

Prime Number Generator with Python

(** With help of CMU 15-110 Class Material)

Prime Numbers

- An integer is “prime” if it is not divisible by any smaller integers except 1.
- 10 is **not** prime because $10 = 2 \times 5$
- 11 **is** prime
- 12 is **not** prime because $12 = 2 \times 6 = 2 \times 2 \times 3$
- 13 **is** prime
- 15 is **not** prime because $15 = 3 \times 5$

Testing Divisibility in Python

- x is “divisible by” y if the remainder is 0 when we divide x by y
- 15 is divisible by 3 and 5, but not by 2:

```
>>> 15 % 3
```

```
0
```

```
>>> 15 % 5
```

```
0
```

```
>> 15 % 2
```

```
1
```

IsPrime(): dumb version

```
def IsPrime_dumb(n):  
    if (n < 2):  
        return False  
    for factor in range(2, n):  
        if (n % factor == 0):    # 모든숫자 n에 대해서 n번의 modulo 계산필요  
            return False  
    return True  
  
for i in range(1,100):  
    if IsPrime_dumb(i):  
        print(i)
```

A cartoon illustration of the ancient Greek mathematician Eratosthenes. He is depicted as an older man with a white beard and hair, wearing a white tunic and a red shawl draped over his left shoulder. He is standing and gesturing with his right hand. Behind him is a bright blue circular halo.

SIEVE OF ERATOSTHENES

A 2000 year old algorithm
(procedure) for generating a table
of prime numbers.

2, 3, 5, 7, 11, 13, 17, 23, 29, 31, ...

What Is a "Sieve" or "Sifter"?

Separates stuff you want from stuff you don't:



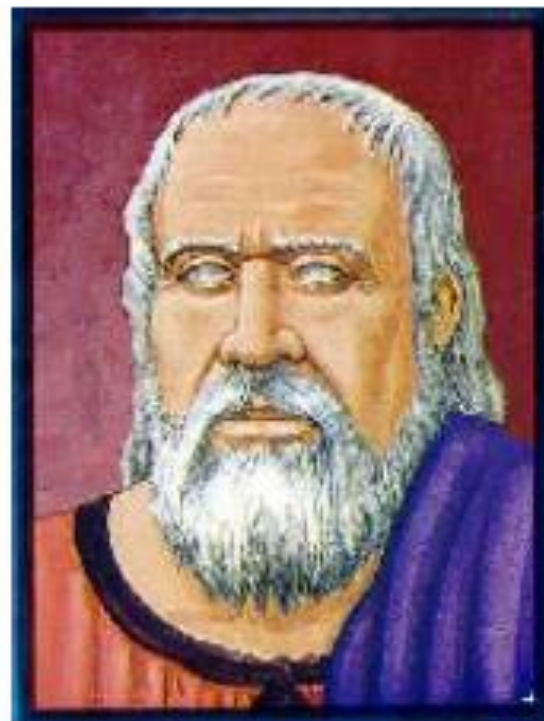
We want to separate prime numbers.

The Sieve of Eratosthenes

Start with a table of integers from 2 to N .

Cross out all the entries that are divisible by the primes known so far.

The first value remaining is the next prime.



Finding Primes Between 2 and 50

2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

2 is the first prime

Finding Primes Between 2 and 50

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Filter out everything divisible by 2.

Now we see that 3 is the next prime.

Finding Primes Between 2 and 50

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Filter out everything divisible by 5.

Now we see that 7 is the next prime.

Finding Primes Between 2 and 50

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Filter out everything divisible by 7.

Now we see that 11 is the next prime.

Finding Primes Between 2 and 50

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Since $11 \times 11 > 50$, all remaining numbers must be primes. Why?

An Algorithm for Sieve of Eratosthenes

Input: A number n :

1. Create a list *numlist* with every integer from 2 to n , in order.
(Assume $n > 1$.)
2. Create an empty list *primes*.
3. For each element in *numlist*
 - a. If element is not marked, copy it to the end of *primes*.
 - b. Mark every number that is a multiple of the most recently discovered prime number.

