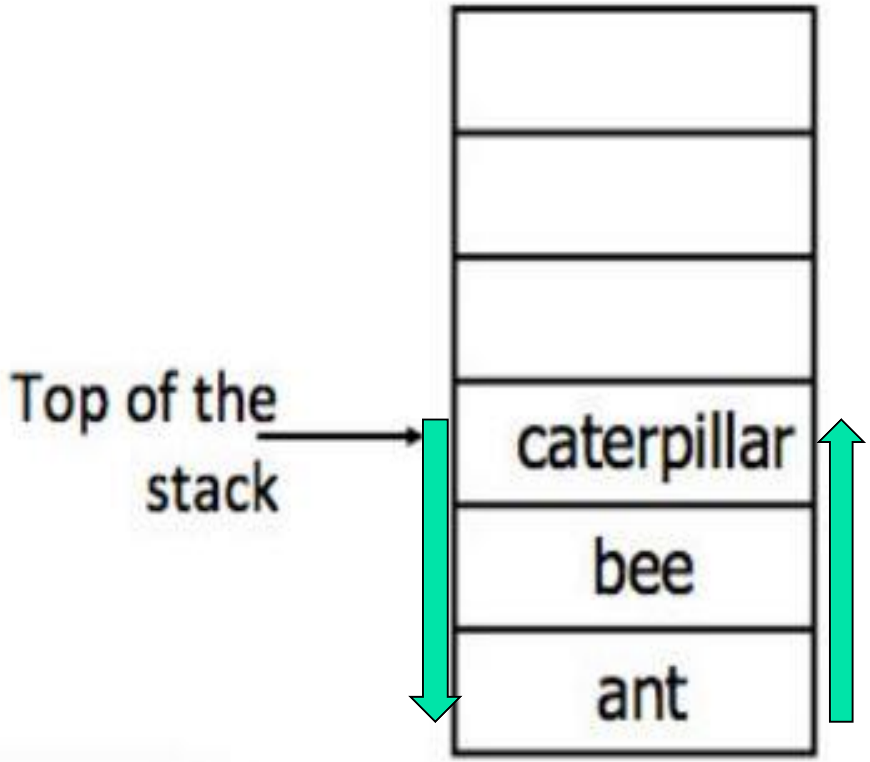
- A kind of Linear list
- One end is called "top" and the other end is called "bottom"
- Insertions and removals take place at the top
- A stack is a LIFO list (Last In First Out)

- Stack of Cups



Stacks

A stack is a last in, first out (LIFO) structure. Items are stored in the stack, but if an item is taken from the stack, it is always the last one that was added.



The ADT Stack

```
AbstactDataType Stack{
    instantces
        linear list of elements;
        bottom;
        top;
    operations
        empty() : Return true if the stack is empty,
                  Return false otherwise;
         peek() :  Return the top element;
        push(x) :  Add element x at the top;
          pop() :  Remove the top element  and return it;
}
```

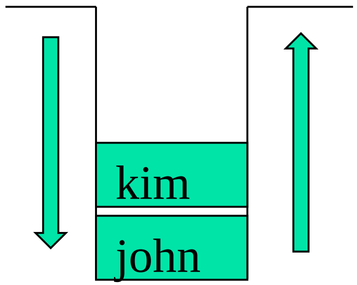
Q: Can we think of any other core operation of the Stack?

Python for Stack?

- Queues standard library
  - LifoQueue class

```
>>> class Stack:
	# the stack class
	def __init__(self):
		self.items = []
	def push(self, item):
		self.items.append(item)
	def pop(self):
		return self.items.pop()
	def isEmpty(self):
	    if self.items == []:
			return True
	    else:
			return False
	def peek(self):
		return self.items[len(self.items)-1]
```

```
>>> myStack = Stack()
>>> myStack.push("john")
>>> myStack.push("kim")
>>> myStack.peek()
'kim'
>>> myStack.pop()
'kim'
>>> myStack.items
['john']
>>>
```



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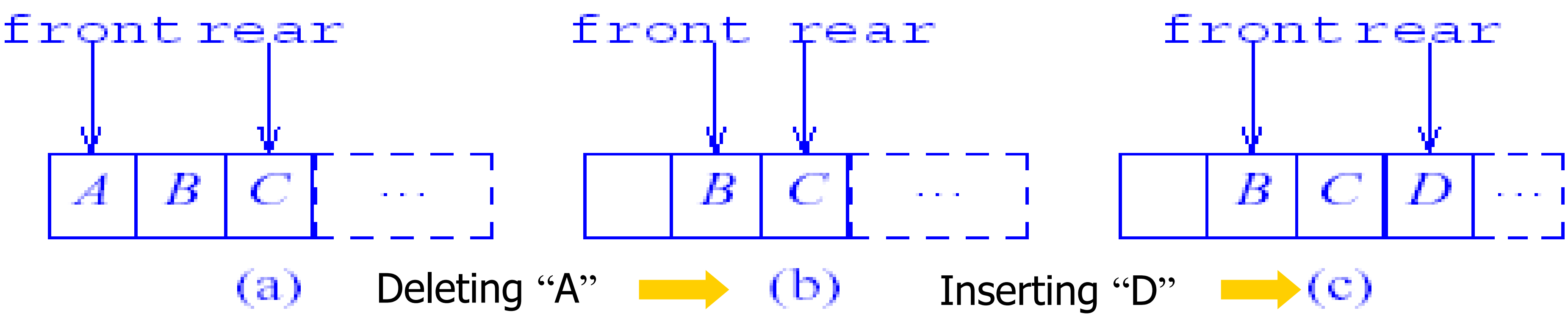
Queue

- The index i for the front element is 0
- front = location(front element)
- rear = location(last element)
- Empty queue has the condition: rear < front
- Insert an element
  - The worst-case time from **θ** (1) to **θ** (queue.length)
- Delete an element: **θ** (1) time
  - Move front to right by 1

