a) $v := v(N) := \begin{cases} \{\emptyset\} : 0 \\ \{S \subset N \mid |S| = 1\} : 0 \\ \{S \subset N \mid |S| = 2\} : 4 \\ \{S \subset N \mid |S| = 3\} : 6 \end{cases}$ "Charakt. Fkt."

$v : 2^N \to \mathbb{R}$

$\boxed{x \in K_{ern} : \quad \forall S \subseteq N : \sum_{i \in S} x_i \geq v(S)}$

$x^A = (2, 2, 2)$ $\{1, 2, 3\}$

$|S| = 0 : \quad 0 \geq 0 \quad \checkmark$
$|S| = 1 : \quad 2 \geq 0 \quad \checkmark$
$|S| = 2 : \quad 4 \geq 4 \quad \checkmark$
$|S| = 4 : \quad 6 \geq 6 \quad \checkmark$

b)

$$\phi_i \left((N, v) = V \right) = \frac{1}{|N|!} \sum_{S \subseteq N \setminus \{i\}} |S|! \, (|N| - |S| - 1)! \left(v(S \cup \{i\}) - v(S) \right)$$

$$= \frac{1}{6} \cdot \left(|0|! \, (3-1)! \left(v(\emptyset \cup \{i\}) - v(\emptyset) \right) \right.$$
$$+ 2 \left(|1|! \, (3-1-1)! \, (4 - 0) \right)$$
$$\left. + |2|! \, (3 - 2 - 1)! \, (6 - 4) \right)$$

$$= \frac{1}{6} \cdot (0 + 2 \cdot 4 + 4) = \frac{12}{6} \quad \checkmark$$

$$\phi(V) = (2, 2, 2)$$

c)

$$N(N) = \begin{cases} \{\emptyset\} : 0 \\ \{S \subset N \mid |S| = 1\} : 0 \\ \{S \subset N \mid |S| = 2\} : 0.8x \\ \{S \subset N \mid |S| = 3\} : 1.0x \end{cases}$$

$$\phi_i(V) \cdot \frac{1}{6} \left(1 \cdot 2 \cdot 0 + 2 \cdot (1 \cdot 1 \cdot (0.8x - 0)) \right)$$

$$2 \cdot 1 \left(x - 0.8x \right)$$

$$= \frac{1.6 + 0.9}{6}x = \frac{2}{6}x = \frac{1}{3}x \qquad \text{✗}$$

$\rightsquigarrow$ Koalition:

d) $V \cdot \left(\{ c, w_1, w_2 \} = N, v \right)$

$v: 2^N \rightarrow \mathbb{R}$

$\quad \{c, w_1\}, \{c, w_2\} \longmapsto 3$

$\quad \{c, w_1, w_2\} \longmapsto 4$

$\quad 0$ sonst

I)
$x_1 = (2, 1, 1) \qquad \text{im Kern} \checkmark$

$x_2 = (2.5, 0.5, 1) \qquad \text{im Kern} \checkmark$

$x_3 = (4, 0, 0) \qquad \text{im Kern} \checkmark$

II)
$$\phi_c = \frac{1}{6} \cdot \Big(\quad 0$$

$\quad \color{green}{S = \{w_1\},} \quad + \; 2 \cdot 1 \cdot 1 \cdot (3 - 0)$
$\quad \color{green}{S = \{w_2\}}$
$\quad \color{green}{S = \{w_1, w_2\}} \quad + \; 2 \cdot 1 \cdot (4 - 0) \quad \Big)$

$$= \frac{1}{6}(6 + 8) \qquad = \frac{14}{6}$$

$$\phi_w = \frac{1}{6} \Big(\quad 0$$

$\quad \color{green}{S = \{w\}} \quad + \quad 1 \cdot 1 \cdot (0 - 0)$
$\quad \color{green}{S = \{c\}} \quad + \quad 1 \cdot 1 \cdot (3 - 0)$
$\quad \color{green}{S = \{w, c\}} + \quad 2 \cdot 1 \cdot (4 - 3) \Big)$

$$= \frac{1}{6}(0 + 0 + 3 + 2) = \frac{5}{6}$$

$$\Rightarrow \quad \phi = \left(\frac{14}{6}, \frac{5}{6}, \frac{5}{6} \right)$$

$14 + 5 + 5 = 24$

$\dfrac{24}{6} = 4$

III)
$$\frac{14}{6} + \frac{5}{6} = \frac{19}{6} > 3 \quad \checkmark$$

$$\frac{14}{6} + 2 \cdot \frac{5}{6} = \qquad 4 \quad \checkmark$$

e) Was die optimale Koalitionsstruktur $C \subseteq N$

$$f(C) := \max_{CS} V(CS)$$

Dann.
$$f(C) = \begin{cases} v(C) & \text{falls } |C| = 1 \\ \max\left\{v(C), \max_{\{C_1, C_2:\, C_1 \cup C_2 = C\}} f(C_1) \cdot f(C_2)\right\} & \text{sonst} \end{cases}$$