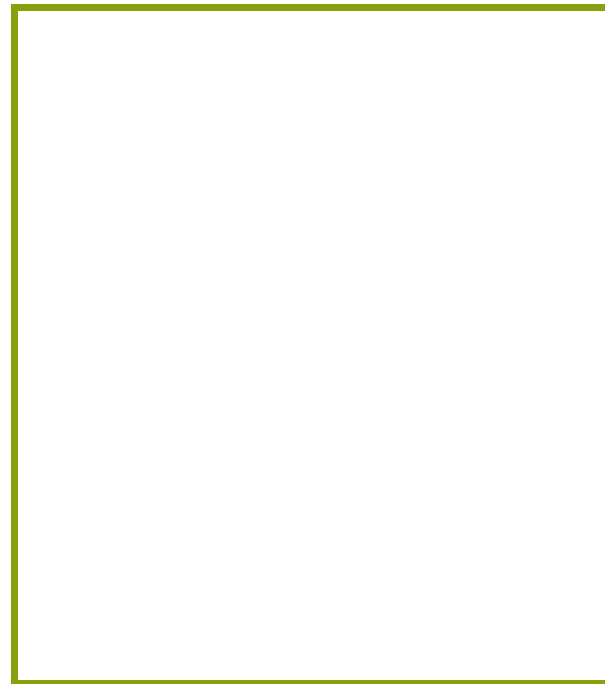
Output: The list of all prime numbers less than or equal to n

Automating the Sieve

numlist

2	3	4	5
6	7	8	9
10	11	12	13
...			

primes



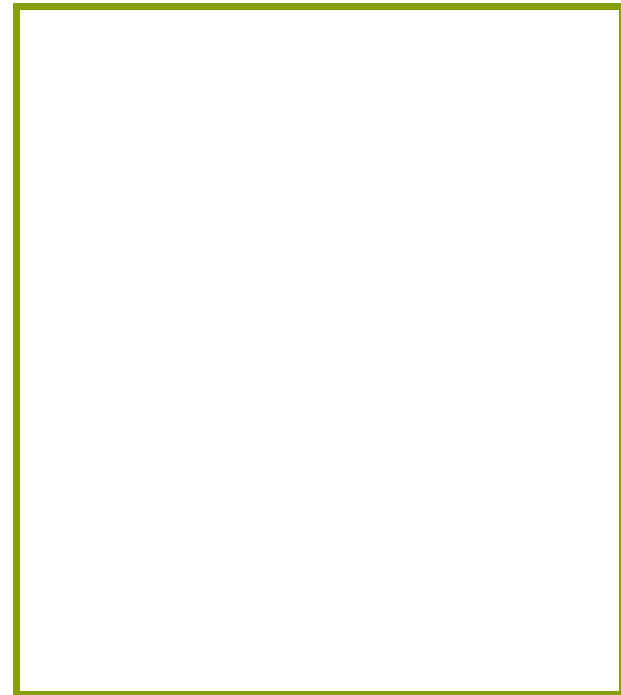
Use *two* lists: candidates, and confirmed primes.

