INDIRECT ADAPTIVE ROBOT CONTROL

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ABSTRACT

Adaptive robot controllers can be classified into three categories: direct, indirect, and composite controllers. While globally convergent direct and composite controllers were derived and extensively demonstrated in our earlier work, this paper provides a discussion of the theoretical issues linked to the development of indirect adaptive robot controllers, and proposes some possible solutions. Several aspects of this paper, such as new parameter estimation structures with desirable properties, and a discussion of the convexity of parameter space, are also of interest in their own right.

1. INTRODUCTION

Advanced manipulator applications often require effective control design to achieve accurate tracking of fast desired motions. If the parameters of a manipulator's links and its load are known *a priori*, the well-known "computed-torque" control design can be used for this purpose, and theoretically guarantees exact tracking. However, for a manipulator handling various loads, the inertial parameters of the load change from time to time without being accurately known by the controller, and the performance of the computed-torque controller degrades substantially or may even go unstable. This parameter sensitivity is particularly severe for direct-drive robots and/or fast manipulator motions (which are dominated by dynamic forces rather than friction). Therefore, there has been active research in adaptive manipulator control (see e.g., the survey in [Hsia, 1986]), which intends to provide stable and consistent performance in spite of large parameter uncertainties.

The more recently developed adaptive robot controllers make use of the full robot dynamic model, and can be classified into three categories: direct, indirect, and composite controllers.

The *direct* adaptive controllers, as in e.g. [Slotine and Li, 1986], use tracking errors of the joint motion to drive the parameter adaptation. In this class of adaptive controllers, the predominant concern of the adaption laws is to reduce the tracking errors. In [Slotine and Li, 1986, 1987a, 1987b], the global tracking convergence of an adaptive feedforward-plus-PD controller, which is computationally simple and requires no acceleration measurement, is established without trajectory excitation requirement, and its performance is demonstrated in both computer simulations and hardware imlementation. The approach avoids the difficulties linked to the SPR (strict positive realness) requirement in traditional adaptive control, by taking advantage of the inherent positive definiteness of the manipulator's inertia matrix. [Craig, *et al*, 1986] proposed an adaptive controller based on computed-torque control and showed its global convergence, but the noise-prone requirement of joint acceleration measurement, and the computationally expensive requirement of inversion of the estimated inertia matrix (which also implies monitoring the matrix to guarantee that it remains invertible in the course of adaptation) limit its practical applicability.

The *indirect* adaptive controllers, like those of [Middleton and Goodwin, 1986; Hsu, *et al*, 1987], on the other hand, use prediction errors on the filtered joint torques to generate parameter estimates to be used in the control law. For these adaptive controllers, the predominant concern of the adaptation is to extract information about the true parameters from the prediction errors, with no *direct* concern to adapt the parameters so that the tracking errors converge to zero. Based on an input-output stability analysis, [Middleton and Goodwin, 1986] show the global tracking convergence of their adaptive controller, which is composed of a modified computed-torque controller and a modified least-square estimator. But the computation of their adaptive controller again requires inversion of the estimated inertia matrix. The indirect adaptive controller in [Hsu, *et al*, 1987], which is composed of a gradient estimator and the unmodified computed-torque controller, currently does not seem to have a complete proof of global convergence.

The *"composite"* adaptive controllers proposed in [Li and Slotine, 1987; Slotine and Li, 1987], on the other hand, use *both* tracking errors in the joint motion and the prediction errors in the predicted filtered torque to drive the parameter adaptation. They are based on the observation that the parameter uncertainty is reflected in both the tracking error and the prediction error and, therefore, it is desirable to extract the parameter information from both sources. This not only allows a full use of available information sources, but also offers an automatic way of modulating the adaptation gain according to the excitation of the desired trajectories.

This paper provides a discussion of the theoretical difficulties linked to the development of purely indirect adaptive robot controllers, and proposes some possible solutions. After reviewing, in Section 2, the prediction models used for robotic parameter estimation, a variety of parameter estimation methods are discussed in Section 3, under a common framework based on an "exact" solution approach. Section 4 discusses a new indirect adaptive controller structure, which consists of a modified computed torque using parameters obtained from any of the estimators of Section 3, under certainty equivalence. It shows that a critical difficulty in using indirect adaptive control is the necessity to explicitely guarantee that the estimated inertia matrix remains positive definite in the course of adaptation, a requirement avoided by both the direct and the composite adaptive controllers. A practically-motivated possible solution to this difficulty is proposed. Section 5 presents brief concluding remarks.

2. MANIPULATOR MODEL AND PREDICTION MODEL

In this section, we first provide the rigid-body model of the manipulator and its properties relevant to controller design. Then, we briefly sketch the prediction error model.

In the absence of friction or other disturbances, the dynamics of a rigid manipulator (with the load considered as part of the last link) can be written as

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau \qquad (1)$$

where $\mathbf{q}$ is the $n\times 1$ vector of joint displacements, τ is the $n\times 1$ vector of applied joint torques (or forces), $\mathbf{H}(\mathbf{q})$ is the $n\times n$ symmetric positive definite manipulator inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$ is the $n\times 1$ vector of centripetal and Coriolis torques, and $\mathbf{G}(\mathbf{q})$ is the $n\times 1$ vector of gravitational torques.

