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Composite Adaptive Control of Robot Manipulators*

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The performance of direct adaptive controllers can be enhanced by incorporating a prediction error on the joint torques or on the power input into the tracking-error-driven adaptation law.

Key Words—Adaptive control; composite adaptation; Lyapunov analysis; exponential convergence; robotics.

Abstract—Adaptive control of linear time-invariant single-input single-output systems has been extensively studied, and a number of globally convergent controllers have been derived. While extensions of the results to non-linear or multivariable systems have rarely been achieved, similar global convergence properties can indeed be obtained in the case of robot manipulators, an important and unique class of non-linear multi-input multi-output dynamic systems. Based on the observation that the parameter uncertainty is reflected in both the tracking error in joint motion and the prediction error in the joint torques or the power input, we recently proposed a new class of adaptive robot controllers, the parameter adaptation of which is driven by both tracking error and prediction error. In this paper, following a brief review of our earlier globally convergent direct adaptive controller, we provide a detailed analysis of these "composite" adaptive controllers. Results on global asymptotic and exponential tracking convergence are established and confirmed by simulation.

1. INTRODUCTION

ADVANCED ROBOTIC applications often require effective controller design in order to achieve accurate tracking of fast desired motions. If the parameters of a manipulator's links and its load are known a priori, the well-known "computed-torque" control approach (see e.g. Asada and Slotine (1986)) can be used for this purpose, and theoretically guarantees exact tracking. However, for a manipulator handling various loads, the inertial parameters of the load change from time to time without being accurately known by the controller, and the performance of the computed-torque controller may degrade substantially or even become unstable. This parameter sensitivity is particularly severe for direct-drive robots and/or fast manipulator motions (which are dominated by dynamic forces rather than friction). Therefore, there has been active research in adaptive manipulator control, which intends to provide stable and consistent performance in spite of large parameter uncertainties.

There are two clear phases in the history of adaptive manipulator control research, which aims at dealing with the non-linear multi-input multi-output robot dynamics effectively: an "approximation" phase (roughly 1979–1985), and a "linear parametrization" phase (after 1985). In the approximation phase, researchers (e.g. survey by Hsia (1986)) made significant contributions by developing a number of algorithms from different perspectives and demonstrating their usefulness in simulations and experiments, but had to rely on various restrictive assumptions or approximations for adaptive control design and analysis, such as linearizing the robot dynamics, approximating the joint motions as decoupled, or assuming "slow" variations of the inertia matrix. The explicit introduction in adaptive robotic control research of the linear parametrization of robot dynamics represents a turning point. In the linear-parametrization phase, based on the possibility of selecting a proper set of equivalent parameters such that the manipulator dynamics depend linearly on these parameters, research on adaptive robot control (Craig *et al.*, 1986; Slotine and Li, 1986, 1987d; Middleton and Goodwin, 1986; Hsu *et al.*, 1987; Sadegh and Horowitz, 1987; Bayard and Wen, 1987; Koditschek, 1987; Li and Slotine, 1988a), on the other hand, takes full consideration of the non-linear, time-varying and coupled nature of robot dynamics. The proposed adaptive tracking controllers can be classified into three categories

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according to their adaptation mechanisms: direct, indirect, and composite adaptive controllers.

The *direct* adaptive controllers, as in Slotine and Li (1986), use tracking errors of the joint tracking motion to drive the parameter adaptation. In Slotine and Li (1986, 1987a,c), the global tracking convergence of an adaptive feedforward-plus-PD controller, which is computationally efficient and requires no acceleration measurement, is established, and its good performance is demonstrated in both computer simulations and experimental implementation. Craig *et al.* (1986) proposed an adaptive controller based on computer-torque control and showed its global convergence, but the noise-prone requirement of joint acceleration measurement, and the computationally expensive requirement of inversion of the estimated inertia matrix (assumed to remain uniformly positive definite in the course of adaptation) may limit its practical applicability.

The *indirect* adaptive controllers, like those of Middleton and Goodwin (1986), Hsu *et al.* (1987) and Li and Slotine (1988b), on the other hand, use prediction errors on the filtered joint torque to generate parameter estimates to be used in the certainty-equivalence control law. For these adaptive controllers, the predominant concern of the adaptation is to extract information about the true parameters from the prediction errors, with no *direct* concern for the convergence of tracking errors to zero. Middleton and Goodwin (1986) showed the global tracking convergence of their indirect adaptive controller, which is composed of a least-squares parameter estimator and a modified computed-torque controller and avoids the measurement of joint acceleration. But the computation of their adaptive controller requires the inversion of the estimated inertia matrix (also assumed to remain uniformly positive definite). The indirect adaptive controllers in Li and Slotine (1988b), obtained by using a different modification of the computed-torque controller, has global tracking convergence and also avoids the requirement of inertia matrix inversion. The indirect adaptive controller in Hsu *et al.* (1987), which is composed of a gradient estimator and the unmodified computed-torque controller, currently does not seem to have a complete proof of global convergence.

The "*composite*" adaptive controllers proposed and preliminarily studied in Li and Slotine (1987) and Slotine and Li (1987d), on the other hand, use *both* tracking errors in the joint motion and the prediction errors in the predicted filtered torque or power to drive the parameter adaptation. They are based on the observation that the parameter uncertainty is reflected in both the tracking error and the prediction error and, therefore, it is desirable to extract the parameter information from both sources. In this paper, we provide a detailed analysis of the convergence and performance properties of the composite adaptive controllers.

In Section 2, the dynamics model of manipulators and its relevant properties are summarized, followed by a description of the joint torque prediction model. In Section 3, we present a brief review of our earlier, direct adaptive controller. Section 4 presents the composite adaptation law, and Section 5 analyzes the properties of the composite adaptive controllers with various gain update schemes. Simulation results in Section 6 demonstrate the improved performances of the new adaptive controllers. Section 7 offers brief concluding remarks.

2. MANIPULATOR MODEL AND PREDICTION MODEL

In this section, we first provide the rigid-body model of the manipulator and its properties relevant to adaptive controller design. Then, we briefly sketch the prediction model for the joint torque, since the prediction error will be included in our composite adaptation law.

2.1. *Manipulator dynamics model*

In the absence of friction or other disturbances, the dynamics of a rigid manipulator (with the load considered as part of the last link) can be written as

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau} \qquad (1)$$

where $\mathbf{q}$ is the $n \times 1$ vector of joint displacements, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied joint torques (or forces), $\mathbf{H}(\mathbf{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ is the $n \times 1$ vector of centripetal and Coriolis torques, and $\mathbf{G}(\mathbf{q})$ is the $n \times 1$ vector of gravitational torques.