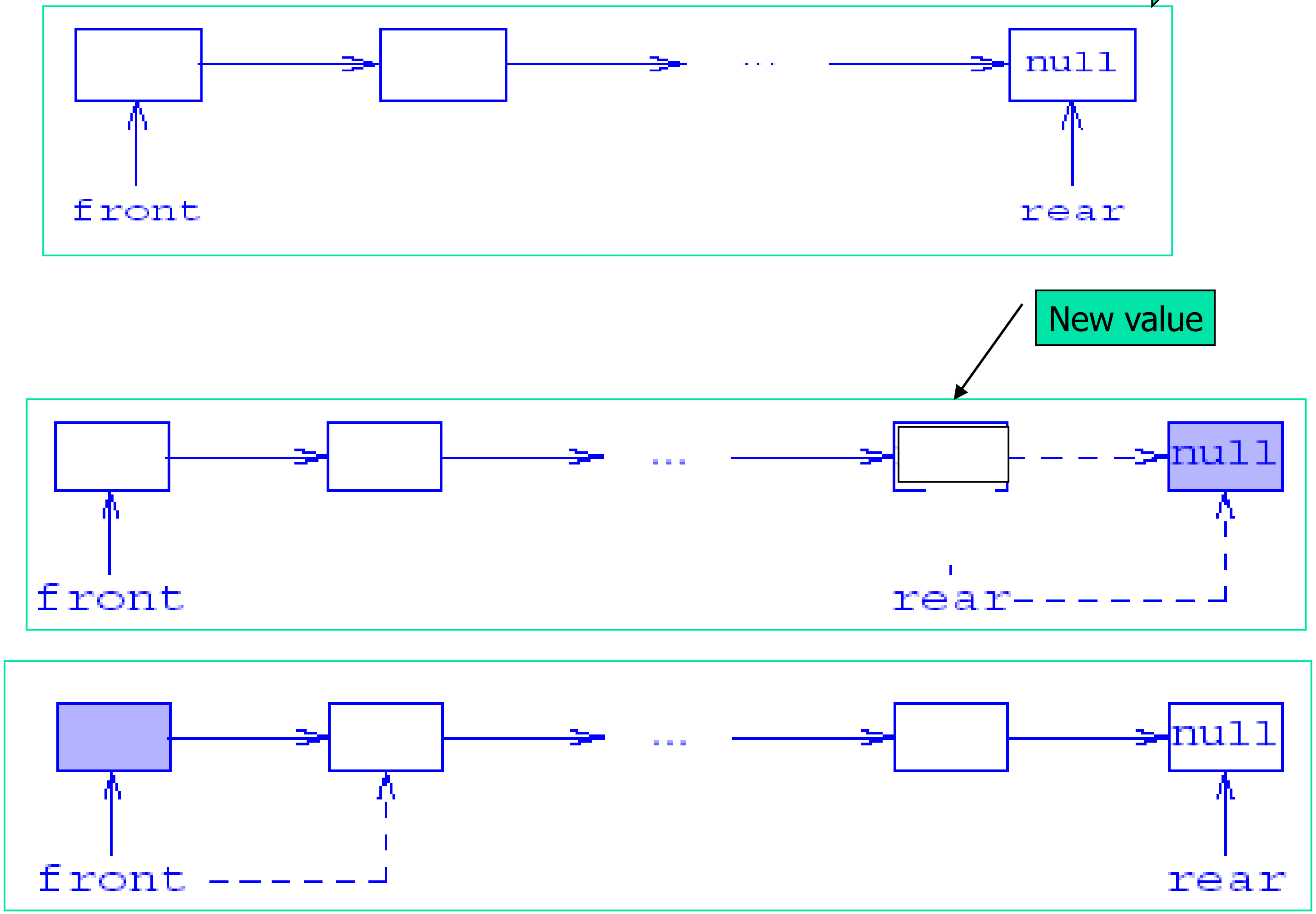


Queues

A queue is a first in, first out (FIFO) structure. This means that the first item to join the queue is the first to leave the queue. A queue can be implemented using an array (called a list in Python), or using OOP techniques.

A list implementation for a linear queue will use an append method to add to the queue and a delete method to remove from the queue.

Queue in Linked-List Structure

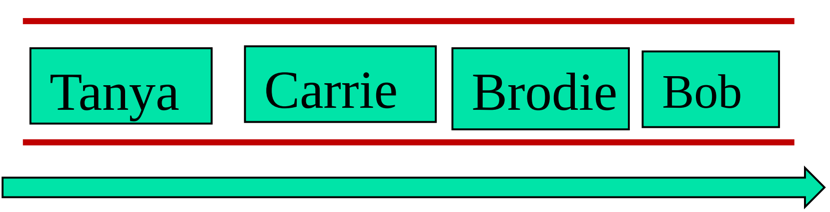


Python for Queue?

- Queues standard library
  - Queue class

```python
# OOP implementation of a queue
# the Queue class

class Queue:
    def __init__(self):
        self.items=[]
    def add(self,item):
        self.items.append(item)
    def delete(self):
        itemToDelete = self.items[0]
        del self.items[0]
        return itemToDelete
    def size(self):
        return len(self.items)
    def report(self):
        return self.items
```

```python
myQueue=Queue()
myQueue.add("Bob")
myQueue.add("Brodie")
myQueue.add("Carrie")
myQueue.add("Tanya")
print(myQueue.size())
print(myQueue.report())
print(myQueue.delete())
print(myQueue.report())
```



Python Standard Library “queue” Module

List, Set, Dict같은것으로 Queue를 지원하는것은 불편하고, 세가지 대표 queue종류을 한번에 지원!!

- The queue module provides a safe implementation of FIFO structure
  - Queue class implemented all the required locking semantics
- There are 3 types of Queue, which differ in the order of the entities retrieved
  - FIFO queue ➔ Queue class
  - LIFO queue (Works like a stack) ➔ LifoQueue class
  - Priority queue ➔ PriorityQueue class

```
import queue

a = queue.Queue(5)
b = queue.LifoQueue(5)
c = queue.PriorityQueue(5)

print("Successfully created 3 queues")
```

Result

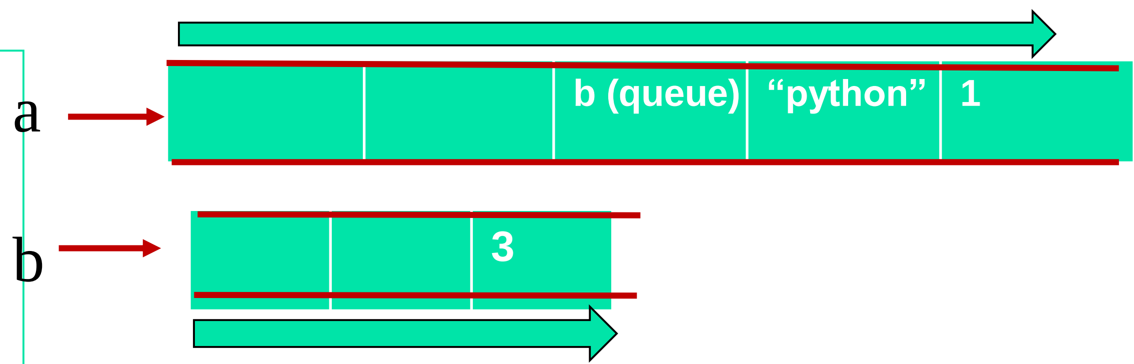
```
>>>
Successfully created 3 queues
```

“queue” Module – Queue Class [1/2]

- queue.**Queue**(x) : Construct a FIFO queue of size ‘x’
- queue.**Queue**() : Construct a FIFO queue of infinite size
- queue.**put**(x) : Put item into the queue. Item can be anything
- queue.**get**(x) : Delete the item and return that item

```python
import queue

a = queue.Queue(5)
b = queue.Queue(3)

a.put(1)
a.put("python")
a.put(b)

b.put(3)

print(a.get())
print(a.get())
print(a.get().get())
```

Return the queue ‘b’

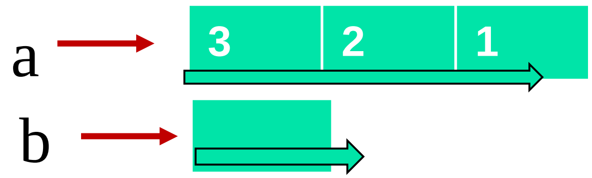


Result

```
>>>
1
python
3
```

“queue” Module – Queue class [2/2]

- queue.**qsize**() : Return the number of items in the queue
- queue.**empty()** : Return True if the queue is empty, False otherwise
- queue.**full**() : Return True if the queue is full, False otherwise




```
import queue

a = queue.Queue(3)
b = queue.Queue()

a.put(1)
a.put(2)
a.put(3)

print("qsize : ")
print(a.qsize())
print(b.qsize())
print()

print("Empty?")
print(a.empty())
print(b.empty())
print()

print("Full?")
print(a.full())
print(b.full())
print()
```

Result

```
>>>
qsize :
3
0

Empty?
False
True

Full?
True
False
```

“queue” Module – LiFoQueue class

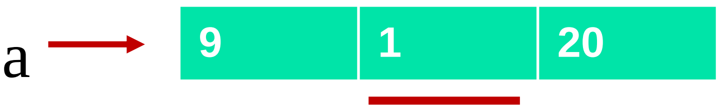
- Subclass of Queue class
- put(x), get(x), qsize(), empty(), full() are all similar with that of Queue class

```
>>> import queue
>>> a = queue.LifoQueue(3)
>>> a.put("Kim")
>>> a.put(55)
>>> a.put("SNU")
>>> a.qsize()
3
>>> a.get()
'SNU'
>>> a.qsize()
2
```

"queue" Module – PriorityQueue class

- A subclass of Queue class, retrieves entries in priority order (lowest first)
- put(x), get(x), qsize(), empty(), full() are all similar with that of Queue class

```
>>> import queue
>>> a = queue.PriorityQueue(3)
>>> a.put(20)
>>> a.put(1)
>>> a.put(9)
>>> a.qsize()
3
>>> a.get()
1
>>> a.qsize()
2
```



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  - Python dictionary data type
  - Python hashlib standard library

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Dictionary in Python [1/4]

Dictionaries in Python are implemented using hash tables. It is an array whose indexes are obtained using a hash function on the keys.