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An important property of the manipulator dynamics is that each term in the right-hand side of (1) can be linearly parameterized in terms of $\mathbf{a}$, a suitably selected set of equivalent link and load parameters, so that

$$\tau = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\,\mathbf{a} \qquad (2)$$

As an example, the dynamic equations of a two-degree-of-freedom manipulator can be written explicitly as

$$a_1 \ddot{q}_1 + (a_3 c_{21} + a_4 s_{21})\ddot{q}_2 - a_3 s_{21} \dot{q}_2^{\,2} + a_4 c_{21} \dot{q}_2^{\,2} = \tau_1$$

$$(a_3 c_{21} + a_4 s_{21})\ddot{q}_1 + a_2 \ddot{q}_2 + a_3 s_{21} \dot{q}_1^{\,2} - a_4 c_{21} \dot{q}_1^{\,2} = \tau_2$$

where $c_{21} = cos(q_2 - q_1)$, $s_{21} = sin(q_2 - q_1)$. It is clearly linear in terms of the four parameters a_1, a_2, a_3, a_4, with the load treated as part of the second link.

Note that the number of equivalent parameters in (2) may be much smaller than that of physical parameters, since the equivalent parameters are nonlinear combinations of physical parameters. This means that the physical parameters themselves may be unidentifiable. This does not represent a difficulty from a control point of view, since only the equivalent parameters affect the dynamics and control.

It is convenient to rewrite equation (2) as

$$\tau = \mathbf{Y}_u \mathbf{a}_u + \mathbf{Y}_k \mathbf{a}_k$$

where $\mathbf{a}_u$ contains the unknown equivalent parameters and $\mathbf{a}_k$ contains the known ones. If the parameters of the manipulator links have been estimated beforehand, $\mathbf{a}_u$ will only contain the 10 unknown load parameters (mass, position of the center of mass, and six independent components of the inertia matrix). In the prediction approach to be used, the second right-hand term associated with the known parameters do not cause any difficulty. Therefore, *we will only treat model (2), which corresponds to assuming that all* components of $\mathbf{a}$ are unknown.

In practice, the joint position $\mathbf{q}$ and joint velocity $\dot{\mathbf{q}}$ can be conveniently meaured by encoders and tachometers at the joints, and the joint torque τ is available either directly from control commands (assuming actuator dynamics to be negligible), or from torque sensors. Direct use of the model in (2) for estimating $\mathbf{a}$ requires the joint acceleration which can only be obtained by numerical differentiation. This noise-prone requirement can be avoided by filtering both sides of (2) through a stable first-order filter as suggested in [Middleton and Goodwin, 1986; Hsu *et al*, 1987]. In fact, any exponentially stable and strictly proper filter of arbitray order can be used to achieve the same objective and, in addition, allows us to place its poles and zeroes. If the frequency range of the measurement noise and link vibration modes are known, proper placement of the filter poles and zeroes may improve the robustness and performance of the estimator by filtering out the undesirable components of the torque signals and appropriately amplify the useful frequency components.

Let $w(s)$ be the impulse response of the filter. Multiplying both sides of (2) by w, and then integrating over [0,t], yields a relation free of $\ddot{\mathbf{q}}$ through partial integration,

$$\int_0^t w(r)\tau(r)\,dr = \int_0^t w(r)[\mathbf{H}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{G}]\,dr \qquad (3)$$

$$\mathbf{y}(t) = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})\,\mathbf{a} \qquad (4)$$

where $\mathbf{y}$ is the filtered torque and $\mathbf{W}$ is the filtered $\mathbf{Y}$.

From (4), a prediction of the filtered torque $\mathbf{y}(t)$ can be generated based on the estimated parameters $\hat{\mathbf{a}}$,

$$\hat{\mathbf{y}}(t) = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})\,\hat{\mathbf{a}}(t)$$

a prediction error $\mathbf{e}$ can be formed

$$\mathbf{e} = \hat{\mathbf{y}} - \mathbf{y} = \mathbf{W}\tilde{\mathbf{a}}$$

The prediction error reflects the error between the currently estimated parameter $\hat{\mathbf{a}}$ (t) and the true parameter $\mathbf{a}$. The estimation methods in Section 3 extract parameter information from the prediction error.

3. ESTIMATION METHODS

In an indirect adaptive controller, the parameter estimators are used as adaptation mechanism. Many estimation methods are eligible to guarantee the global convergence of the adaptive controller. We briefly describe four types of such mechanisms, and provide a few comments about the comparative performance of the estimators.

All the estimators which we consider have a common form of parameter update, namely

$$\dot{\hat{\mathbf{a}}} = -\mathbf{P}\mathbf{W}\mathbf{e} \qquad (5)$$

where $\mathbf{P}(t)$ is a constant or time-varying positive definite gain matrix. The gain matrix $\mathbf{P}$ may be generated in different forms and from different perspectives. The performances of the estimators are different from each other, as a result of the difference in gain matrix generation. However they share a common theoretical property - the prediction-error $\mathbf{e}$ is square integrable (mathematically, $\mathbf{e} \in L^2$), which will be used later to guarantee the global tracking convergence of our indirect adaptive controller. The common form of the parameter update in (5) allows the estimators to be analyzed using the same Lyapunov function

$$V(t) = \tilde{\mathbf{a}}^T \mathbf{P}^{-1} \tilde{\mathbf{a}} \qquad (6)$$

where $\tilde{\mathbf{a}} = \hat{\mathbf{a}}(t) - \mathbf{a}$ is the parameter estimation error.

In this section, we briefly sketch the gradient, least-square and two newly proposed estimators [Li and Slotine, 1988] with fast convergence and ability to estimate time-varying parameters.