Two specific properties of the above dynamics can be used for adaptive control design. The first is the linear parametrization property, i.e. each of the individual terms on the left-hand side of (1), and therefore the whole robot dynamics, is linear in terms of a suitably selected set of robot and load parameters (Khosla and Kanade, 1985; An *et al.*, 1985). As the parameters of the load change each time a new payload is picked up while the parameters of manipulator links are constant, in practice only the parameters of the load (ten parameters in general motion, namely, load mass, three parameters for mass center location, six moments of inertia) need to be estimated on-line. Secondly, as pointed out by Takegaki and Arimoto (1981) and Koditschek (1984), the matrices **H** and **C** in (1) are not independent; specifically, using a proper definition of the matrix **C** (only the *vector* $\mathbf{C}\dot{\mathbf{q}}$ is uniquely defined) the matrix ($\dot{\mathbf{H}}$−2**C**) is *skew-symmetric*, as can be shown simply (Slotine and Li, 1987e).

As an example, the dynamic equations of the two-degrees-of-freedom manipulator used in our earlier experimental implementation of direct adaptive controllers (Slotine and Li, 1987a, 1988), can be written explicitly as

$$a_1\ddot{q}_1 + (a_3c_{21} + a_4s_{21})\ddot{q}_2 - a_3s_{21}\dot{q}_2^2 + a_4c_{21}\dot{q}_2^2 = \tau_1$$
$$(a_3c_{21} + a_4s_{21})\ddot{q}_1 + a_2\ddot{q}_2 + a_3s_{21}\dot{q}_1^2 - a_4c_{21}\dot{q}_1^2 = \tau_2$$

where $c_{21} = \cos(q_2 - q_1)$, $s_{21} = \sin(q_2 - q_1)$. It is clearly linear in terms of the four parameters a_1, a_2, a_3, a_4, which are nonlinearly related to the physical link parameters like link masses, lengths, mass centers, and moments of inertia. The skew-symmetry property can also be confirmed on this specific example.

The adaptive robot controller design problem is as follows: given the desired trajectories $\mathbf{q}_d(t)$, $\dot{\mathbf{q}}_d(t)$, $\ddot{\mathbf{q}}_d(t)$, measurements of the joint position $\mathbf{q}$ and velocity $\dot{\mathbf{q}}$, and with some or all the manipulator parameters being unknown, derive a control law for the actuator torque $\boldsymbol{\tau}$, and an adaptation law for the unknown parameters, such that the manipulator joint position $\mathbf{q}(t)$ closely track the desired position $\mathbf{q}_d(t)$. If the adaptive controller is designed such that for any unknown load/link parameters and any errors in initial joint position and velocity, the tracking error $\mathbf{q}(t) - \mathbf{q}_d(t)$ converges to zero, then the adaptive controller is said to have global tracking convergence.

2.2. *Dynamics filtering and torque prediction*

The robot dynamics model (1) can be simply written as

$$\boldsymbol{\tau} = \mathbf{Y}_1(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\mathbf{a} \tag{2}$$

based on the linear-parametrization property, with $\mathbf{Y}_1$ being a non-linear matrix function of $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$, and $\mathbf{a}$ being a $m \times 1$ vector of equivalent parameters. As the torque-parameter relation (2) involves the joint acceleration $\ddot{\mathbf{q}}$, which generally cannot be measured, it is desirable, for parameter estimation purposes, to eliminate $\ddot{\mathbf{q}}$ from (2) by filtering both sides of the equation through an exponentially stable and strictly proper filter (Middleton and Goodwin, 1986; Hsu *et al.*, 1987). The filter should be shaped to effectively filter out high frequency and low frequency disturbances, such as vibrational modes, measurement noise, and biases.

Specifically, convolving both sides of (2) or (1) by $w(t)$, the impulse response of the filter, yields

$$\int_0^t w(t-r)\boldsymbol{\tau}(r)\,\mathrm{d}r = \int_0^t w(t-r)[\mathbf{H}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{G}]\,\mathrm{d}r \tag{3}$$

which we shall write as

$$\mathbf{y}(t) = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{a}$$

where $\mathbf{y}$ is the filtered torque and $\mathbf{W}$ the filtered $\mathbf{Y}_1$. Note that the matrix $\mathbf{W}$ can be computed from measurements of $\mathbf{q}$ and $\dot{\mathbf{q}}$ only, as the term $\ddot{\mathbf{q}}$ on the right-hand side of (3) can be eliminated by integrating by parts. From (4), a prediction of the filtered torque $\mathbf{y}(t)$ and a prediction error can be generated based on the estimated parameters $\hat{\mathbf{a}}$ from the adaptation law

$$\hat{\mathbf{y}}(t) = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})\hat{\mathbf{a}}(t)$$
$$\mathbf{e} = \hat{\mathbf{y}} - \mathbf{y} = \mathbf{W}\tilde{\mathbf{a}} \tag{4}$$

with $\tilde{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{a}$ being the estimation error.

Note that a computationally efficient alternative to filtering the full dynamics (2) is to filter the energy conservation relation

$$\dot{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\tau} = \frac{\mathrm{d}}{\mathrm{d}t}[\tfrac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{H}\dot{\mathbf{q}}] + \dot{\mathbf{q}}^{\mathrm{T}}\mathbf{G}$$

where the left-hand term represents the power input from the actuator. The above scalar relation and its filtered version avoid the computationally complex Coriolis and centripetal terms.

For convenience of later reference, we first state the following lemmas.

Lemma 1. Consider an exponentially stable and strictly proper linear system with input u and output y. Then, if u asymptotically converges to zero, so does y; and if u exponentially converges to zero, so does y.

The asymptotic convergence result is intuitively obvious, and is detailed analytically in Slotine and Li (1987c). The exponential convergence result can be easily shown using the convolution relation between y and u.

Lemma 2. Let $f(.)$ be a scalar function. If

(1) f is lower bounded, i.e. $\exists c, \forall t \geq 0$, $f(t) \geq c$;

(2) f has a uniformly continuous, non-positive derivative $\dot{f}$;

then $\dot{f}(t) \rightarrow 0$ as $t \rightarrow \infty$.

The meaning of Lemma 2 is geometrically clear, implying that a lower-bounded, non-increasing and smooth curve necessarily has vanishing slope. It is a special case of Barbalat's lemma (Popov, 1973).