We declare an empty dictionary like this:

```
>>> D = {}
```

Then, we can add its elements:

```
>>> D['a'] = 1
>>> D['b'] = 2
>>> D['c'] = 3
>>> D
{'a': 1, 'c': 3, 'b': 2}
```

Dictionary in Python [2/4]

It's a structure with (key, value) pair:

```
D[key] = value
```

The string used to "index" the hash table D is called the "key". To access the data stored in the table, we need to know the key:

```
>>> D['b']
2
```

Dictionary in Python [3/4]

How we loop through the hash table?

```
>>> for k in D.keys():
...     print D[k]
...
1
3
2
```

If we want to print the (key, value) pair:

```
>>> for k,v in D.items():
...     print k,':',v
...
a : 1
c : 3
b : 2
```

Dictionary in Python [4/4]

Create a dictionary from two arrays

Using two Arrays of equal length, create a Hash object where the elements from one array (the keys) are associated with the elements of the other (the values):

```
>>> keys = ['a', 'b', 'c']
>>> values = [1, 2, 3]
>>> hash = {k:v for k, v in zip(keys, values)}
>>> hash
{'a': 1, 'c': 3, 'b': 2}
```

Hashing in Python [1/2]

Hashing

Here are some hashing samples using built-in **hash** function:

```
>>> map(hash, [0, 1, 2, 3])
[0, 1, 2, 3]
>>> map(hash, ['0','1','2','3'])
[6144018481, 6272018864, 6400019251, 6528019634]
>>> hash('0')
6144018481
```

As we can see from the example, Python is using different hash() function depending on the type of data.

Example of Hashing

Hashing based on Dept_name

- E.g. h(Music) = 1 h(History) = 2 h(Physics) = 3 h(Elec. Eng.) = 3

Id	name	Dept_name	salary
10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	80000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	60000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

bucket 0

bucket 1

15151	Mozart	Music	40000

bucket 2

32343	El Said	History	80000
58583	Califieri	History	60000

bucket 3

22222	Einstein	Physics	95000
33456	Gold	Physics	87000
98345	Kim	Elec. Eng.	80000

bucket 4

12121	Wu	Finance	90000
76543	Singh	Finance	80000

bucket 5

76766	Crick	Biology	72000

bucket 6

10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000

bucket 7

Hashing in Python [2/2]

Python provides **hashlib** for secure hashes and message digests:

md5(), sha*():

```
>>> import hashlib

>>> hashlib.md5('a')

>>> hashlib.md5('a').digest()
'\x0c\xc1u\xb9\xc0\xf1\xb6\xa81\xc3\x99\xe2iw&a;'
>>> hashlib.md5('a').hexdigest()
'0cc175b9c0f1b6a831c399e269772661'

>>> hashlib.sha512('a')

>>> hashlib.sha512('a').digest()
'\x1f@\xfc\x92\xda$\x16\x94u\ty\xee1\xf5\x82\xf2\xd5\xd7\xd2\x8e\x183]\xe0Z\xbcT\xd0\
>>> hashlib.sha512('a').hexdigest()
'1f40fc92da241694750979ee6cf582f2d5d7d28e18335de05abc54d0560e0f5302860c652bf08d56025
>>>
```

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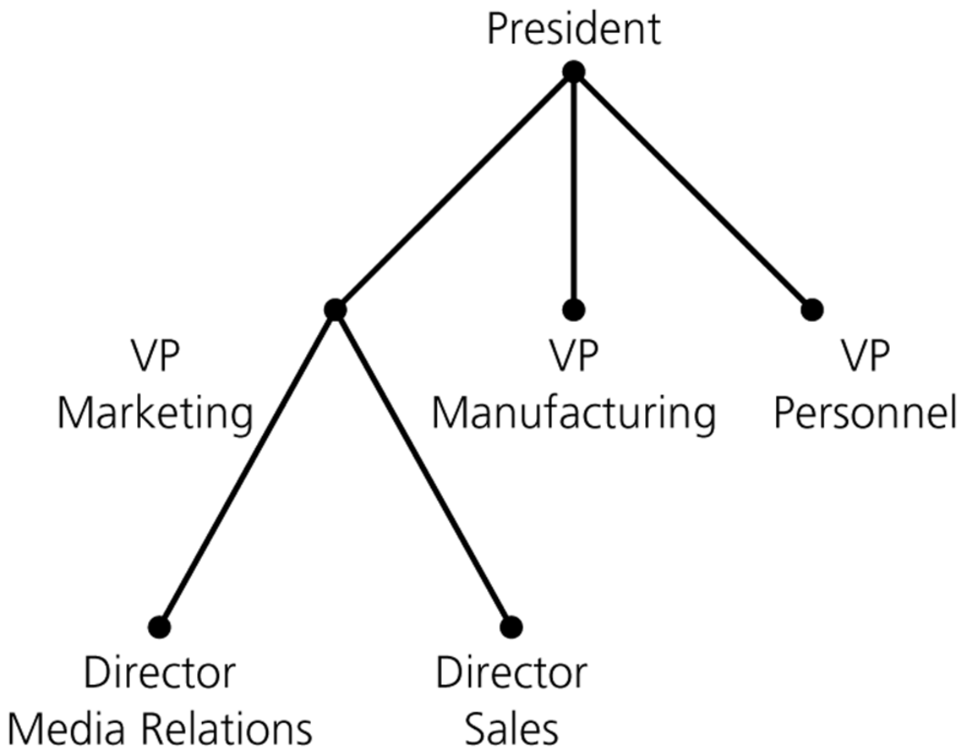
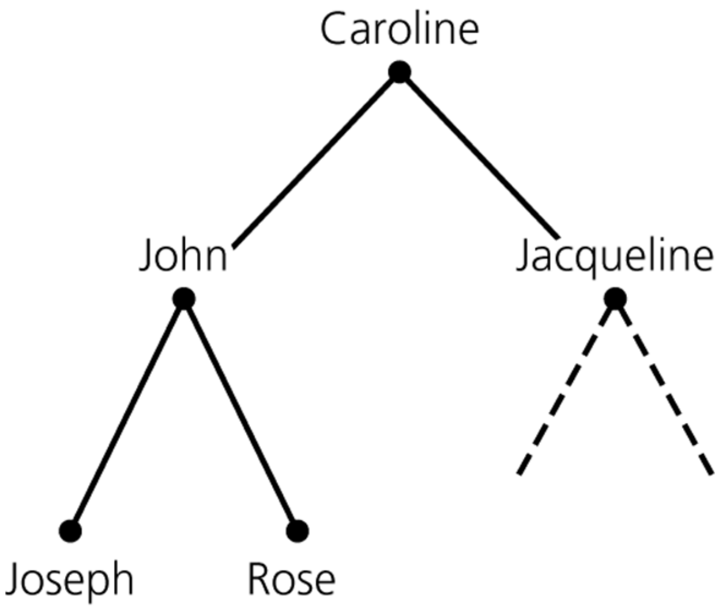
Open source code를
이용하는 경우도
있고, 본인이 개발을
해도 되고!

