UNIVERSITY OF PORTSMOUTH

Radio Interferometry – the return of the Fourier Transform

HAMIDREZA KHALEGHZADEH

Introduction

This image made the news in 2019. It is the shadow cast by a supermassive black hole at the center of the giant galaxy M87.

How are such images constructed?

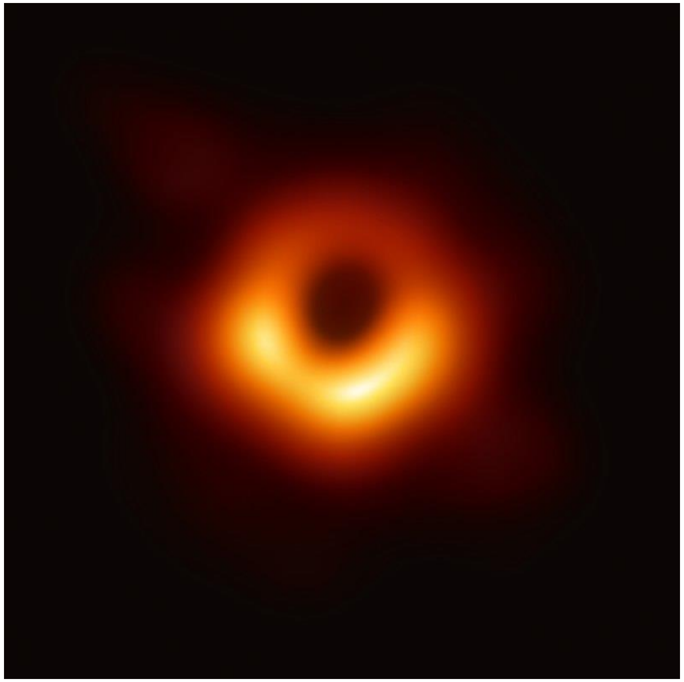


Image from Wikipedia

The Problem

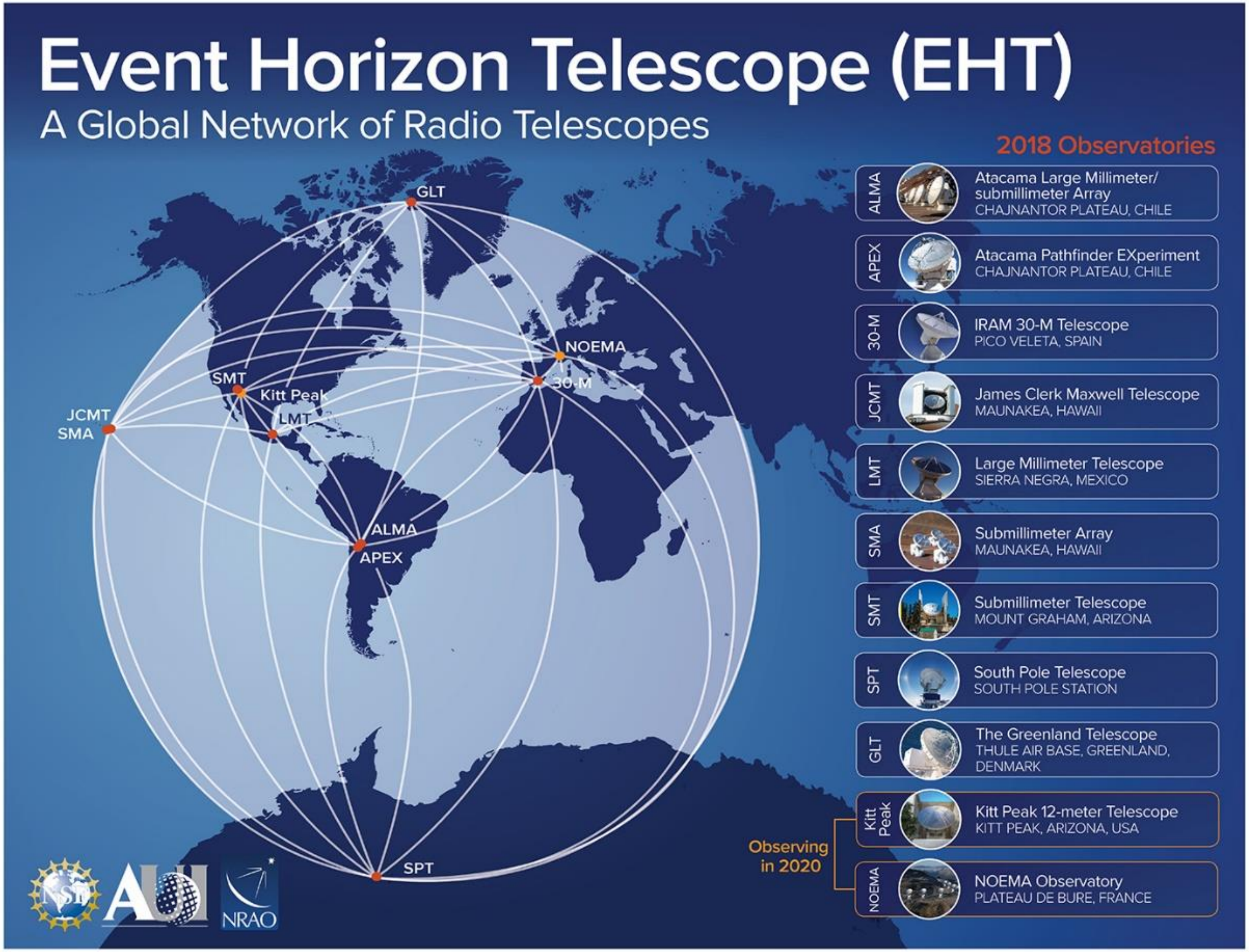
The black hole M87* weighs in at about 6 billion solar masses, but its event horizon is “only” about 40 billion km across.