Steps 1 and 2

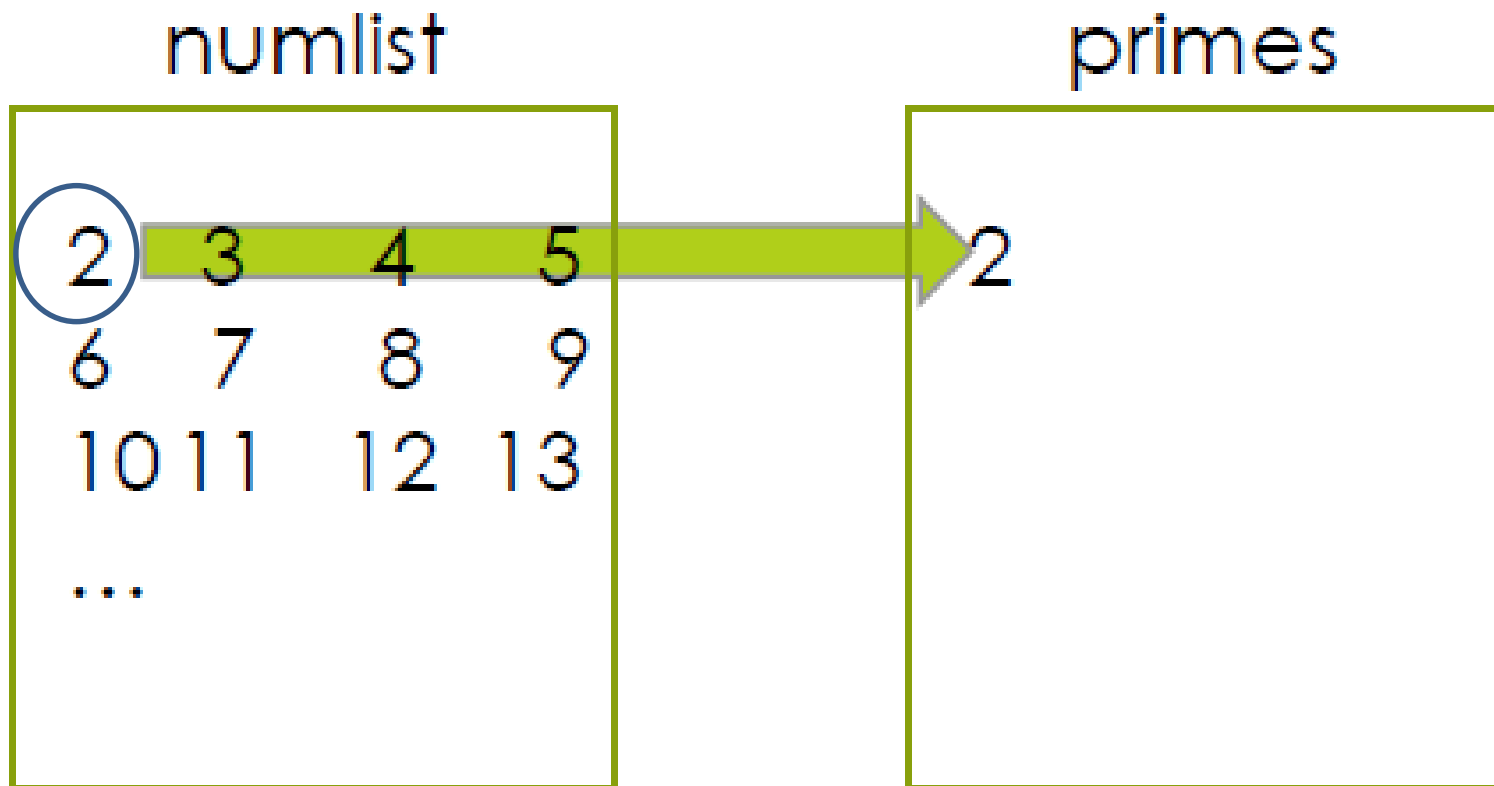
numlist

2	3	4	5
6	7	8	9
10	11	12	13
...			

primes



Step 3a



Append the current number in numlist to the end of primes.