Real-World Tree Structures

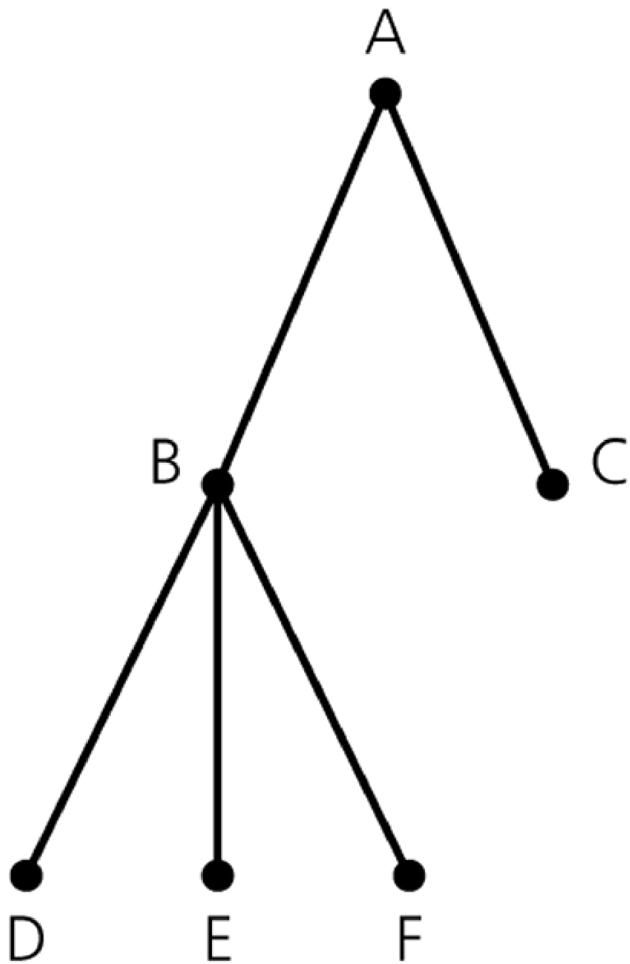
An organization chart



A family tree



A general tree

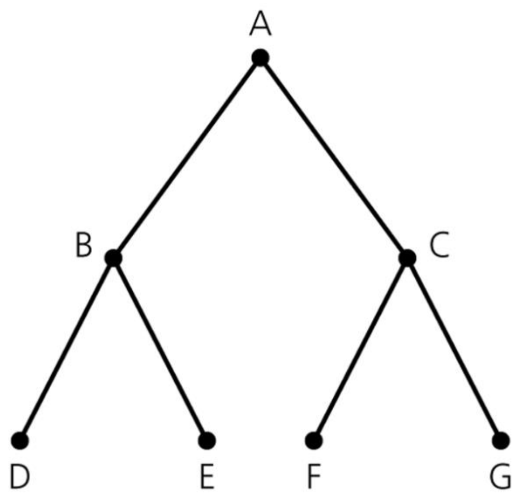


Terminology

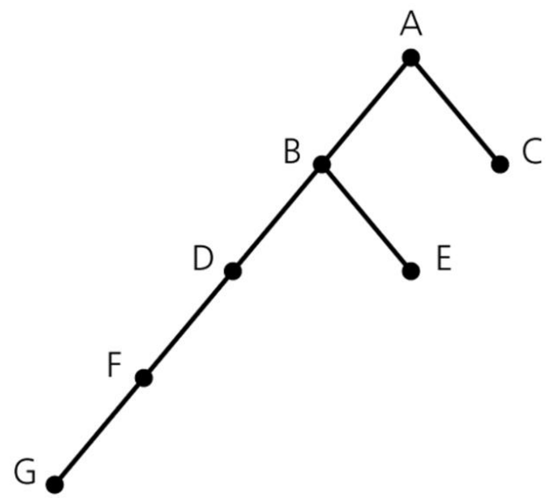
node or vertex
edge
parent
child
siblings
root
leaf
ancestor
descendant
subtree

Height of a Tree

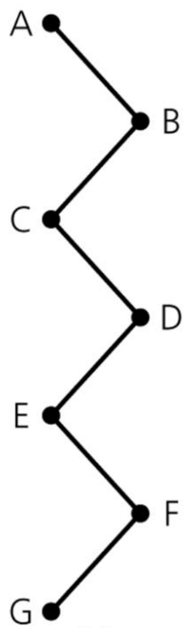
- The number of nodes on the longest path from the root to a leaf



Height 3



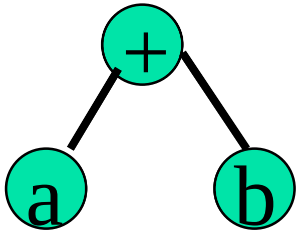
Height 5



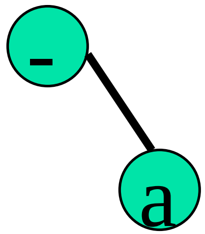
Height 7

Binary Tree Form of Arithmetic Expression

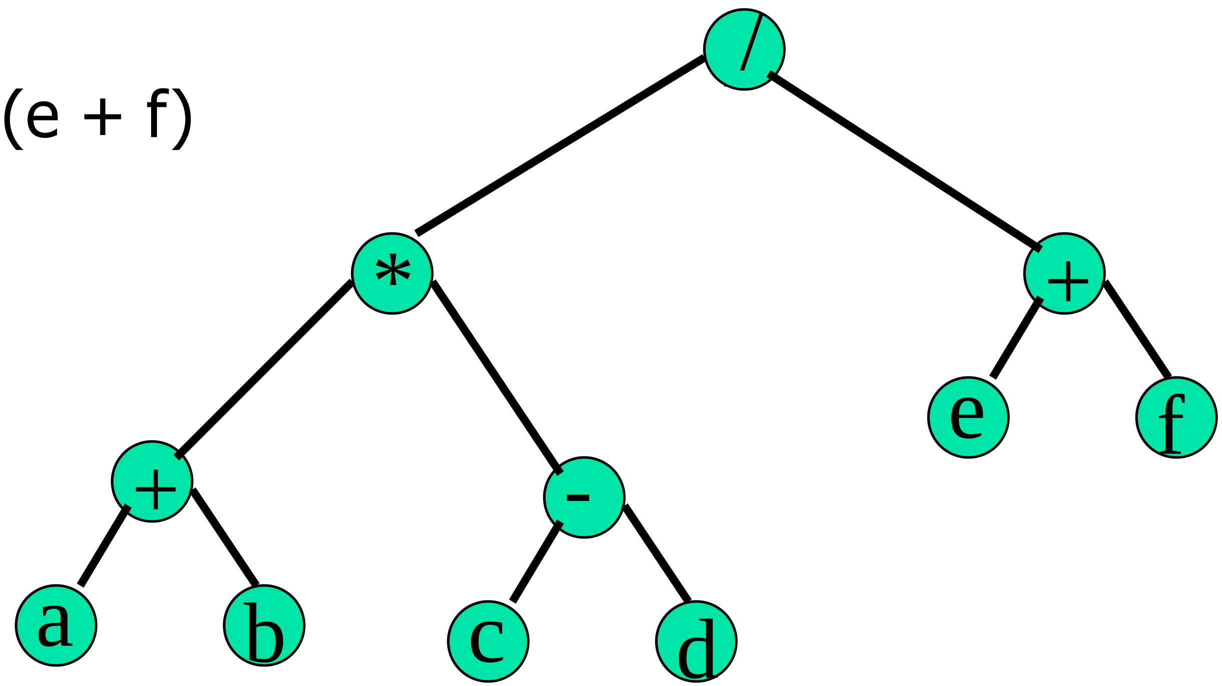
- a + b



- \- a



- (a + b) * (c – d) / (e + f)

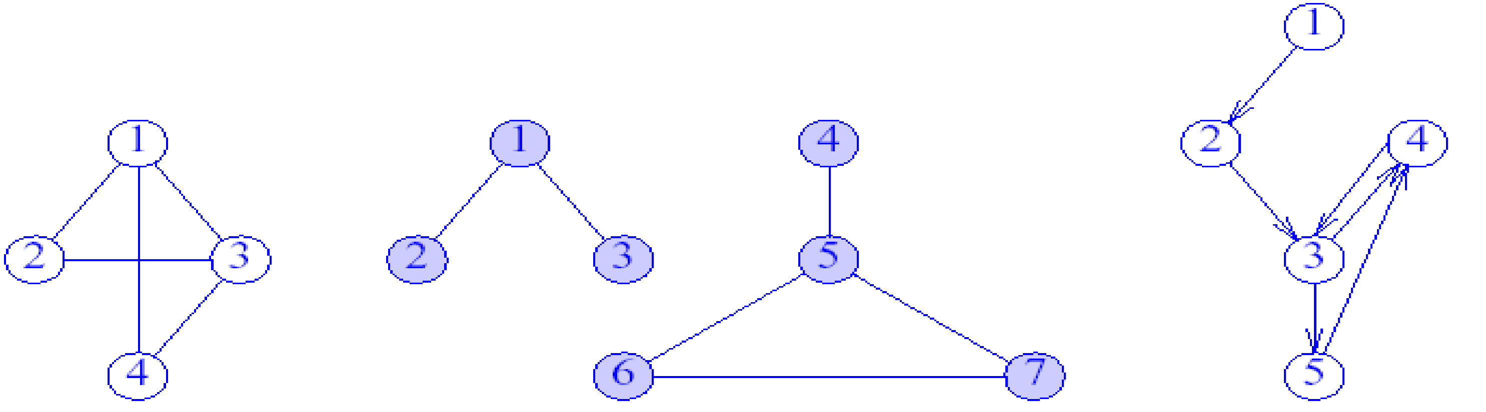


Graph Definition [1/2]

- Graph G = (V, E)
  - Finite set V (=vertices, nodes, points)
  - Finite set E (=edges, arcs, lines)