- $1\ solar\ mass\ \cong 2 * 10^{30}\ kg$
- An event horizon is a boundary beyond which events cannot affect an observer

M87 is 53 million light years away - about 500 *billion billion* km.

The angle M87* subtends from an observer on Earth is 8×10^{-11} radians (or about 16 micro arc seconds).

As somebody said, that is more than enough resolution to read a newspaper in New York from London or Paris (better than mm resolution, at that distance).


Event Horizon Telescope (EHT)
A Global Network of Radio Telescopes
2018 Observatories
GLT
NOEMA
SMT
Kitt Peak
30-M
LMT
JCMT
SMA
ALMA
APEX
SPT
ALMA Atacama Large Millimeter/ submillimeter Array CHAJNANTOR PLATEAU, CHILE
APEX Atacama Pathfinder EXperiment CHAJNANTOR PLATEAU, CHILE
30-M IRAM 30-M Telescope PICO VELETA, SPAIN
JCMT James Clerk Maxwell Telescope MAUNAKEA, HAWAII
LMT Large Millimeter Telescope SIERRA NEGRA, MEXICO
SMA Submillimeter Array MAUNAKEA, HAWAII
SMT Submillimeter Telescope MOUNT GRAHAM, ARIZONA
SPT South Pole Telescope SOUTH POLE STATION
GLT The Greenland Telescope THULE AIR BASE, GREENLAND, DENMARK
Observing in 2020
Kitt Peak Kitt Peak 12-meter Telescope KITT PEAK, ARIZONA, USA
NOEMA NOEMA Observatory PLATEAU DE BURE, FRANCE
NRAO

Telescope Resolution

The angular resolution of a telescope is given by the approximate formula:

$$R = {}^{\lambda}\!/_{D}$$

where R is the resolution in radians, λ is the wavelength of the observed radiation, D is aperture (diameter) of the telescope.

Hubble Space Telescope, $\lambda = 500\text{nm}$, $D = 2.4\text{m}$, gives resolution of 2×10^{-7} radians, or 0.04 arc seconds.

Event Horizon Telescope, $\lambda = 1.3\text{mm}$, $D = 12{,}700\text{km}$ (diameter of Earth), gives resolution of 10^{-10} radians, or 20 *micro* arc seconds.

Note: 1 arc second = 4.85 * 10^{-6} radian = 1/3600 degree.

But how do we get images?

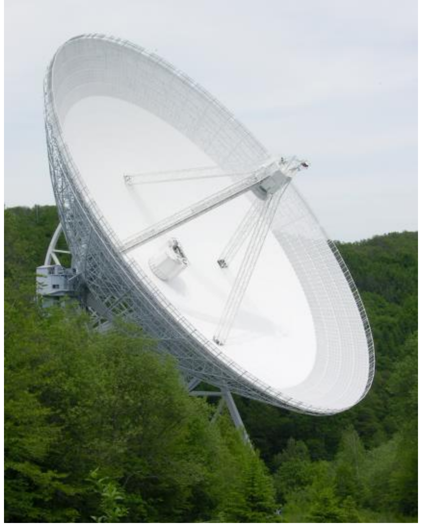
Presumably we have some idea of how a lens or parabolic mirror creates an image in a light telescope.

For a conventional radio telescope, the dish can take a similar role to a mirror.

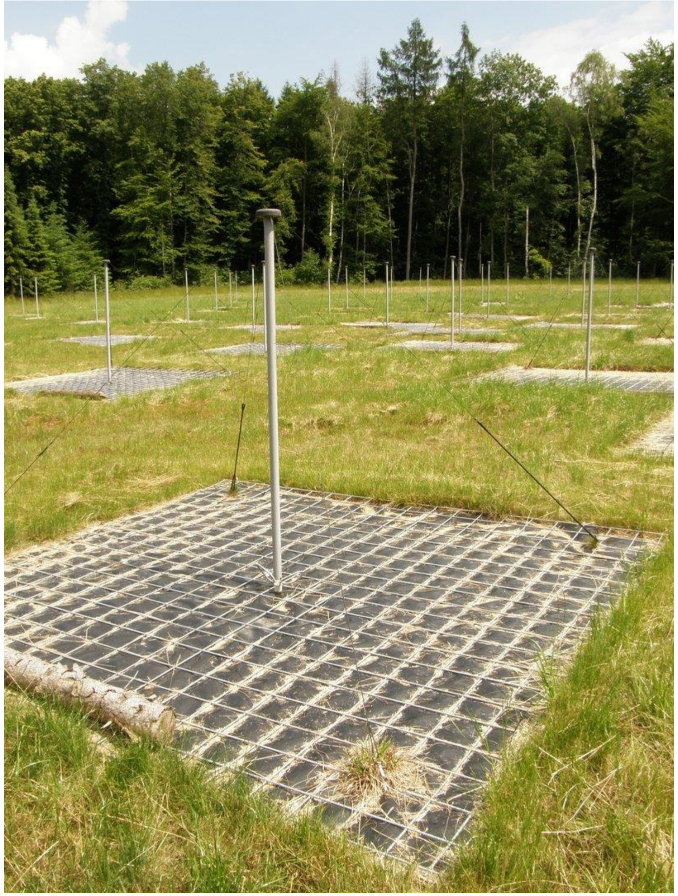
But how are images reconstructed from multiple radio telescopes at remote locations (Very Large Baseline Arrays)?