We also make some nomenclature conventions for later convenience. We define "uniform positive definiteness" of a time-varying $i \times i$ matrix $\mathbf{M}(t)$ as meaning that there exists a positive constant ζ such that $\mathbf{M}(t) \geq \zeta\mathbf{I}$, where $\mathbf{I}$ is the $i \times i$ unity matrix. By persistent excitation (p.e.) of a matrix $\mathbf{M}$, we mean that their exist strictly positive constants β_1, β_2 and δ_0 such that

$$\forall t \geq 0, \quad \beta_1\mathbf{I} \leq \int_t^{t+\delta_0} \mathbf{M}^{\mathrm{T}}(r)\mathbf{M}(r)\,\mathrm{d}r \leq \beta_2\mathbf{I}. \quad (5)$$

Also, by "the desired trajectory", we shall imply the desired joint position, $\mathbf{q}_d$, velocity $\dot{\mathbf{q}}_d$ and acceleration $\ddot{\mathbf{q}}_d$. Finally, we shall refer to our later convergence analysis as a "Lyapunov analysis" because of its obvious similarity with Lyapunov approaches, even though it is not directly based on standard Lyapunov stability results, but stands on its own mathematical rigor.

3. THE DIRECT ADAPTIVE ROBOT CONTROLLER

A globally asymptotically convergent direct adaptive controller was developed in Slotine and Li (1986) to control manipulators with uncertain inertial parameters in the payloads or the links. The good performance of the scheme was demonstrated experimentally in Slotine and Li (1987a, 1988), while explicit conditions on the desired trajectories for parameter convergence were derived in Slotine and Li (1987c). The method, which was developed in joint-space, has also been extended to task-space control and the control of mobile environments (Slotine and Li, 1987b). In the following, we sketch the joint-space direct adaptive controller because of its immediate relevance to the composite controllers in terms of theory and formulation.

3.1. *Globally convergent direct adaptive controller*

Let $\hat{\mathbf{H}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{G}}$ be obtained by substituting the estimated parameters $\hat{\mathbf{a}}(t)$ into $\mathbf{H}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ and $\mathbf{G}(\mathbf{q})$. Then the control and adaptation laws in the direct adaptive controller can be written as

$$\boldsymbol{\tau} = \hat{\mathbf{H}}(\mathbf{q})\ddot{\mathbf{q}}_r + \hat{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \hat{\mathbf{G}}(\mathbf{q}) - \mathbf{K}_D\mathbf{s} \quad (6)$$

$$\dot{\hat{\mathbf{a}}}(t) = -\mathbf{P}_0\mathbf{Y}^{\mathrm{T}}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\mathbf{s} \quad (7)$$

where $\mathbf{K}_D$ is a (perhaps time-varying) uniformly positive definite (p.d.) matrix, the adaptation gain $\mathbf{P}_0$ is a *constant* symmetric p.d. matrix, and

$$\dot{\mathbf{q}}_r = \dot{\mathbf{q}}_d - \mathbf{\Lambda}\tilde{\mathbf{q}}$$
$$\mathbf{s} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r = \dot{\tilde{\mathbf{q}}} + \mathbf{\Lambda}\tilde{\mathbf{q}} \quad (8)$$

with $\tilde{\mathbf{q}} = \mathbf{q}(t) - \mathbf{q}_d(t)$ denoting the error in tracking the desired joint position $\mathbf{q}_d(t)$ and $\mathbf{\Lambda}$ a constant p.d. matrix. The matrix $\mathbf{Y}$ of (7) is defined by the following linearity relation associated with the dynamics model (1):

$$\tilde{\mathbf{H}}(\mathbf{q})\ddot{\mathbf{q}}_r + \tilde{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \tilde{\mathbf{G}}(\mathbf{q}) = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\tilde{\mathbf{a}}$$

where $\tilde{\mathbf{H}} = \hat{\mathbf{H}} - \mathbf{H}$, $\tilde{\mathbf{C}} = \hat{\mathbf{C}} - \mathbf{C}$ and $\tilde{\mathbf{G}} = \hat{\mathbf{G}} - \mathbf{G}$. As $\ddot{\mathbf{q}}_r = \ddot{\mathbf{q}}_d - \mathbf{\Lambda}\dot{\tilde{\mathbf{q}}}$, the matrix $\mathbf{Y}$ can be computed from measurements of only $\mathbf{q}$ and $\dot{\mathbf{q}}$.

Equation (6), which can also be written as

$$\boldsymbol{\tau} = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\hat{\mathbf{a}} - \mathbf{K}_D\mathbf{s}$$

represents an adaptive feedforward-plus-PD controller, with the feedforward action adaptively cancelling the robot dynamic forces, and the PD action

$$\mathbf{K}_D\mathbf{s} = \mathbf{K}_D\dot{\tilde{\mathbf{q}}} + \mathbf{K}_D\mathbf{\Lambda}\tilde{\mathbf{q}}$$

regulating the tracking error to zero. The vector $\dot{\mathbf{q}}_r$ (called "reference velocity" for convenience), obtained by shifting the desired joint velocity according to the current position tracking error $\tilde{\mathbf{q}}$, is introduced to guarantee convergence of the tracking errors, rather than merely of the velocity errors. The vector $\mathbf{s}$, a compact measure of tracking accuracy, is used to drive the parameter adaptation.

Substituting the control law (6) into the manipulator dynamics (1), we obtain the closed-loop dynamics

$$\mathbf{H}\dot{\mathbf{s}} + (\mathbf{K}_D + \mathbf{C})\mathbf{s} = \mathbf{Y}\tilde{\mathbf{a}}. \quad (9)$$

To show the global tracking convergence of the adaptive controller, we consider the Lyapunov function candidate

$$V(t) = \tfrac{1}{2}[\mathbf{s}^{\mathrm{T}}\mathbf{H}\mathbf{s} + \tilde{\mathbf{a}}^{\mathrm{T}}\mathbf{P}_0^{-1}\tilde{\mathbf{a}}]. \quad (10)$$

Using the dynamics in (1), its skew-symmetry property, the control and adaptation laws above, it can be shown from (9) and (10) and the skew-symmetry of $(\dot{\mathbf{H}} - 2\mathbf{C})$ that

$$\dot{V}(t) = -\mathbf{s}^{\mathrm{T}}\mathbf{K}_D\mathbf{s} \leq 0 \quad (11)$$

Therefore $0 \leq V(t) \leq V(0)$. As $\mathbf{K}_D$ is uniformly p.d., the upper boundedness of $V(t)$ implies the boundedness of $\mathbf{s}$ and $\tilde{\mathbf{a}}$. By showing, similarly to the proof in Section 5, that $\dot{V}(t)$ is uniformly continuous provided that the desired trajectories are bounded, $\dot{V}(t)$ and accordingly $\mathbf{s}$ can be shown to converge to zero, using (10) and Lemma 2. In view of the formal filter relation between $\tilde{\mathbf{q}}$ and $\mathbf{s}$ as defined in (8), the convergence of $\mathbf{s}$ to zero in turn implies that of both position error $\tilde{\mathbf{q}}$ and velocity error $\dot{\tilde{\mathbf{q}}}$, according to Lemma 1. Note that the *exponential* convergence of the tracking errors in the presence of persistent excitation has not been shown, because (11) does not contain a quadratic term in $\tilde{\mathbf{a}}$; this will be remedied by the composite adaptive controller studied in Section 4.

