4) Se tiene la función $y = a \log b + bc^2$.

Calcular las derivadas $\frac{\partial y}{\partial a}, \frac{\partial y}{\partial b}, \frac{\partial y}{\partial c}$ evaluando

$a = 2,\ b = 10,\ c = 0{,}5$

- Derivadas analíticas (symbolic differentiation)

$$\frac{\partial y}{\partial a} = \frac{\partial(a \log b + bc^2)}{\partial a} = \frac{\partial(a \log b)}{\partial a} = \log b \frac{\partial a}{\partial a} = \boxed{\log b}$$

$$\frac{\partial y}{\partial b} = \frac{\partial(a \log b + bc^2)}{\partial b} = a\frac{\partial(\log b)}{\partial b} + c^2\frac{\partial(b)}{\partial b} = a\frac{1}{b} + c^2 = \boxed{\frac{a}{b} + c^2}$$

$$\frac{\partial y}{\partial c} = \frac{\partial(a \log b + bc^2)}{\partial c} = \frac{\partial(bc^2)}{\partial c} = b\frac{\partial(c^2)}{\partial c} = \boxed{2bc}$$

- Límite de delta numérico pequeño (Numerical differentiation)

$$\frac{\partial y}{\partial a} = \lim_{\Delta h \to 0} \frac{(a+\Delta h)\log b + \cancel{bc^2} - (a \log b + \cancel{bc^2})}{\Delta h}$$

$$\frac{\partial y}{\partial a} = \lim_{\Delta h \to 0} \frac{\log b\,(\cancel{a} + \Delta h - \cancel{a})}{\Delta h} = \lim_{\Delta h \to 0} \log b \frac{\cancel{\Delta h}}{\cancel{\Delta h}} = \boxed{\log b}$$

$$\frac{\partial y}{\partial b} = \lim_{\Delta h \to 0} \frac{f(b+\Delta h) - f(b)}{\Delta h} = \lim_{\Delta h \to 0} \frac{a \log(b+\Delta h) + (b+\Delta h)c^2 - a \log b - bc^2}{\Delta h}$$

$$\frac{\partial y}{\partial b} = \lim_{\Delta h \to 0} \frac{a(\log(b+\Delta h) - \log b) + c^2(\cancel{b} + \Delta h - \cancel{b})}{\Delta h}$$

$$\frac{dy}{db} = \lim_{\Delta h \to 0} \frac{a\left(\log\left(\frac{b+\Delta h}{b}\right)\right) + c^2 \Delta h}{\Delta h}$$

$$\frac{dy}{db} = \lim_{\Delta h \to 0} \frac{a \log\left(1 + \frac{\Delta h}{b}\right)}{\Delta h} + c^2 \frac{\cancel{\Delta h}}{\cancel{\Delta h}}$$

$$\frac{dy}{db} = \lim_{\Delta h \to 0} a\left(\frac{1}{\Delta h}\left(\log 1 + \frac{\Delta h}{b}\right)\right) + c^2 = \lim_{\Delta h \to 0} a\left(\log\left(1+\frac{\Delta h}{b}\right)^{1/\Delta h}\right) + c^2$$

$$\frac{dy}{db} = a \log\left(\lim_{\Delta h \to 0} \left(\frac{1+\Delta h}{b}\right)^{1/\Delta h}\right) + c^2 = a \log e^{1/b} + c^2$$

$$\frac{dy}{db} = a \log e^{1/b} + c^2 = a \frac{1}{b} + c^2 = \boxed{\frac{a}{b} + c^2}$$

$$\frac{dy}{dc} = \lim_{\Delta h \to 0} \frac{a \cancel{\log} b + b(c+\Delta h)^2 - a \cancel{\log} b - bc^2}{\Delta h}$$

$$\frac{dy}{dc} = \lim_{\Delta h \to 0} b\left(\frac{(c+\Delta h)^2}{\Delta h} - \frac{c^2}{\Delta h}\right) = \lim_{\Delta h \to 0} b\frac{\left((c^2 + 2c\Delta h + \Delta h^2) - c^2\right)}{\Delta h}$$