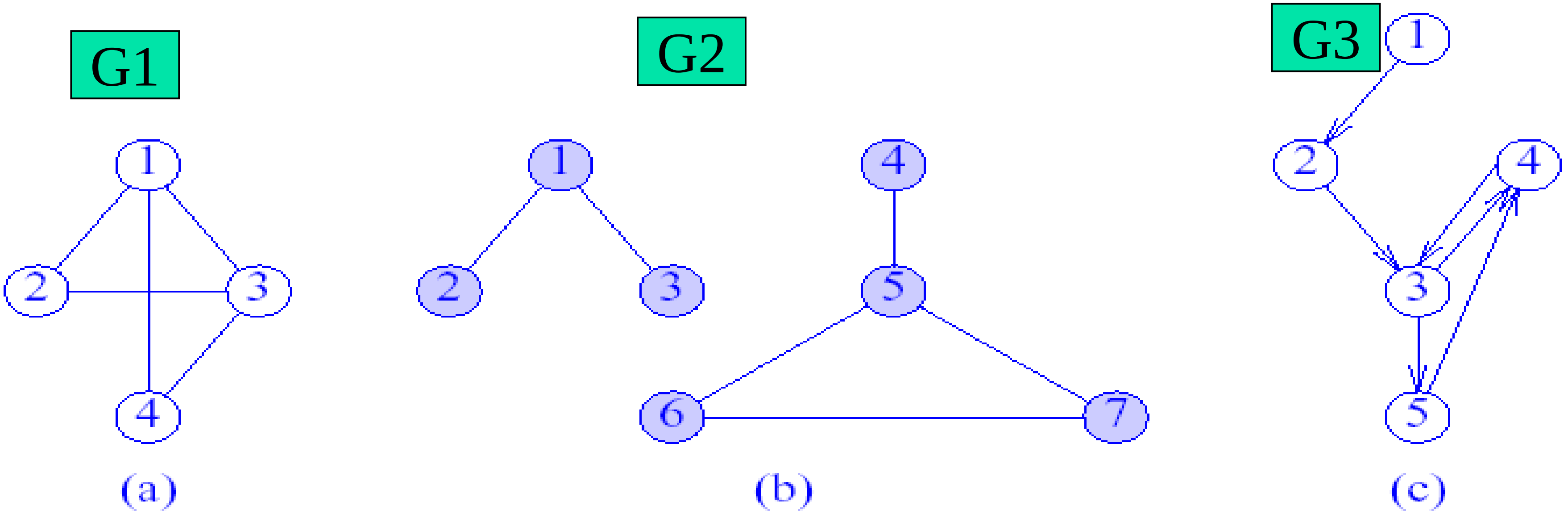
- Directed edge: orientation
- Undirected edge: no orientation

- Vertices i and j are **adjacent** vertices iff (i,j) is an edge in the graph
- Edge(i,j) is **incident** on the vertices i and j



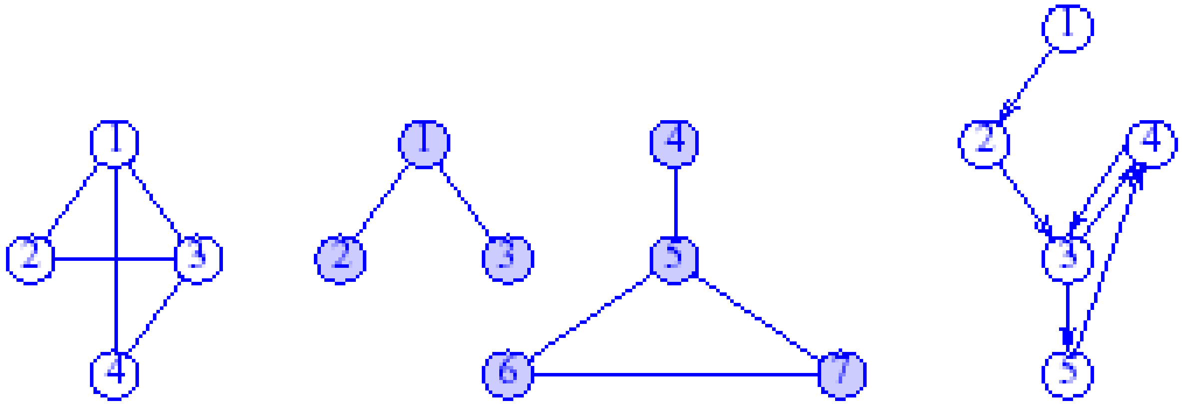
(a) (b) (c)

Graph Definition [2/2]



- $G_1 = (V_1, E_1)$:
  $V_1 = \{1, 2, 3, 4\}$, $E_1 = \{(1,2), (1,3), (2,3), (1,4), (3,4)\}$
- $G_2 = (V_2, E_2)$:
  $V_2 = \{1, 2, 3, 4, 5, 6, 7\}$, $E_2 = \{(1,2), (1,3), (4,5), (5,6), (5,7),(6,7)\}$
- $G_3 = (V_3, E_3)$:
  $V_3 = \{1, 2, 3, 4, 5\}$, $E_3 = \{(1,2), (2,3), (3,4), (4,3), (3,5), (5,4)\}$

Adjacency Matrix Representation for Graph



(a) (b) (c)

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(a)

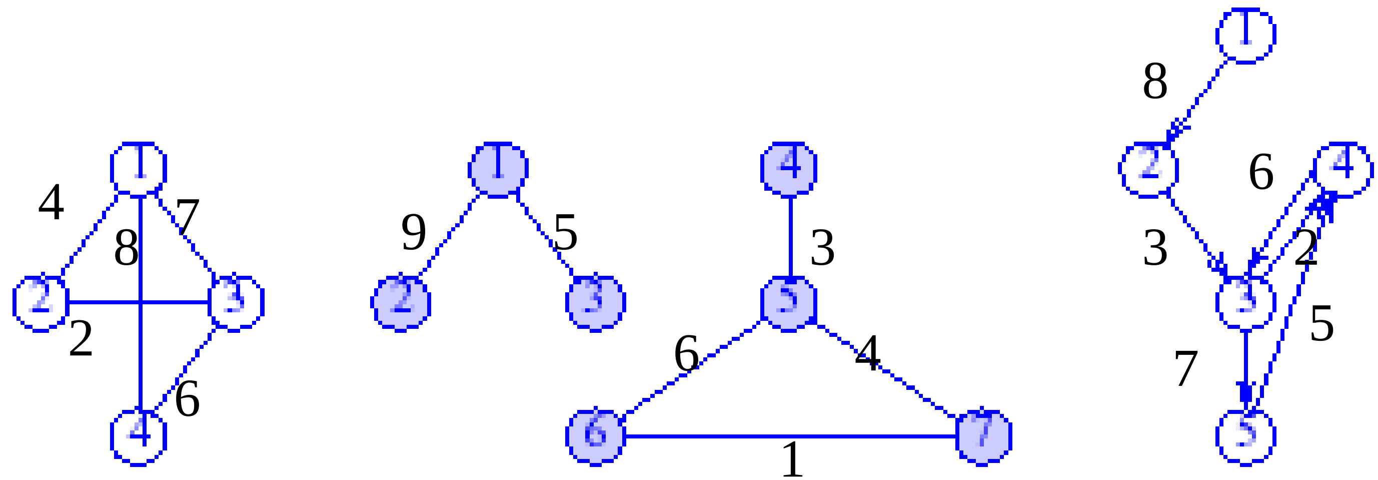
$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(b)

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(c)

Cost-Adjacency Matrix Represenation for Weighted Graph



(a) (b) (c)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & - & 4 & 7 & 8 \\ 2 & 4 & - & 2 & - \\ 3 & 7 & 2 & - & 6 \\ 4 & 8 & - & 6 & - \end{array}$$

(a)

$$\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & - & 9 & 5 & - & - & - & - \\ 2 & 9 & - & - & - & - & - & - \\ 3 & 5 & - & - & - & - & - & - \\ 4 & - & - & - & - & 3 & - & - \\ 5 & - & - & - & 3 & - & 6 & 4 \\ 6 & - & - & - & - & 6 & - & 1 \\ 7 & - & - & - & - & 4 & 1 & - \end{array}$$

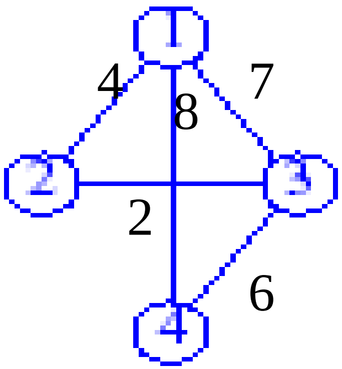
(b)