3.1.1 Gradient Estimator

In the gradient estimator, the gain matrix is simply held constant

$$\mathbf{P}(t) = \mathbf{P}_o$$

where $\mathbf{P}_o$ is a constant *p.d.* matrix. Then from (5) and (6),

$$\dot{V}(t) = -\tilde{\mathbf{a}}^T \mathbf{W}^T \mathbf{W} \tilde{\mathbf{a}} = -\mathbf{e}^T \mathbf{e} \le 0$$

This implies that $0 \le V(t) \le V(0)$. Since $V(0)$ is a finite positive constant, integration of $\dot{V}$ leads to

$$\int_0^\infty \mathbf{e}^T(r)\mathbf{e}(r)\,dr = V(0) - V(t) \le V(0)$$

so that $\mathbf{e}$ belongs to L^2. The gradient estimator is is computationally simple but its convergence is usually slow. It leads to exponential convergence of the estimated parameters if the trajectories are persistently exciting, by which we mean that there exist positive constant α_1, α_2 and δ, such that

$$\forall t \ge 0, \quad \alpha_1 \mathbf{I} \le \int_t^{t+\delta} \mathbf{W}^T(r)\mathbf{W}(r)\,dr \le \alpha_2 \mathbf{I} \qquad (7)$$

3.2 Least-Square Estimators

The gain update equation in least-square estimation with time-varying forgetting factor λ is

$$\frac{d}{dt}[\mathbf{P}^{-1}] = -\lambda(t)\mathbf{P}^{-1} + \mathbf{W}^T\mathbf{W} \qquad (8)$$

where $\lambda(t) \ge 0$. Using the Lyapunov function in (6),

$$\dot{V}(t) = -\lambda(t)V(t) - \mathbf{e}^T\mathbf{e} \le 0$$

This leads to the boundedness of parameter error and to $\mathbf{e}$ belonging to L^2.

Equation (8) represents a whole class of estimators in the literature, including the standard least-square method (corresponding to $\lambda(r) = 0$), constant-forgetting-factor estimator, constant-trace-forgetting factor, and so on. The gain matrix $\mathbf{P}$ can be easily found from (8) to satisfy

$$\mathbf{P}^{-1}(t) = \mathbf{P}^{-1}(0)\,e^{-\int_0^t \lambda(r)dr} + \int_0^t e^{-\int_r^t \lambda(v)dv}\mathbf{W}^T(r)\mathbf{W}(r)\,dr$$

The parameter error in the least-square estimator with time-varying factor can be explicitly solved to be

$$\tilde{\mathbf{a}} = [\mathbf{P}^{-1}(0) + \int_0^t e^{\int_0^r \lambda(v)\,dv} \mathbf{W}^T(r)\,\mathbf{W}(r)\,dr]^{-1} \mathbf{P}^{-1}(0)\,\tilde{\mathbf{a}}(0)$$

This "exact" solution approach is systematically exploited in [Li and Slotine, 1988] to provide a common framework in the discussion and comparison of various least-square estimators. It is interesting to notice that the parameter error of the exponentially-forgetting estimator with not-always-zero forgetting factor is always smaller than the SLS estimator. It is easy to show that, if $\mathbf{W}$ is such that the smallest eigenvalue of the matrix $\mathbf{Z}(t) = \int_0^t \mathbf{W}^T\mathbf{W}\,dr$ is infinitely large, the estimated parameters of all the least-square estimators asymptotically converge to the true parameters. Persistent excitation is easily shown to be a stronger condition than the above-mentioned one.

Choosing the forgetting factor $\lambda(t)$ has been a difficult and largely unresolved issue in the parameter estimation literature. If λ is chosen as zero (SLS method), the gain matrix converges to zero in the presence of trajectory *p.e.* and the estimator is essentially turned off after a while. If $\lambda(t)$ is chosen to be a positive constant λ_o, the estiamted parameters will converge exponentially with the rate λ_o in the presence of *p.e.*. This is a desirable convergence property. However, constant choice of forgetting factor is not acceptable because the gain matrix will go unbounded if $\mathbf{W}$ is not *p.e.*. In [Irving, 1979; Lozano-Leal and Goodwin, 1985], a variable-forgetting-factor which guarantees a contant-trace gain is suggested and studied. But the analysis of the algorithm does not provide exponential convergence results, and the computation of the forgetting factor in the multi-output case is complex.

3.3 The Gain-Adjusted-Forgetting Estimator

We recently formulated a novel way of varying the forgetting facor $\lambda(t)$, as an improved version of the switching factor estimator in [Li and Slotine, 1987], namely,

$$\lambda(t) = (\lambda_1/k_1)(k_1 - \|\mathbf{P}\|) \qquad (9)$$

with $\lambda(0)$ and k_0 being positive constants, which implies forgetting at a maximum factor λ_0 if the norm of $\mathbf{P}$ is zero (which indicates infinitely strong *p.e.*) and stops forgetting if the norm reaches the specified upper bound (indicating weak excitation). In this way, the forgetting factor automatically adjusts itself to the trajectory excitation so that the estimator behaves like constant-forgetting-factor estimator if the *p.e.* is strong and like a standard least-square estimator if the trajectory is weakly or non-persistently exciting. We will refer this estimator as the gain-adjusted-forgetting (GAF) estimator.