Note that the exploitation of the physical properties of the controlled plant is essential in the development of this and also the composite adaptive robot controllers. In the literature on direct adaptive control of linear time-invariant systems, proofs of global tracking convergence generally require the satisfaction of a certain SPR (strictly positive real) condition. In our robot control problem, however, we simply use the physical fact that the inertia matrix is inherently positive definite. Furthermore, we have directly dealt with the natural second-order equation of the mechanical system and resisted the temptation to transform the dynamics into the stylish state-space form, which would have made impossible the simple derivation of the adaptive controller.

3.2. *Choice of* $\mathbf{K}_D$ *and* $\mathbf{\Lambda}$

The performance of the adaptive controller, in terms of accuracy and robustness, depends much on the choice of the gain matrices $\mathbf{K}_D$ and $\mathbf{\Lambda}$, although theoretically any uniformly p.d. matrices can maintain the eventual convergence of the tracking errors. The choice of the two matrices involves a trade-off between tracking accuracy in the practically important transient period (in view of the fact that real tasks last only seconds while theoretical convergence takes infinitely long) and robustness to noise, unmodelled dynamics, and disturbances. While one way of determining these design parameters is by trial and error (namely, tuning $\mathbf{K}_D$ and $\mathbf{\Lambda}$ as large as possible while ensuring stable joint response), more systematic approaches are desirable as $\mathbf{K}_D$ and $\mathbf{\Lambda}$ are matrices containing many independent elements. A reasonable choice consists in using

$$\mathbf{K}_D = \lambda_c \hat{\mathbf{H}}$$

with λ_c being a positive constant, so that the choice of $\mathbf{K}_D$ reduces to the selection of the single design parameter λ_c. Physically, this structure of $\mathbf{K}_D$ intends to use higher gain for joints with higher inertia. The matrix $\mathbf{\Lambda}$ may then be chosen to be diagonal, with each diagonal element reflecting a desired speed of position convergence. A convenient choice is to take $\mathbf{\Lambda} = \lambda_c \mathbf{I}$. With the above choices of $\mathbf{K}_D$ and $\mathbf{\Lambda}$, we obtain the simple form of control law

$$\boldsymbol{\tau} = \hat{\mathbf{H}}(\ddot{\mathbf{q}}_d - 2\lambda_c \dot{\tilde{\mathbf{q}}} - \lambda_c^2 \tilde{\mathbf{q}}) + \hat{\mathbf{C}}\dot{\mathbf{q}}_r + \hat{\mathbf{G}}. \quad (12)$$

This control law is very similar to a "computed torque" controller with critically-damped tracking error dynamics, except for a small difference in the second term on the right-hand side. Now, as $\mathbf{K}_D$ is required to be uniformly p.d. for global stability and convergence of the adaptive controller, $\hat{\mathbf{H}}$ should be uniformly p.d. in order for the above choice of $\mathbf{K}_D$ to be valid. However, the adaptive law (7) does not necessarily guarantee the uniform positive definiteness of $\hat{\mathbf{H}}$. The physically based technique in Li and Slotine (1988b) can be used to slightly modify the adaptive law in (7) so as to guarantee the uniform positive definiteness of $\hat{\mathbf{H}}$, with the global convergence of the adaptive controller retained because of the convexity result associated with the technique.

Furthermore, it is interesting to notice that control law (12) can guarantee the global tracking convergence regardless of the positive definiteness of $\hat{\mathbf{H}}$, provided that a slight modification of the adaptation law (7) is made, namely, substituting

$$\ddot{\mathbf{q}}_c = \ddot{\mathbf{q}}_r - \lambda_c \mathbf{s} \quad (13)$$

in place of $\ddot{\mathbf{q}}_r$ in the matrix $\mathbf{Y}$ in adaptation law (7). This leads to [instead of (11)]

$$\dot{V}(t) = -\lambda_c \mathbf{s}^{\mathrm{T}} \mathbf{H} \mathbf{s} \le 0$$

where the actual inertia matrix $\mathbf{H}$ is inherently uniformly positive definite.

3.3. *Parameter convergence*

The issue of parameter error convergence is studied in Slotine and Li (1987c), and the condition for the parameter estimates to converge to the true parameters is found to be the p.e. of $\mathbf{Y}_d$, where $\mathbf{Y}_d = \mathbf{Y}(\mathbf{q}_d, \dot{\mathbf{q}}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)$. The matrix $\mathbf{Y}_d$ is an intricate non-linear matrix function of the desired trajectories $\mathbf{q}_d$, $\dot{\mathbf{q}}_d$ and $\ddot{\mathbf{q}}_d$. This condition reflects another similarity to the adaptive control of linear time-invariant systems (LTI), where p.e. is a well-known condition for parameter convergence. But a major difference exists—in LTI systems, the p.e. condition for $2n_1$ parameters can be simply guaranteed by having n_1 frequency components in the desired trajectory, while no such simple results have been shown for the p.e. condition of $\mathbf{Y}_d$. The issues of persistent excitation and robustness in adaptive robot control are far from being resolved, and represent an important topic of future research. In order to prevent parameter drift of the estimated parameters in the absence of persistent excitation, the parameter update should be stopped if the tracking error gets into a dead-zone predetermined based on the a priori knowledge about the disturbance and noise characteristics.

4. THE COMPOSITE ADAPTIVE CONTROLLER

In the direct adaptive controller just reviewed, the parameter adaptation is driven by the tracking error $\mathbf{s}$ in joint motion. But in indirect adaptive control, which uses pure estimators for parameter adaptation, the parameter estimation is driven by the prediction error $\mathbf{e}$ in filtered joint torque (or power input). This motivates us to consider a new adaptation law, called composite adaptation law, which extracts information from *both* the tracking error $\mathbf{s}$ and prediction error $\mathbf{e}$. The resulting adaptive controller will be shown to have faster parameter convergence and, accordingly, better tracking performance as compared with the direct adaptive controller.

The composite adaptation law has the following form:

$$\dot{\hat{\mathbf{a}}}(t) = -\mathbf{P}(t)(\mathbf{Y}^{\mathrm{T}}\mathbf{s} + \mathbf{W}^{\mathrm{T}}\mathbf{R}(t)\mathbf{e}) \qquad (14)$$

where $\mathbf{R}(t)$ is a uniformly p.d. weighting matrix indicating how much attention the adaptation law should pay to the parameter information in the prediction error, and the adaptation gain $\mathbf{P}(t)$ is a uniformly p.d. gain matrix determined by the techniques to be described below. We thus obtain a new class of adaptive controllers. Note that the form of control law and closed-loop dynamics are still the same as the direct adaptive controller, i.e. (6) and (9), and that $\mathbf{R}(t) = \mathbf{0}$ and a constant $\mathbf{P}$ would correspond to the direct adaptive law. For simplicity of presentation, we shall take $\mathbf{R}(t)$ to be the unity matrix.