To get a feeling for this let’s consider a radio telescope with an even simpler individual receiving element.

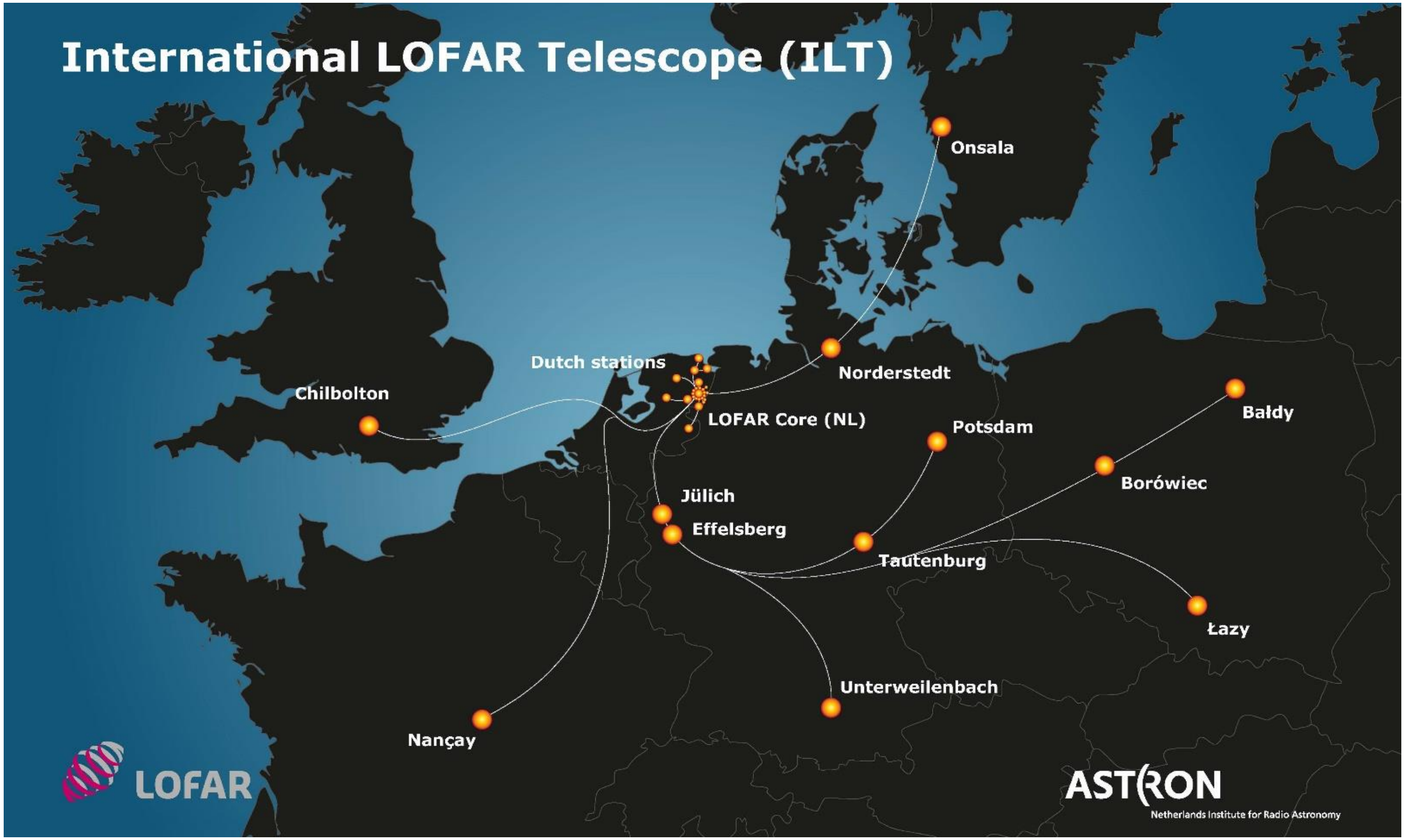
LOFAR (Low-Frequency Array) is a radio telescope run by Astron in the Netherlands, with base stations spread across Europe.



LOFAR Antennae (in a field)



LOFAR Stations



A Software Telescope

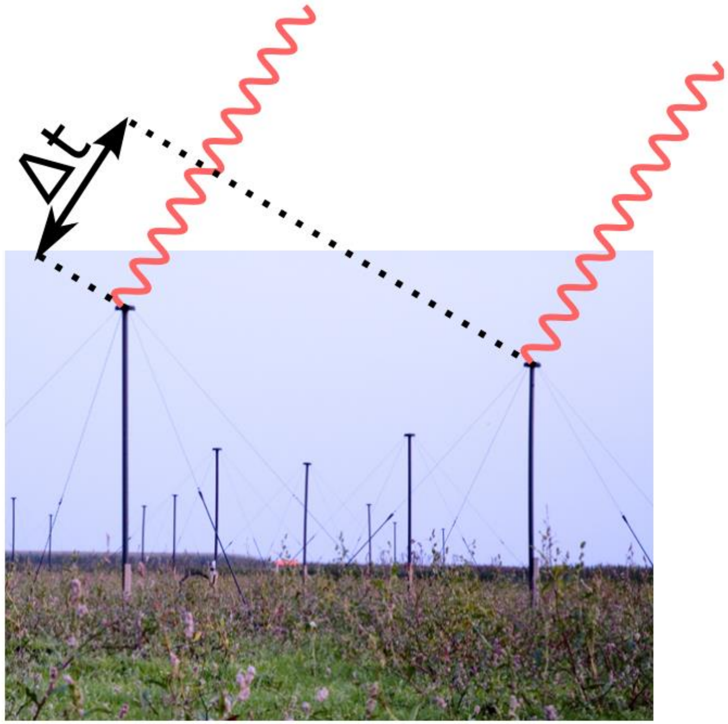
LOFAR bills itself as a “software telescope”.