The gain update equation leads to

$$\mathbf{P}^{-1}(t) \;\geq\; (\mathbf{P}^{-1}(0) - k_1^{-1}\mathbf{I})\,e^{-\lambda_1 t} + (1/k_1)\mathbf{I} + \int_0^t e^{-\lambda_1 (t-r)}\mathbf{W}^T\mathbf{W}\,dr$$

where we used $\|\mathbf{P}(t)\|\,\mathbf{P}^{-1}(t) \geq \mathbf{I}$. Note that $\|\mathbf{P}(0)\| \leq k_1$ guarantees the *p.d.* of $(\mathbf{P}(0)^{-1} - k_1^{-1}\mathbf{I})$ and, accordingly, of $\mathbf{P}$(t). Violation of this initial gain requirement has only transient effect (decaying with rate λ_1). Therefore, $\forall t \geq 0$

$$\mathbf{P}^{-1}(t) \geq (1/k_1)\mathbf{I}$$

i.e., $\|\mathbf{P}(t)\| \leq k_1$ and $\lambda(t) \geq 0$. If $\mathbf{W}$ is *p.e.* as specified by (7), $\mathbf{P}^{-1}(t) \geq [\,1/k_1) + e^{-\lambda\delta}\alpha_1\,]\mathbf{I}$, and $\mathbf{P}(t) \leq k_1/(1 + k_1\alpha_1 e^{-\lambda_1\delta})\mathbf{I}$. Therefore, $\lambda(t) = (\lambda_1/k_1)(k_1 - \|\mathbf{P}\|) \geq \lambda_1 k_1 \alpha_1 e^{-\lambda_1\delta}/(1 + k_1\alpha_1 e^{-\lambda_1\delta})$

This fact can be used to show that $\mathbf{P}(t)$ is uniformly lower bounded by a positive constant matrix. In summary,

Theorem 1: *In the gain-adjusted-forgetting estimator, the estimated parameters are always bounded and the gain matrix is always uniformly upper bounded, i.e.,* $\mathbf{P}(t) \leq k_1\mathbf{I}$; *if* $\mathbf{W}$ *is persistently exciting, the estimated parameters converge exponentially with a rate dependent on the persistent excitation strength and the gain matrix is uniformly bounded from both below and above by positive constant matrices, namely,*

$$k_0\mathbf{I} \leq \mathbf{P}(t) \leq k_1\mathbf{I}$$

with $k_0 \geq 0$.

3.4 Inherently-Bounded-Gain Estimators

Another way of preventing $\mathbf{P}$ from becoming too large, or equivalently $\mathbf{P}^{-1}$ from vanishing, is to consider the alternative gain update law

$$\dot{\mathbf{P}}^{-1}(t) = -\lambda(t)\,[\mathbf{P}^{-1} - \mathbf{K}_o^{-1}] + \mathbf{W}^T\mathbf{W} \qquad (11)$$

instead of (8), where $\mathbf{K}_o$ is a constant *p.d.* matrix and $\lambda(t)$ is a constant or time-varying non-negative forgetting factor independant of $\mathbf{P}$. Note that the first term is somewhat similar to a spring effect in a mechanical system, reflecting our intention of limiting $\mathbf{P}^{-1}$ from below. Unlike the GAF estimator, this estimator resulting from this gain update law is not a special case of least-square estimation, but, on the contrary, includes least-square-type estimators as special cases, with $\lambda(t) = 0$ corresponding to the SLS method, and $\mathbf{K}_o^{-1} \to \mathbf{0}$ corresponding to the exponentially-forgetting least-square method. The solution of (11) is

$$\mathbf{P}^{-1}(t) - \mathbf{K}_o^{-1} = [\mathbf{P}^{-1}(0) - \mathbf{K}_o^{1}]\,e^{-\int_0^t \lambda(r)\,dr} + \int_0^t e^{-\int_r^t \lambda(v)\,dv}\,\mathbf{W}^T(r)\,\mathbf{W}(r)\,dr \qquad (12)$$

Therefore, $\mathbf{P}^{-1}(t) \geq \mathbf{K}_o^{-1}$, and accordingly, $\mathbf{P}(t) \leq \mathbf{K}_o$. We call the estimator with gain update law (11) the inherently-bounded-gain (IBG) estimator.

For the IBG estimator,

$$\dot{V}(t) = -\tilde{\mathbf{a}}^T[\mathbf{P}^{-1} - \mathbf{K}_o^{-1}]\,\tilde{\mathbf{a}} - \mathbf{e}^T\mathbf{e}$$

This implies the boundedness of the parameters and $\mathbf{e} \in L^2$.

If the joint velocities are bounded, which leads to bounded $\mathbf{W}$, the filtering relation (11) will imply the upper boundedness of $\mathbf{P}^{-1}$ and, correspondingly, the lower boundedness of $\mathbf{P}(t)$.

Theorem 2: *The estimated parameters from the IBG estimators are guaranteed to be bounded, and the gain matrix is guaranteed to be uniformly upper bounded. If* $\mathbf{W}$ *is bounded, then*

$$\mathbf{K}_1 \leq \mathbf{P}(t) \leq \mathbf{K}_0$$

for constant positive matrices $\mathbf{K}_o$ *and* $\mathbf{K}_1$; *if* $\mathbf{W}$ *is p.e., the estimated parameters converge exponentially with a rate depending on the p.e. strength.*

The exponential parameter convergence in the theorem can be proven from the (6), (11) and the fact

$$\forall t \geq \delta \quad \mathbf{P}^{-1} - \mathbf{K}_o^{-1} \geq \alpha_1 e^{-\lambda\delta}$$

derivable from (7) and (12).

A natural choice of $\mathbf{P}_o$ is $\mathbf{K}_o$, *i.e.*, starting the estimator with the largest gain allowed. If $\mathbf{W}$ is only weakly *p.e.*, we expect the estimator to behave similarly to a gradient estimator or a constant forgetting factor least-square estimator.

We have formulated two exponentially convergent estimators with bounded gains: the GAF estimator and the IBG estimator. While we do not attempt here to make conclusive performance comparisons, which should be obtained by experimental implementations, let us make a few remarks.