For the composite adaptive controller, we may use the gain update techniques of various parameter estimators in order to generate the gain matrix $\mathbf{P}$. The gradient method is computationally simple but leads to relatively slow convergence. The standard least-squares method is good for constant unknown parameters but its gain vanishing tendency makes it unsuitable for time-varying parameters. We recently studied two parameter estimators, called bounded-gain-forgetting method and cushioned-floor method (Li and Slotine, 1987, 1988c), with the desirable convergence and robustness properties that, under persistent excitation, the parameter estimation errors are exponentially convergent if the unknown parameters are constant and no disturbance is present, and uniformly bounded if the unknown parameters vary with bounded derivative and if disturbances are bounded. Their update laws represent good choices for use in composite adaptive control.

Let us summarize some gain update laws and their relevant properties (detailed in Li and Slotine (1988c)).

4.1. *The gradient method*

$$\mathbf{P}(t) = \mathbf{P}_0 \qquad (15)$$

where $\mathbf{P}_0$ is a constant symmetric p.d. matrix.

4.2. *The "bounded-gain-forgetting" (BGF) method*

The BGF method uses the least-squares gain update

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}^{-1}(t) = -\lambda(t)\mathbf{P}^{-1} + \mathbf{W}^{\mathrm{T}}\mathbf{W} \qquad (16)$$

with the following variable forgetting factor:

$$\lambda(t) = \lambda_0(1 - \|\mathbf{P}\|/k_0) \qquad (17)$$

where k_0 and λ_0 are two positive constants specifying the upper bound of the gain matrix norm and the maximum forgetting rate. From (16) and (17), one can show that $\forall t \geq 0,\ \lambda(t) \geq 0$, and $\mathbf{P}(t) \leq k_0\mathbf{I}$; and that, if $\mathbf{W}$ is p.e., then $\exists \lambda_1 > 0,\ \forall t \geq 0,\ \lambda(t) \geq \lambda_1$.

4.3. *The "cushioned-floor" (CF) method*

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{P}^{-1})(t) = -\lambda(t)(\mathbf{P}^{-1} - \mathbf{K}_0^{-1}) + \mathbf{W}^{\mathrm{T}}\mathbf{W} \qquad (18)$$

where $\mathbf{K}_0$ is a constant p.d. matrix specifying an upper bound of the gain matrix, the $\lambda(t)$ is a uniformly positive design parameter, i.e. $\lambda(t) \geq \lambda_2 > 0$. The first right-hand term in (18) produces a "cushion" on the floor $\mathbf{P}^{-1} = \mathbf{0}$, so that $\mathbf{P}^{-1}$ is maintained larger than $\mathbf{K}_0^{-1}$, i.e. $\mathbf{P}^{-1} - \mathbf{K}_0^{-1} \geq \mathbf{0}$, and, accordingly, $\mathbf{P}(t) \leq \mathbf{K}_0$, as can be seen from the solution of (18). One can also show that, if $\mathbf{W}$ is p.e. as defined by (5), then for all $t \geq \delta_0$

$$\mathbf{P}^{-1}(t) \geq \mathbf{K}_0^{-1} + \beta_1\, \mathrm{e}^{-\lambda_2\delta_0}\mathbf{I}. \qquad (19)$$

Let us also point out some results on persistency of excitation relevant to our later discussion. If tracking errors $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$ converge to zero, then the p.e. of $\mathbf{W}_d = \mathbf{W}(\mathbf{q}_d, \dot{\mathbf{q}}_d)$ imply the p.e. of $\mathbf{W}$, as shown simply in Slotine and Li (1987c). Also, note that when the prediction model of the filtered torque (rather than the power input) is used, the theorem in Li and Slotine (1988c) on the effects of filtering on signal p.e. shows that p.e. of $\mathbf{W}_d$ is itself guaranteed by the p.e. and uniform continuity of $\mathbf{Y}_d$.

5. GLOBAL ASYMPTOTIC OR EXPONENTIAL TRACKING CONVERGENCE

In the following, we show the global asymptotic and exponential convergence of the tracking errors and parameter errors for our composite adaptation law (14), using a Lyapunov analysis.

We first note that the inertia matrix $\mathbf{H(q)}$ is uniformly p.d. regardless of $\mathbf{q}$

$$\mathbf{H(q)} \geq h_1\mathbf{I} \tag{20}$$

(where h_1 is a positive constant), as the kinetic energy $\dot{\mathbf{q}}^T\mathbf{H}\dot{\mathbf{q}}/2$ for any joint velocity vector of unit magnitude cannot possibly be zero.

The convergence analysis for the composite adaptive controllers based on various gain-update techniques will all rely on the following Lyapunov function candidate:

$$V(t) = \tfrac{1}{2}[\mathbf{s}^T\mathbf{H}\mathbf{s} + \tilde{\mathbf{a}}^T\mathbf{P}^{-1}\tilde{\mathbf{a}}]. \tag{21}$$

5.1. *The gradient adaptive controller*

The gradient adaptive controller is defined by (6), (14) and (15). The derivative of Lyapunov function can be obtained, using (1), (6), (14), (15) and the skew-symmetry of $(\dot{\mathbf{H}} - 2\mathbf{C})$, as

$$\dot{V}(t) = -\mathbf{s}^T\mathbf{K}_D\mathbf{s} - \tilde{\mathbf{a}}^T\mathbf{W}^T\mathbf{W}\tilde{\mathbf{a}}. \tag{22}$$

This implies that $V(t) \leq V(0)$ and, therefore, that $\mathbf{s}$ and $\tilde{\mathbf{a}}$ are bounded from the construction of V, as $\mathbf{H}$ is uniformly p.d. It is interesting to note that the Lyapunov function here is the same as that used for the direct adaptive controller, while its derivative now contains an additional negative term $(-\mathbf{e}^T\mathbf{e})$, indicating that $V(t)$ will decrease as long as either the tracking error or the prediction error is not zero.

Theorem 1. If the desired joint trajectories are bounded, then the tracking errors $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$ and the prediction error $\mathbf{e}$ all globally converge to zero, and the parameter error $\tilde{\mathbf{a}}$ remains bounded. If, in addition, $\mathbf{W}_d$ is persistently exciting, then the estimated parameters asymptotically converge to the true parameters.