Thousands of simple antennae at widespread locations record radio signals arriving from all directions in space.

The only way to combine these signals to provide directionality or imaging is under control of software algorithms.

Pointing the array†

Before signals from different antennae are combined, a carefully calculated delay can be inserted into all signals, so that all signals from one particular direction are in phase with one another.



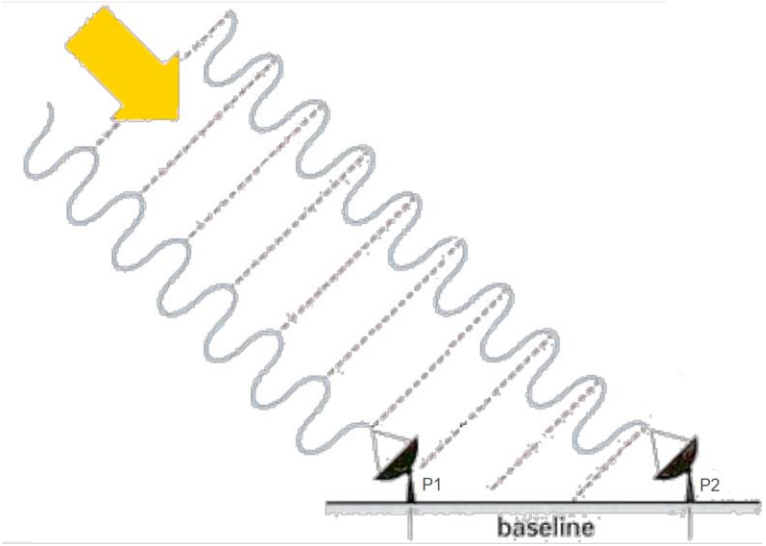
†Image from *The LOFAR Beam Former: Implementation an Performance Analysis*, Mol and Romein, 2011

Visibilities

Visibilities have a central role in image reconstruction for radio interferometers.

A visibility is a *correlation function* between the signals arriving at two different antennae, say P_1 and P_2.

The two signals can be encoded by treating the signals and resulting visibilities as *complex numbers*.



Visibilities

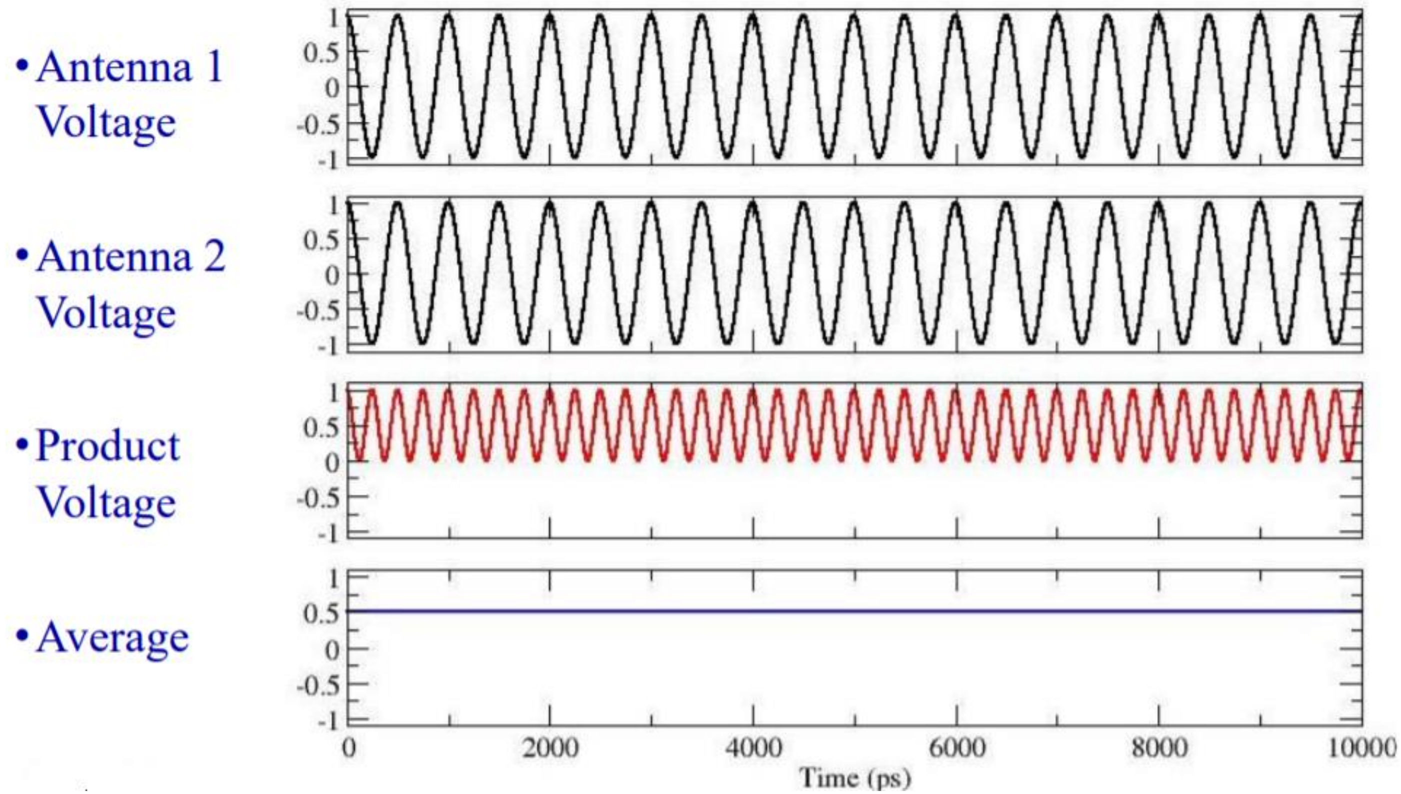
To obtain the visibility between two antennas P1 and P2