First, the IBG approach includes the GAF as a special case, and therefore allows more flexibility in the estimator design. Second, the GAF estimator has a minimization interpretation from a least-square perspective but IBG approach in general does not seem to correspond to index minimization, although the Lyapunov analysis of the estimator are very similar. Third, the IBG estimator allows forgetting in the strongly exciting directions of $\mathbf{W}^T\mathbf{W}$ even though the excition in other directions is weak, but the GAF estimator, representing a worst-case safety approach, stops or slows the forgetting in all directions if the excitation in certain direction is weak. To see this point, consider the non-*p.e.* case $\mathbf{W} = \{sin(t) \;\; 0\}$, $k_1 = 1$ and $\mathbf{P}(0) = diag\{\,1\,,\,1\,\}$. λ(t) in GAF estimator will be always zero, and $\mathbf{P}(t)$ will approach to $diag\{0, 1\}$ as a result of the SLS estimation, implying the estimator "sleeps" in the strongly exciting direction. The gain matrix in the IBG estimator, however, will not vanish. In this sense, IBG has a "directional" forgetting feature, which is also seen by writing the gain update using (11),

$$\frac{d}{dt}\mathbf{P}^{-1} = -(\lambda_1/k_1)(k_1\mathbf{I}-\mathbf{P})\mathbf{P}^{-1} + \mathbf{W}^T\mathbf{W}$$

where the "forgetting matrix" varies according the the value of $\mathbf{P}$ itself, instead of the largest eigenvalue of $\mathbf{P}$. We point out that the gain matrix in the constant trace estimator of [Irving, 1979; Lozano-Leal and Goodwin, 1985] will also "sleep" in the exciting direction because one can show that its gain matrix approaches $diag\{0, 2\}$.

4. INDIRECT ADAPTIVE CONTROLLER

The adaptive controller design problem is as follows: given the desired trajectory $\mathbf{q}_d(t)$, and with some or all the manipulator parameters being unknown, derive a control law for the actuator torques, and an estimation law for the unknown parameters, such that the manipulator output $\mathbf{q}(t)$ closely tracks the desired trajectory.

4.1 The Indirect Adaptive Controller

The indirect adaptive controller uses a first-order filter $\alpha/(p+\alpha)$ (p is Laplace operator) to avoid the joint acceleration measurement. The adaptive controller is composed of a control law and an parameter estimation law. The controller part is a modified computed torque law in the form

$$\tau = \hat{\mathbf{H}}(\mathbf{q})\mathbf{u}_1 + \hat{\mathbf{B}}(\mathbf{q},\dot{\mathbf{q}}) + \hat{\mathbf{G}}(\mathbf{q}) - \dot{\hat{\mathbf{H}}}\,\mathbf{s} - (\mathbf{e} - (1/\alpha)\mathbf{W}\dot{\hat{\mathbf{a}}}) \qquad (13)$$

with

$$\mathbf{u}_1 = \ddot{\mathbf{q}}_d - 2\lambda\dot{\tilde{\mathbf{q}}} - \lambda^2\tilde{\mathbf{q}}$$

where $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$ and λ a positive constant. The parameter estimator is given by (5), with $\mathbf{P}$ generated by any of the estimators discussed in Section 3. Using the modification of the estimators as described in 4.3, the estimated inertia matrix remain strictly positive definite, *i.e.*,

$$\exists\, \alpha_o > 0, \qquad \hat{\mathbf{H}} \geq \alpha_o \mathbf{I}$$

The need for modifying the computed torque controller is intuitively understandable because the estimator itself does not attempt to drive the tracking error to zero, so that the tracking convergence has to be taken care of by the control law, unlike the situation in derect adaptive control [Slotine and Li, 1986]. The first three terms correspond to computed torque compensation of inertial, Coriolis and centripetal, and gravity torques using the estimated parameters. The fourth term accounts for the time variation of the inertia matrix. The major difference between the controllers (13) and computed-torque control is the term $(\mathbf{e} - \mathbf{W}\dot{\hat{\mathbf{a}}})$. This difference can be intuitively explained as follows. Ideally, we should use the *torque* prediction error $(\tau - \hat{\tau})$ to compensate for the error in the inverse dynamics computation (first few terms in left-side of (13)) due to the parameter estimation error. Since the joint acceleration $\ddot{\mathbf{q}}$ is assumed to be unavailable and, hence, $\hat{\tau}$ can not be computed, $\mathbf{e}$, the error in the predicted filtered torque is used instead. Due to the delaying effect of the filtering, $\mathbf{e}$ does not really reflect the error of the inverse dynamics computation at the present instant, but rather reflects that at some time ago, roughly $(1/\alpha)$, the time constant of the filter. The anticipative part $(1/\alpha)\mathbf{W}\dot{\hat{\mathbf{a}}}$ intends to account for this delaying effect. This corrective part is smaller if larger bandwidth α is used, reflecting the fact that the $\mathbf{e}$ is closer to $(\tau - \hat{\tau})$ and therefore requires less correction.