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 8 & - & - & - \\ 2 & - & - & 3 & - & - \\ 3 & - & - & - & 2 & 7 \\ 4 & - & - & 6 & - & - \\ 5 & - & - & - & 5 & - \end{array}$$

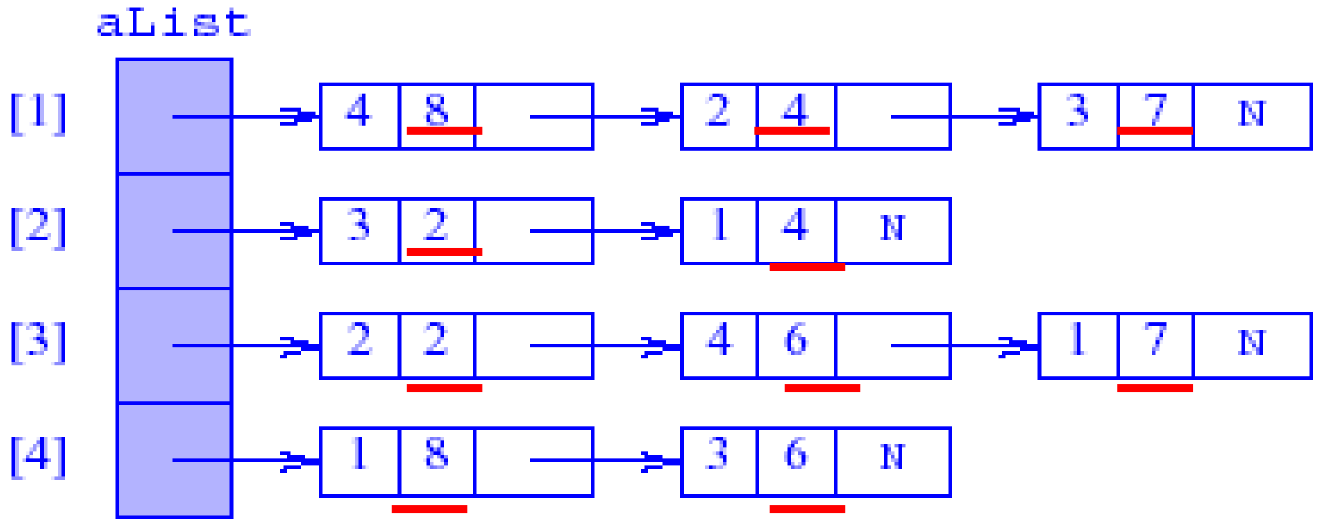
(c)

- denotes a `null` value

Linked-Adjacency List Representation for Weighted Graph



(a)



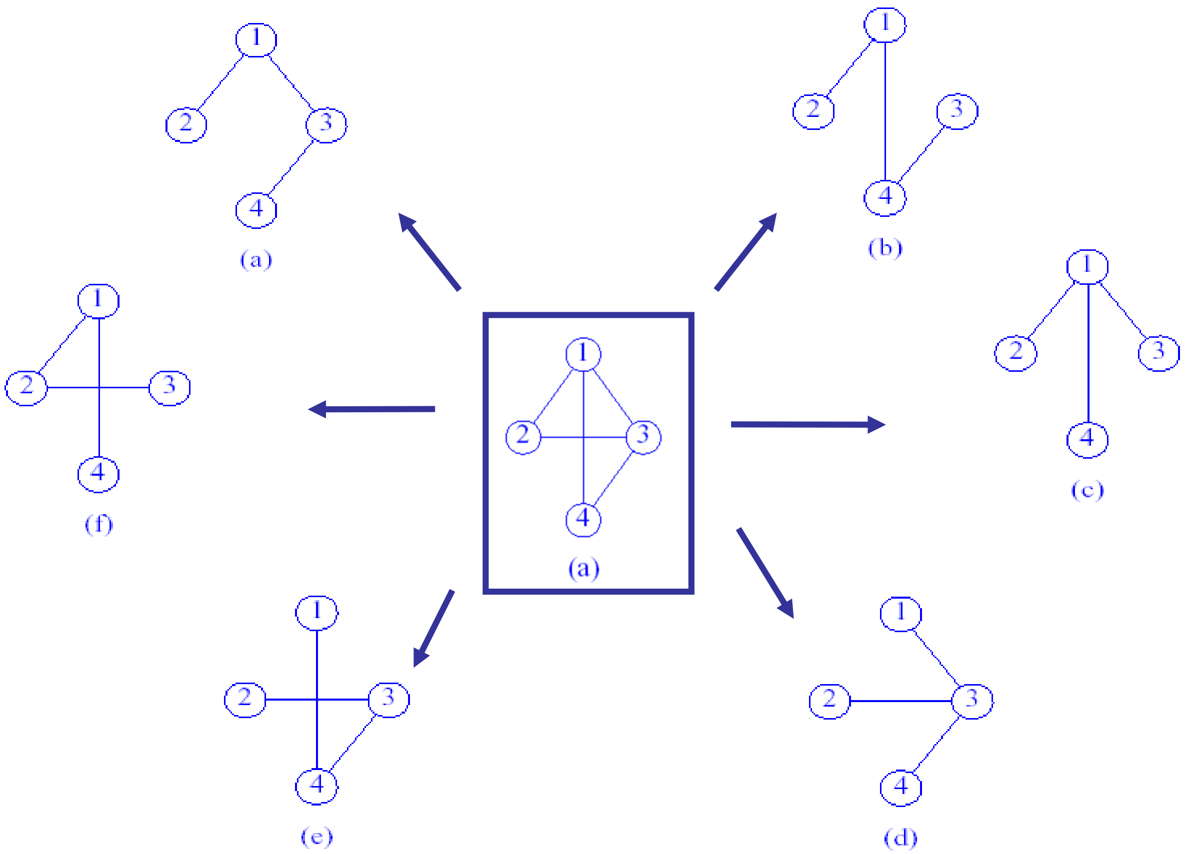
N denotes a null link

$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & - & 4 & 7 & 8 \\
2 & 4 & - & 2 & - \\
3 & 7 & 2 & - & 6 \\
4 & 8 & - & 6 & -
\end{array}
$$

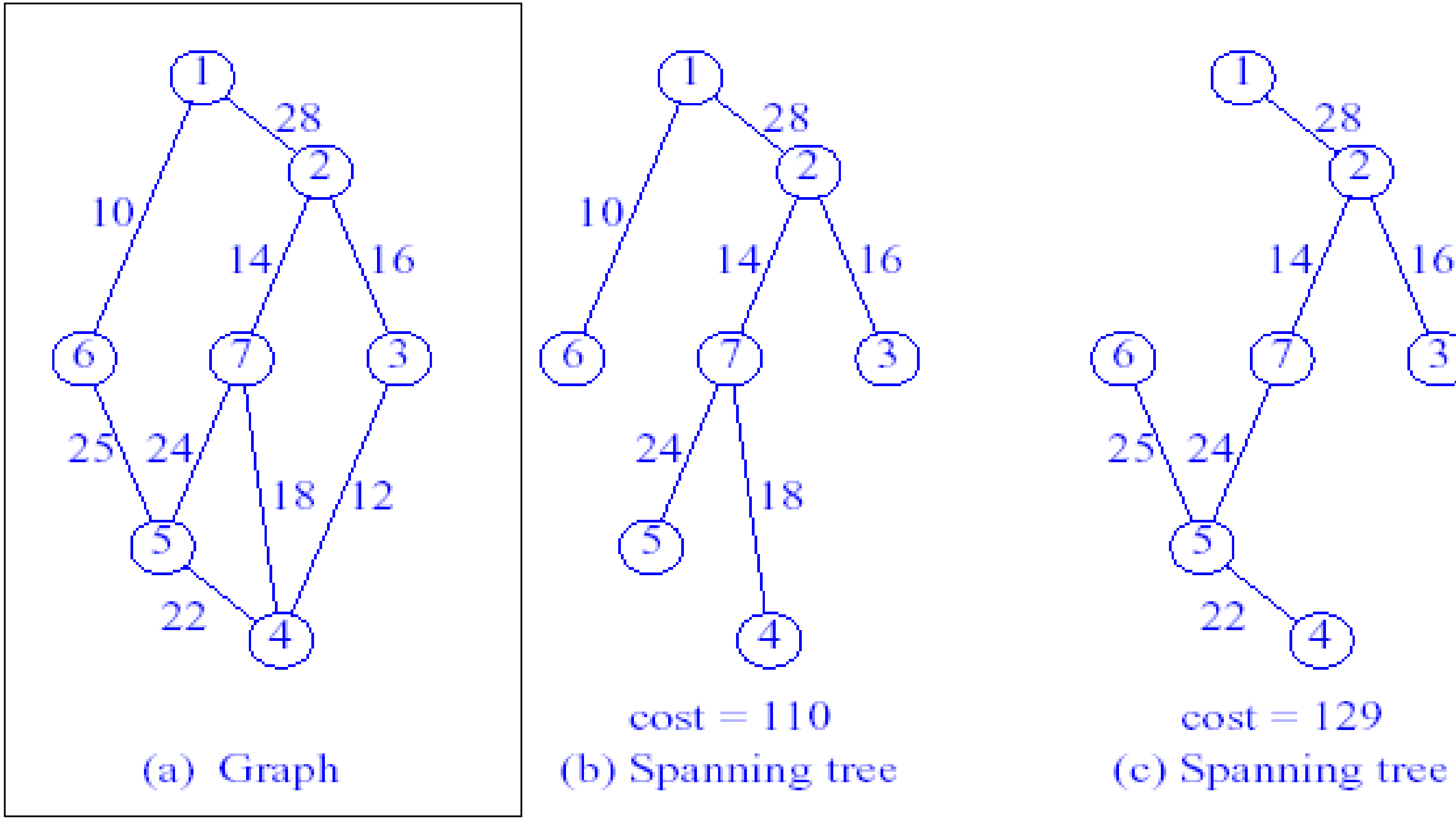
(a)