1. Multiply together the two signals received by P1 and P2 at corresponding times. Now average the result over some fixed interval of time, to obtain the cosine correlator R_c as a single number (for a single frequency of interest).
2. Inserting a 90 degree phase shift in one of the signal paths. Then, Multiply together the two signals at corresponding times and average the result over some fixed interval of time, to obtain the Sine correlator R_s a single number.
3. The complex visibility between antennas P1 and P2 is

$$V = R_c - iR_s$$

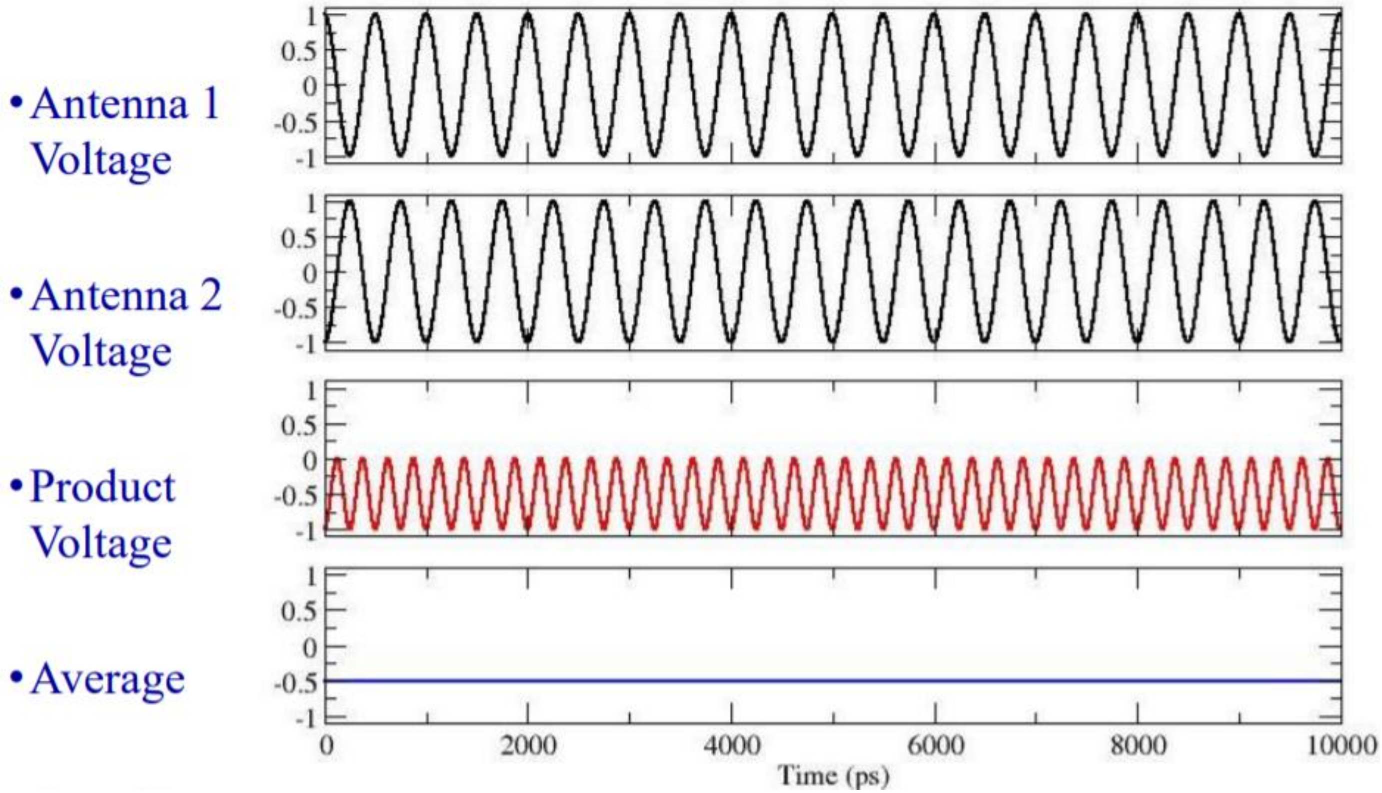
Visibilities - Example

Obtaining the cosine correlator R_c



Visibilities - Example

Obtaining the cosine correlator R_s by a 90 degree shift



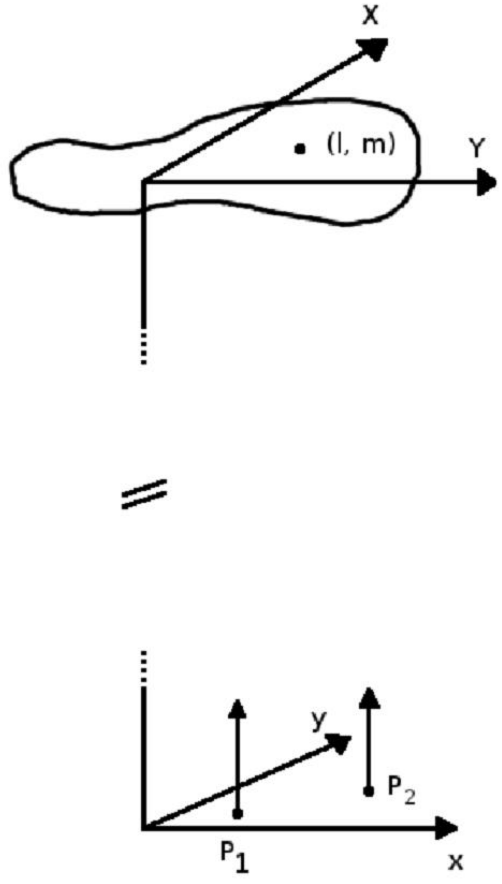
The Sky Intensity

The picture envisages some object high in the sky, over some antenna on the ground.