To establish the global tracking convergence of this indirect adaptive controller, let us obtain the closed-loop dynamics by substituting (13) into (1), and rearranging terms

$$\hat{\mathbf{H}}[\dot{\mathbf{s}} - \lambda\mathbf{s}] + \dot{\hat{\mathbf{H}}}\,\mathbf{s} = \mathbf{Y}\tilde{\mathbf{a}} - \mathbf{e} + (1/\alpha)\mathbf{W}\dot{\hat{\mathbf{a}}}$$

with

$$\mathbf{s} = \dot{\tilde{\mathbf{q}}} + \lambda\tilde{\mathbf{q}} \qquad (15)$$

and

$$\mathbf{Y}\tilde{\mathbf{a}} = \mathbf{H}\ddot{\mathbf{q}} + \mathbf{B} + \mathbf{G} - \hat{\mathbf{H}}\ddot{\mathbf{q}} - \hat{\mathbf{B}} - \hat{\mathbf{G}}$$

and $\mathbf{Y} = [\,\mathbf{W} + (1/\alpha)\dot{\mathbf{W}}]$. The tracking error measure $\mathbf{s}$ is therefore related to the prediction error $\mathbf{e}$ by

$$\frac{d}{dt}(\hat{\mathbf{H}}\,\mathbf{s}) + \lambda\hat{\mathbf{H}}\,\mathbf{s} = \dot{\mathbf{e}}/\alpha$$

It is easy to show that

$$\hat{\mathbf{H}}\,\mathbf{s}(t) = (\mathbf{e}(t) - \mathbf{v}(t))/\alpha \qquad (16)$$

where, in the frequency domain,

$$\mathbf{v} = \frac{\lambda}{p+\lambda}\mathbf{e}$$

Now we first state a simple lemma which holds for both the scalar case and the vector case.

Lemma 1: *Consider the filter relation $y = \mathbf{L}(p)u$, with y being the output, u the input, and $L(p)$ an exponentially stable and strictly proper transfer function. 1: If $u \in L^2$, then $y \in L^2$, $y \in L_\infty$ and $y(t) \rightarrow 0$ as $t \rightarrow 0$. 2: If u is exponentially convergent to zero, so is y.*

The first result is shown in [Desoer and Vidyasagar, 1975], implying that filtering a square-integrable function not only maintains its square-integrability but also leads to a convergent output. The second part can be easily shown by the convolution expression of the output and the exponential properties of the input and the impulse function.

It is now easy to prove the global convergence of the tracking error. Using the first part of lemma 1, $\mathbf{v}(t) \in L^2$ and $\mathbf{v}$ converges. Since multiplying a square-integrable function by a upper bounded function retain the square-integrable property, $\tilde{\mathbf{q}}$ must be square integrable and asymptotically convergent to zero by noting right-hand terms in

$$\dot{\tilde{\mathbf{q}}} + \Lambda\tilde{\mathbf{q}} = \hat{\mathbf{H}}^{-1}(\mathbf{e} - \mathbf{v})/\alpha \qquad (17)$$

are both square-integrable. Since $\dot{\tilde{\mathbf{q}}}$ is square-integrable from (17), and a similar reasoning as in [Middleton and Goodwin, 1986] shows that $\ddot{\mathbf{q}}$ is bounded, we conclude the $\dot{\tilde{\mathbf{q}}}$ is globally convergent to zero. In summary,

Theorem 3: *The indirect adaptive controller given in (5) and (13) guarantees the convergence of prediction error $\mathbf{e}$, position tracking error $\tilde{\mathbf{q}}$ and velocity tracking error $\dot{\tilde{\mathbf{q}}}$ if the desired trajectories $\mathbf{q}_d$, $\dot{\mathbf{q}}_d$ and $\ddot{\mathbf{q}}_d$ are bounded.*

Note that the condition for tracking convergence is quite weak. In fact, the condition is the same as that of the direct adaptive control of [Slotine and Li, 1986].

4.2 Exponential Convergence of Tracking Errors and Estimated Parameters

As shown, the boundedness of the desired trajectories $\mathbf{q}_d$, $\dot{\mathbf{q}}_d$ and $\ddot{\mathbf{q}}_d$ is enough for the global tracking convergence of the above adaptive controllers and the boundedness of the estimated parameters. But it is not enough for the convergence of the estimated parameters to the true parameters, which is physically obvious since the accurate identification of the parameters requires the signals to contain enough information on the parameters. In this section, we show that the persistent excitation of $\mathbf{W}$ can guarantee the exponential convergence of the estimated parameters and tracking errors, for the indirect adaptive controllers based on most of the estimators described in section 2.

The results to be derived use the following two lemmas:

Lemma 2: *If* $\mathbf{W}(\mathbf{q}_d, \dot{\mathbf{q}}_d)$ *is p.e., and* $\mathbf{q} \to \mathbf{q}_d$ $\dot{\mathbf{q}} \to \dot{\mathbf{q}}_d$ *as* $t \to \infty$, *then* $\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})$ *is also p.e.*

Lemma 3: *If the input matrix of an exponentially stable and strictly proper filter is p.e. and u.c. (uniformly continuous), then the output matrix is also p.e. and u.c.*

The proof of lemma 2 can be found in [Slotine and Li, 1988]. Intuitively, this result is reasonable since $\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})$ is almost the same as $\mathbf{W}(\mathbf{q}_d, \dot{\mathbf{q}}_d)$ after some time, and therefore, their excitation should be very similar. The proof of lemma 3 is presented in [Li and Slotine, 1988]. It means that if the signals are *p.e.* and smooth, passing it through an exponentially stable filter preserves its persistent excitation.