Proof. The boundedness of $\mathbf{s}$ and $\tilde{\mathbf{a}}$ has already been pointed out. As (8) can be viewed as an exponentially stable first-order filter relation, the boundedness of $\mathbf{s}$ guarantees the boundedness of $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$. We now prove the convergence of the tracking error measure $\mathbf{s}$ and prediction error $\mathbf{e}$ by showing the convergence of $\dot{V}$ to zero for bounded desired trajectories through the use of Lemma 2.

Let us first show the boundedness of $\ddot{V}(t)$, as this in turn guarantees the uniform continuity of $\dot{V}(t)$. The boundedness of $\tilde{\mathbf{q}}$, $\dot{\tilde{\mathbf{q}}}$, $\mathbf{q}_d$, $\dot{\mathbf{q}}_d$ and $\ddot{\mathbf{q}}_d$ implies that of $\mathbf{q}$, $\dot{\mathbf{q}}$, $\dot{\mathbf{q}}_r$ and $\ddot{\mathbf{q}}_r$. Examination of the terms in $\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)$ and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ reveals that they are all bounded, reflecting the physical fact that, for a mechanical manipulator, bounded motion quantities cannot correspond to unbounded forces. Given the closed-loop dynamics (10)

$$\dot{\mathbf{s}} = \mathbf{H}^{-1}[\mathbf{Y}\tilde{\mathbf{a}} - (\mathbf{K}_D + \mathbf{C})\mathbf{s}] \tag{23}$$

and the upper boundedness of $\mathbf{H}^{-1}$ from (20), $\dot{\mathbf{s}}$ is bounded. This also implies that $\ddot{\mathbf{q}}$ is bounded.

From (4), we have

$$\dot{\mathbf{e}} = \dot{\mathbf{W}}\tilde{\mathbf{a}} + \mathbf{W}\dot{\tilde{\mathbf{a}}}.$$

The second right-hand term is bounded because $\mathbf{W}$, $\mathbf{Y}$, $\tilde{\mathbf{a}}$ and $\mathbf{s}$ are all bounded. Note that the boundedness of $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ implies the boundedness of the torque $\boldsymbol{\tau}$ and that of the matrix $\mathbf{Y}_1$. Based on the facts that the filter in (3) is exponentially stable and strictly proper, and that $\mathbf{Y}_1$ is bounded, one can easily show the boundedness of $\dot{\mathbf{W}}$ and, accordingly, that of $\dot{\mathbf{e}}$.

The boundedness of $\mathbf{s}, \mathbf{e}, \dot{\mathbf{e}}$ and $\dot{\mathbf{s}}$ implies the boundedness of $\ddot{V}(t)$. Straightforward application of Lemma 2 then leads to $\dot{V}(t) \to 0$ as $t \to \infty$. Therefore, both the tracking error $\mathbf{s}$ and the prediction error $\mathbf{e}$ asymptotically converge to zero. The convergence of $\mathbf{s}$ to zero in turn guarantees the convergence of $\dot{\tilde{\mathbf{q}}}$ and $\tilde{\mathbf{q}}$ to zero, according to Lemma 1.

If $\mathbf{W}_d$ is persistently exciting, then $\mathbf{W}$ is also p.e., as noticed in Section 4. The convergence of the estimated parameters to the true parameters can then be shown easily by noting that the adaptation law

$$\dot{\tilde{\mathbf{a}}} = -\mathbf{P}_0\mathbf{W}^T\mathbf{W}\tilde{\mathbf{a}} - \mathbf{P}_0\mathbf{Y}^T\mathbf{s} \tag{24}$$

represents an exponentially stable dynamics (Anderson, 1977; Morgan and Narendra, 1977) with convergent input $\mathbf{Y}^T\mathbf{s}$. QED

It is interesting to note that composite adaptive control guarantees the convergence to zero of *both* tracking error and prediction error, while direct adaptive control only guarantees that of the tracking error. This is a reflection of the fact that composite adaptation explicitly pays attention to both tracking error and prediction error.

5.2. *The bounded-gain-forgetting adaptive controllers*

The adaptive controller with $\mathbf{P}$ determined by expressions (16) and (17) is called the BGF adaptive controller. For convenience of exponential convergence analysis, we require the simple modification corresponding to (13) be made.

Theorem 2. The BGF adaptive controller has globally convergent tracking errors $\tilde{\mathbf{q}}$ *and* $\dot{\tilde{\mathbf{q}}}$ and prediction error $\mathbf{e}$ if the desired trajectories are bounded. Furthermore, if $\mathbf{W}_d$ is persistently exciting, then the parameter estimation errors and tracking errors are globally exponentially convergent.

Proof. For the BGF adaptive controller defined by (6), (14), (16) and (17), one has

$$\dot{V}(t) = -\lambda_c \mathbf{s}^{\mathrm{T}}\mathbf{H}\mathbf{s} - (\lambda(t)/2)\tilde{\mathbf{a}}^{\mathrm{T}}\mathbf{P}^{-1}\tilde{\mathbf{a}} - (1/2)\tilde{\mathbf{a}}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}\mathbf{W}\tilde{\mathbf{a}} \leq 0. \quad (25)$$

The convergence of $\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}$ and $\mathbf{e}$ can be shown as before. Note that, additionally, the convergence of $\dot{V}(t)$ to zero leads to that of $\lambda(t)\tilde{\mathbf{a}}^{\mathrm{T}}\mathbf{P}^{-1}\tilde{\mathbf{a}}$.

As pointed out in Section 4, $\mathbf{P}(t) \leq k_0\mathbf{I}$, and the persistent excitation of $\mathbf{W}_d$ (and consequently, that of $\mathbf{W}$) guarantees that $\lambda(t) \geq \lambda_1$. These imply

$$\lambda(t)\tilde{\mathbf{a}}^{\mathrm{T}}\mathbf{P}^{-1}\tilde{\mathbf{a}} \geq \lambda_1\tilde{\mathbf{a}}^{\mathrm{T}}\tilde{\mathbf{a}}/k_0. \quad (26)$$

Therefore the convergence of $\lambda(t)\tilde{\mathbf{a}}^{\mathrm{T}}\mathbf{P}^{-1}\tilde{\mathbf{a}}$ to zero implies that of $\tilde{\mathbf{a}}$. In fact, we can more precisely show the *exponential* convergence of the tracking and estimation errors. Indeed, let γ_0 be the strictly positive constant defined by $\gamma_0 = \min(2\lambda_c, \lambda_1)$. In view of (25), we can write

$$\dot{V}(t) + \gamma_0 V(t) \leq 0.$$

Therefore, $V(t) \leq V(0)\,\mathrm{e}^{-\gamma_0 t}$. This, in turn, implies the exponential convergence of $\mathbf{s}$ and $\tilde{\mathbf{a}}$ to zero. The exponential convergence of $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$ to zero follows as a result of exponential convergence of $\mathbf{s}$, according to Lemma 1. QED

The exponential convergence of an adaptive controller is an attractive property, since it favors robustness to noise and other disturbances, as pointed out by Anderson and Johnson (1982). Note that only global asymptotic convergence can be shown for the standard least squares gain update.