l, m are angular measures of position in the sky, from some centre of a field of view. Usually referred to a *direction cosines*, but can just think of them as angles in radians†.

The radio intensity of the object we are viewing, as a function of position on the sky, is the *sky intensity*, denoted:

$$I(l, m)$$



†Image from Wikipedia *Van Cittert-Zernike Theorem*

The van Cittert-Zernicke Theorem

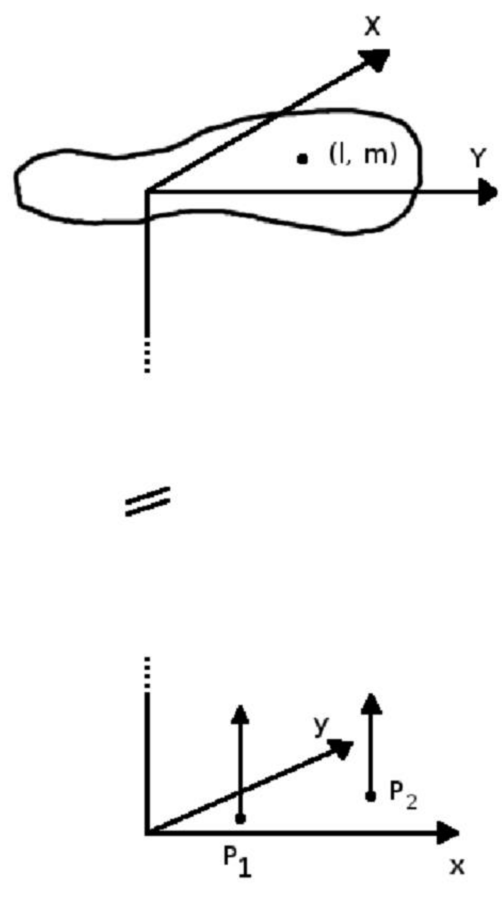
Let's denote by (u, v) the two dimensional displacement between the antennae P1 and P2. This vector should be measured in units of the wavelength of the signal we observe:

$$u = (x_{P_1} - x_{P_2})/\lambda, \qquad v = (y_{P_1} - y_{P_2})/\lambda$$

In the 1930s the physicists van Cittert and Zernicke showed the correlation or visibility between antennae P_1 and P_2 can be calculated as:

$$V(u, v) \propto \sum_{l=\cdots}^{\cdots} \sum_{m=\cdots}^{\cdots} I(l, m) \cdot e^{-2\pi i(ul+vm)}$$

By now you may perhaps recognize this as a Fourier Transform!



Recovering the Sky Intensity

The van Cittert-Zernicke theorem gives us the measurable visibilities as a function of the sky intensity $I(l,m)$. This latter could be readily interpreted as an image of the target object, in terms of its radio emissions.

Since the expression for $V(u,v)$ is in the form of a Fourier transform we can in principle invert it as follows:

$$I(l,m) \propto \sum_{u=-\infty}^{+\infty} \sum_{v=-\infty}^{+\infty} V(u,v) \cdot e^{2\pi i(ul+vm)}$$

The values of u and v can be calculated for each signal as explained in slide 16.

