

$$\begin{aligned}
f_1(n) &= C(0) \\
f_2(n) &= C(1) + \sum_{i=1}^n f_1(i) \\
&= C(1) + C(0)n \\
f_3(n) &= C(2) + \sum_{i=1}^n f_2(i) \\
&= C(2) + C(1)n + \sum_{i=1}^n C(0)i \\
&= C(2) + C(1)n + \frac{C(0)}{2}n(n+1) \\
&= C(2) + \left(C(1) + \frac{C(0)}{2}\right)n + \frac{C(0)}{2}n^2 \\
f_4(n) &= C(3) + \sum_{i=1}^n f_3(i) \\
&= C(3) + C(2)n + \left(C(1) + \frac{C(0)}{2}\right)\frac{n(n+1)}{2} + \frac{C(0)}{2}\sum_{i=1}^n i^2 \\
&= C(3) + C(2)n + \left(\frac{C(1)}{2} + \frac{C(0)}{4}\right)(n^2 + n) +
\end{aligned}$$

I didn't bother to solve the rest, but apparently it is a polynomial.