5.3. *The CF adaptive controller*

The CF adaptive controller, obtained by using the CF gain update (18), has properties similar to those of the BGF adaptive controller.

Theorem 3. The CF adaptive controller has globally asymptotically convergent tracking errors $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$ and prediction error $\mathbf{e}$ if the desired trajectories are bounded. If $\mathbf{W}_d$ is persistently exciting, then the tracking errors and parameter estimation errors exponentially converge to zero.

Proof. For the CF adaptive controller defined by (6), (14) and (18), we can obtain

$$\dot{V}(t) = -\lambda_c \mathbf{s}^{\mathrm{T}}\mathbf{H}\mathbf{s} - (\lambda(t)/2)\tilde{\mathbf{a}}^{\mathrm{T}}[\mathbf{P}^{-1} - \mathbf{K}_0^{-1}]\tilde{\mathbf{a}} - (1/2)\tilde{\mathbf{a}}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}\mathbf{W}\tilde{\mathbf{a}} \leq 0 \quad (27)$$

as the CF gain update equation guarantees that $\mathbf{P}^{-1} - \mathbf{K}_0^{-1} \geq \mathbf{0}$. Similar reasoning as before shows the global asymptotic convergence of $\dot{V}(t)$, and accordingly, of the tracking errors $\tilde{\mathbf{q}}$, $\dot{\tilde{\mathbf{q}}}$, and the prediction error $\mathbf{e}$.

If $\mathbf{W}_d$ is persistently exciting, then the parameter estimation errors and tracking errors can also be shown, by using (27) and (19), to be exponentially convergent to zero with a rate equal to $\min(2\lambda_c, \lambda_2\beta_1\,\mathrm{e}^{-\lambda_2\delta_0})$. QED

6. SIMULATION RESULTS

Extensive computer simulations have been carried out to examine the performances of the new adaptive controllers, using the 2-DOF robot model of Section 2.1. The four parameters to be estimated are assumed to have the following true values $\mathbf{a} = \{0.15, 0.04, 0.03, 0.025\}^{\mathrm{T}}\,\mathrm{kg\,m^2}$. In the simple example below, the arm is required to follow the desired position $\mathbf{q}_d(t)$

$$q_{d1}(t) = \pi/4 + 2(1 - \cos 3t)$$
$$q_{d2}(t) = \pi/6 + (1 - \cos 5t)$$

as shown in Fig. 1. A conventional PD controller (integral feedback is not needed because of the absence of gravity), the direct adaptive controller and the BGF composite adaptive controllers are compared. The PD controller, used here for comparison purposes, is

$$\tau_1 = -3(\dot{q}_1 - \dot{q}_{d1}) - 10(q_1 - q_{d1})(\mathrm{N\,m})$$
$$\tau_2 = -3(\dot{q}_2 - \dot{q}_{d2}) - 10(q_2 - q_{d2})(\mathrm{N\,m}).$$

The adaptive controllers have the same PD part for simplicity. In order to highlight the effect of adaptation, the initial parameter estimates are assumed to be zero, implying that we have no a priori knowledge about the parameters, and thus that the adaptive controller is initially identical to the PD controller, as the feedforward part starts from zero. The gain matrix for the direct

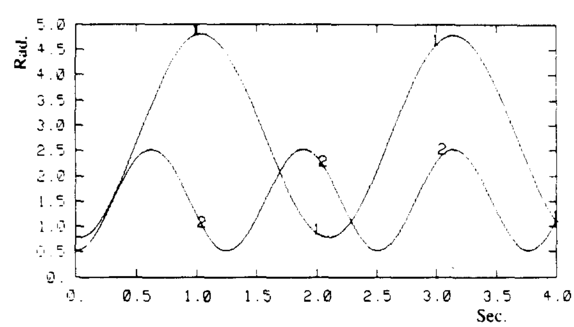

FIG. 1. Desired joint positions.

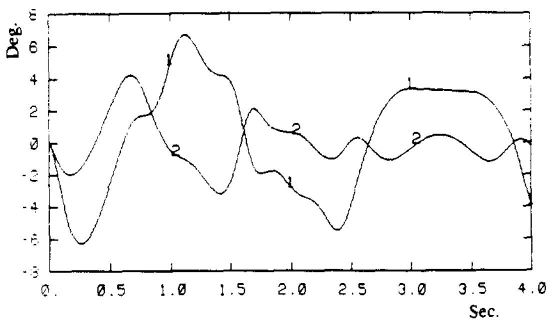

FIG. 2. Position errors of the PD controller.

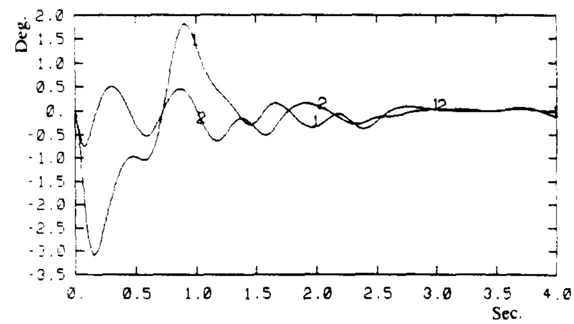

FIG. 4. Position errors of the direct adaptive controller.

adaptive controller is chosen to be constant, $\mathbf{P}_0 = 0.03\mathbf{I}$. The initial gain of BGF gain update is the same as $\mathbf{P}_0$, and $k_0 = 0.03$, $\lambda_0 = 4$. Therefore, the gain matrix in the BGF method will be time-varying and always smaller than $\mathbf{P}_0$. The robot is initially at rest at the desired initial position.

The position tracking errors and the joint torques of the PD controller are plotted in Figs 2 and 3. The tracking errors, joint torques and parameter estimates of the direct adaptive controller are plotted in Figs 4–7. The tracking errors of the adaptive controller in the initial period are close to those of the PD control as a result of using zero initial parameter estimates, but the tracking errors improve as the adaptive law extracts parameter information from the tracking errors. The maximum tracking errors of the direct adaptive controller are seen to be significantly smaller than those of the PD controller while the joint torques are comparable (in fact the adaptive controller input is smaller after a while of adaptation, indicating more efficient use of energy by the model-based control law). The tracking errors of the direct adaptive controller is somewhat oscillatory and converges after about 3 s. The above simulation comparison results are consistent with the experimental data in Slotine and Li (1987a, 1988). The position tracking error and estimated parameters of the BGF composite adaptive controller are plotted in Figs 8–10, which show considerable improvement over the direct adaptive controller in both tracking convergence and estimation precision. All the errors converge in about 1.5 s and have little oscillation. The joint torques of the direct and BGF composite adaptive controllers are hardly distinguishable.