Graph Application: Spanning Trees

- A spanning tree is a tree and a subgraph of G that contains all the vertices of G

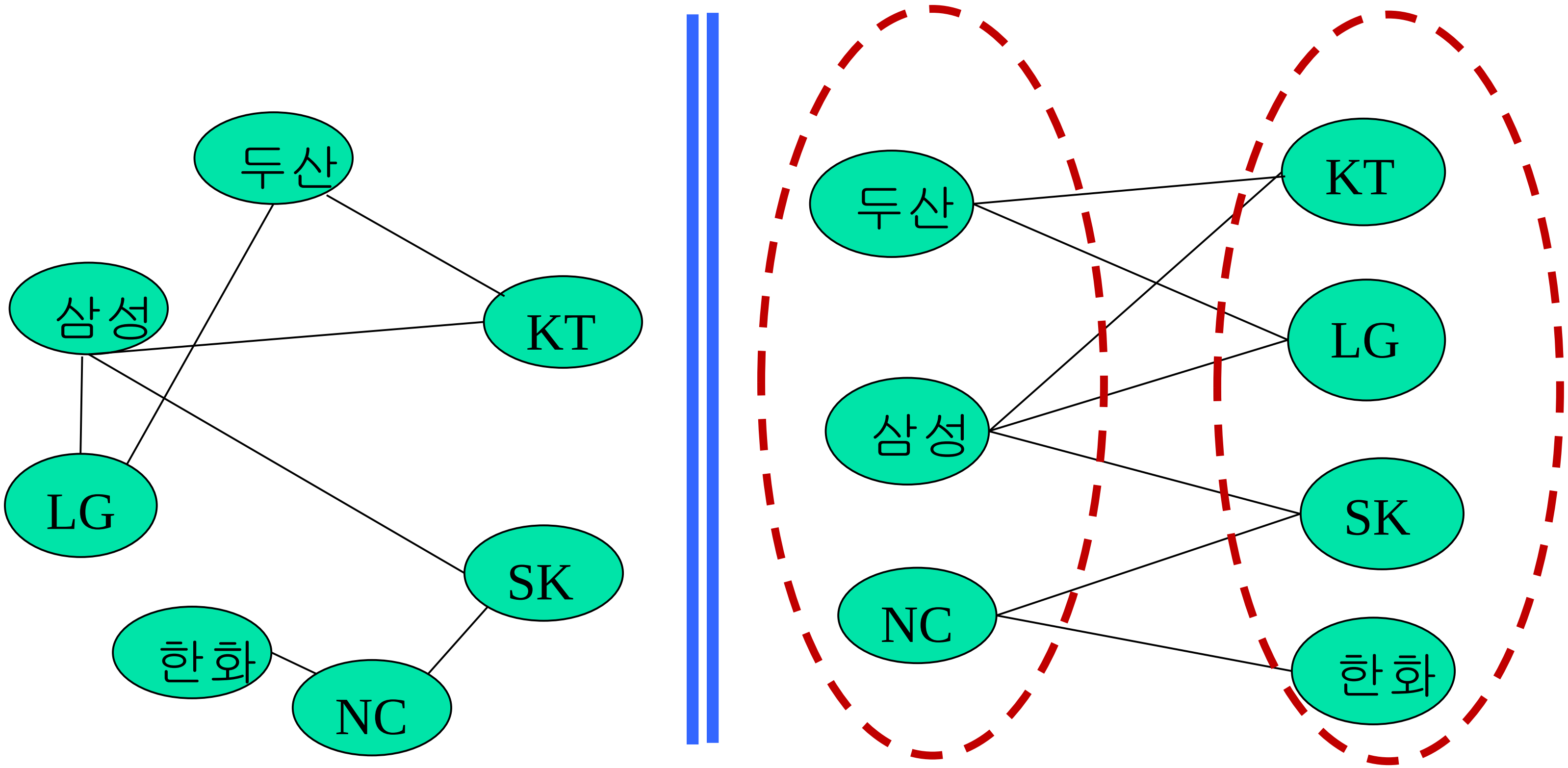


Graph Application:
Weighted Graph and its Spanning Trees
** Minimum Cost Spanning Tree **



Graph Application: Bipartite Graphs

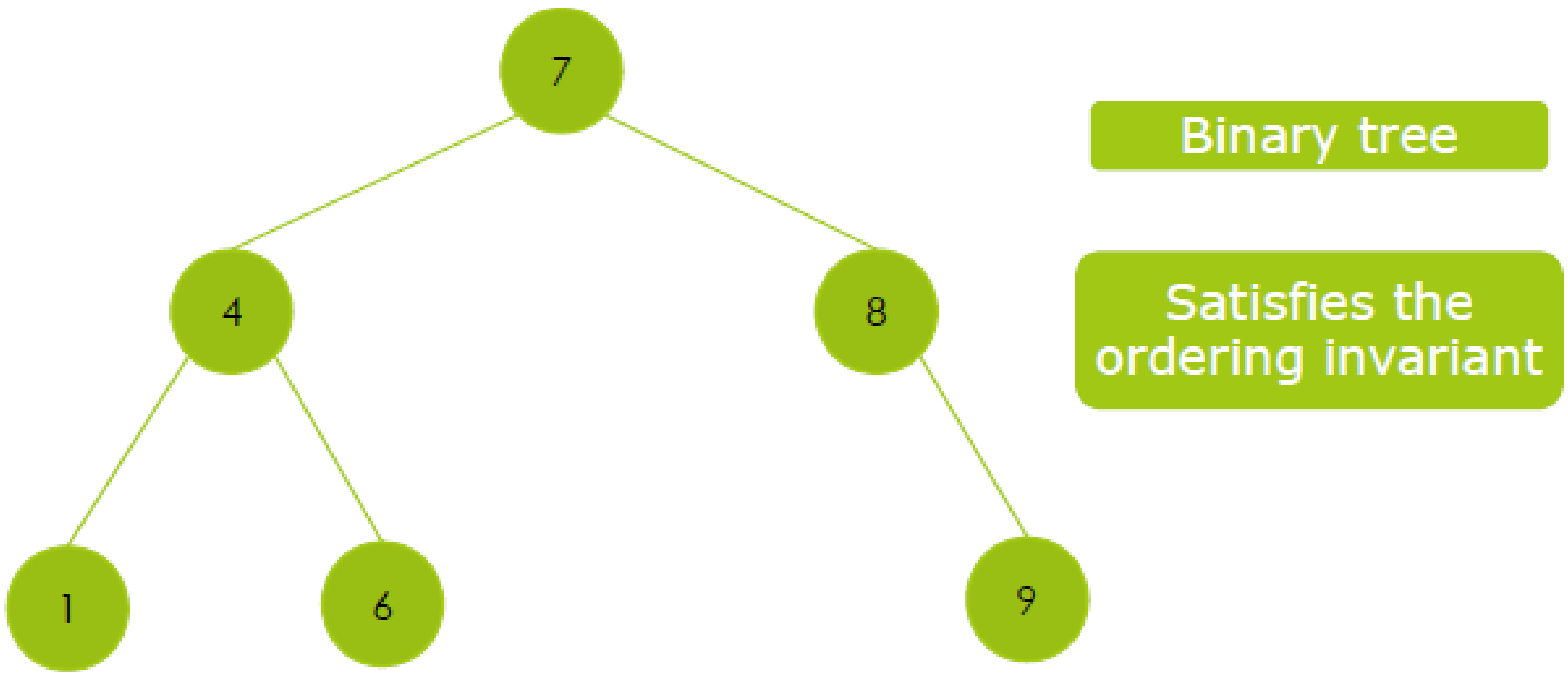
- Partition the vertex set into two subsets A and B so that every edge has one endpoint in A and the other in B
  - Example: 프로야구 개막후 3일동안 (두산, LG), (삼성, 한화), (LG, 삼성), (두산, KT), (NC,SK), (NC, 한화), (SK, 삼성) 의 경기가 있었다. 아직 한번도 서로 경기를 안한 팀들을 2 group을 묶어라? (즉, 이 경기Graph에서 Bipartite Graph를 찾아라?)