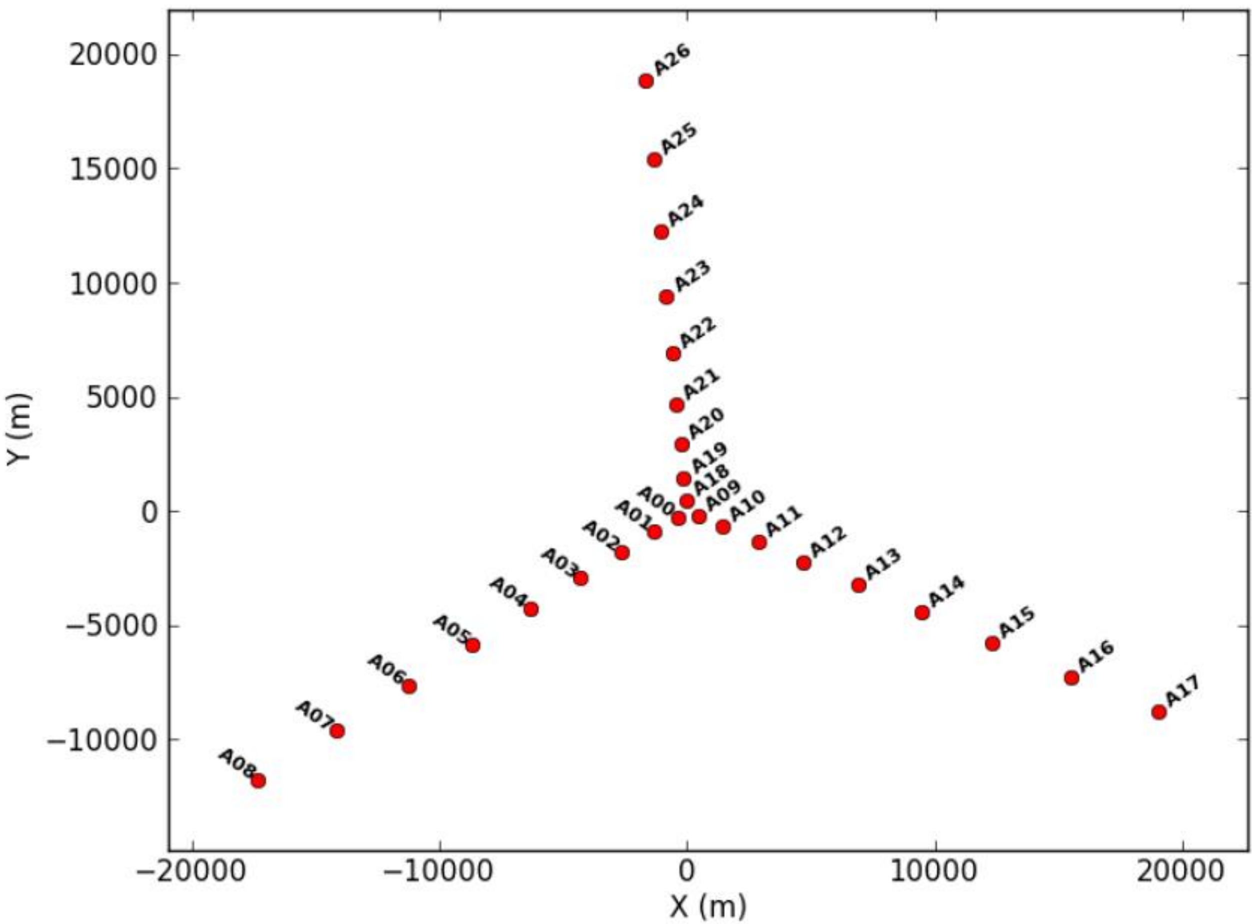
u,v sampling (coverage)

At first sight the limited, fixed number of antennae between which we are calculating correlations (visibilities) may seem a serious obstacle to inverting the FT.

- A limited number of u, v points can be obtained using a limited number of antennas.

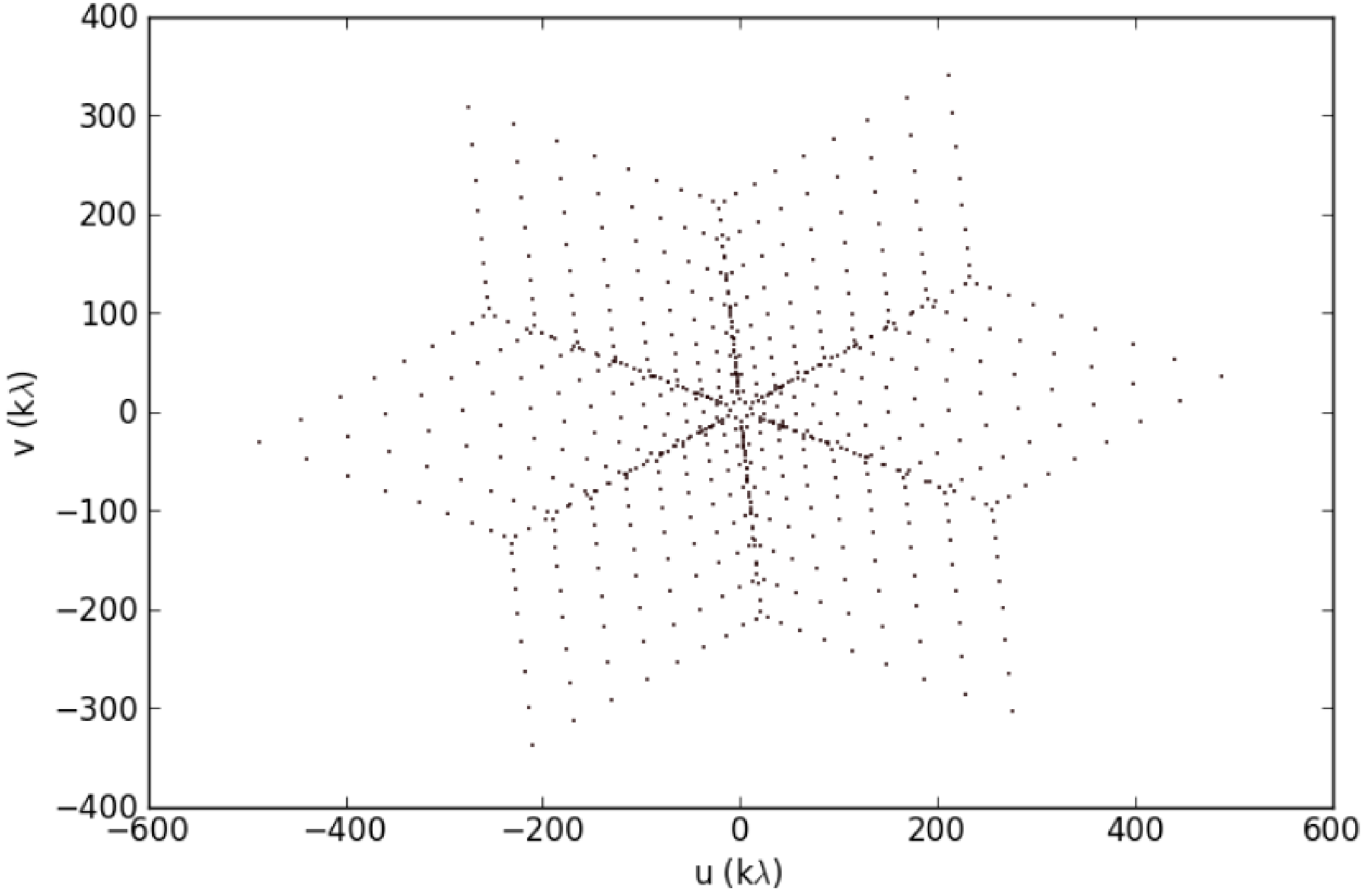
The examples on the next few slides are borrowed† from the doctoral thesis of Haoyang Ye, *Accurate image reconstruction in radio interferometry*, 2019, which you can find online.