Step 3b

numlist

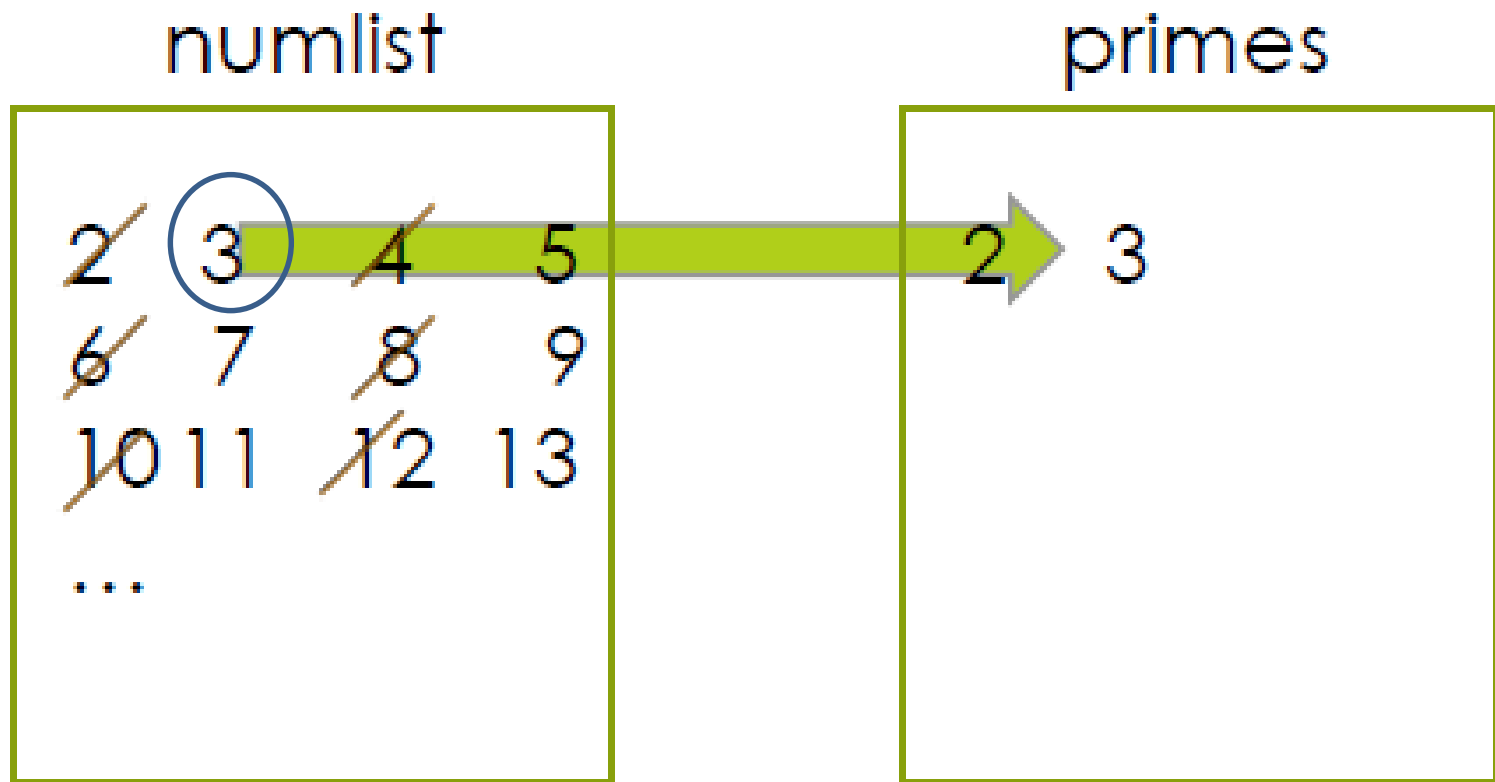
2	3	4	5
6	7	8	9
10	11	12	13
...			

primes

2

Cross out all the multiples of the last number in primes.

Iterations



Append the current number in numlist to the end of primes.