Theorem 4: *If* $\mathbf{Y}(\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)$ *are p.e. and uniformly continuous, or* $\mathbf{W}(\mathbf{q}_d, \dot{\mathbf{q}}_d)$ *is p.e., the tracking errors, parameter errors and prediction errors in the indirect adaptive controllers based on the gradient, gain-resetting LS, GAF and IBG estimators will exponentially converge to zero.*

Proof: From lemmas 2 and 3, the *p.e.* and *u.c.* of $\mathbf{Y}_d$ guarantees the *p.e.* of $\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})$, which implies the exponential convergence of the estimated parameters as explained in section 3. The exponential convergence of parameter error implies the exponential convergence of prediction error $\mathbf{e}$. The exponential convergence of $\mathbf{e}$ in turn leads to the exponential convergence of $\mathbf{s}$, $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$ from (17) and lemma 2. **q.e.d.**

4.3 Implementation and computation Issues

In the analysis of this adaptive controller, as well as those of [Craig, *et al.*, 1986; Middleton and Goodwin, 1986; Hsu *et al*, 1987], it is assumed that $\hat{\mathbf{H}}$ remains positive definite in the course of adaptation. However, this condition is not automatically guaranteed. Therefore, the estimators should be modified to constrain estimated parameters so that $\hat{\mathbf{H}}$ is *p.d.*, while retaining the square-integrable property of $\mathbf{e}$. If we know that the true parameters lie in a certain *convex* region in which $\hat{\mathbf{H}}$ is positive definite then, as remarked in [Middleton and Goodwin, 1986], the use of a projection method will prevent the estimated parameters from getting out of it, and will also retain the square-integrability property of $\mathbf{e}$. Yet it appears that no result has been derived on the existence of such a convex region and, if there does exist one, how to find it. This is by no means a trivial issue, since $\hat{\mathbf{H}}$ is a complicated non-linear matrix which depends on both $\hat{\mathbf{a}}$ and $\mathbf{q}$. In the following, we propose a physically-motived technique of guaranteeing the *p.d.* of $\hat{\mathbf{H}}$ in the course of adaptation. The maintenance of the $\mathbf{e}$ square-integrability remains to be shown in the general case. But by assuming the mass of the load is estimated beforehand, while the other nine load parameters are estimated on-line, we can show that the constraint surfaces are convex, and consequently that $\mathbf{e}$ is square integrable. The assumption on the availability of the load mass is not too restrictive considering that an advanced robot usually has a force sensor at the hand, and it is immediate to obtain the load mass from the force sensor measurements (while the mass center and moments parameters are difficult to measure but can be handled by the adaptive controller). It is also easy, without a force sensor, to obtain the load mass if gravitational forces are present. The discussion is followed by some remarks concerning the computational efficiency of the indirect adaptive controller.

4.3.1 Guaranteeing the Positive Definiteness of $\hat{\mathbf{H}}$

We make the realistic assumption that the manipulator's link parameters have been accurately estimated *a priori* by off-line or on-line methods, so that the only unknown inertial parameters are the ten equivalent parameters (corresponding to previous linearity property) of the load in robot hand, namely, mass m (load mass), mx, my, mz (with x, y, z describing the location of the center of mass of the load in hand coordinates), and $I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}$ (the moments of inertias around the origin of the hand coordinate). Therefore, the estimated inertia matrix $\hat{\mathbf{H}}(\mathbf{q})$ can be written as

$$\hat{\mathbf{H}} = \mathbf{H}_m(\mathbf{q}) + \hat{\mathbf{H}}_l(\hat{\mathbf{a}}, \mathbf{q}))$$

where $\mathbf{H}_m$ is the known inertial matrix of the manipulator without load, and $\hat{\mathbf{H}}_l(\hat{\mathbf{a}}, \mathbf{q}))$ is the estimated inertia matrix of the load with $\hat{\mathbf{a}}$ included to stress the fact that the estimated inertia matrix depends on both joint position $\mathbf{q}$ and estimated load parameters $\hat{\mathbf{a}}$.

We now make two physical observations:

1: the inertia matrix $\mathbf{H}_m$ of any manipulator must be strictly positive definite for any joint position $\mathbf{q}$;

2. if the equivalent parameters in $\hat{\mathbf{a}}$ is such that m, the estimated load mass, is positive, and $\hat{\mathbf{M}}_c$, the corresponding estimated moment of inertia matrix *about the center of mass* is *p.d.*, then the estimated inertia matrix of the load $\hat{\mathbf{H}}_l(\hat{\mathbf{a}}, \mathbf{q})$ is positive semi-definite (the "semi" accounting for the case when there is no load), where, according to classical mechanics, $\hat{\mathbf{M}}_c$ is related to the equivalent parameters by

$$\hat{\mathbf{M}}_c = \hat{\mathbf{M}} - \hat{m}[\hat{\mathbf{r}}^T\hat{\mathbf{r}}\,\mathbf{I} - \hat{\mathbf{r}}\,\hat{\mathbf{r}}^T]$$

with $\hat{\mathbf{r}} = [\hat{x}, \hat{y}, \hat{z}]^T$ being the estimated mass center location and $\hat{\mathbf{M}}$ the estimated moment of inertia matrix about the hand coordinate origin. The first observation comes from the fact that, at any position $\mathbf{q}$, the kinetic energy of the manipulator corresponding to any unit magnitude joint velocity must be positive, implying the lower boundedness of all $\mathbf{H}_m(\mathbf{q})$ eigenvalues. The second observation is due to the existence of a physical load which has the same inertial parameters as the estimated ones, and to the fact the kinetic energy of the object must be positive.