Layout of an antenna array



Ye, Fig 1.3

Sampling of u,v plane



Ye, Fig 1.2a

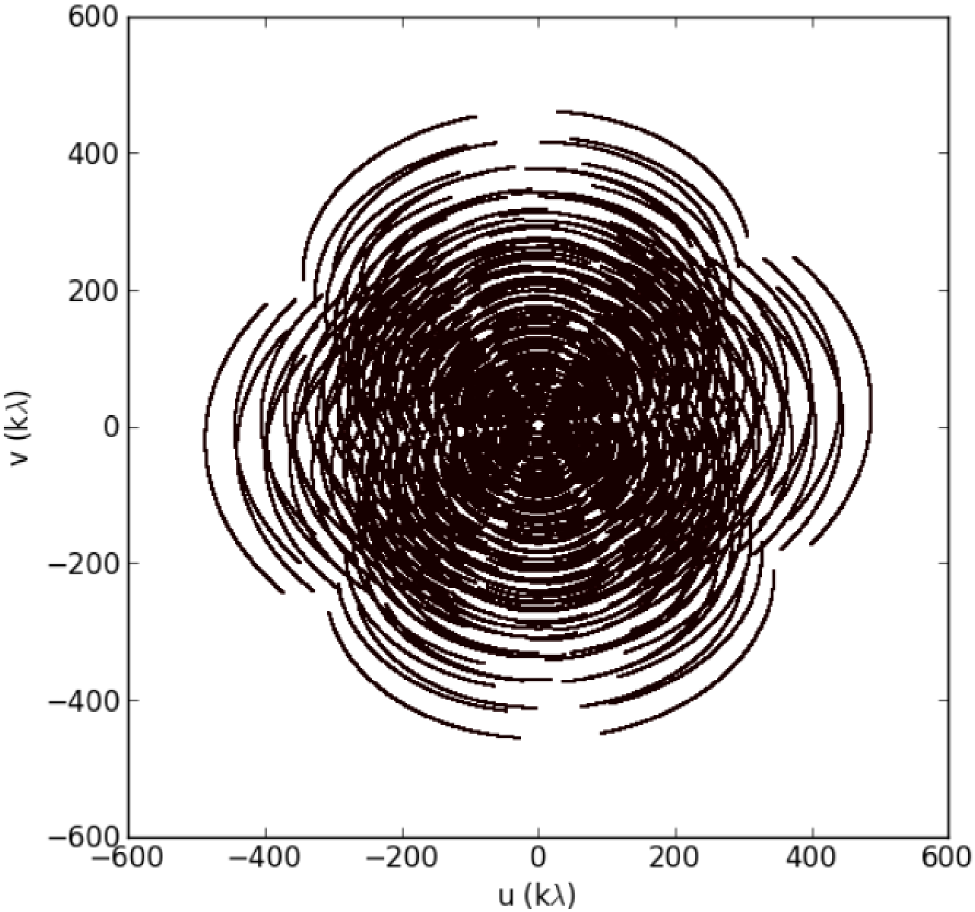
Rotating Earth to the rescue

It would indeed be hard to invert the Fourier transform with any kind of accuracy with such a sparse coverage of the u, v plane.

What comes to our rescue is the fact that although our antennae are fixed to the ground, giving us limited displacements at any given moment, the ground is rotating with the Earth and thus the whole array is rotating with respect to the fixed stars.

If we repeat measurements at many times throughout the day, orientation of baselines relative to the plane of observation (perpendicular to direction to object in fixed space) changes from measurement to measurement.

Example u,v coverage over 5 hours of observations.



Ye, Fig 1.4

Approximate Fourier inversion

Now we can approximate the integral over u,v by:

$$I(l,m) \approx \sum_{u,v}^{all\ observations} V(u,v) \cdot e^{2\pi i(ul+vm)}$$

This approximation to the true value of $I(l,m)$ is technically known as the *dirty image*.

The sum here can be evaluated as written, but that can lead to poor efficiency – it is analogous to inverting a discrete Fourier transform by the naïve algorithm.

Production quality astronomical imaging software will usual go through an interpolation stage, *gridding* the visibilities in the u, v plane, then use an FFT.

Dirty Beam

The dirty image $I(l, m)$ cerates a blurred image of the desired target. It is because of using non-complete coverage of the u,v plane.

Usually an attempt is made to obtain *dirty beam* from the dirty image, to recover a “clean” image.

One of the oldest but still popular methods of to obtain the clean sky images is called *CLEAN* (or *Högbom CLEAN*).

- Cornwell, T. J. "*Hogbom's CLEAN algorithm. Impact on astronomy and beyond-Commentary on*: Högbom JA, 1974, A&AS, 15, 417." Astronomy & Astrophysics 500.1 (2009): 65-66.