Iterations

numlist

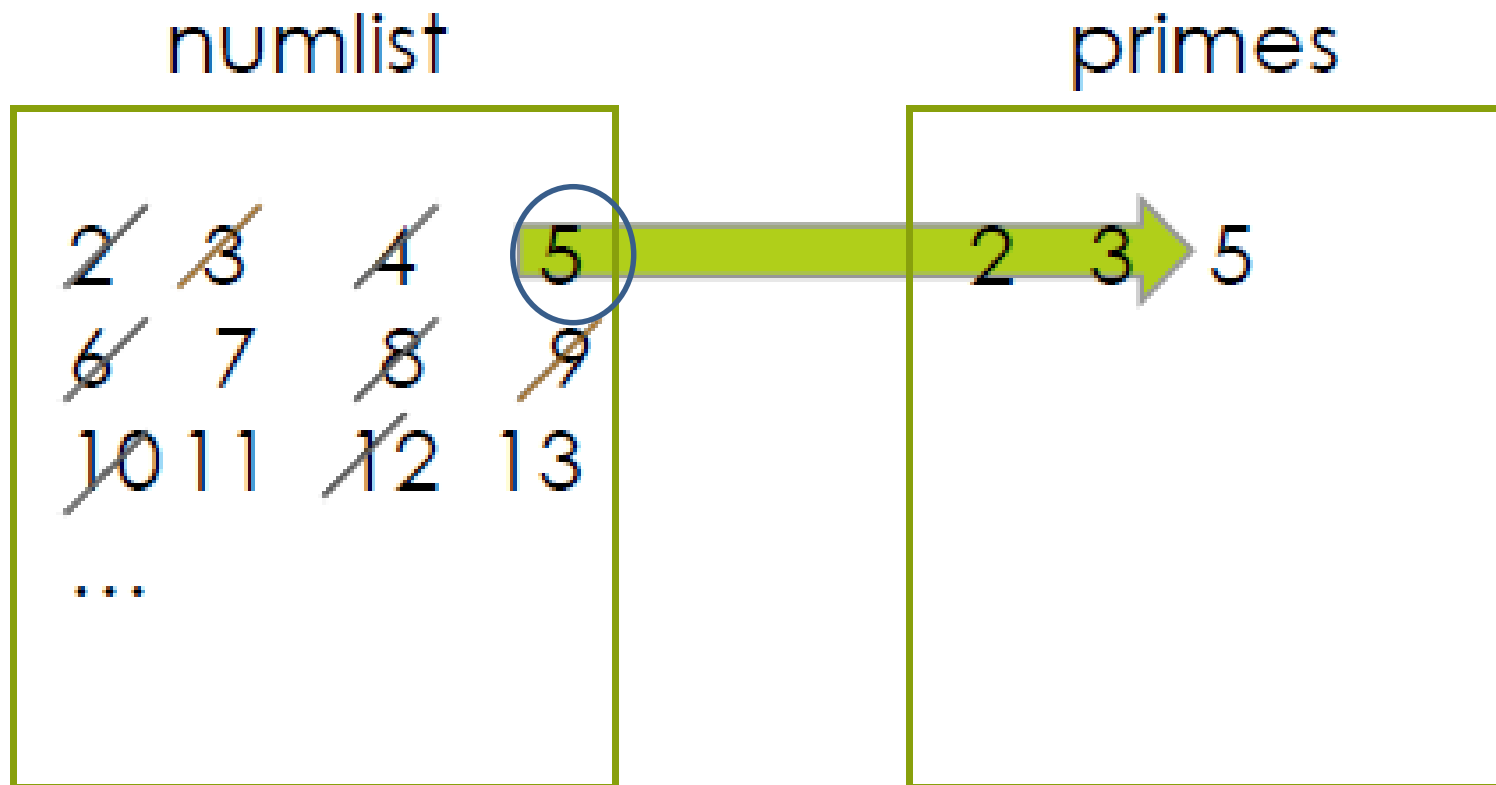
2	3	4	5
6	7	8	9
10	11	12	13
...			

primes

2	3
---	---

Cross out all the multiples of the last number in primes.

Iterations



Append the current number in numlist to the end of primes.

Iterations

numlist

2	3	4	5
6	7	8	9
10	11	12	13
...			

primes

2	3	5
---	---	---

Cross out all the multiples of the last number in primes.

An Algorithm for Sieve of Eratosthenes

Input: A number n :

1. Create a list *numlist* with every integer from 2 to n , in order.
(Assume $n > 1$.)
2. Create an empty list *primes*.
3. For each element in *numlist*
 - a. If element is not marked, copy it to the end of *primes*.
 - b. Mark every number that is a multiple of the most recently discovered prime number.

Output: The list of all prime numbers less than or equal to n

Implementation Decisions

- How to implement *numlist* and *primes*?
 - For *numlist* we will use a list in which crossed out elements are marked with the special value `None`. For example,
`[None, 3, None, 5, None, 7, None]`
- Use a helper function for step 3.b. We will call it `sift`.