OOP of Binary Search Tree

Binary Search Tree

BST ordering invariant: At any node with key *k*, all keys of elements in the left subtree are strictly less than *k* and all keys of elements in the rightsubtree are strictly greater than *k* (assume that there are no duplicates in the tree)



BST TreeNode Class [1/3]

```
class TreeNode:

  def __init__(self, key, val, left=None,
               right=None, parent=None):
          self.key = key
          self.payload = val
          self.leftChild = left
          self.rightChild = right
          self.parent = parent

  def hasLeftChild(self):        # return True or False

  def hasRightChild(self):       # return True or False

  def isLeftChild(self):         # return True or False

  def isRightChild(self):        # return True or False
```

```
def isRoot(self):          # return True or False

def isLeaf(self):          # return True or False

def hasAnyChildren(self): # return True or False

def hasBothChildren(self): # return True or False

def replaceNodeData(self, key, value, lc, rc):
```

BST TreeNode Class [2/3]

```
def findSuccessor(self):
    # self node의 next key value를 가진 node를 찾아서 self node가 delete 되면
    #  그자리에 넣기 위한 작업
    self가 leaf node이면 return no-successor

    self가 right child를 가지고 있으면 return right child side’s minimum value

    self가 parent가 있고 left child를 가지고 있으면 return parent
```

```
def findMin(self):
        if self.hasLeftChild():
            return self.leftChild.findMin()
        else:
            return self
```

BST Tree Node [3/3]

```python
def sliceOut(self):
    """ move node's child to its own position """
    if self.parent and self.hasRightChild():
        if self.isLeftChild():
            self.parent.leftChild = self.rightChild
        else:
            self.parent.rightChild = self.rightChild
    # !!! the successor node never has a left child.
```

```python
def inorder_traverse(self):
    # ! in-order traverse prints out an sorted list.
    if self.hasLeftChild():
        self.leftChild.inorder_traverse()
    print(self.payload)
    if self.hasRightChild():
        self.rightChild.inorder_traverse()
```

BinarySearchTree Class [1/7]

```
class BinarySearchTree:

    def __init__(self):
        self.root = None
        self.size = 0

    def length(self):

    def __len__(self):

    def __iter__(self):

```

BinarySearchTree Class [2/7]

```
def put(self, key, val):
        if self.root:
            self._put(key, val, self.root)
        else:
            self.root = TreeNode(key, val)
        self.size += 1
```

def _put(self, key, val, currentNode):

key 값이 currentNode.key 보다 작으면

currentNode에 left child가 있으면 _put() recursion with the left child

currentNode에 left child가 없으면 key값을 가진 node를 생성하여 currentNode의 left child로 만듬

key 값이 currentNode.key 보다 크면 (key이므로 값이 같을수는 없다)

currentNode에 right child가 있으면 _put() recursion with the right child

currentNode에 right child가 없으면 key값을 가진 node를 생성하여 currentNode에 right child로 만듬

BinarySearchTree Class [3/7]

```
def get(self, key):
    if self.root:
        res = self._get(key, self.root)
        if res:
            return res.payload
        else:
            return None
    return None
```

```
def _get(self, key, currentNode):
    current node가 없으면 return None
    current node의 key가 원하는 key면 return current node
    current node의 key가 원하는 key보다 크면
        current node의 left child를 가지고 _get() recursion
    current node의 key가 원하는 key보다 작으면
        current node의 right child를 가지고 _get() recursion
```

```
def __setitem__(self, k, v):
    self.put(k, v)
```

```
def __getitem__(self, key):

def __contains__(self, key):
```

BinarySearchTree Class [4/7]

```
def delete(self, key):
    if self.size > 1:
        nodeToRemove = self._get(key, self.root)
        if nodeToRemove:
            self.remove(nodeToRemove)
            self.size -= 1
        else:   raise KeyError('Error, key is not in tree')
    elif self.size == 1 and self.root.key == key:
        self.root = None
        self.size = 0
    else:   raise KeyError('Error, key not in tree')
```

BinarySearchTree Class [5/7]

def remove(self, currentNode):

current node가 leaf node이면

current node가 parent node의 left child이면 parent node의 left child part를 none으로 변경

current node가 parent node의 right child이면 parent node의 right child part를 none으로 변경

current node가 leaf node가 아니면 child가 1개 or 2개가 있는 경우

(A) left child와 right child를 다 가지고 있는 경우

replace current node with next largest node (only key and payload)

successor's right child move to its parent's position. This is done with 'node.sliceOut()'

(B) left child를 가지고 있는데

(B-1: LL type) current node가 parent node의 left child이면

parent node의 left child part를 current node의 left child로 변경

(B-2: RL type) current node가 parent node의 right child이면

parent node의 right child part를 current node의 left child로 변경

(C) right child를 가지고 있는데

(C-1: LR type) current node가 parent node의 left child이면

parent node의 right child part를 current node의 right child로 변경

(C-2: RR type) current node가 parent node의 right child이면

parent node의 right child part를 current node의 left child로 변경

Case Analysis of Remove [6/7]

A

```
#          parent
#            |
#        current node << REMOVE
#       /            \
#  left-child      right-child
#------------------------------------------------------------
# REPLACE CURRENT NODE with NEXT LARGEST NODE (only key and payload)
#
#        current node << REMOVE
#       /            \
#  left-child      right-child << NEXT?
#                    /
#           next-left-child
#           /
#   next-next-left-child  <<< NEXT!!! = (SUCCESSOR)
#                        \
#                         Successor's rightChild
#------------------------------------------------------------
# Successor's right child move to its parent's position.
# This is done with 'node.sliceOut()'
#
#         (SUCCESSOR)
#       /            \
#  left-child      right-child << NEXT?
#                    /
#           next-left-child
#           /
#        Successor's right child
```

B-1: LL case

```
# current node is left child of its parent node.
#                    parent
#                   /
#              currentnode << remove
#             /
#    childnode
```

B-2: RL case

```
# current node is right child of its parent node.
#                  parent
#                     \
#                    currentNode <<< REMOVE
#                     /
#                childnode
```

C-1: LR case

```
# current node is left child of its parent node.
#             parent
#            /
#        currentNode <<< REMOVE
#                  \
#                 childnode
```

C-2: RR case

```
# current node is right child of its parent node.
#       parent
#            \
#           currentNode <<< REMOVE
#                     \
#                   childnode
```

BinarySearchTree Class [7/7]

```
def main():
    bst = BinarySearchTree()
    input_data = (17, 5, 25, 2, 11, 29, 38, 9, 16, 7, 8)
    for i in input_data:
        bst.put(i, i)
    bst.root.inorder_traverse()
    #
    print('remove 5')
    bst.delete(5)
    bst.root.inorder_traverse()
    #
    print(‘put 39')
    bst.put(39, 39)
    bst.root.inorder_traverse()
```