$$
\begin{aligned}
\int \frac{\mathrm{d}x}{a(p+x)^2+1} &= \frac{1}{\sqrt{a}} \int \frac{\mathrm{d}\sqrt{a}(p+x)}{\left(\sqrt{a}(p+x)\right)^2+1} \\
&= \frac{1}{\sqrt{a}} \arctan(\sqrt{a}(p+x)) + \text{constant}
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{\mathrm{d}x}{a(p+x)^2+b(q+x)^2+c(r+x)^2+1} &= \int \frac{\mathrm{d}x}{(a+b+c)(D+x)^2+F+1} \\
&= \int \frac{\mathrm{d}x}{\frac{a+b+c}{F+1}(D+x)^2+1} \\
&= \sqrt{\frac{F+1}{a+b+c}} \arctan\left(\sqrt{\frac{a+b+c}{F+1}}(D+x)\right) + \text{constant} \\
&=: \frac{1}{R} \arctan(R(D+x)) + \text{constant} \\
\int_0^1 \frac{\mathrm{d}x}{(a+b+c)(D+x)^2+F+1} &= \frac{\arctan\left((D+1)R\right) - \arctan(DR)}{R}
\end{aligned}
$$

where D, F are given by

$$
\begin{aligned}
a(p+x)^2+b(q+x)^2+c(r+x)^2 &= ap^2+2apx+ax^2+bq^2+2bqx+bx^2+cr^2+2crx+cx^2 \\
&= (a+b+c)x^2+2(ap+bq+cr)x+ap^2+bq^2+cr^2 \\
&= (a+b+c)\left(x^2+2\frac{ap+bq+cr}{a+b+c}x+\frac{ap^2+bq^2+cr^2}{a+b+c}\right) \\
&= (a+b+c)\left(x^2+2\frac{ap+bq+cr}{a+b+c}x+\left(\frac{ap+bq+cr}{a+b+c}\right)^2-\left(\frac{ap+bq+cr}{a+b+c}\right)^2+\frac{ap^2+bq^2+cr^2}{a+b+c}\right) \\
&= (a+b+c)\left(\left(x+\frac{ap+bq+cr}{a+b+c}\right)^2+\frac{(a+b+c)\left(ap^2+bq^2+cr^2\right)-(ap+bq+cr)^2}{(a+b+c)^2}\right) \\
&= (a+b+c)\left(\left(x+\frac{ap+bq+cr}{a+b+c}\right)^2+\frac{(a+b+c)\left(ap^2+bq^2+cr^2\right)-a^2p^2-b^2q^2-c^2r^2-2abpq-2acpr-2bcqr}{(a+b+c)^2}\right) \\
&= (a+b+c)\left(\left(x+\frac{ap+bq+cr}{a+b+c}\right)^2+\frac{a^2p^2+abq^2+acr^2+abp^2+b^2q^2+cbr^2+acp^2+bcq^2+c^2r^2-a^2p^2-b^2q^2-c^2r^2-2abpq-2acpr-2bcqr}{(a+b+c)^2}\right) \\
&= (a+b+c)\left(\left(x+\frac{ap+bq+cr}{a+b+c}\right)^2+\frac{abq^2+acr^2+abp^2+cbr^2+acp^2+bcq^2-2abpq-2acpr-2bcqr}{(a+b+c)^2}\right) \\
&= (a+b+c)\left(\left(x+\frac{ap+bq+cr}{a+b+c}\right)^2+\frac{ab\left(p^2+q^2-2pq\right)+ac\left(p^2+r^2-2pr\right)+bc\left(q^2+r^2-2qr\right)}{(a+b+c)^2}\right) \\
&=: (a+b+c)\left((x+D)^2+F\right)
\end{aligned}
$$

Conclusion:

$$
\int_0^1 \frac{k}{\sum_{i=1}^3 s_i\left(P_{i3}h+P_{i1}u+P_{i2}v+P_{i4}\right)^2+1}\mathrm{d}h = \frac{k}{R}\left(\arctan((D+1)R)-\arctan(DR)\right)
$$

where

$$
\begin{aligned}
C_i &= P_{i1}u+P_{i2}v+P_{i4}-b_i \\
D &= \frac{\sum_{i=1}^3 s_iC_i}{\sum_{i=1}^3 s_iP_{i3}} \\
F &= \frac{1}{\left(\sum_{i=1}^3 s_iP_{i3}\right)^2} \sum_{(i,j)=(1,2),(1,3),(2,3)} P_{i3}P_{j3}s_is_j\left(\frac{C_i}{P_{i3}}-\frac{C_j}{P_{j3}}\right)^2 \\
R &= \sqrt{\frac{s_1+s_2+s_3}{F+1}}
\end{aligned}
$$

with the parameters:

- P_{ij} is the inverse of the perspective matrix (from view space to world)
- k is the magnitude of a signal
- s_i is the scale of a signal
- b_i is the center of a signal
- u, v are the screen coordinates of a pixel

where each signal is

$$
f(x,y,z) = \frac{k}{s_1(x-b_1)^2+s_2(y-b_2)^2+s_3(z-b_3)^2+1}
$$