The performance of the CF adaptive controller in simulation is comparable to that of the

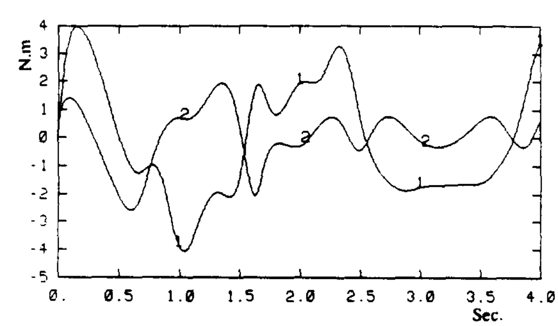

FIG. 3. Joint torques of the PD controller.

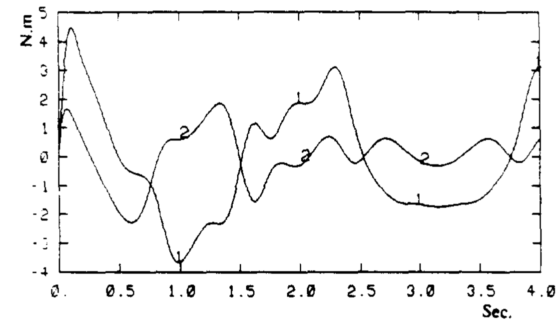

FIG. 5. Joint torques of the direct adaptive controller.

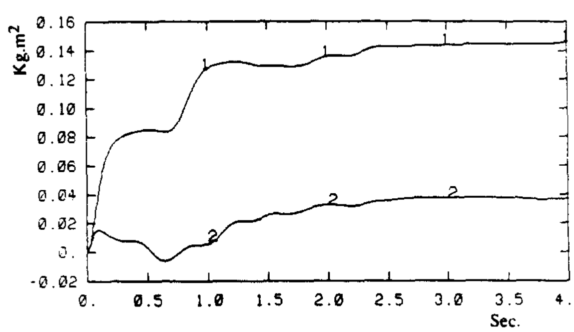

FIG. 6. Estimates $\hat{a}_1$, $\hat{a}_2$ from the direct adaptive controller.

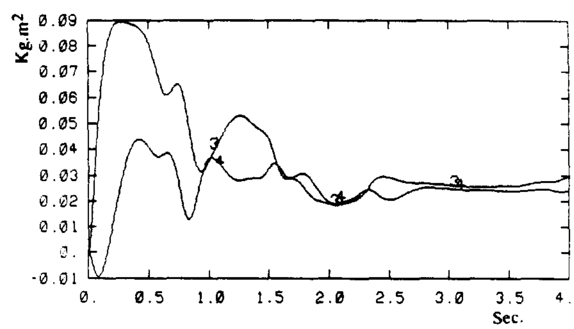

FIG. 7. $\hat{a}_3$, $\hat{a}_4$ from the direct adaptive controller.

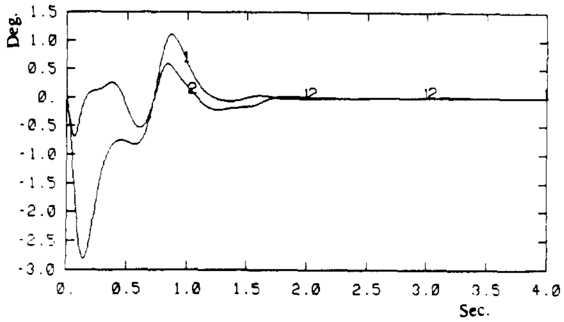

FIG. 8. Position errors of the BGF composite adaptive controller.

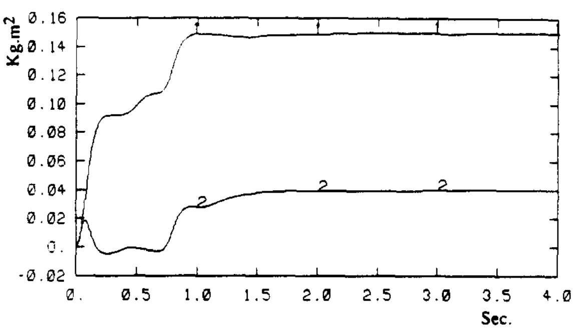

FIG. 9. Estimates $\hat{a}_1$, $\hat{a}_2$, from the BGF adaptive controller.

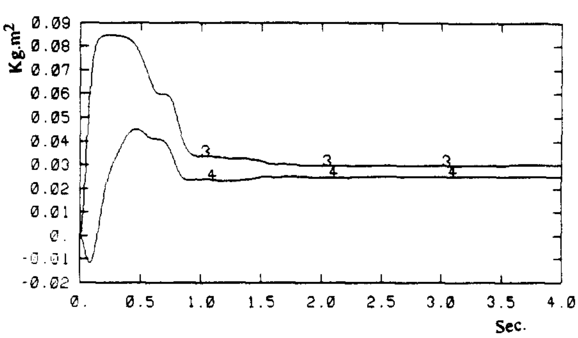

FIG. 10. $\hat{a}_3$, $\hat{a}_4$ from the BGF adaptive controller.

BGF adaptive controller. Further simulations using this desired trajectory, with significant levels of unmodelled friction and measurement noise, and unmodelled motor dynamics, indicate that the direct and composite adaptive controllers with a small dead zone in adaptation retain stability and maintain good tracking accuracy.

Composite adaptive manipulator control is illustrated experimentally by Niemeyer and Slotine (1988).

7. CONCLUSIONS

The composite adaptive controller is obtained by properly combining the tracking error and prediction error for parameter adaptation. It is shown to retain the desirable characteristics of our earlier direct adaptive controller, such as global asymptotic convergence of tracking error and avoidance of measuring joint acceleration or of inverting the estimated inertia matrix. Furthermore, the composite adaptive controller has global *exponential* convergence in the presence of persistently exciting trajectories, and its improved performance is reflected by faster parameter convergence and better tracking accuracy in simulations. The robustness and persistent excitation issues and comparisons for various adaptive robot controllers deserve further experimental and analytical research.

Finally, we remark that the composite adaptive controller should be viewed as an extension of the direct adaptive controller, rather than a unification of direct and indirect adaptive controllers. Indeed, the presence of the tracking error in the composite adaptive law is essential to the Lyapunov convergence analysis, and the indirect adaptive controller obtained by omitting the tracking error term in the composite law cannot be shown to be convergent.

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