Therefore, it is clear that $\hat{\mathbf{H}}$ will be strictly positive definite if $\hat{\mathbf{a}}$ is restricted to be in the region defined by

$$\hat{m} \geq 0 \quad \text{and} \quad \hat{\mathbf{M}}_c \geq 0$$

The semi-positiveness of m can easily be guaranteed by stopping the adaptation on $\hat{m}$ as soon as $\hat{m}$ reaches zero, and resuming its adaptation as $\dot{\hat{m}}$ becomes positive. In addition, we restrict the parameter update so that $\hat{m}\hat{\mathbf{M}}_c$ is *p.d.* or semi-*p.d.* instead of directly dealing with $\hat{\mathbf{M}}_c$. This parameter restriction can be achieved by updating the parameters only along the constraint surfaces once the estimated parameters reachs either of the three constraint surfaces of *p.d.* region of $\hat{\mathbf{M}}_c$. Specifically, we should compute the values of the determinants of the three primary submatrices of

$\hat{m}\hat{\mathbf{M}}_c$ on-line and, when either of the determinants become zero, use the projection method in [Goodwin and Mayne, 1987] to modify the parameter update direction so that the estimated parameters do not get out of the *p.d.* region (in parameter space) of $\hat{m}\hat{\mathbf{M}}_c$ which depends of estimated equivalent parameters in $\hat{\mathbf{a}}$, denoted by $\mathbf{F}(\hat{\mathbf{a}})$,

$$\mathbf{F}(\hat{\mathbf{a}}) = \hat{m}\hat{\mathbf{M}}_c = \hat{m}\hat{\mathbf{M}} - [(\hat{m}\hat{\mathbf{r}}^T(\hat{m}\hat{\mathbf{r}})\mathbf{I} - (\hat{m}\hat{\mathbf{r}})(\hat{m}\hat{\mathbf{r}}^T) \tag{18}$$

This indicates that the elements of $\hat{m}\hat{\mathbf{M}}_c$ are all quadratic polynomials of the estimated parameters, so that three determinants and the normal directions of the three constraint surfaces can be computed efficiently. If the *p.d.* region of $\hat{\mathbf{M}}_c$ in parameter space is *convex*, then the projection method can be shown [Goodwin and Mayne, 1987] to retain the crucial prediction error square-integrable property. However, this convexity remains to be shown if all ten load parameters are estimated simultaneously (or, of course, if all the manipulator parameters have to be estimated).

If we make the *assumption that the load mass is estimated beforehand*, while the other nine parameters are estimated on-line by the proposed indirect adaptive controller, then the *p.d.* region of $\hat{\mathbf{M}}_c$ can be shown to be convex due to the unique structure of (18), as follows. Let $\mathbf{z}_1 = \{\mathbf{y}_1, \mathbf{J}_1\}^T$ be a point in the *p.d.* region of $\hat{\mathbf{M}}_c$ (implying $\mathbf{F}(\mathbf{z}_1) \geq \mathbf{0}$) with $\mathbf{y}_1$ being the row vector corresponding the equivalent mass center parameters and $\mathbf{J}_1$ corresponding to equivalent moments, and $\mathbf{z}_2 = \{\mathbf{y}_2, \mathbf{J}_2\}^T$ be another point in the region. Let $\underline{\mathbf{z}}$ be the middle point between $\mathbf{z}_1$ and $\mathbf{z}_2$,

$$\underline{\mathbf{z}} = \{\frac{\mathbf{y}_1 + \mathbf{y}_2}{2}, \frac{\mathbf{J}_1 + \mathbf{J}_2}{2}\}^T$$

Then one easily shows that

$$\mathbf{F}(\underline{\mathbf{z}}) = (1/2)(\mathbf{F}(\mathbf{z}_1) + \mathbf{F}(\mathbf{z}_2)) + (1/4)(\mathbf{x}^T\mathbf{x}\,\mathbf{I} - \mathbf{x}\mathbf{x}^T)$$

where $\mathbf{x}^T = \mathbf{y}_2 - \mathbf{y}_1$, *i.e.*, $\mathbf{F}(\underline{\mathbf{z}})$ is *p.d.* since the third term is *p.d.* by Schwarz inequality, and therefore, $\underline{\mathbf{z}}$ is in the region. As a result, the *p.d.* region of $\hat{\mathbf{M}}_c$ in parameter space is convex.

4.3.2 Computational Efficiency

While further research is needed on computational aspects, we believe that the indirect adaptive controller can be implemented relatively efficiently. Much of the computation involved in the estimation part is the generation of the regression matrix **W**, which may possibly be computed recursively with efficiency using the similarity of its terms to those in inverse-dynamics computation. The computation of the control law is not as complicated as it looks at a first glance. The first three terms in left-side of (13) can be efficiently computed by recursive Newton-Euler methods. The final two terms do not require much computation since **e** is already computed in the estimation part, and $\mathbf{W}\hat{\mathbf{a}}$ can be computed like $\hat{\mathbf{y}}$. Note that

$$\hat{\dot{\mathbf{H}}} = \dot{\mathbf{H}}(\hat{\mathbf{a}}, \mathbf{q}) + \hat{\dot{\mathbf{H}}}$$

where $\hat{\dot{\mathbf{H}}}$ is the estimated value of $\dot{\mathbf{H}}$. The first part may be recursively computed as if it were the inertial joint torques with parameters $\hat{\mathbf{a}}$ and joint acceleration **s**. The second part is also suitable to recursive computation since

$$\hat{\dot{\mathbf{H}}} = \sum_{i=1} \frac{\partial \hat{\mathbf{H}}}{\partial \mathbf{q}} \dot{q}_i$$

5. CONCLUDING REMARKS

In direct and composite adaptive robot control, the known positive definiteness of the manipulator's inertia matrix is used as an asset to simplify computations and avoid the difficulties linked to the SPR requirement in traditional adaptive control. It becomes in effect a liability in adaptive "computed-torque" approaches, which requires special procedures to be overcome, even when the inversion of the estimated inertia matrix is avoided as in the proposed indirect controller.

The GAF and IBG parameter estimators proposed in Section 3 can be applied to any parameter estimation problem in the basic linear form, and are actually most suitable in the (generally non-robotic) case of time-varying parameters. The discussion of convexity is also of interest in its own right.

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