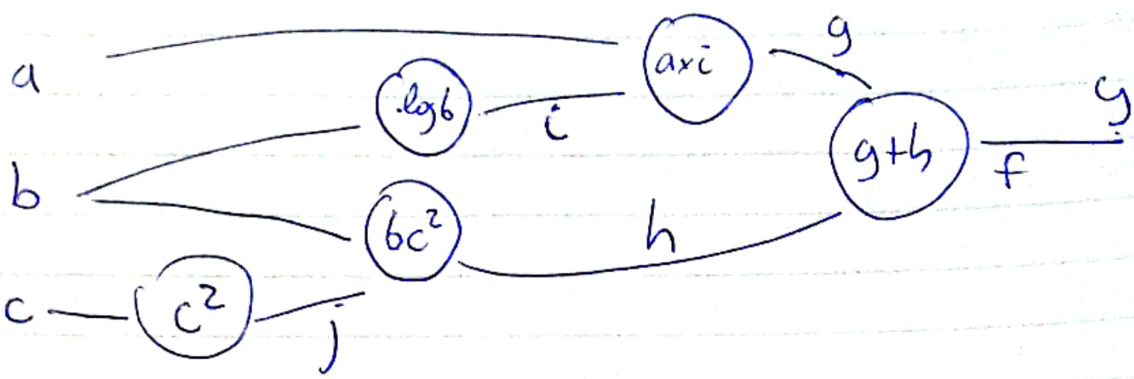
$$\frac{dy}{dc} = \lim_{\Delta h \to 0} b\left(\frac{\cancel{c^2} + 2c\Delta h + \Delta h^2 - \cancel{c^2}}{\Delta h}\right) = \lim_{\Delta h \to 0} b\left(\frac{2\cancel{\Delta h}c}{\cancel{\Delta h}} + \frac{\Delta h^2}{\cancel{\Delta h}}\right)$$

$$\frac{dy}{dc} = \lim_{\Delta h \to 0} b(2c + \Delta h) = \lim_{\Delta h \to 0} 2bc + b\Delta h = 2bc + \lim_{\Delta h \to 0} b\Delta h$$

$$\frac{dy}{dc} = \boxed{2bc}$$

Escaneado con CamScanner

grafos y regla de la cadena (chain rule differentiation)



Forward

$j = c^2$

$h = bc^2 = b \times j$

$i = \log b$

$g = a \log b = a \times i$

$f = g + h = y$

backward

$$\frac{\partial y}{\partial a} = \frac{\partial y}{\partial g}\frac{\partial g}{\partial a} = 1 \times i = \boxed{\log b}$$

$$\frac{\partial y}{\partial b} = \frac{\partial y}{\partial g}\frac{\partial g}{\partial i}\frac{\partial i}{\partial b} + \frac{\partial y}{\partial h}\frac{\partial h}{\partial b} = 1 \times a \times \frac{1}{b} + c^2$$

$$\frac{\partial y}{\partial b} = \boxed{\frac{a}{b} + c^2}$$

$$\frac{\partial y}{\partial c} = \frac{\partial y}{\partial h}\frac{\partial h}{\partial j}\frac{\partial j}{\partial c} = 1 \times b \times 2c$$

$$\frac{\partial y}{\partial c} = \boxed{2bc}$$

Evaluando las derivadas en $a=2$, $b=10$ y $c=0{,}5$

$\boxed{\frac{dy}{da}(a,b,c) = \log(10) = 2{,}3}$; $\boxed{\frac{\partial y}{\partial b} = \frac{2}{10} + (0{,}5)^2 = 0{,}45}$; $\boxed{\frac{dy}{dc} = 2(10 \times 0{,}5) = 10}$