Relational Operators

- If we want to compare two integers to determine their relationship, we can use these relational operators:

<	less than	<=	less than or equal to
>	greater than	>=	greater than or equal to
==	equal to	!=	not equal to

- We can also write compound expressions using the Boolean operators **and** and **or**.

$x \geq 1$ and $x \leq 1$

Sifting: Removing Multiples of a Number

```
def sift(lst,k):  
    # marks multiples of k with None  
    i = 0  
    while i < len(lst):  
        if (lst[i] != None) and lst[i] % k == 0:  
            lst[i] = None  
        i = i + 1  
    return lst
```

Filters out the multiples of the number k from list by marking them with the special value None (greyed out ones).

Sifting: Removing Multiples of a Number (Alternative version)

```
def sift2(lst,k):  
    i = 0  
    while i < len(lst):  
        if lst[i] % k == 0:  
            lst.remove(lst[i])  
        else:  
            i = i + 1  
    return lst
```

Filters out the multiples of the number k from list by modifying the list. Be careful in handling indices.

A Working Sieve

```
def sieve(n):  
    numlist = list(range(2,n+1))  
    primes = []  
    for i in range(0,len(numlist)):  
        if numlist[i] != None:  
            primes.append(numlist[i])  
            sift(numlist,numlist[i])  
    return primes
```

Use the first version of sift
in this function, which does
the filtering using Nones.

We could have used
`primes[len(primes)-1]` instead.

Helper function that we defined before

Observation for a Better Sieve

We stopped at 11 because all the remaining entries must be prime since $11 \times 11 > 50$.

2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
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41	42	43	44	45	46	47	48	49	50

Jonathan Sorenson (1990, Univ Wisconsin)는 sieve algorithm에서 n 까지 확인을 할 필요없이 $\text{square_root}(n)$ 까지만 확인하면 남아있는 숫자들은 전부 prime이라는것을 증명

A Better Sieve

```
def better_sieve(n):  
    numlist = list(range(2, n+1))  
    primes = [ ]  
    i = 0 # index 0 contains number 2  
    while (i + 2) <= math.sqrt(n):  
        if numlist[i] != None;  
            primes.append(numlist[i])  
            sift(numlist, numlist[i])  
        i = i + 1  
    numlist.remove(None)  
    return primes + numlist
```

Algorithm-Inspired Sculpture



The Sieve of Eratosthenes,
1999 sculpture by Mark di
Suvero. Displayed at
Stanford University.

Prime Number Checker

IsPrime(): dumb version

```
def IsPrime_dumb(n):  
    if (n < 2):  
        return False  
    for factor in range(2, n):  
        if (n % factor == 0):    # 모든숫자 n에 대해서 n번의 modulo 계산필요  
            return False  
    return True
```

```
for i in range(1,100):  
    if IsPrime_dumb(i):  
        print(i)
```

- 앞에 있었던 sieve(n)은 n까지의 모든 prime number를 생성하는 algorithm
- IsPrime(i)는 i가 prime인지 아닌지만 결정하는 algorithm

Prime Number Checker

IsPrime_better(): better version

```
def IsPrime_better(n):  
    if (n < 2):  
        return False  
    if (n == 2):  
        return True  
    if (n % 2 == 0):  
        return False  
    for factor in range(3, n, 2): # 2의 배수는 다 skip하므로 효과적!  
        if (n % factor == 0):  
            return False  
    return True  
  
for i in range(1,100):  
    if IsPrime_better(i):  
        print(i)
```

Prime Number Checker

IsPrime_best(): best version

```
def IsPrime_best(n):
    if (n < 2):
        return False
    if (n == 2):
        return True
    if (n % 2 == 0):
        return False
    maxFactor = round(n**0.5)
    for factor in range(3, maxFactor+1, 2):
        if (n % factor == 0):          # Modulo 계산횟수가 획기적으로 줄어듬
            return False
    return True

for i in range(1,100):
    if IsPrime_best(i):
        print(i)
```